Computational modeling of the Hybrid procedure in hypoplastic left heart syndrome: a comparison of zero-dimensional and three-dimensional approach.

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Abstract

Previous studies have employed generic 3D-multiscale models to predict haemodynamic effects of the hybrid procedure in hypoplastic left heart syndrome. Patient-specific models, derived from image data, may allow a more clinically relevant model. However, such models require long computation times and employ internal pulmonary artery band \(d_{\text{int}}\) dimension, which limits clinical application. Simpler, zero-dimensional models utilize external PAB diameters \(d_{\text{ext}}\) and provide rapid analysis, which may better guide intervention. This study compared 0-D and 3-D modeling from a single patient dataset and investigated the relationship \(d_{\text{int}}\) versus \(d_{\text{ext}}\) and hemodynamic outputs of the two models. Optimum oxygen delivery defined at \(d_{\text{int}} = 2\) mm corresponded to \(d_{\text{ext}} = 3.1\) mm and 3.4 mm when models were matched for cardiac output or systemic pressure, respectively. 0-D and 3-D models when matched for PAB dimension produced close equivalence of hemodynamics and ventricular energetics. From this study we conclude that 0-D model can provide a valid alternative to 3D-multiscale in the Hybrid-HLHS circulation.

Abbreviations

HLHS = Hypoplastic left heart syndrome
0-D = zero-dimensional
3-D = three-dimensional
\(Q_p\) = pulmonary flow
\(Q_s\) = systemic flow
PAB = pulmonary artery band
PDA = patent Ductus Arteriosus
\(d_{\text{int}}\) = internal diameter of PAB
\(d_{\text{ext}}\) = external diameter of PAB

Introduction
In hypoplastic left heart syndrome there is developmental failure of left heart structure and the right ventricle must supply both systemic and pulmonary circulations. This is achieved in the newborn with the Hybrid procedure which stabilizes the circulation by regulating the correct flow distribution between the pulmonary and systemic circulation, \( Q_p : Q_s \), using surgically placed bilateral PA bands. A stent placed in the PDA permits unrestricted blood flow from the single ventricle to the systemic circulation. [Fig. 1]. Defining the correct PAB dimension is critical as this determines ventricular workload, systemic oxygen delivery and patient outcome. During Hybrid procedure the PAB dimension is empirically based on patient’s body weight and calibrated during surgery to achieve desired systemic oxygen saturations and pressure. Because the method is imprecise and the clinical parameters used to inform the PAB size reflect poorly the ventricular workload and circulation, the condition still carries a significant risk [1,2].

Computational models have been used to inform congenital circulations and surgical intervention including Norwood and Hybrid procedure [3-7]. They have provided a theoretical analysis of how the circulation might be influenced by varying PAB dimension, stent size and aortic obstruction. Multiscale models construct an idealized, generic 3-D geometry to represent the surgical region from which regional flow profiles can be calculated. Alternatively the geometry of the surgical region can be obtained from the patient’s image dataset, thus providing a patient-specific model. Such models define PAB and stent-PDA size by internal luminal dimensions which is in contra-distinction to the surgical procedure, being based on calibration of the external PAB dimension [1,2]. This difference in quantifying PAB dimension and the fact that these models are computationally demanding limit their clinical application.

An alternative approach is to represent the surgical region by lumped parameter method. Using regional pressure data obtained during surgery or cardiac catheterization parameters of resistance can be defined and related to flow [8,9]. 0-D models because of their simplicity can provide fast and reliable solutions and potentially greater clinical application compared with the 3D-multiscale approach. However simplicity should not compromise accurate description of the physiology.

In this study we compared 0-D and 3D-multiscale patient-specific models of the surgical region constructed from a single patient dataset. This allowed a comparative analysis of the predicted physiological outcomes of the two modeling approaches. Furthermore by comparing the models under
equivalent hemodynamic conditions, corresponding external and internal PA band diameters, defining
the PAB dimension in the 0-D and 3-D models respectively, were determined.

