SIMULATION OF A TWELVE PULSE DIODE RECTIFIER WITH SATURABLE REACTORS

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ABSTRACT

A twelve pulse diode rectifier with saturable reactors to allow a degree of phase control is simulated using mathematical software. First order differential equations are solved numerically to allow calculation of line current. Application of the fast Fourier transform to line current provides harmonic current magnitudes. Line current waveforms and harmonic magnitudes from the simulation agree with expected results. The simulation is to be used as part of a harmonic analysis of multiple parallel connected diode rectifiers.

Keywords: twelve pulse; harmonics; diode rectifiers; saturable reactors, Runge Kutta, simulation

INTRODUCTION

The aluminium smelting company situated in the Scottish highlands is in the process of increasing its output. This is a long term project which will take it from having hydro direct current (dc) generation with an additional 16MW fed from grid connected diode rectifiers, referred to as rectifiers, required for power management to having 90 MW of installed rectification, essential for production, supplied from hydro alternating current (ac) generation and the grid. At present the installed rectification is two 11 MW and one 15 MW rectifier. Given the differing ages of the rectifiers and a recent up date to the relevant Engineering Recommendation, ER G5/4, [1] along with the proposed increase in rectification, a harmonic study was deemed necessary to help ascertain levels of harmonic pollution and any necessary filtration. While it is recognised that software exists to aid harmonic study, given a lack of detailed system parameters, the method of approach decided upon was to build a mathematical model of a twelve pulse 11 MW rectifier which could be applied to the site supply.

RECTIFICATION

Rectifiers are a member of a family of electrical apparatus that are referred to as non linear loads. The classical approach to rectifier analysis assumes that their switching action chops blocks of current sequentially from each phase of the ac supply to provide a constant, ripple free, dc output feeding an infinite load inductance. The resulting ac waveform is a non sinusoidal step like waveform. In terms of power flow the output of the rectifier is only real power. The input power from the ac side is the output real power plus real power losses and reactive power required to produce the step like waveform. This reactive power has to be dissipated and is done so by producing higher order harmonics which are transmitted back into the ac system. Hence the term non linear load and the label assigned to a rectifier of being a 'harmonic generator'.

Ideal Rectifier

The non sinusoidal rectifier response is complex and may be separated into fundamental and higher order harmonics with the application of Fourier transform. Idealised harmonic response of a rectifier is given by the equation:

$$h = kq \pm 1 \qquad (1)$$

where

h	is the harmonic order
k	is an integer
q	is the pulse number of the rectifier.

The magnitudes of the harmonic currents of such a rectifier are obtained by:

$$I_h = I_1 / h \qquad (2)$$

where

I_h	is the harmonic current
I_1	is the magnitude of the fundamental current
h	is the harmonic order

With reference to equation (1) it is apparent that the harmonics generated occur one harmonic order lower and one higher around multiples of the pulse number. Thus a twelve pulse rectifier will have the first harmonic, after the fundamental, at eleven times the fundamental frequency and the next at thirteen times the fundamental frequency. The magnitude of a harmonic is proportional to the reciprocal of its order which in turn is equivalent to the per unit harmonic current.

It can be shown [2] that the phase current of a star – star six pulse connected system is obtained from the following expression:

$$i_a = 2\sqrt{3}/\pi I_d(\cos\omega t - (1/5)\cos 5\omega t + (1/7)\cos 7\omega t - (1/11)\cos 11\omega t + (1/13)\cos 13\omega t - \ldots) \qquad (3)$$

Where I_d is diode current.

And the phase current of a star – delta six pulse connected system is obtained from:

$$i_a = 2\sqrt{3}/\pi I_d(\cos\omega t + (1/5)\cos 5\omega t - (1/7)\cos 7\omega t - (1/11)\cos 11\omega t + (1/13)\cos 13\omega t - \ldots) \qquad (4)$$

The summation of equations (3) and (4) provide an expression for the phase current of a twelve pulse connected system which, for the frequency range fundamental to fiftieth harmonic inclusive is:

$$i_a = 4\sqrt{3}/\pi I_d(\cos\omega t - (1/11)\cos 11\omega t + (1/13)\cos 13\omega t - (1/23)\cos 23\omega t + (1/25)\cos 25\omega t - (1/35)\cos 35\omega t + (1/37)\cos 37\omega t - (1/47)\cos 47\omega t + (1/49)\cos 49\omega t) \qquad (5)$$

Equation (5) contains all the harmonics associated with an ideal twelve pulse rectifier over the frequency range covered in Engineering Recommendation G5/4.
A normalised plot of the waveform produced by equation (5), expanded to the 385th harmonic, is shown in Figure 1.

