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Optimization of the Receiving Orientation Angle for Zero-Forcing Precoding in VLC

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***Abstract*—We study the performance of linear zero-forcing (ZF) precoding in multiuser multiple-input single-output visible light communications (VLC) when we are able to select the receiving orientation angle (ROA) of each user. For radio-frequency communications, the non-line-of-sight rich scattering environment usually ensures the linear independence among user's channels. However, this condition is less likely to happen in VLC systems, degrading the performance of ZF precoding. In this work, we propose a variable ROA (vROA) photodetector able to modify its orientation vector in order to generate semi-orthogonal channel responses among users. We derive the algorithm for determining the orientation of the vROA photodetector of each user, obtaining optimal and suboptimal solutions with high and low complexity, respectively. Simulation results show that the performance of ZF precoding is improved considerably by managing the users ROA.**

***Index Terms*—Visible light communications, multiuser multiple-input single-output, zero forcing, optical channel.**

I. Introduction

Visible light communication (VLC) has received significant attention as a means of moving part of the indoor wireless data traffic to the optical spectrum. Given the multiple advantages of VLC such as avoiding interference with radio-frequency (RF) deployments, the availability of a wide and unregulated spectrum, the high energy efficiency and low cost implementation, VLC is considered a key component for the evolution of wireless communications [1].

Typically, multiple light emitting diodes (LED) sources are deployed in a single room for providing satisfactory illumination. As a consequence, downlink VLC lends itself naturally to multiple-input multiple-output (MIMO) systems. Therefore, it is possible to apply the well-known MIMO signal processing to VLC. Transmit precoding schemes such as zero-forcing (ZF) [2], can be implemented in VLC to manage the multiuser interference (MUI). Moreover, in [3], the optimality of ZF precoding is demonstrated for a large number of users in rich scattering environments by exploiting the multiuser diversity. However, there exist several issues that negatively impact the implementation of ZF precoding in VLC such as:

- The transmitted signal must be real and non-negative.
- Closed-form expression of the capacity is not available for the amplitude constrained channels. However, the capacity can be upper and lower bounded [4], [5].
- Precoding schemes such as ZF typically consider uncorrelated channel responses among users. For RF communications, this condition is naturally satisfied in rich scattering environments, which does not occur for VLC.

Several works analyze the implementation of precoding techniques for VLC considering the aforementioned issues. In [6], the precoding vectors for minimizing the maximum mean square error (MMSE) are derived taking into consideration the optical power constraints. The design of ZF precoding considering the upper and lower bounds of the capacity is analyzed in [5]. However, these works assume that the photodiode (PD) of each user is pointing perpendicularly to the ceiling. In this sense, angle diversity receivers (ADRs) have been proposed for improving the performance of multiuser MIMO systems [7]. In [8], the use of ADRs where the PDs are arranged following a geometrical pattern is analyzed. It is shown that this approach improves the bit error rate (BER) in comparison with traditional approaches. With the same purpose, a mirror ADR for 2×2 MIMO channel is proposed in [9]. It is shown that the BER can be improved by optimizing the tilt angle and the height of the mirror. In [10], the impact of the ROA on the data rate performance is analyzed. However, these works do not focus on determining the users ROA for improving the performance of precoding techniques.

In this letter, we propose a variable ROA (vROA) photodetector so that each user can select a specific channel response by choosing one from a set of possible orientations. We derive the semi-orthogonal ROA selection (SRS) algorithm for VLC exploiting the *multi ROA diversity* given by the orientations of the vROA photodetector. However, this approach requires to evaluate each of the orientations for each user, which may result in an excessive computational complexity. We devise a heuristic algorithm based on the geometrically-dependent characteristics of the optical channel that reduces the complexity. Simulation results show that the proposed approach increases considerably the capacity of ZF precoding for VLC.

