Aerodynamic Design Optimization of Wind Turbine Airfoils under Aleatory and Epistemic Uncertainty

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Abstract. This paper presents different approaches to optimize wind turbine airfoils in an uncertain scenario. The approaches are specifically applied to the aerodynamic design optimization of a wind turbine airfoil accounting for the uncertainty in setting up the XFOIL's NCRIT constant: a parameter that is considered affected by a chain of aleatory and epistemic uncertainty. Subject to a set of aerodynamic and structural constraints, the uncertain response of the airfoil is optimized by means of both probability- and imprecise probability-based approaches. These solutions are compared with a reference airfoil optimized with a conventional design approach, in which the treatment of uncertainty is carried out in a simplistic fashion. Once evaluated in the probabilistic scenario, the airfoil designed with the conventional approach still achieves the largest aerodynamic efficiency mean. This airfoil is however affected by the largest performance sensitivity to NCRIT variations. The airfoils optimized by means of uncertainty-based approaches instead achieve larger performance robustness and reliability than the airfoil optimized with the conventional approach.

1. Introduction

The aerodynamic design of wind turbine airfoils is largely affected by uncertainty. The sources of this uncertainty are of different nature, and they are mainly related to the highly stochastic nature of the wind, to manufacturing errors, to leading edge erosion, to the limited fidelity of the aerodynamic models that are used to perform the simulations, and to these models' parameters setup. Based on the current industrial practice, the aerodynamic design of wind turbine airfoils is performed deterministically or, in the best cases, following arbitrary and rather unmethodical engineering approaches to include the effect of the different sources of uncertainty. When designing wind turbine airfoils, the inadequate treatment of uncertainty may lead to inefficient airfoils, having poor aerodynamic performance or not meeting the design requirements under off-design conditions. Properly quantifying and including uncertainty in the aerodynamic design of wind turbine airfoils allows designers to achieve more robust and reliable airfoils, improving the overall performance of wind turbines and reducing the cost of wind energy.

In recent years, a number of papers on wind turbine airfoil and rotor design optimization have dealt with uncertainty in a more rigorous way [1, 2, 3, 4, 5]. A common approach to formulate uncertainty is through the classic probability theory, representing uncertain parameters by means of probability distributions. This approach is reasonable if enough information are available to define the distributions. However, this probability approach is weak when little or no information is available on the uncertain parameters [6]. Based on the level of knowledge, uncertainty is classified in two categories: aleatory uncertainty and epistemic uncertainty. Aleatory uncertainty is related to the intrinsic variability of the physical system, and experimental data are normally available to define it. Epistemic uncertainty is instead related to the lack of knowledge or incomplete information, typically tied to the analysis models. Hence, since sufficient data is available, aleatory uncertainty can be represented mathematically through a probability distribution function, and therefore the classic probability theory is a sound approach. On the other side representing epistemic uncertainty by a probability distribution is questionable because there are not enough elements to choose a function over another, and different methods need to be used. One of the approach developed to deal with epistemic uncertainty is the evidence (Dempster-Shafer) theory [6].

The aim of the paper is twofold. The first goal is to present and compare three reliability-based design optimization (RBDO) approaches to include uncertainty in wind turbine airfoil design (RBDO here indicates the probabilistic treatment of both objective function and constraints). To this aim, we have devised a simple probabilistic design scenario in which uncertainty incorporates an aleatory component, related to the inflow characteristics, and an epistemic source, related to the definition of a key parameter in the airfoil aerodynamic model, namely XFOIL's NCRIT [7]. The three approaches proposed by this paper deal with uncertainty in different way. The first approach considers uncertainty in a simplistic way, somehow resembling the current industrial practice. The second one uses classical probability theory to model both aleatory and epistemic uncertainty, although not enough data is available to confidently define a distribution function for the epistemic component. The third approach models epistemic uncertainty by means of the evidence theory. The second goal of the paper is on the one hand to understand how different airfoil shapes can be achieved by including a proper treatment of uncertainty, and on the other hand to understand how different probabilistic optimization objectives affect the airfoils' shape and the aerodynamic performance.

