Structure of turbulence prior to rapid-distortion

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Abstract

Rapid-distortion theory (RDT) uses linear analysis to study the interaction of turbulence with solid surfaces. It applies whenever the turbulence intensity is small and the length (or time) scale over which the interaction takes place is short compared to the length (or time) scale over which the turbulent eddies evolve. When both of these conditions are interpreted asymptotically, it implies that the upstream boundary condition that enters as an input to a scattering problem (given by the term, ω_c, below) may be specified at an infinite distance from the surface discontinuity on the scale of the interaction but one that is still short on the scale of the turbulence evolution. In this paper we develop the mathematical theory for the leading edge interaction of turbulence by considering a canonical problem of a jet flow interacting with a semi-infinite infinitesimally thin flat plate positioned parallel to the level curves of the mean flow field. To fix ideas we consider a constant shear flow in which $U(y_2) = \gamma y_2$ where $\gamma = O(1)$ is the shear rate. We show in the accompanying talk that the Fourier transform of a turbulence correlation of the form $R_{2,2}(\boldsymbol{x}, \boldsymbol{x}' - \boldsymbol{x}, t' - t)$ and which is stationary is given by the following exact integral formula

$$R_{22}(x_2, x_2' - x_2, \omega) = \int_0^\infty dy_2 \int_0^\infty d\tilde{y}_2 \left[\overline{G}_0(x_2|y_2)\overline{G}_0^*(x_2'|\tilde{y}_2)S(y_2, \tilde{y}_2, k_3 = 0, \omega)\right] \qquad (1)$$

where $S(y_2, \tilde{y}_2, k_3 = 0, \omega)$ is the spanwise independent spectrum of the convected quantity $\omega_c(t - x_1/\lambda y_2, y_2)$ that enters the Rapid-distortion theory equations as an upstream boundary condition. This spectrum is given by (6.13) in Goldstein *et al.* (JFM, vol. 824, pp.477-512, 2017). (Note that the spanwise wavenumber, k_3, is equal to zero in the above formula for R_{22} because for the constant shear flow $U = \gamma y_2$ is independent of the spanwise co-ordinate, y_3). $\overline{G}_0(x_2|y_2)$ is the Fourier transform of the Green's function of the two-dimensional Laplace equation and is given by $\overline{G}_0(x_2|y_2) = \frac{i}{2}\text{sgn}(y_2)\text{sgn}(\omega)e^{-|\omega||x_2-y_2|/\gamma|y_2|}$, where ω is the temporal frequency. In Figure 1 we show that the inverse Fourier transform of the computation of (1) results in a space-time form of the transverse correlation function R_{22} that is realistic in the sense that the it possesses (a). a cusp at zero values of the time interval $(t' - t)$ (b). a finite anti (i.e. negative)-correlation and (c). decays to zero as $(t' - t) \to \infty$. The spatial difference $(x_2' - x_2)$ is set to zero in the computation below. We shall discuss how this theory can be used in the problem of turbulence scattering.

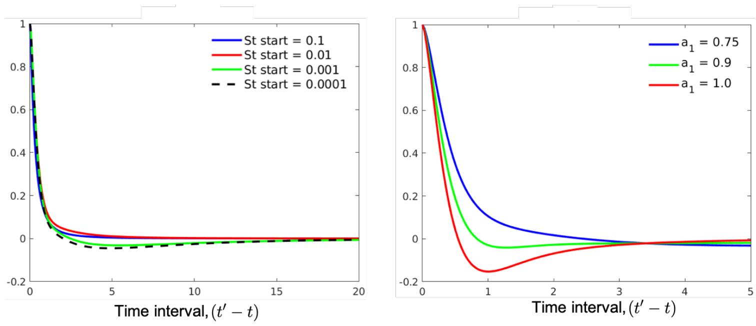

Figure 1: Inverse Fourier transform of (1) computed using Matlab's FFT routine. $\gamma = 2$. Left figure shows effect of different starting values for temporal frequency, ω (here refered to as St). Right figure, show effect of parameter a_1 in Eq. (6.13) on p.494 of GLA17.