The remainder of the paper is organized as follows. Section II describes the system model. In Section III, we present the ZF precoding and the capacity bounds for VLC systems. The role of the vROA photodetector and the algorithms for selecting the proper orientation are derived in Section IV. Simulation results are presented in Section V, and finally, concluding remarks are provided in Section VI.

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II. System model

We consider an indoor VLC system composed of L, $l \in \{1, \ldots, L\}$ optical transmitters serving K users, $k \in \{1, \ldots, K\}$. Each user is located in a fixed position and equipped with a vROA photodetector that can modify its orientation so that the resulting channel depends on the selected azimuthal and elevation angles. The position in Cartesian coordinates of transmitter l and user k are (x_l, y_l, z_l) and $(x^{[k]}, y^{[k]}, z^{[k]})$, respectively, as is shown in Fig. 1. The transmitted signal is given by the vector $\mathbf{u} = [u_1, \ldots, u_L]^T \in \mathbb{R}^{L\times 1}$ where u_l is the signal corresponding to the l-th transmitter. Thus, the signal received by user k is

$$y^{[k]} = \mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\mathbf{u} + z^{[k]}, \tag{1}$$

where $\mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right) \in \mathbb{R}^{1\times L}$ is the row vector that contains the channel response between the L transmitters and user k, $\alpha^{[k]}$ and $\theta^{[k]}$ are the azimuthal and elevation angles, respectively, that determine the orientation of user k and $z^{[k]}$ is real additive white Gaussian noise with variance σ_z^2. The L transmitters are connected to a central unit (CU) enabling cooperation among them. The CU knows the position of the transmitters and users and communicates the desired azimuthal and elevation angles to each user. Moreover, the optical power of each transmitter is denoted by P_{LED}.

Similarly to other works, e.g., [5], [6], we consider that the predominant contribution to the optical channel corresponds to the line-of-sight (LoS) component. This component is determined by the geometry between transmitter and receiver as is shown in Fig. 1. The distance between transmitter l and user k is denoted as d_{kl} and the irradiance and incidence angles are denoted as $\phi_l^{[k]}$ and $\varphi_l^{[k]}$, respectively. Thus, the channel between transmitter l and user k is given by

$$h_l^{[k]} = \begin{cases} \frac{\gamma A}{d_{kl}^2} R\left(\phi_l^{[k]}\right) T\left(\varphi_l^{[k]}\right)\cos\left(\varphi_l^{[k]}\right) & \varphi_l^{[k]} \le \Psi_c \\ 0 & \varphi_l^{[k]} > \Psi_c \end{cases} \tag{2}$$

where γ and A are the responsivity and physical area of detection of the PD, respectively, Ψ_c denotes the field of view (FoV) of the PD, $T\left(\varphi_l^{[k]}\right)$ is the gain of the optical filter and $R\left(\phi_l^{[k]}\right) = \frac{m+1}{2\pi}\cos^m\left(\phi_l^{[k]}\right)$ is the Lambertian beam distribution, where m is the radiation index for the radiation semi-angle $\phi_{1/2}$ given by $\frac{-\log(2)}{\log(\phi_{1/2})}$.

The orientation vector of the PD of user k is determined by its azimuthal and elevation angles. This orientation vector in Cartesian coordinates is given by

$$\hat{\mathbf{n}}^{[k]} = \left[\sin(\theta^{[k]})\cos(\alpha^{[k]}),\ \sin\theta^{[k]})\sin(\alpha^{[k]}),\ \cos(\theta^{[k]})\right].$$

We assume that the optical transmitters are pointing perpendicularly to the floor, which corresponds to the vector $\hat{\mathbf{n}}_l = [0, 0, -1]$. Thus, the irradiance and incidence angles are $\phi_l^{[k]} = \arccos\left(\frac{\hat{\mathbf{n}}_l \cdot \mathbf{r}_l^{[k]}}{\|\hat{\mathbf{n}}_l\|\|\mathbf{v}_{tk}\|}\right)$ and $\varphi_l^{[k]} = \arccos\left(\frac{\mathbf{r}_l^{[k]} \cdot \hat{\mathbf{n}}^{[k]}}{\|\mathbf{r}_l^{[k]}\|\|\hat{\mathbf{n}}^{[k]}\|}\right)$, respectively, where $\mathbf{r}_l^{[k]}$ is the vector stemming from transmitter l to user k as is shown in Fig. 1. Notice that the orientation vector $\hat{\mathbf{n}}^{[k]}$ determines the ROA, which has a direct impact on the optical channel (see (2)).