2. Methods

This paper aims to compare the airfoils designed by means of three different optimization approaches: a pseudo-deterministic optimization approach and two different uncertainty-based design optimization approaches: one based on probability uncertainty quantification and the other based on evidence theory uncertainty quantification. The pseudo-deterministic optimization resembles the common practice approach in which airfoil design is performed considering uncertainty in a rather simplistic way. The first uncertainty-based design optimization approach, based on probability uncertainty quantification, improves the pseudo-deterministic approach by considering uncertainty in a more rigorous fashion though probability distributions, although not sufficient information might be available for certain uncertain variables. The second uncertainty-based design optimization approach, based on evidence theory uncertainty quantification, further improves the previous approach by properly modeling uncertainty based on the actual amount of information available, avoiding assuming probability distributions when not enough information are available. It should be noted that the design optimization problems shown below are intentionally not equivalent (i.e, they do not have the same objectives), and they were devised to show how the optimized airfoil shapes are influenced by different probabilistic optimization formulations.

2.1. Problem definition

In order to illustrate the aforementioned optimization techniques, this paper considers a practical design optimization problem under uncertainty. The objective is to design a new airfoil for the outboard part of the blades of the 10 MW INNWIND reference turbine to improve their aerodynamic performance. The airfoil used at the outer part of the INNWIND reference turbine's blades is the 24% thick FFA-W3-241 airfoil. We assumed that the only structural requirement of the new airfoil is related to its relative thickness. That is, we assumed that the new airfoil should have at least a relative thickness of 24%, just like that of the reference airfoil. Moreover, in this design exercise we optimized the new airfoil with the same design lift coefficient as that of the reference one. This would allow us to implement the new airfoil on the blade without the need to redesign the blade planform. We also assumed to keep the same design angle of attack (7°) and design Reynolds number ($12{\cdot}10^6$).

2.1.1. Probabilistic scenario The aerodynamic design of wind turbine airfoils is here performed by using the two-dimensional panel code XFOIL. In this code, laminar-turbulent transition can be simulated by the e^N method, through the parameter NCRIT, depending on the disturbance level (DL) in which the airfoil operates. The DL is related to freestream turbulence, wall roughness and vibration level. Based on a given DL, evaluating the NCRIT parameter is not an easy task, and assumptions about its values need to be made, leading to a certain level of uncertainty affecting the code output. Moreover, it should be also considered that the DL changes based on the operational conditions and then it should be considered as an uncertainty too. This paper assumes that the DL is an aleatory uncertainty, and therefore enough data are available to define it through a probability distribution. NCRIT is instead considered as epistemic uncertainty. The global uncertainty here is simplified with a chain of aleatory uncertainty (DL) and epistemic uncertainty (NCRIT). The environment DL is assumed to be lognormally distributed (with $\mu = 0$ and $\sigma = 1$) from a minimum value of 0 to a maximum value of 1. The epistemic uncertainty related to NCRIT is modeled differently as discussed below.

2.1.2. Design variables The airfoil parametrization is here achieved by a composite Bézier curve shown in Figure 1. The 11 degrees of freedom of the control points (in the x- and y-direction) represent the design variables denoted by $\boldsymbol{b} = \{b_1, ..., b_{11}\}$.

2.1.3. Figure of merit The figure of merit considered in this study is the mean of the lift-to-drag ratio calculated at the angles of attack equal to 5°, 7° and 10° (Eq. 1). In this paper the value of f is considered as the airfoil aerodynamic efficiency.

$$f(\boldsymbol{b}, NCRIT) = \frac{1}{3}\left[\left(\frac{C_l(\boldsymbol{b}, NCRIT)}{C_d(\boldsymbol{b}, NCRIT)}\right)_{AoA=5} + \left(\frac{C_l(\boldsymbol{b}, NCRIT)}{C_d(\boldsymbol{b}, NCRIT)}\right)_{AoA=7} + \left(\frac{C_l(\boldsymbol{b}, NCRIT)}{C_d(\boldsymbol{b}, NCRIT)}\right)_{AoA=10}\right] \quad (1)$$

Optimizing the airfoil efficiency over an operative range of angles of attack, rather than a single one, allows the airfoil's performance to be less sensitive to variations of the angle of attack due to, for example, sudden variations of the wind which cannot be followed by the rotor controller.