Methods
The analysis was based on a 3kg patient with HLHS with aortic atresia. Hybrid palliation included
3mm bilateral PA bands and 10mm PDA stent.
The surgical region [main pulmonary artery, PABs, PDA-ductal stent] was represented by either
equations-based 0-D model or 3-D model derived from the patient’s CT scan. The remaining
cardiovascular system was described as lumped parameter network [LPN]. In order to compute the
entire circulation [surgical region +LPN] with the 0-D model the equations representing the surgical
region where incorporated as part of the LPN [fig 2a] whereas in the multiscale model the 3D geometry
region was coupled to LPN [figure 2b]. The 0D-LPN has been previously described in detail [8]. A
brief outline of the methods is described.

Heart
Right ventricle, and atrial chambers are represented by the time-varying elastance model [3]. Equations
1 and 2 describe the pressure-volume relationships of the three cardiac chambers.

1. \[a(t) = \begin{cases} \frac{1}{2} \left(1 - \cos \left( \frac{\pi t}{T_{ps}} \right) \right) & 0 < t \leq 2T_{ps} \\ 0 & 2T_{ps} < t \leq T_c \end{cases}\]

2. \[P = a(t) \cdot E(V - V_0) + [1 - a(t)] \cdot A(e^{B(V - V_0)} - 1)\]

where \(a(t)\) = activation function switching between systole and diastole, \(E\) = end systolic elastance, \(A\) and \(B\) =
linear and exponent scaling factor of the end-diastolic pressure-volume relationship respectively, \(V_0\) =
unstressed chamber volume, \(T_c\) = duration of the cardiac cycle and \(T_{ps}\) = time to peak systole. The delay
in ventricular systole is accounted for by a temporal translation of Equation (1) by \(\Delta T\).
The valves are modeled as ideal diodes and an orifice resistance model such that there is no flow when
the pressure gradient across the valve is reversed:
3. \( Q = \begin{cases} 0 & \Delta P \leq 0 \\ \sqrt{\frac{\Delta P}{R}} & \Delta P > 0 \end{cases} \)

The atrial septal defect is described as a constant resistance.

Systemic and Pulmonary circulation

The circulation was modeled by a multi-compartmental Windkessel method. Each vascular system, pulmonary and systemic, is modeled by a lumped arterial and venous capacitance, and resistance.

In each compliant chamber of the circulation, the pressure was determined by assuming a constant compliance:

4. \( P(t) = \frac{V(t)}{c} \)

Flows were calculated using a linear resistance model:

5. \( Q = \frac{\Delta P}{R} \)

The Surgical region

[1] 0-D Model

Symmetry in left and right pulmonary artery flow was assumed in the 0-D model because the same band dimensions were applied to right and left branch PAs during surgery. The pulmonary circulation was therefore modeled as a single unit rather than a left-right lung distribution (figure 2). A reference value \( R_{\text{ref}} \) was identified using post-hybrid catheterization data and pulmonary flow values were obtained from literature [3]. This reference value was then varied as a function of the external diameter of the PA band (d), adopting a Poiseuille relationship:

6. \( R_{\text{band}} = R_{\text{ref}} \cdot \left( \frac{d}{D} \right)^4 \)
Stent flow was described using an empirically derived equation of shunt flow \[9\] in which the diameter \(D\) was scaled to match the pressure difference measured at catheterization:

\[
\Delta P = \frac{k_1 Q + k_2 Q^2}{D^4}
\]

Conservation of Flow

The conservation of flow dictates that the change in volume of a compliant chamber must equal the difference of the flow in and out of that specific chamber. This leads to a set of differential equations, which are used to determine a solution. By summing all the individual differential equations for the volume of each compliant chamber it is shown that \(\frac{dV_T}{dt} = 0\) where \(V_T\) is the total stressed blood volume defined as:

\[
V_T = V_{RA} + V_{RV} + V_{MPA} + V_{SA} + V_{SV} + V_{PA} + V_{PV} + V_{LA}
\]

Thus the total stressed blood volume is a constant and is employed as an input parameter, with the diameters of the band and stent, \(d\) and \(D\) respectively, to the model to solve the set of ordinary differential equations using Euler’s Method.

