Investigating the effectiveness of Padé-type approximations

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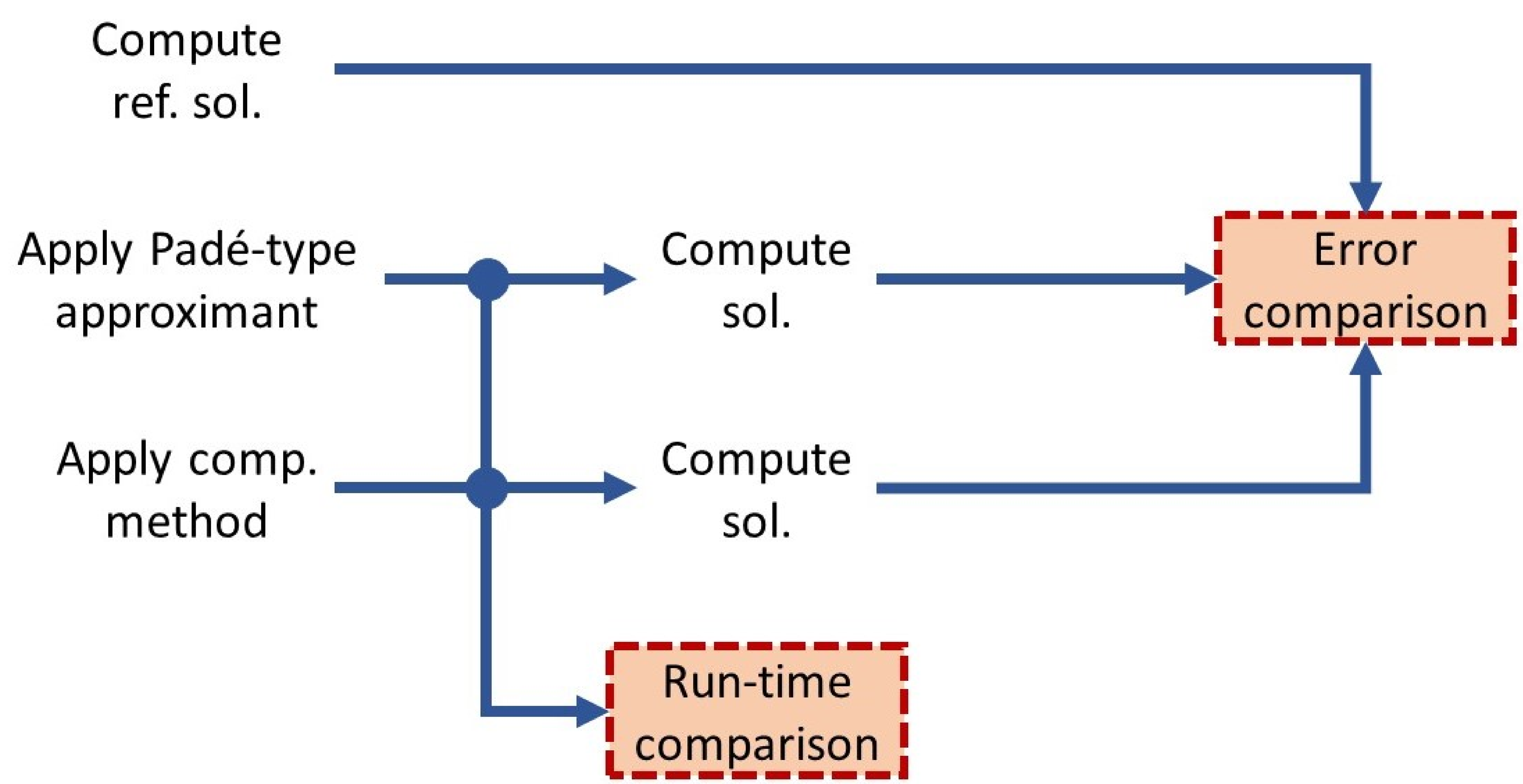
Abstract

Padé-type approximants represent improvements on more common approaches as far as accuracy is concerned. However, the computation of such approximations typically demands higher computational expense. We quantify the improvements in accuracy alongside the associated increases in expense. We do so for a set of problems chosen to depict differing behaviours in a range of target functions. We apply the compact schemes introduced by Lele (which have since been used in problems requiring their "spectral-like" resolution) to various PDEs intended to represent a range of phenomena in fluid dynamics.

Method



To appropriately quantify and qualify the numerical error in a given Padé approximation, we compute it first by comparison to some reference solution (ref. sol.). This, where available, is the given problem's analytical solution. Additionally, we compare the Padé approximation to that of a comparison (comp.) method - at least one more commonly-used approximant.

In comparing the run-times of the approximants, the error in MATLAB's *tic-toc* measure had to be accounted for. We did this by repeating the experiment until the standard deviation of the results fell below 1% of the mean; with each new "run", we varied the step-size according to a random scattering Design of Experiment.

Results

Scheme	Mean increase in run-time (vs. central)	Median reduction in error (vs. central)
Padé-4	51.1%	1.81%
Padé-6	50.4%	2.03%
Padé-8	81.6%	2.10%
Padé-10	79.3%	2.12%

Table 1: Average increases in run-time, and decreases in error, for each scheme for the 1D heat equation are considered.