2.1.4. Constraints Each airfoil generated during the course of the optimization process was subject to a number of feasibility checks, aiming to avoid unphysical or unconventional shapes, for which XFOIL likely incurs in convergence problems. Airfoil relative thicknesses was also subject to feasibility checks. Relative thickness was enforced to vary within the ranges [24%, 35%], respectively. The design lift coefficient (at the design angle of attack of 7°) was set lower or equal to 1.3. With respect to the design angle of attack, during the course of the optimization, the stall margin was set larger or equal to 4°. The stall angle is here defined as the angle of attack in which the drag coefficient becomes the double of that at 0°. The maximum lift coefficient in

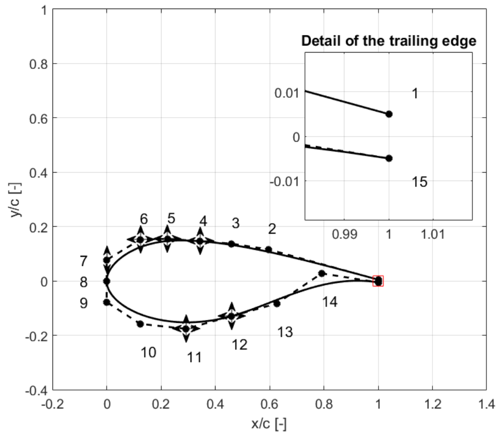

Figure 1. Airfoil parametrization. The airfoil shape is defined by a composite Bézier curve controlled by 15 points. Horizontal and vertical arrows denote the actual degrees of freedom.

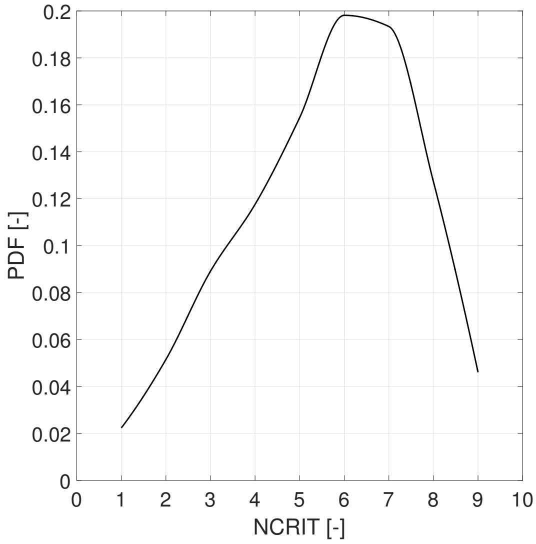

Figure 2. In the RBDO based on probability theory, both aleatory and epistemic uncertainties are modeled by continuous distribution functions. The chain of these sources results in the NCRIT's PDF depicted in this figure.

clean conditions is constrained to be lower or equal to 2.07. The maximum lift is constrained to avoid too large aerodynamic loads in case of sudden increase of the angle of attack over the outboard part of the blade, potentially resulting in structural issues.

2.2. Pseudo-deterministic optimization

The goal of the pseudo-deterministic design optimization was to maximize the mean of the efficiency (Eq. 1) evaluated at two NCRIT values, namely at 0.5 and 9.5. These are respectively the lower and upper bounds of the NCRIT's variability range. This optimization was denoted by "pseudo-deterministic" because the objective function and constraints were valuated through two deterministic calculations. The pseudo-deterministic optimization was formulated as follows:

$$\begin{aligned}
\text{Find: } & \boldsymbol{b} = \{b_1, ..., b_{11}\} \\
\text{to maximize: } & f_{\text{det}}(\boldsymbol{b}, NCRIT = 0.5, 9.5) \\
\text{subject to: } & 0.24 \leq t(\boldsymbol{b}) \leq 0.35 \\
\text{and: } & C_{l,\text{des}}(\boldsymbol{b}, NCRIT = 0.5, 9.5) \leq 1.3 \\
\text{and: } & S(\boldsymbol{b}, NCRIT = 0.5, 9.5) \leq 4^\circ \\
\text{and: } & C_{l,\max}(\boldsymbol{b}, NCRIT = 0.5, 9.5) \leq 2.07 \\
\text{and: } & \boldsymbol{b_L} \leq \boldsymbol{b} \leq \boldsymbol{b_U}
\end{aligned} \tag{2}$$

where f_{det} is the objective function of the pseudo-deterministic problem expressed by Eq. 3, t is the airfoil relative thickness, while $C_{L,\text{des}}$, S and $C_{L,\max}$ are respectively the design lift coefficient, stall margin and maximum lift coefficient. $\boldsymbol{b_L}$ and $\boldsymbol{b_U}$ are respectively the lower and upper bounds of the design variables' variability ranges.

$$f_{\text{det}}(\boldsymbol{b}, NCRIT = 0.5, 9.5) = 0.5f(\boldsymbol{b}, NCRIT = 0.5) + 0.5f(\boldsymbol{b}, NCRIT = 9.5) \quad (3)$$