Using Mimics (Materialize, Leuven, Belgium) a 3-D geometry of the surgical region was constructed from CT scan dataset of patient post-hybrid. Following construction of the geometries the region of the banded areas was manipulated by inserting cylinders of known diameter and length into the banded region and merged into one geometry. The diameter of the cylinders, or virtual bands, was varied under study. Internal meshes were developed in Gambit (Fluent v13, Ansys, Canonsburg, PA) for
computational fluid dynamic simulations. Equations (6) and (7) are replaced with the CFD model. The 3-D geometry was coupled with the remaining circulation [figure 2b] using the following interface conditions:

9. \[ Q_{0D} = \int \frac{m_i}{\rho} dA_{3D,i} \]

10. \[ P_{3D,i} = P_{0D} \]

The Pressure is set from the 0D model, \( P_{0D} \) and applied to each face (i) of the boundary in the 3D model, \( P_{3D,i} \). The CFD model then determines a solution from which the mass flow rate of each face \( m_i \) is divided by the constant density \( \rho \) and integrated over the area of the boundary \( A \) to determine the instantaneous volumetric flow rate. [blood density \( \rho = 1060 \text{Kg/m}^3 \); viscosity \( \mu = 0.005 \text{Kg m}^{-1} \text{s}^{-1} \)].

With the flows obtained from simulations, oxygen delivery was calculated as previously described by Bove [10].

Protocols

Simulations were run in the 3-D model for a range of virtual internal PAB diameters [1.5mm to 4.0 mm, 0.5mm increments) with band length = 2 mm. The stent diameter, \( D \), was set at 10mm. For each band size 4 cycles were simulated to converge to a stable solution. The False Position Method was used to determine the value of \( d_{ext} \) that corresponded to the equivalent \( d_{int} \) simulated in the 3-D model. For comparison between 0-D and 3-D models, two circulation conditions were matched: [1] mean Pulmonary artery pressure and [2] cardiac output.

Results

Equivalent external PA band diameters in the 0-D model for each of the six internal diameters 3-D simulations are presented in Table 1. Larger \( d_{ext} \) were required to match \( d_{int} \) under conditions of
matched mean MPA pressure compared to cardiac output. Previous published multiscale simulations identified systemic oxygen delivery was highest with $d_{\text{int}} = 2.0$ mm [3,5] which equated to $d_{\text{ext}} = 3.1$ mm (matched for CO) or 3.4 mm (matched for mean MPA pressure) in the 0-D model.

As the internal diameter of PAB increased in the 3D geometries, a decrease in the difference between $d_{\text{ext}}$ and $d_{\text{int}}$ was observed, particularly at $d_{\text{int}} 3.5, 4$ mm (figure 4). However, assuming a circular cross-section, the difference between external and internal luminal area, $\Delta A$, remained relatively constant at $\Delta A \approx 4$ mm$^2$ (figure 4).

The overall hemodynamic results, including oxygen delivery and ventricular energetics, for 3D at $d_{\text{int}} = 2$ mm and the equivalent $d_{\text{int}}$ for 0D model are presented in figure 3, table 2. The 3-D and 0-D modeling correlated well with matching for cardiac output producing the closest equivalence.

Discussion

Mathematical modeling has the potential to inform surgical decision-making and optimize the Hybrid procedure in HLHS. With the multiscale approach 3-D patient-specific geometries of the surgical region are constructed, and hemodynamic profiles determined by computational fluid dynamics. This provides an analysis of the Hybrid circulation but long computational times limits clinical application. Alternatively a simpler equation-based 0-D model incorporating external stent and PA band diameters is computer efficient and could provide rapid clinical applicable solutions.

