Robust Design Optimisation of Dynamical Space Systems

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ABSTRACT

In this paper we present a novel approach to the optimisation of complex systems affected by epistemic uncertainty when system and uncertainty evolve dynamically with time; we propose a new modelling approach that uses Evidence Theory to capture epistemic uncertainty

A system is considered which is affected by the time during the operational life (failure rate, performance degradation, function degradation, etc.). The goal is to obtain a resilient design: robust with respect to performance variability and reliable against possible partial failures of one or more components.

We propose to enhance the Evidence Network Model (ENM) with time-dependent reliability functions and decompose the problem into subproblems of smaller complexity. Through this decomposition uncertainty quantification of complex systems becomes affordable for a range of real-world applications.

The method is here applied to a simple resource allocation problem where the goal is to optimally position subsystems within a spacecraft [1].

1. INTRODUCTION

On-orbit space systems are subjected to a hostile environment where maintenance is impossible or very limited. Therefore, lifetime reliability is an inevitable factor to consider during space system design. Longer service lifetime or higher confidence in lifetime length is commonly reached by introducing redundancy into the system employing safety margins and safety factors. This traditional method typically misses the proper estimation of the uncertainties which can lead to an unnecessary high number of redundant systems and an increase of development expenses. To reduce the unnecessary costs of the space system well-designated uncertainty quantification method must be employed. A comprehensive review of uncertainty-based design techniques for aerospace engineering can be found in [2]. These techniques are highly relevant for space system design as well. The review discusses the most popular uncertainty modelling approaches including probability theory, evidence theory, possibility theory, interval analysis, and convex modelling. As a conclusion, evidence theory is proposed to use if the available information is conflicting. The authors also provide an overview of uncertainty classification schemes. Following their definitions, we adopt the taxonomy of risk assessment in this paper. Accordingly, uncertainties can be divided into two categories. *Aleatory uncertainty* which stems from an inherently random natural process. Hence, this type of uncertainty is irreducible and can be well described by the probability theory. *Epistemic uncertainty* is due to the lack of knowledge or incomplete information and can be eliminated by obtaining more knowledge about the investigated problem. This type of uncertainty is typical in early design phases when multiple experts provide different opinions, models are low fidelity, or in the case of poor quality and incomplete data. Evidence Theory, also known as Dempster-Shafer theory, provides a valid mathematical tool to model this type of uncertainty [2,3,4,5] though it is computationally expensive and difficult to handle.

A new, called Evidence Network Model (ENM), was introduced in [6] and extended in [7] to model engineering systems that can be decomposed in a number of subsystems or functions. ENMs are undirected and connected graphs where each node is a sub-system and each link an information pathway. In this work, ENMs are extended to include time-dependent uncertainty and a time-varying performance criterion.

The proposed method is then used to solve a simple resource allocation problem where the subsystems are subjected to uncertain time-dependent failure rates and sizing parameters are also uncertain. The goal is to optimally position the subsystems within a spacecraft [1].

2. EVIDENCE THEORY

In Evidence theory, expert opinions are expressed by assigning confidence levels to certain sets of intervals. The confidence levels are called basic probability assignment (bpa) and represent the probability that an uncertain variable is within its corresponding interval. All the Cartesian products of these intervals provide a power set, the uncertain space U. An element of this power set with non-zero bpa is the so-called Focal Element (FE), where the bpa of the element is the product of the bpas of the constructing intervals. After identifying all the FEs, the lower and upper boundary, or belief (Bel) and plausibility (Pl), of a proposition A can be evaluated as:

$$Bel(A) = \sum_{FE \subset A, FE \in U} bpa(FE) \quad (1)$$

$$Pl(A) = \sum_{FE \cap A \neq 0, FE \in U} bpa(FE) \quad (2)$$

In this paper, a proposition "the state of the system loaded by uncertainty is not greater than a threshold value" is considered:

$$A = \{u \in U | f(u) \leq v\} \quad (3)$$

By calculating the Bel and Pl values for each possible threshold value two curves can be calculated: the Cumulative Belief Function (CBF) and the Cumulative Plausibility Function (CPF). The unknown precise cumulative distribution function (CDF) is enveloped by these two curves.