2.3. RBDO based on probability theory uncertainty quantification

As seen above, the pseudo-deterministic optimization considered only the lower and upper bounds of NCRIT's variable range. This approach neglects the actual probability or likelihood of NCRIT to assume values between these two extreme bounds. The approach presented in this section characterizes uncertainty in a more rigorous way by assigning NCRIT a continuous probability distribution function (PDF). This PDF represents the probability of NCRIT resulting from the chain the aleatory uncertainty (DL) and the epistemic one (NCRIT), both quantified through continuous PDFs. As seen above, DL was lognormally distributed whereas the epistemic uncertainty related to NCRIT was modeled with a uniform distribution spanning a given range (not shown here) of NCRIT values depending on the value of DL. The combination of these two sources of uncertainty resulted in the NCRIT's PDF depicted in Fig. 2.

Various formulations can be adopted to define RBDO problems based on probability theory [8]. This paper follows a RBDO approach aiming to maximize the mean of efficiency (Eq. 1) by enforcing its standard deviation lower or equal to 7 (we enforced this value once we determined the standard deviation of the pseudo-deterministic optimum, which we aimed to reduce). Through this approach we aimed at designing an airfoil to have the highest mean efficiency, but at the same time we limited the efficiency sensitivity to NCRIT variations (i.e., we ask for a certain performance robustness). This design optimization approach is formulated as follows:

$$\begin{aligned}
\text{Find: } & \boldsymbol{b} = \{b_1, ..., b_{11}\} \\
\text{to maximize: } & \mu_f(\boldsymbol{b}, NCRIT) \\
\text{subject to: } & \sigma_f(\boldsymbol{b}, NCRIT) \leq 7 \\
\text{and: } & 0.24 \leq t(\boldsymbol{b}) \leq 0.35 \\
\text{and: } & P[C_{l,\text{des}}(\boldsymbol{b}, NCRIT) \leq 1.3] = 1 \\
\text{and: } & P[S(\boldsymbol{b}, NCRIT) \leq 4^\circ] = 0 \\
\text{and: } & P[C_{l,\text{max}}(\boldsymbol{b}, NCRIT) \leq 2.07] = 1 \\
\text{and: } & \boldsymbol{b_L} \leq \boldsymbol{b} \leq \boldsymbol{b_U}
\end{aligned} \quad (4)$$

In this optimization, we enforced the probability that each constraint was verified to 1 (i.e., constraints always satisfied). Mean, standard deviation and constraints in Eq. 4 were obtained through Monte Carlo sampling based on the NCRIT's PDF in 2

2.4. RBDO based on evidence theory uncertainty quantification

Evidence theory provides a representation of uncertainty which is different but related to that given by the traditional probability theory. Uncertainty is indeed represented by two measures of likelihood denoted by belief and plausibility. The belief provides a lower bound on the likelihood, and the plausibility provides an upper bound on the likelihood. For a given uncertain input variable, the continuous PDF defined in probability theory is associated to a cumulative distribution function (CDF) or complementary cumulative distribution function (CCDF). CCDF is a single and continuous function. In evidence theory CCDF is replaced by a range (which includes CCFD) between two curves, the complementary cumulative belief function (CCBF) and the complementary cumulative plausibility function (CCPF). Basically evidence theory provides a range of uncertainty related to the CCDF. Or, in other words, it provides an uncertain bound on the description of a given uncertain variable. The PDF determined following a fully probability approach is replaced with a much simpler representation of uncertainty based assigning likelihood in intervals. This basic measurement, is denoted by basic probability assignment (BPA). In this section the NCRIT's PDF shown in Fig. 2 is replaced with the BPAs shown in Tab. 1.

NCRIT's interval [-]	BPA [-]	NCRIT's interval [-]	BPA [-]	NCRIT's interval [-]	BPA [-]
$[0.5, 1.5]$	0.0223	$[1.5, 2.5]$	0.0515	$[2.5, 3.5]$	0.0891
$[3.5, 4.5]$	0.1176	$[4.5, 5.5]$	0.1546	$[5.5, 6.5]$	0.1981
$[6.5, 7.5]$	0.1934	$[7.5, 8.5]$	0.1273	$[8.5, 9.5]$	0.0461

Table 1. In the RBDO based on evidence theory, aleatory uncertainty is model through a continuous distribution function, while epistemic epistemic uncertainty is characterized through BPAs. The chain of these sources results in the BPA's depicted in this figure.