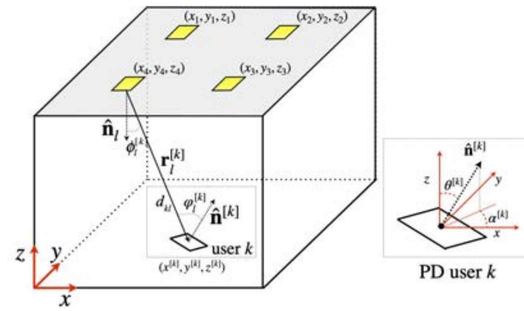

Fig. 1. Scenario and geometry of the pair transmitter-receiver.

III. Capacity and ZF precoding

The symbols are transmitted following a DC-biased pulse amplitude modulation (PAM) scheme. Thus, the symbol intended to user k is denoted by s_k and $\mathbf{s} = \left[s^{[1]}, \ldots, s^{[K]}\right]^T \in \mathbb{C}^{K\times 1}$ is the vector that contains the symbols to the K users. For a M-PAM scheme, each symbol s_k is zero-mean corresponding to M possible values within $[-1, 1]$.

We consider linear transmit precoding to mitigate MUI. Denoting the precoding vector associated to user k as $\mathbf{w}^{[k]} \in \mathbb{R}^{L\times 1}$, the signal received at user k is

$$\begin{aligned} y^{[k]} =& \mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\mathbf{w}^{[k]}s^{[k]} + \mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\mathbf{I}_{\mathrm{DC}} \\ &+ \mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\sum\nolimits_{i=1, i\neq k}^{K}\mathbf{w}^{[i]}s^{[i]} + z^{[k]}, \end{aligned} \tag{3}$$

where $\mathbf{I}_{\mathrm{DC}}$ contains the DC-bias current at each optical transmitter that provides the desired illumination. For ZF precoding MUI is completely canceled. Denoting the resulting channel matrix of the system as

$$\mathbf{H} = \left[\mathbf{h}^{[1]}\left(\alpha^{[1]}, \theta^{[1]}\right)^T \quad \ldots \quad \mathbf{h}^{[K]}\left(\alpha^{[K]}, \theta^{[K]}\right)^T\right]^T \in \mathbb{R}^{K\times L} \tag{4}$$

and the precoding matrix as $\mathbf{W} = \left[\mathbf{w}^{[1]}, \ldots, \mathbf{w}^{[K]}\right] \in \mathbb{R}^{L\times K}$, this condition implies $\mathbf{HW} = \mathrm{diag}\left(\sqrt{\lambda_k}\right)$, where λ_k is the channel gain of user k after ZF precoding.

Although the closed-form expression of the capacity for VLC does not exist, in [4] a lower bound is derived for multiuser MIMO systems considering transmit precoding schemes. Specifically, omitting the azimuthal and elevation angles, the lower bound of the capacity for user k is

$$C^{[k]} \ge \frac{1}{2}\log\left(1 + \frac{2|\mathbf{h}^{[k]}\mathbf{w}^{[k]}|^2}{\pi e\left(\frac{1}{3}\sum_{i\neq k}|\mathbf{h}^{[k]}\mathbf{w}^{[i]}|^2 + \sigma_z^2\right)}\right). \tag{5}$$