In practice, if the maximum value within a FE is not greater than the threshold value than the evidences in the FE fully support the proposition and thus its bpa contributes to Eq. 1. If the minimum value of the FE is less than or equal to the threshold value than the FE only partially supports the proposition and thus the bpa of the FE contributes to Eq. 2.

This direct method suffers from the curse of dimensionality as the number of FEs increases exponentially with the number of uncertain variables because it results from the Cartesian product of all the intervals of all the uncertain parameters.

3. EVIDENCE NETWORK MODELS

The computational burden of obtaining the exact Bel and Pl curves motivates us to calculate approximations of the two curves. In this section only the approximation of the Bel curve is presented, the Pl curve can be approximated similarly.

A global quantity of interest F is considered:

$$F(d,u) : D \times U \subseteq \mathbb{R}^{n+m} \to \mathbb{R} \quad (4)$$

where F depends on some design parameters $d \in D \subset \mathbb{R}^n$ and uncertain parameters $u \in U \subset \mathbb{R}^m$. The set D is the available design space and U the uncertain space.

ENM separates the uncertain space by defining two types of uncertain variables: *uncoupled* uncertain variable u_i which has an influence only on one subsystem i and *coupled* uncertain variable u_{ij} which influences two subsystems i and j.

A generic complex system can be represented as a network, where each node is a subsystem and information is shared through links between subsystems. We can then define the function F as:

$$F(d,u) = \sum_{i=1}^{N} g_i(d,\ u_i,\ h_i(d,\ u_i,\ u_{ij})) \quad (5)$$

where N is the number of subsystems involved, $h_i(d,\ u_i,\ u_{ij})$ is the vector of scalar *linking* functions $h_{ij}(d,\ u_i,\ u_{ij})$. The functions $g_i(\cdot,\cdot,\cdot)$ represent quantities computed by the governing equations of the different subsystems.

As mentioned above, the exact Bel curve is not calculated in this approach; instead, an approximation of it is estimated with a decomposition approach. It reconstructs an approximated belief curve through three steps, for a given design vector $\bar{d}$ and the corresponding worst-case scenario that refers to the uncertain variable input which maximises F for the given design vector $\bar{d}$.

The first step freezes the uncoupled variables and calculate a (partial) belief curve for each h_{ij} function, that expresses the contribution of each coupled variable.

In the second step the partial Bel curves are sampled N_S times at different levels q, by taking a succession of $\{v_q = v | q = 1 \ldots N_S\}$ values. Corresponding to each sample a coupled vector $\hat{u}^q_{ij}$ is obtained:

$$\{\hat{u}^q_{ij} | \hat{u}^q_{k,ij} = argmax_{u_{ij} \in FE_{k,ij}} F(u_{ij})\} \quad (6)$$

Finally, in the last step, for each sample the decomposition approach constructs an approximation to the value of the Bel using only the FEs of the uncoupled variables scaled by the belief of the samples.

The g_i functions are decoupled from the coupling variables by fixing the value of the linking functions at their maximum $\widehat{u}^q_{ij} = argmax_k F(\bar{u}^q_{k,ij})$ values resulting in Eq. 7:

$$g_i(u_i) = g_i(\bar{d},\ u_i,\ \widehat{h}_i(\bar{d},\ u_i,\ \widehat{u}^q_{ij}) \quad (7)$$