RBDO problems based on evidence theory can be formulated in different ways [9, 10, 11]. Typically, as a multi-objective problem such as for this case it would be:

$$\begin{aligned} &\text{maximise } Belief(f > f_{ref}) \quad \& && \text{maximise } f_{ref} \\ &\text{subject to: } 0.24 \leq t(\boldsymbol{b}) \leq 0.35 && \text{and: } P[C_{l,\text{des}}(\boldsymbol{b}, NCRIT) \leq 1.3] = 1 \\ &\text{and: } P[S(\boldsymbol{b}, NCRIT) \leq 4°] = 0 && \text{and: } P[C_{l,\max}(\boldsymbol{b}, NCRIT) \leq 2.07] = 1 \\ &\text{and: } \boldsymbol{b_L} \leq \boldsymbol{b} \leq \boldsymbol{b_U} \end{aligned} \tag{5}$$

For this work, the adopted formulation aimed at determining only one point of the Pareto front, i.e. an airfoil shape that maximizes the belief to deliver an efficiency (Eq. 1), which is at least 190 (we defined this arbitrary based on the pseudo-deterministic optimization). Basically this approach aims to increase our confidence that the airfoil deliver at least an efficiency of 190. This approach differs that the probability one in the sense that here we ask for more performance reliability (to have efficiency at least 190) rather than a general robustness.

$$\begin{aligned} &\text{Find: } \boldsymbol{b} = \{b_1, ..., b_{11}\} \\ &\text{to maximize: } Belief(f > f_{ref} = 190) \\ &\text{subject to: } 0.24 \leq t(\boldsymbol{b}) \leq 0.35 && \text{and: } P[C_{l,\text{des}}(\boldsymbol{b}, NCRIT) \leq 1.3] = 1 \\ &\text{and: } P[S(\boldsymbol{b}, NCRIT) \leq 4°] = 0 && \text{and: } P[C_{l,\max}(\boldsymbol{b}, NCRIT) \leq 2.07] = 1 \\ &\text{and: } \boldsymbol{b_L} \leq \boldsymbol{b} \leq \boldsymbol{b_U} \end{aligned} \tag{6}$$

3. Results

The pseudo-deterministic optimization and the two RBDO processes based on probability and evidence theories, all solved with the Multi-Population Adaptive Inflationary Differential Evolution Algorithm (MP-AIDEA) [12], determined the three different airfoils depicted in Figure 4, denoted respectively by det., pr., and ev.. The top part of Table 2 compares these three solutions based on the pseudo-deterministic objective function and constraints (Eq. 2). As expected, the pseudo-deterministic solution scores the highest pseudo-deterministic objective function, as this airfoil was indeed optimize to maximize this function. It is also noted that the ev. airfoil achieves the highest value of the efficiency at the lowest NCRIT. This is also an expected result as this airfoil was optimized to increase the reliability of efficiency at values around 190. The bottom part of Table 2 evaluates the optimized airfoils based on the mean and standard deviation of the efficiency. The pseudo-deterministic airfoil achieves the highest mean but also the largest standard deviation. The other two airfoils achieve around 25% lower standard deviation at the cost of around 1.4% reduction in the mean value.

	det.	pr.	ev.
$f_{\text{det}}(\boldsymbol{b}, NCRIT = 0.5, 9.5)[-]$	192.7	187.9	189.1
$f(\boldsymbol{b}, NCRIT = 0.5)[-]$	165.9	166.0	168.1
$f(\boldsymbol{b}, NCRIT = 9.5)[-]$	219.4	210.0	210.1
$t(\boldsymbol{b})[-]$	0.249	0.281	0.265
$C_{l,\text{des}}(\boldsymbol{b}, NCRIT = 0.5)[-]$	1.28	1.28	1.28
$C_{l,\text{des}}(\boldsymbol{b}, NCRIT = 9.5)[-]$	1.29	1.30	1.29
$S(\boldsymbol{b}, NCRIT = 0.5)[^\circ]$	4	4	4
$S(\boldsymbol{b}, NCRIT = 9.5)[^\circ]$	8	7	6
$C_{l,\max}(\boldsymbol{b}, NCRIT = 0.5)[-]$	1.62	1.62	1.62
$C_{l,\max}(\boldsymbol{b}, NCRIT = 9.5)[-]$	1.96	1.94	1.90
$\mu_f(\boldsymbol{b}, NCRIT)[-]$	207.1	204.3	204.2
$\sigma_f(\boldsymbol{b}, NCRIT)[-]$	8.8	6.7	6.2

Table 2. Comparison of the optimized airfoils' pseudo-deterministic objective function and constraints (top part), and the mean and standard deviation the airfoils' efficiency (bottom part).