For ZF precoding, the constraint $\mathbf{h}^{[k]}\mathbf{w}^{[i]} = 0$ holds for $i \neq k$. Moreover, in contrast to RF systems, the pseudo-inverse $\mathbf{H}^\dagger = \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H\right)^{-1}$ is not necessarily the optimal solution [3]. Thus, the precoding vectors that maximize the lower bound of the capacity can be determined by solving,

$$\begin{aligned} \underset{\mathbf{w}^{[k]}}{\text{maximize}} \quad & \sum\nolimits_{k=1}^{K}\frac{1}{2}\log\left(1 + \frac{2|\mathbf{h}^{[k]}\mathbf{w}^{[k]}|^2}{\pi e \sigma_z^2}\right) \\ \text{subject to} \quad & \mathbf{HW} = \mathrm{diag}\left(\sqrt{\lambda_k}\right) \\ & \sum_{k=1}^{K}|\mathbf{e}_l\mathbf{w}^{[k]}| < \Delta I_{\mathrm{tx}}, \quad l = 1, \ldots, L \end{aligned} \tag{6}$$

where $\mathbf{e}_l$ is the unit row vector whose l-th entry is 1 and $\Delta I_{\rm tx} = \min\left(I_{\rm DC}, I_{\max} - I_{\rm DC}\right)$, where $I_{\max}$ is the maximum current of the optical transmitter. In [5], several methodologies are proposed for solving this optimization problem using the convex optimization tool CVX [11].

IV. Semi-orthogonal ROA selection for ZF Precoding

We propose the concept of vROA photodetector that can modify its orientation vector. The implementation in a hand-held device such as a smartphone, which currently implements a gyroscope and an accelerometer [10], can be carried out in a small package using current technology such as micro-electromechanical systems (MEMS) [12]. In this sense, an omnidirectional receiver is proposed in [13] considering several PDs around the six faces of a hand-held device taking advantage of the natural hand movements of rotation and elevation. This configuration can be also managed as a vROA photodetector. It is assumed that the actuators that compose the vROA photodetector vary the azimuthal and elevation angles subject to an accuracy $\Delta\alpha$ and $\Delta\theta$, respectively. Therefore, each user can select $N_\nu = \frac{360^\circ}{\Delta\alpha} \cdot \frac{180^\circ}{\Delta\theta}$ possible orientations.

Assuming that $\mathbf{H}$ (see (4)) is a full rank matrix, the generalized inverse can be written as $\mathbf{H}^- = \mathbf{H}^\dagger + \left(\mathbf{I} - \mathbf{H}^\dagger\mathbf{H}\right)\mathbf{Q}$ where $\mathbf{H}^\dagger = \mathbf{H}^T\left(\mathbf{H}\mathbf{H}^T\right)^{-1}$ and $\mathbf{Q}$ is an arbitrary matrix [5]. At this point, notice that the vROA photodetector aims at providing semi-orthogonal channel responses among users, and therefore, $\mathbf{H}^\dagger\mathbf{H} \approx \mathbf{I}$. As a consequence, the solution based on the pseudo-inverse $\mathbf{H}^\dagger$ is a good approximation [2], [3]. Thus, the channel gain after ZF precoding depends on both the channel strength of $\mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)$ and the orthogonality among channel responses. In the following, we derive a SRS methodology that provides a trade-off between these concepts.

A. User ordering

The proposed SRS algorithm is sensitive to the user ordering. In this sense, there are $K!$ possible permutations of the user ordering. In order to relax the complexity, we propose a greedy ordering based on the received signal strength [14]. Since the CU knows the position of the users and assuming a constant response of the optical filter within the FoV, let us define the channel between transmitter l and user k without considering the effect of the incidence angle as

$$\tilde{h}_l^{[k]} = \frac{\gamma A(m+1)}{2\pi d_{kl}^2} T \cos^m\left(\phi_l^{[k]}\right), \tag{7}$$

and $\tilde{\mathbf{h}}^{[k]} = \begin{bmatrix}\tilde{h}_1^{[k]} & \dots & \tilde{h}_L^{[k]}\end{bmatrix} \in \mathbb{R}^{1\times L}$. Thus, the following user ordering is proposed,

$$\|\tilde{\mathbf{h}}^{[1]}\| \geq \dots \geq \|\tilde{\mathbf{h}}^{[k]}\| \geq \dots \geq \|\tilde{\mathbf{h}}^{[K]}\|. \tag{8}$$

B. Optimal orientation of the vROA photodetectors

Each orientation of the photodetector can be managed as a potentially selectable user [3]. In the following, we derive a SRS algorithm that provides a trade-off between orthogonality and strength of the selected channel responses.