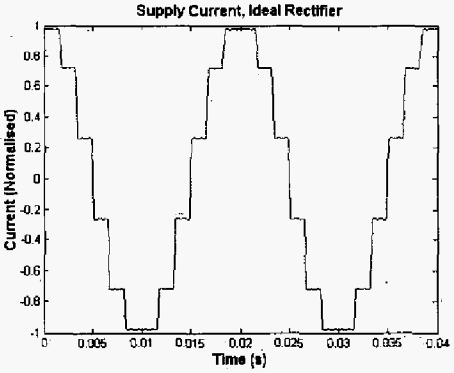


Figure 1 Normalised Ideal Rectifier input current

Realistic rectifier

In reality the effects of commutation (overlap) and delay angle have to be taken into account. It has been found [3] that overlap reduces the mean output voltage as well as rounding off steep sides of the waveform shown in Figure 1. This has the effect of decreasing the harmonic current magnitudes, an effect that is more noticeable the higher the harmonic order.

Delay angle variation is achieved by means of saturable reactors. These operate on the principle that the inductance of a core is directly proportional to its absolute permeability. The absolute permeability of a magnetic circuit is not constant and this allows it to have, in this instance, the useful characteristic of saturation. If a core is not saturated its permeability is high and hence inductance is high, if saturated then the permeability is very low and inductance is negligible. Thus application of a varying voltage to a saturable reactor will cause very low current to flow if the voltage is not great enough to saturate the core, supply voltage predominantly being across the reactor inductance. Conversely, if the voltage is great enough to cause saturation then current will flow in proportion to the load circuit impedance given reactor inductance is negligible. If a rectifier output is connected to a saturable reactor then a positive half cycle will be applied. Prior to saturation the voltage will be present across the reactor, once saturation is reached the reactor will remain saturated for the rest of the half cycle and the voltage will be present across the remainder of the circuit. The angle at which saturation occurs may be controlled by including a control coil wound onto the reactor core. This coil is fed from a variable voltage dc source and allows a variable unidirectional flux to be induced in the core. The greater this flux the lower the magnitude of half cycle applied voltage that will cause saturation. It is this that provides means to control commutation from one diode to the next. The theoretical delay angle control range being 0 to 90^0.

11 MW RECTIFIER MODEL

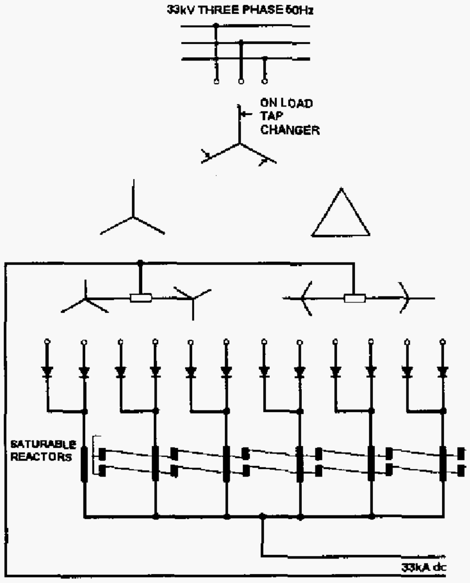


Figure 2 Rectifier Circuit Diagram

A simplified circuit diagram of an 11 MW halfwave rectifier is shown in Figure 2. Voltage control is achieved with an on load tap changer. Rectification is achieved by means of a twelve phase half wave connection. The outputs of these are fed through saturable reactors and then onto the dc bus. Information provided stated that the saturable reactors provide a delay angle range of 8^0 to 17.5^0 and that overlap is 20^0. The output of the rectifier is 33 kA at 350 V.
Twelve sinusoidal voltage waveforms each differing 30^0 in phase are present on the low voltage side of the transformer. If these are presented to diodes of varying resistance and inductance dependent on whether forward or reverse biased and a load of resistance and inductance then the resulting current flow in any diode may be represented by equation 6.

$$di/dt = v/L - (R/L)*i \qquad (6)$$

Where: v is the sinusoidal voltage, i the instantaneous current, R the total circuit resistance and L the total circuit inductance. Both R and L vary depending on whether the relevant diode is conducting or not. Solution of this differential equation for each voltage will provide the complex current associated with each phase which, when summed appropriately and transformed through the transformer, will provide the complex current on the high voltage feeder. The Fourier transform of this current will yield its harmonic components.

Equation 6 is stiff and to ensure stability of the numerical solution the step size must be small relative to the interval over which the equation is to be solved. It was decided that the best solution would be gained by implementing the fourth order Runge Kutta method. Exact details of the algorithm used are available [4].
In addition the stability criterion of the Runge Kutta method meant that careful selection of the diode off parameters was required. It is known [5] that the stability of the fourth order Runge Kutta is related to the product of the real pole of the system being solved, s, and the solution step time, h, which must be greater than -2.75 and not too close to this boundary to avoid inaccuracies. The value sighted as being acceptable is -2. The pole of the series R, L circuit is -R/L and the step size is the inverse of the sampling frequency, f_s. In a diode model the dominant values are the off resistance and off inductance which model the diode in reverse bias. These were set as $R_{off} = 9000\ \Omega$ and $L_{off} = 0.5$ H. With $f_s = 9$ kHz the h*s result is -2. Variation of R_{off} and L_{off} to decrease the h*s product did stop a viable solution after –2.75. Making the h*s product greater than -2 would mean a higher diode off inductance, already unrealistically high at 0.5 H, or a lower off resistance which would result in unrealistically high diode leakage current. The forward biased diode resistance and inductance were set to 0 Ω.