Figure 3 shows the CCDF, CCBF and CCPF for the three optimum airfoils. These curves represent the probability that a selected airfoil delivers at least a given efficiency. For example, the probability that the three optimized airfoils deliver an efficiency which is at least (or larger than) 190 is around 95%. By looking at these curves one can notice the fundamental difference between the probability and the evidence theory uncertainty quantification. In the probabilistic approach, the characterization of the input uncertainty (i.e., NCRIT) through a continuous distribution function resulted in a continuous CCDF (of the output f). Following the evidence approach, we instead characterize NCRIT's uncertainty through discontinuous intervals, and this resulted in discontinuous f's CCBF and CCPF. The region between CCBF and CCPF represent the uncertainty in the probabilistic representation of the uncertain output function. As mentioned above, the ev. airfoil has been optimized to maximize the CCBF for values of the efficiency larger than 190. The result is an airfoil which is indeed more reliable than the other two airfoils for value of f larger than 190 (CCBF more on the right).

Looking at the airfoil geometries in Figure 4 one can notice that the optimized airfoils have increasing relative thickness. The pseudo-deterministic airfoil is the thinner one (25% thick), followed by the airfoil optimized by means of the evidence theory (27% thick) and then the airfoil optimized through the probability theory (28% thick). The lift-to-drag ratio of the three optimized airfoils is shown in Figure 5. In the operative angle of attack range between 5° and 10°, for the largest NCRIT value of 9.5 (lowest DL) the deterministic airfoil is characterized by larger lift-to-drag ratio than that of the other two other airfoils. At the lowest NCRIT value of 0.5 (highest DL) the two airfoils optimized through uncertainty-based approaches instead achieve better lift-to-drag ratio (up to 9°). The reasons for this will be given below.

The airfoils' lift and drag coefficients are depicted in Figures 6 and 7. As expected, all three optimizations have converged towards solutions which maximize the lift coefficient as much as possible. Indeed, in all cases the lift coefficient at the design angle of attack of 7° has reached its maximum allowed value (which is also the sought design one). Given that the three airfoil have the same lift coefficient, the difference in the airfoils' lift-to-drag ratios can be explained by looking at drag. In the operative region, in clean conditions, the pseudo-deterministic solution has the lowest drag, explaining the higher efficiency under these conditions. Under low NCRIT conditions, the ev. optimum has instead the lowest drag, closely followed by the pr. solution.

The drag coefficient behavior can be further investigated by looking at the pressure

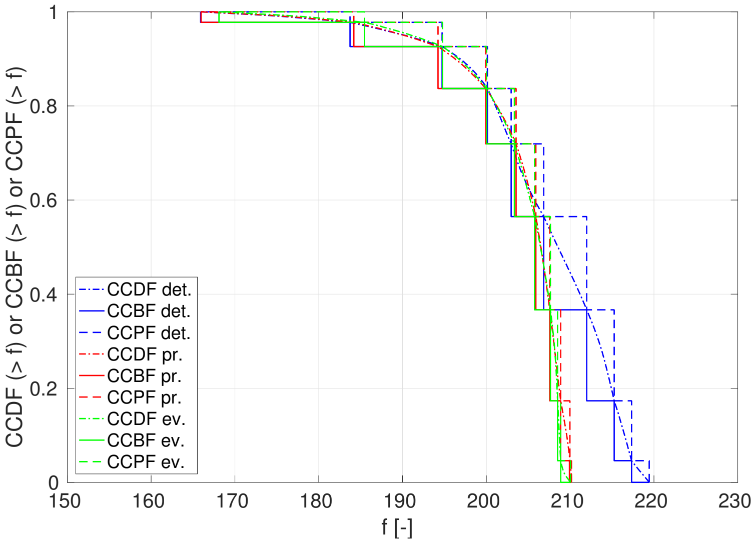

Figure 3. Comparison of the airfoil efficiency's complementary cumulative distribution function (relative to the probability theory uncertainty quantification) and the complementary cumulative belief and plausibility functions (relative to the evidence theory uncertainty quantification).