Step 1) Initialization. $k=1$, $\mathcal{G} = \emptyset$, where $\mathcal{G}$ contains the vectors that define the orthogonal subspace of the $k-1$ users.

Step 2) Determine the orientations of the vROA photodetector of user k semi-orthogonal to $\mathcal{G} = \left\{\mathbf{g}^{[1]}, \dots, \mathbf{g}^{[k-1]}\right\}$. The orientations are given by the azimuthal and elevation angles, denoted as $\alpha_\perp$ and $\theta_\perp$, respectively, that satisfy

$$\left\{\alpha_\perp^{[k]}, \theta_\perp^{[k]} : \frac{|\mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\mathbf{g}^{[j]}|}{\|\mathbf{h}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\|\|\mathbf{g}^{[j]}\|} < \beta_{\rm th}\right\}, \tag{9}$$

where $j = \{1, \dots, k-1\}$ and $\beta_{\rm th}$ is a positive constant[1] between 0 and 1, which takes small values to ensure the semi-orthogonality of the orientations regarding the subspace $\mathcal{G}$. If none of the orientations of the vROA photodetector satisfies this condition user k is not considered, the following steps are not carried out and the algorithm skips to user $k+1$.

Step 3) For user k calculate the components of the possible channel vectors $\mathbf{h}^{[k]}\left(\alpha_\perp^{[k]}, \theta_\perp^{[k]}\right)$ orthogonal to the subspace spanned by $\mathcal{G} = \left\{\mathbf{g}^{[1]}, \dots, \mathbf{g}^{[k-1]}\right\}$ that satisfy (9). That is,

$$\mathbf{g}^{[k]} = \mathbf{h}^{[k]}\left(\alpha_\perp^{[k]}, \theta_\perp^{[k]}\right)\left(\mathbf{I} - \sum_{j=1}^{k-1}\frac{\mathbf{g}^{[j]T}\mathbf{g}^{[j]}}{\|\mathbf{g}^{[j]}\|^2}\right). \tag{10}$$

Note that $\mathbf{g}^{[k]} = \mathbf{h}^{[k]}\left(\alpha_\perp^{[k]}, \theta_\perp^{[k]}\right)$ for $k=1$.

Step 4) Directly minimizing the projection of the selected channel onto subspace $\mathcal{G}$ provides the most orthogonal channel to that subspace. However, this criterion does not consider the channel strength. In order to obtain a trade-off between channel strength and orthogonality, user k selects the ROA of user k that provides the largest projected norm within the angles determined in step 2), which is denoted as $\left\{\alpha_{\rm opt}^{[k]}, \theta_{\rm opt}^{[k]}\right\}$, and update the subspace $\mathcal{G} = \left\{\mathbf{g}^{[1]}, \dots, \mathbf{g}^{[k]}\right\}$ with the orthogonal component of $\mathbf{h}^{[k]}\left(\alpha_{\rm opt}^{[k]}, \theta_{\rm opt}^{[k]}\right)$,

$$\left\{\alpha_{\rm opt}^{[k]}, \theta_{\rm opt}^{[k]}\right\} = \max_{\alpha^{[k]}, \theta^{[k]}} \left\|\mathbf{g}^{[k]}\left(\alpha^{[k]}, \theta^{[k]}\right)\right\| \tag{11}$$

$$\mathbf{g}^{[k]} = \mathbf{h}^{[k]}\left(\alpha_{\rm opt}^{[k]}, \theta_{\rm opt}^{[k]}\right)\left(\mathbf{I} - \sum_{j=1}^{k-1}\frac{\mathbf{g}^{[j]T}\mathbf{g}^{[j]}}{\|\mathbf{g}^{[j]}\|^2}\right) \tag{12}$$

and $k = k+1$. Notice that (11) requires a linear search over the set of possible orientations. Hence, considering a reasonable accuracy, the evaluation of all these orientations may result prohibitive because of computational complexity issues.