This study compared the 0-D and 3-D models, and determined the external PA band diameters of the 0D model that corresponded to a range of internal diameters simulated in the 3D model. The difference in diameter between equivalent internal and external band dimensions was not consistent but varied over the band range. Potentially this was due to the minor degrees of alignment error associated with insertion of the virtual bands within the 3-D geometries.

Ideal hybrid palliation aims to maximize systemic oxygen delivery within the workload capacity of the single ventricle by optimizing $Q_P/Q_S$ by PAB calibration.[2,7]. Previous studies have demonstrated an internal diameter of 2 mm provides the optimum systemic oxygen delivery for a 3kg neonate [4,6]. In this study 2 mm internal PAB diameter [3-D model] corresponded to an external diameter of 3.1/3.4 mm in the 0-D model. This finding is consistent with that observed clinically in which 3-3.5 mm is the typical external band diameter applied in 3kg neonate.
The study further demonstrated that 0-D and 3-D models, with matched boundary conditions and corresponding PA band dimensions, demonstrated equivalent ventricular energetics and hemodynamic outcomes.

The implications of the study are two-fold. Firstly, the study confirms that in comparison with 3D-multiscale modeling, the 0-D approach can provide a valid representation of the hybrid circulation. Secondly, there is the potential for 0-D and 3-D models to be used interchangeably to inform clinical management. Initial patient-specific 3-D geometry with virtual internal PA band and stent dimension calibration can be used to define the optimal hemodynamics for the individual patient’s anatomy. The corresponding external PA band diameter, as determined by this study, can be applied to configure the hybrid procedure and also used to input the 0-D model. Any subsequent circulation analyses [e.g., due to subsequent stent obstruction] could be evaluated via the efficient 0-D model.

In conclusion the study compared two modeling approaches, 0-D and 3-D in the computational analysis of the hybrid palliation of HLHS. The models demonstrated close equivalence of predicted hemodynamics. Internal PA band diameter of 2 mm corresponded to external band diameter of 3.1/3.4 mm in the 0D model, consistent with clinical observation. From this study we conclude that 0-D modeling can provide a valid clinically applicable alternative to 3D-multiscale in the Hybrid-HLHS circulation.

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Conflict of Interest: None


5. Hsia TY, Cosentino D, Corsini C, Pennati G, Migliavacca F. Use of mathematical modeling to compare and predict haemodynamic effects between hybrid and surgical Norwood palliations for hypoplastic left heart syndrome. Circulation 2011;124:S204-10


figure 1. Illustration of the Hybrid procedure reprinted from Galantowicz et al [1] reproduced with author’s permission
Figure 2. Analogous electric circuit diagrams of zero-dimensional and 3D-multiscale models
Figure 4. Difference in diameter and area of 0D-external versus 3D-internal pulmonary artery band dimensions, matched for cardiac output.
Table 1. Equivalent external band diameter of 0-D model for internal band diameter in 3-D model when matched for cardiac output and mean MPA pressure.