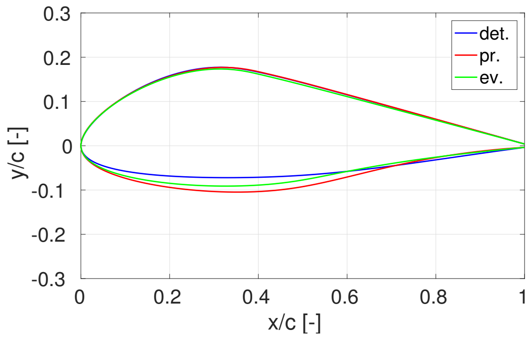

Figure 4. Geometries of the optimized airfoils.

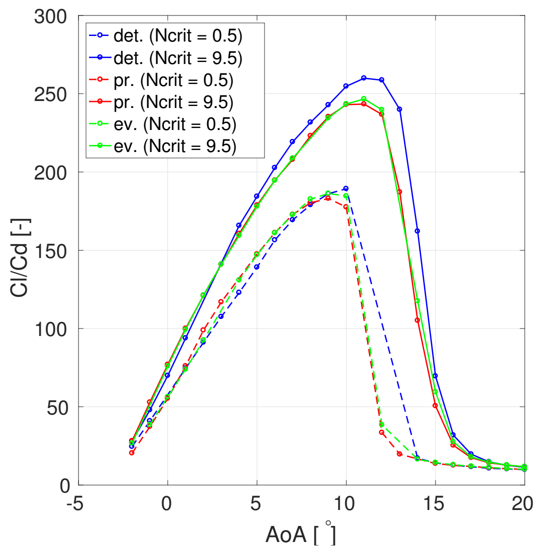

Figure 5. Airfoils' lift-to-drag ratios evaluated at the lower and upper bounds of the NCRIT's variability range.

distribution and the skin friction coefficient depicted in Figures 8 and 9 (both generated at the design angle of attack). Under high NCRIT conditions, the pseudo-deterministic airfoil minimizes drag by maximizing the extension of laminar flow on the pressure sides. This is achieved with a rather small curvature, which limits the pressure gradient, delaying transition. Under low NCRIT conditions, the onset of the pseudo-deterministic airfoil's transition is largely anticipated, increasing the skin friction drag due to a greater amount of turbulent flow. The

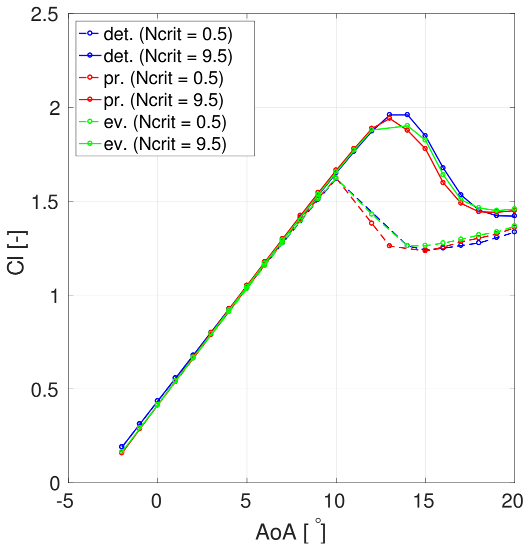

Figure 6. Lift coefficient.

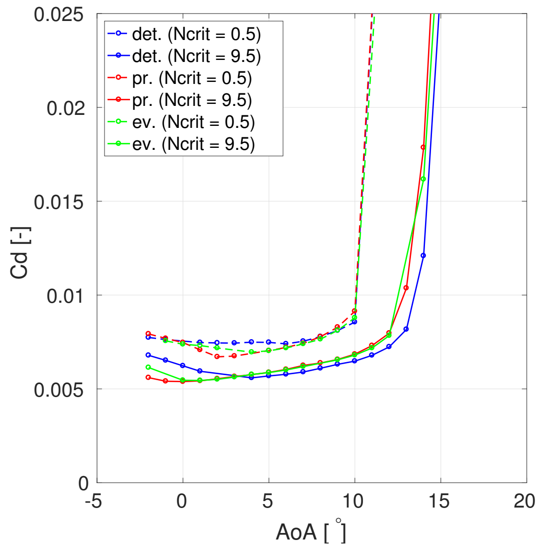

Figure 7. Drag coefficient.

two airfoils optimized with uncertainty-based approaches limit this situation through a larger relative thickness which allows for a larger curvature in the aft part of the pressure side. This geometry induces a less steep pressure gradient delaying transition, and having more laminar flow that that of the pseudo-deterministic airfoil. Under these conditions, despite the earlier occurrence of transition (larger skin friction drag due to larger amount of turbulent flow), the ev. airfoil has lower overall skin friction drag than that of the pr. airfoil due to the lower relative thickness, which reduces the airfoil's cross-section (and hence reduces the skin friction drag).