C. Heuristic orientation of the vROA photodetectors

We derive a heuristic algorithm that obtains a near-optimal orientation for each user without the need for evaluating each of all the possible orientations. Interestingly, the proposed scheme simply exploits the geometric characteristics of the optimal channel (see (2)). Considering the definition of $\tilde{\mathbf{h}}^{[k]}$ and defining the matrix that contains the incidence angles of user k from the L transmitters as

$$\mathbf{\Phi}_\nu\left(\alpha^{[k]}, \theta^{[k]}\right) = \operatorname{diag}\left\{\cos(\varphi_1^{[k]}), \dots, \cos(\varphi_L^{[k]})\right\}, \tag{13}$$

[1]The value $\beta_{\rm th} = 0.35$ is selected empirically. This value ensures at least 69.5° between vectors.

the channel of user k can be written as

$$\mathbf{h}^{[k]}\left(\alpha^{[k]},\theta^{[k]}\right)=\tilde{\mathbf{h}}^{[k]}\mathbf{\Phi}_{\nu}. \tag{14}$$

Although $\mathbf{\Phi}_{\nu}$ contains L values, they only depends on the two variables that define the ROA, i.e., $\alpha^{[k]}$ and $\theta^{[k]}$. According to (13), the components of user k orthogonal to the subspace spanned by $\mathcal{G}=\{\mathbf{g}^{[1]},\ldots,\mathbf{g}^{[k-1]}\}$ can be written as

$$\mathbf{g}^{[k]}\left(\alpha^{[k]},\theta^{[k]}\right)=\tilde{\mathbf{h}}^{[k]}\mathbf{\Phi}_{\nu}\left(\alpha^{[k]},\theta^{[k]}\right)\mathbf{P}^{[k]}, \tag{15}$$

where $\mathbf{P}^{[k]}=\left(\mathbf{I}-\sum_{j=1}^{k-1}\frac{\mathbf{g}^{[j]T}\mathbf{g}^{[j]}}{\|\mathbf{g}^{[j]}\|^2}\right)$. Since $\tilde{\mathbf{h}}^{[k]}\in\mathbb{R}^{L\times1}$ is a constant vector and $\mathbf{\Phi}_{\nu}\left(\alpha^{[k]},\theta^{[k]}\right)\in\mathbb{R}^{L\times L}$, i.e., these matrices exclusively contain positive and real values, the optimization problem (11) can be written as

$$\arg\max_{\alpha^{[k]},\theta^{[k]}}\|\mathbf{p}^{[k]}\,\mathbf{\Phi}_{\nu}\left(\alpha^{[k]},\theta^{[k]}\right)\|, \tag{16}$$

where $\mathbf{p}^{[k]}=\begin{bmatrix}p_1 & \ldots & p_L\end{bmatrix}\in\mathbb{R}^{1\times L}$ and p_l corresponds to the sum of the elements in the l-th column of matrix $\mathbf{P}^{[k]}$. Notice that the weight vector $\mathbf{p}$ is determined by the subspace $\mathcal{G}=\{\mathbf{g}^{[1]},\ldots,\mathbf{g}^{[k-1]}\}$ and $\mathbf{\Phi}_{\nu}$ exclusively depends on the orientation of the vROA photodetector of user k.