The decoupled governing functions $g_i(u_i)$ are maximised w.r.t. each $FE_{k,i}$ and the corresponding values $\widehat{g}^q_{k,i}$ are obtained:

$$max_{(u_{i,i=1...N} \in FE_K)} F(\bar{d}, u_i, \widehat{u}^q_{ij}) = \sum_{i=1}^{N} \widehat{g}^q_{k,i}$$
(8)

where $FE_K = (\Box_{k_i=1}^{k} FE_{k_N,N})$

with the associated bpa:

$$bpa^q(FE_K) = \prod_{i=1}^{N} bpa(FE_{k_i,i}) \prod_{ij} \Delta Bel_{ij}^q \quad (9)$$

where $\prod_{ij} \Delta Bel_{ij}^q$ is the contribution of the partial belief curves of coupled variables and $\prod_{i=1}^{N} bpa(FE_{k_i,i})$ is the product of all the bpa's of the focal elements of the uncoupled variables $FE_{k_i,i}$.
The approximation of the belief can then be expressed as:

$$\overline{Bel}(F(d,u) \le v) = \sum_q \sum_K bpa^q(FE_K)$$
(10)

The described approach reduces the computational load by reducing the number of maximizations to be linear with the problem dimension. For more details about definitions and proofs please refer to [6] and [7].

4. CONSTRAINED MIN-MAX APPROACH

The approach to the design of complex systems under uncertainty proposed in this paper, requires the solution of one or more constrained min-max optimisation problems. The solution to this class of problems is here approached with a constrained variant of MPAIDEA, an adaptive version of Inflationary Differential Evolution [13]. This section describes only the strategy to handle constraints in the min-max version of MPAIDEA. More details on the approach to the solution of unconstrained min-max problems with Inflationary Differential Evolution can be found in [14].

The min-max algorithm proposed in this paper iteratively solves a bi-level optimisation, first minimising over the design vector **d** (outer loop) and then maximising over the uncertainty vector **u** (inner loop). The inner loop provides solutions that satisfy the constraint, while the outer loop maintains the constraint satisfaction while minimising the cost function F.

The constraint handling procedure, implements the following steps:

- Initialisation of a population of **d** and **u** vectors;
- While the number function evaluations is lower than the maximum allowed, do the following
  - [Outer-Loop]Constrained minimisation of the objective function over the design space, evaluating the cost function F over all the uncertainty vectors stored in the archive **Ar = Ar_u** ∪ **Ar_c** updated during the inner loop;
  - [Inner-Loop]Constrained maximisation of the cost function F over the uncertain parameters **u** (updating the archive **Ar_u**) and parallel maximisation of the constraint violation over the uncertainty space U (updating the archive **Ar_c**). If a feasible solution cannot be found, the constraints are relaxed such that a small violation is accepted.

For a more detailed description of the constrained min-max approach please refer to [7].

5. RELIABILITY OF SPACE SYSTEMS

Space system maintenance is highly expensive and sometimes impossible due to the operational environment. The useful service time of a space system, therefore, highly depends on the reliability of the system and of its components. For this reason, Reliability-based Design Optimisation (RBDO) has been recognised as an essential tool for space system design [8].
Various approaches have been proposed to model the reliability of space systems; however, they lack of any statistical support from on-orbit data or they are focusing on a particular space vehicle of a manufacturer. Recently, Castet and Saleh [8-9] have presented a reliability model based on actual on-orbit data of 1584 Earth-orbiting satellites successfully launched between January 1990 and October 2008. They employed the Kaplan-Meier estimator [10] to build a nonparametric model which was used as a reference reliability function. A parametric Weibull distribution was fitted on this reference reliability function with a maximum likelihood estimation. The authors demonstrated that infant mortality (decreasing failure rate) is a valid phenomenon for satellites and the Weibull distribution with shape parameter less than 1 well approximates this failure behaviour of satellites and their components.
According to the Weibull distribution, the time-dependent reliability function is:

$$R(t) = e^{-(t/\theta)^{\beta}} \quad (11)$$

where θ and β are the scale and shape parameters respectively; β is dimensionless and θ is expressed in units of time and the shape parameter influence R(t) as follow.