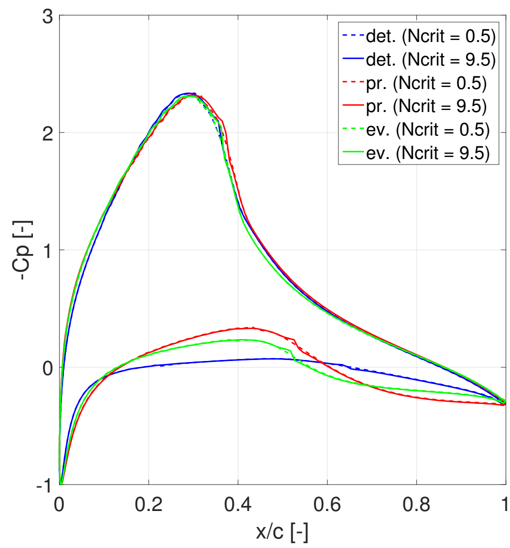

Figure 8. Pressure coefficient at the angle of attack of 7° and Re of $12{\cdot}10^6$.

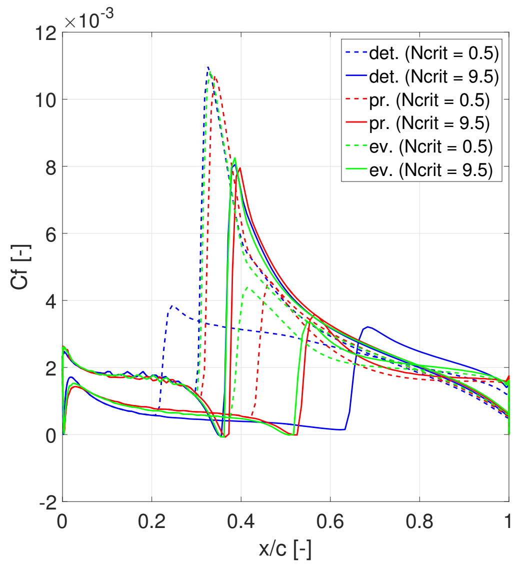

Figure 9. Skin friction coefficient at the angle of attack of 7° and Re of $12{\cdot}10^6$.

4. Conclusions

This paper has presented three approaches to optimize wind turbine airfoils in an uncertain design scenario. Uncertainty has been related to the definition of the XFOIL's NCRIT value: a parameter we have considered affected by both aleatory and epistemic uncertainty. The first optimization approach, denoted by pseudo-deterministic, resembles the common practice to treat uncertainty in a rather simplistic way. The pseudo-deterministic airfoil has been optimized by maximizing the average of its efficiency at the upper and lower bounds of the NCRIT's variability range. The second and the third approaches have taken into account uncertainty in a more rigorous way. One through probability theory and the other by means of evidence theory. The optimization using probability theory characterized NCRIT's uncertainty through a distribution function, and aimed at maximizing the mean efficiency by constraining its standard deviation. The optimization using evidence theory instead defined uncertainty with less information through likelihood intervals, and aimed at maximizing the reliability of the airfoil efficiency.

The resulting airfoils have shown different characteristics based on the proposed optimization approaches. The pseudo-deterministic airfoil, once evaluated in the probabilistic scenario, has achieved the largest mean value, but highest standard deviation. The uncertainty-based design optimization approaches have allowed us to achieve more robust airfoils, limiting the efficiency's standard deviation with small penalty in term of its mean. The largest robustness achieved by these airfoils is mainly tied to a larger relative thickness, which in turn has reduced efficiency at high NCRIT values and increased performance at low NCRIT values. The largest thickness indeed has allowed these airfoils to have a more gentle curvature on the aft part of the pressure side (than that of the pseudo-deterministic one), which delays transition under low NCRIT conditions. Between the two uncertainty-based approaches, the most promising one appears to be the one using evidence theory as it has led to an airfoil having also more reliable performance.

Future work will investigate the design of airfoils under more complex design scenarios, and assess and improve the uncertainty propagation algorithms in terms of computational cost.

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