The evaluation of the N_{ν} orientations of the vROA photodetector can be reduced to an optimization problem that maximizes (16), i.e.,

$$\begin{aligned}&\underset{\varphi_1^{[k]},\ldots,\varphi_L^{[k]}}{\text{maximize}} && \sum\nolimits_{l=1}^{L}p_l^2\cos^2(\varphi_l^{[k]})\\ &\text{subject to} && \varphi_l^{[k]}\le\Psi_c,\end{aligned} \tag{17}$$

where the cosine can be written as a function of the geometry of the pair transmitter-receiver

$$\begin{aligned}\cos\left(\varphi_l^{[k]}\right)=\frac{1}{d_{kl}}\Big[&(x_l-x^{[k]})\cos(\alpha^{[k]})\sin(\theta^{[k]})\\ &+(y_l-y^{[k]})\sin(\alpha^{[k]})\sin(\theta^{[k]})+(z_l-z^{[k]})\cos(\theta^{[k]})\Big].\end{aligned} \tag{18}$$

The cosine function in (17) can be approximated[2] as $\cos^2(\varphi_l^{[k]})\approx\mathcal{C}\cos(\varphi_l^{[k]})$ within the FoV range, where $\mathcal{C}$ is the correction factor. The value of $\mathcal{C}$ aims at minimizing the error between the derivates of $\cos^2(\varphi_l^{[k]})$ and $\cos(\varphi_l^{[k]})$. Omitting the indexes of the incidence angle,

$$\min_{\mathcal{C}}\int_{-\Psi_c}^{\Psi_c}|\mathcal{C}\sin(\varphi)-2\cos(\varphi)\sin(\varphi)|d\varphi\Rightarrow\mathcal{C}=1+\cos(\Psi_c) \tag{19}$$

Thus, applying (18) in the problem formulated in (17),

$$\begin{aligned}&\underset{\alpha^{[k]},\theta^{[k]}}{\text{maximize}} && f\left(\alpha^{[k]},\theta^{[k]}\right)=Z_{\Sigma}\cos(\theta^{[k]})\\ &&& +\sin(\theta^{[k]})\left(X_{\Sigma}\cos(\alpha^{[k]})+Y_{\Sigma}\sin(\alpha^{[k]})\right)\\ &\text{subject to} && \alpha^{[k]}\in(0,2\pi],\theta^{[k]}\in(-\tfrac{\pi}{2},\tfrac{\pi}{2}],\end{aligned} \tag{20}$$

and $X_{\Sigma}=\mathcal{C}\sum_{l=1}^{L}p_l^2\left(\frac{x_l-x^{[k]}}{d_{kl}}\right)$, $Y_{\Sigma}=\mathcal{C}\sum_{l=1}^{L}p_l^2\left(\frac{y_l-y^{[k]}}{d_{kl}}\right)$ and $Z_{\Sigma}=\mathcal{C}\sum_{l=1}^{L}p_l^2\left(\frac{z_l-z^{[k]}}{d_{kl}}\right)=\mathcal{C}\frac{aL}{d_{kl}}\|\mathbf{p}\|^2$, where a is the height between the optical transmitters and the users. The procedure to solve (20) is described in the following.

The objective function of (20) can be written as

$$f(\alpha^{[k]},\theta^{[k]})=\sin(\theta^{[k]})f_2(\alpha^{[k]})+Z_{\Sigma}\cos(\theta^{[k]}). \tag{21}$$

where $f_2(\alpha^{[k]})=X_{\Sigma}\cos(\alpha^{[k]})+Y_{\Sigma}\sin(\alpha^{[k]})$. Since $Z_{\Sigma}\cos(\theta^{[k]})$ is strictly positive for $\theta^{[k]}\in\left[\frac{-\pi}{2},\frac{\pi}{2}\right]$, maximizing $f(\alpha^{[k]},\theta^{[k]})$ comprises two options, either $\sin(\theta^{[k]})\ge0$ and $f_2(\alpha^{[k]})\ge0$ or $\sin(\theta^{[k]})<0$ and $f_2(\alpha^{[k]})<0$.