1. For 0<**β**<1, the failure rate is decreasing (infant mortality).
2. For **β**=1, the failure rate is constant, and in this case the Weibull distribution is equivalent to the exponential distribution.
3. For **β**>1, the failure rate is increasing (wearout):
   a. For 1<**β**<2, we have an increasing concave failure rate.
   b. For **β**=2, we have a linear failure rate, and in this case the Weibull distribution is equivalent to the Rayleigh distribution.
   c. For **β**>2, we have an increasing convex failure rate.
   d. For 3<**β**<4, the Weibull distribution approaches the normal distribution.

6. PROBLEM FORMULATION

The statistical analysis of Castet and Saleh [8-9] has revealed that the failure of telemetry, tracking and command (TTC) and the attitude and orbit control (AOCS) (particularly the gyros, sensors, and reaction wheels) subsystems are the major drivers of satellite failure. Considering only small scale satellites (under 500 kg), it has been shown that TTC and the electrical power system (POWER) are the major contributors of space system failure [11].
The positioning problem of a three-subsystem satellite containing these critical subsystems is presented in this paper to demonstrate the use of the ENM on a time-dependent problem. The space system is then modelled as a network with three nodes (AOCS, TTC and POWER) and two links between TTC-POWER and AOCS-POWER.

The objective function we consider is the weighted function of the mass and the moment of inertia with respect to the vertical axes z:

$$F = w_m \sum_{i=1}^{3} m_i + w_i \sum_{i=1}^{3} m_i r_i^2 \qquad (12)$$

where m_i is the mass of the subsystem i and r_i is the horizontal distance of Center of Mass (CoM) from the vertical axes z. The w_m and w_i denote the weights and for the sake of simplicity they are considered to be equal to one.
The way the subsystems and components are allocated influences the centre of gravity of the whole system. The mass and size of each component is affected by uncertainty in system design parameters and system degradation. A failure rate (function of time and affected by epistemic uncertainty) is used to quantify the performance degradation of power, TTC and AOCS systems.

Our goal is the evaluation of the constrained worst-case scenario:

$$min_{d\in D} max_{u\in U} F(d, u) \ s.t. \ C(d,u) < 0 \qquad (13)$$

and the reconstruction of the CBF.

The problem is influenced by 24 design parameters and 17 uncertain parameters. Mass, moment of inertia and reliability of the systems are all affected by both design and uncertainty variables.

In the problem there are one uncoupled parameter for AOCS, one for TTC and six for the POWER. Regarding the coupled components, there are five uncertain parameters that influence both AOCS and POWER and six that influence TTC and POWER. This separation allows to apply the decomposition method for the belief reconstruction.

Two approaches, explained in the next sections, have been used to model the reliability of the system.

6.1. RELIABILITY IN THE OBJECTIVE FUNCTION

OBJECTIVE FUNCTION:
With regard to equation [12] r_i depends only on design parameters while the mass m depends on both design and uncertain variables.
Weibull distributions are used to model failure of all the subsystems where the shape and scale parameters are defined as epistemic uncertainty [8].
The failure in the POWER system can occur in the battery cells or in the solar panel cells; the failure rate is then used to modify the number and type of cells of the solar panels and of the batteries.
Failure in the TTC and AOCS, instead, influences the power that the two subsystems require from the POWER subsystem.
The variation of these parameters induces a change of the mass and size of the component and as a result the change of the barycentre and moment of inertia of the whole system.

CONSTRAINT:
There is only a geometric constraint: the subsystems, modelled as boxes, can neither intersect each other nor the spacecraft body.

6.2. RELIABILITY IN THE CONSTRAINT

OBJECTIVE FUNCTION:
The same functions as the previous ones are here used to model the mass and the volume of the subsystems, but the failure of the components does not influence, here, the objective function: it is a constraint.