Equivalent circuit

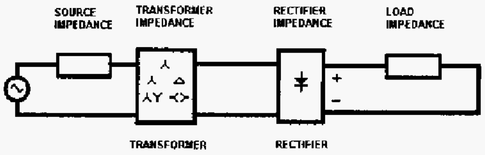


Figure 3 Equivalent Circuit

The equivalent circuit is shown in Figure 3. The supply network has been replaced by a Thevenin equivalent circuit consisting of a voltage source at rated voltage, 33 kV, on the HV side of the transformer and a system impedance. All impedances were transposed to the secondary side of the transformer.

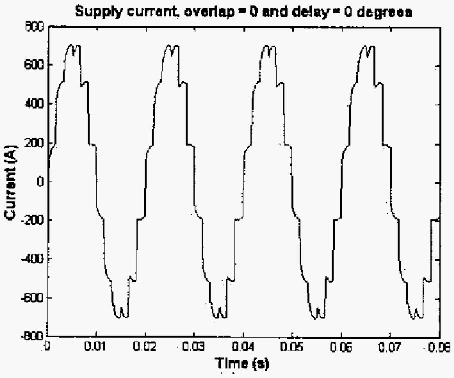
Model Results

Table 1

		I_h / I_1		
h	1/h (%)	Alpha = 0^0 Overlap = 0^0 (%)	Alpha = 8^0 Overlap = 20^0 (%)	Alpha = 17.5^0 Overlap = 20^0 (%)
11	9.09	9.09	8.07	7.79
13	7.69	5.99	6.99	7.09
23	4.35	3.60	3.56	3.55
25	4.00	3.50	3.68	3.52
35	2.86	2.24	2.59	2.74
37	2.70	2.48	2.25	2.01
47	2.13	1.72	1.84	1.90
49	2.04	1.95	1.93	1.92

The model results in table one contain three sets of results. The first result is for no overlap or delay to allow a control reference between model output and expected theoretical output. The remaining results are for minimum and maximum delay at rated voltage input.

The current waveforms for all outputs are shown in Figure 4.



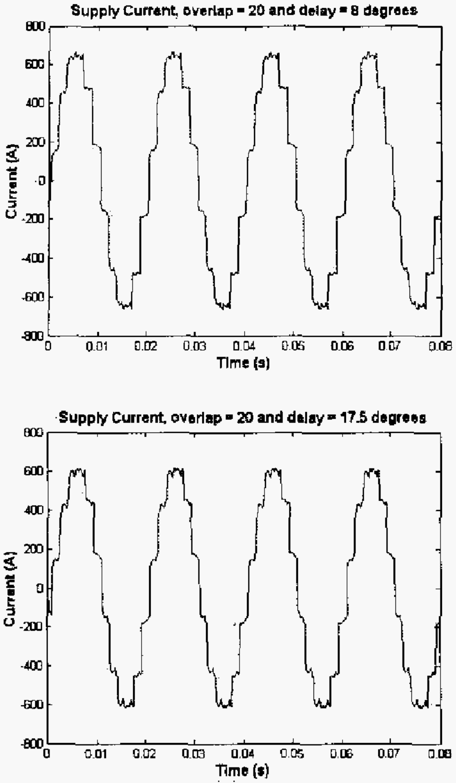


Figure 4 Output Current Waveforms

The control results do differ from the theoretical (1/h) results given that they show a decrease in magnitude for the higher order harmonics. This is as expected given the non infinite nature of the model inductance. Comparison of the current waveform in Figure 1 and the relevant waveform in Figure 4 clearly shows the difference this causes. The remaining results do show a variation in harmonic magnitude over the small range of delay afforded by saturable reactors and the decrease in peak current magnitude is seen in the corresponding current waveforms. Overall the results are in line with other findings to date [3] and demonstrate the successful operation of the model.

CONCLUSIONS

A twelve pulse rectifier has been simulated using a mathematical model. The model has been used to ascertain the harmonic currents generated in a large power rectifier over the range of control angle available. The results of this, when compared with known theory and other work, indicate that the model operation is successful.

FURTHER WORK

To date it has been assumed that the supply voltage is an ideal sinusoid. When water reserves and works schedule allow it is planned to take actual voltage measurements, which will be used in the model. In addition harmonic spectrum measurements will be taken to allow comparison between model and actual results. Thereafter the Total Harmonic Distortion (THD) will be calculated and compared with recommended values [1]. Indication at present is that, although THD is low, harmonic filters are required. The last step will be to investigate the application of active filters to the plant.

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