<table>
<thead>
<tr>
<th>3D Diameter (mm)</th>
<th>0D Diameter to match cardiac output (mm)</th>
<th>0D Diameter to match MPA pressure (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5 mm</td>
<td>2.67 mm</td>
<td>2.97 mm</td>
</tr>
<tr>
<td>2.0 mm</td>
<td>3.10 mm</td>
<td>3.40 mm</td>
</tr>
<tr>
<td>2.5 mm</td>
<td>3.47 mm</td>
<td>3.81 mm</td>
</tr>
<tr>
<td>3.0 mm</td>
<td>4.05 mm</td>
<td>4.55 mm</td>
</tr>
<tr>
<td>3.5 mm</td>
<td>3.98 mm</td>
<td>4.45 mm</td>
</tr>
<tr>
<td>4.0 mm</td>
<td>4.58 mm</td>
<td>5.37 mm</td>
</tr>
<tr>
<td>Outcome</td>
<td>3D model</td>
<td>0D model matching MPA pressure</td>
</tr>
<tr>
<td>--------------------------------------</td>
<td>--------------</td>
<td>--------------------------------</td>
</tr>
<tr>
<td>Band Diameter (mm)</td>
<td>2.00</td>
<td>3.40</td>
</tr>
<tr>
<td>Systolic MPA Pressure (mmHg)</td>
<td>71.04</td>
<td>77.86, 8.7%</td>
</tr>
<tr>
<td>Diastolic MPA Pressure (mmHg)</td>
<td>42.42</td>
<td>40.25, 5.1%</td>
</tr>
<tr>
<td>Mean MPA Pressure (mmHg)</td>
<td>53.41</td>
<td>53.41</td>
</tr>
<tr>
<td>Systolic Systemic Pressure (mmHg)</td>
<td>58.80</td>
<td>73.24, 19.7%</td>
</tr>
<tr>
<td>Diastolic Systemic Pressure (mmHg)</td>
<td>42.53</td>
<td>40.31, 5.2%</td>
</tr>
<tr>
<td>Mean Systemic Pressure (mmHg)</td>
<td>50.31</td>
<td>52.73, 4.5%</td>
</tr>
<tr>
<td>Systolic Pulmonary Pressure (mmHg)</td>
<td>14.50</td>
<td>17.68, 17.9%</td>
</tr>
<tr>
<td>Diastolic Pulmonary Pressure (mmHg)</td>
<td>13.17</td>
<td>14.37, 8.3%</td>
</tr>
<tr>
<td>Mean Pulmonary Pressure (mmHg)</td>
<td>13.91</td>
<td>16.10, 13.6%</td>
</tr>
<tr>
<td>Cardiac Output (l/min)</td>
<td>1.80</td>
<td>1.98, 9%</td>
</tr>
<tr>
<td>Pulmonary Flow (l/min)</td>
<td>1.07</td>
<td>1.24, 13.7%</td>
</tr>
<tr>
<td>Systemic Flow (l/min)</td>
<td>0.73</td>
<td>0.74, 1%</td>
</tr>
<tr>
<td>Pulmonary-Systemic Flow Ratio</td>
<td>1.47</td>
<td>1.68, 12.5%</td>
</tr>
<tr>
<td>Stent Backflow (l/min)</td>
<td>-0.64</td>
<td>-0.55, 14%</td>
</tr>
<tr>
<td>Arterial Oxygen Saturation (%)</td>
<td>72.22</td>
<td>75.77, 4.6%</td>
</tr>
<tr>
<td>Venous Oxygen Saturation (%)</td>
<td>34.35</td>
<td>38.32, 10%</td>
</tr>
<tr>
<td>Systemic Oxygen Delivery (ml O₂/min/m²)</td>
<td>352.77</td>
<td>374.30, 5.7%</td>
</tr>
<tr>
<td>Total Stressed Blood Volume (ml)</td>
<td>72.50</td>
<td>72.50</td>
</tr>
<tr>
<td>Right Ventricle End Diastolic Volume (ml)</td>
<td>21.81</td>
<td>23.62, 7.6%</td>
</tr>
<tr>
<td>Stroke Work (ml·mmHg)</td>
<td>782.87</td>
<td>833.78, 6.1%</td>
</tr>
<tr>
<td>Systolic Pressure-Volume Area (ml·mmHg)</td>
<td>962.98</td>
<td>1057.87, 8.9%</td>
</tr>
<tr>
<td>Mechanical Efficiency (%)</td>
<td>81.30</td>
<td>78.82, 3%</td>
</tr>
</tbody>
</table>

Table 2. Haemodynamic outcomes of 3D versus OD models with equivalent pulmonary artery band dimensions when matched for mean MPA pressure and cardiac output, with % difference between 3D and 0D outputs.