CONSTRAINT:
Two constraints are here considered: a geometric one, as in the previous approach, and a reliability constraint:

$$R(t, d, u) > 0.95 \quad (14)$$

7. RESULTS

The algorithm described in section 4 has been applied to the considered problem in order to find the worst-case scenario; the parameters have been set as follow:

- 500000 function evaluation for the whole min-max algorithm;
- 5000 function evaluation for the inner loop;
- 5000 function evaluation for the outer loop;

Then the decomposition approach has been applied to reconstruct the belief curve with 1000 function evaluation for each maximisation.

Figure 1 and Figure 2 show the results obtained with the first approach (section 6.1) and second approach (section 6.2) respectively. The belief (blue) and plausibility (green) curves are evaluated with the Decomposition approach; the min-max (belief equal to one), the min-min solution (plausibility equal to zero) are evaluated with the constrained approach described in section 4. Finally there is a comparison with the classical margin approach. The first approach is more restrictive and brings to a higher value for the objective function.

In Figure 1 the nominal solution (black line) has zero belief to occur, while the margin would bring to a robust solution but with an objective function bigger than the necessary. 10% margins have been applied to the subsystems, the exchange functions and the final result. In Figure 2, that shows the results for a different design vector, 10% margins (red) and 20% margins (purple) are not enough to cover the unexpected uncertainty. ENM, instead, allows to do a rigorous analysis of the uncertainty associated to the mass of the spacecraft.

For more details about the convergence of the method, please refer to [6] and [7].

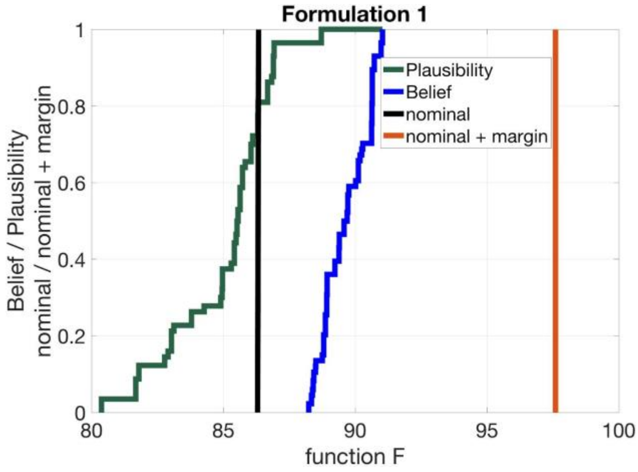

Figure 1. Belief, nominal solution and margin for the first approach described in paragraph 6.1

8. CONCLUSIONS

In this paper we described a new approach to do design for resilience by taking into account both robustness and reliability. The method is the Evidence Network Model that is able to model complex systems varying in time and affected by epistemic uncertainty. The approach has been validated with a realistic test case regarding the resource allocation problem in a spacecraft and finally compared with the classical approach of margins.

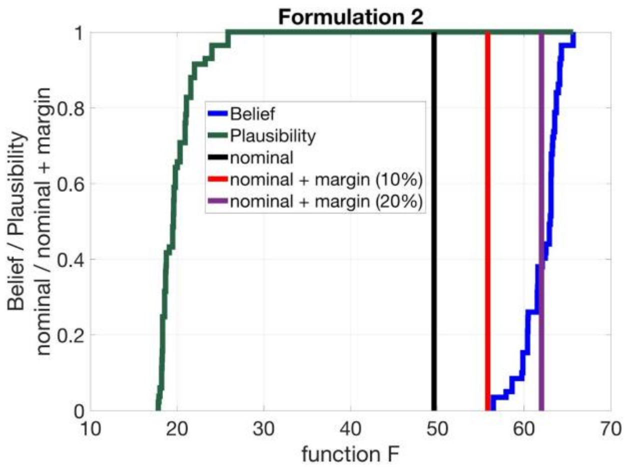

Figure 2. Belief, nominal solution and margin for the second approach described in paragraph 6.2

9. ACKNOWLEDGMENT

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