The function $f_2(\alpha^{[k]})$ contains a maximum and a minimum value within the range $\alpha^{[k]}\in[-\pi,\pi]$. Calculating the first derivate regarding $\alpha^{[k]}$ and equaling zero

$$\tilde{\alpha}^{[k]}=\arctan\left(\frac{Y_{\Sigma}}{X_{\Sigma}}\right)\pm a\pi, \tag{22}$$

where $a\in\mathbb{N}$. We denote the positive and negative values that satisfy (22) as $\tilde{\alpha}_+^{[k]}$ and $\tilde{\alpha}_-^{[k]}$, respectively. For each of these values of $\alpha^{[k]}$, we reformulate the objective function as

$$f(\tilde{\alpha}^{[k]},\theta^{[k]})=\sin(\theta^{[k]})f_2\left(\alpha^{[k]}\in\{\tilde{\alpha}_+^{[k]},\tilde{\alpha}_-^{[k]}\}\right)+Z_{\Sigma}\cos(\theta^{[k]}). \tag{23}$$

Similarly to the previous step, we calculate the first derivate for the elevation angle and equaling zero,

$$\tilde{\theta}^{[k]}=\arctan\left(\frac{f_2\left(\alpha^{[k]}\in\{\tilde{\alpha}_+^{[k]},\tilde{\alpha}_-^{[k]}\}\right)}{Z_{\Sigma}}\right)\pm b\pi, \tag{24}$$

where $b\in\mathbb{N}$.

For $\tilde{\alpha}_+^{[k]}$ we evaluate the solution that satisfies $\tilde{\theta}^{[k]}\in[0,\frac{\pi}{2}]$, i.e., $\sin(\theta^{[k]})\ge0$, denoted as $\tilde{\theta}_+^{[k]}$ while the negative value of f_2 given by $\tilde{\alpha}_-^{[k]}$ is considered for the cases where $\theta^{[k]}\in[\frac{-\pi}{2},0)$, i.e., $\sin(\theta^{[k]})<0$, denoted as $\tilde{\theta}_-^{[k]}$. The heuristic values $\hat{\alpha}^{[k]}$, $\hat{\theta}^{[k]}$ that maximize (21) are given by $\max_{\hat{\alpha}^{[k]},\hat{\theta}^{[k]}}\left\{f\left(\tilde{\alpha}_+^{[k]},\tilde{\theta}_+^{[k]}\right),f\left(\tilde{\alpha}_-^{[k]},\tilde{\theta}_-^{[k]}\right)\right\}$. Finally, it is checked if the obtained solution satisfies (9). If not, the closest angles that satisfy this condition are selected.

D. Complexity

The complexity of the proposed algorithms is now analyzed. First, determining the optimal user ordering requires to compute $K!$ possible permutations. We propose an user ordering based on the channel strength without considering the influence of the ROA. This approach provides a close-to-optimal performance as can be seen in Section V.

The proposed SRS algorithm first calculates an inner product for N_{ν} possible orientations of the vROA photodetector. After that, it requires $(1\times L)\times(L\times L)$ vector-matrix multiplications for the selected orientations, which are upper bounded by N_{ν}. To conclude, a linear search is carried out over the possible orientations. The computational complexity of the inner product, the vector-matrix multiplication and the linear search calculation are denoted as C_{in}, C_{x} and C_{ls}, respectively. Since these steps are carried out for the K users, the complexity is $C_{\text{SRS}}=(C_{\text{in}}+C_{\text{x}}+C_{\text{ls}})\,N_{\nu}K$.

For the heuristic scheme, only a vector-matrix multiplication (see (15)) and the sum of the columns in the resulting matrix

[2]The formula $\cos\varphi=\frac{1+\cos(2\varphi)}{2}$, which is always true, is not considered since $\cos(2\varphi)$ cannot be related with the geometry of the pair transmitter-user.

$\mathbf{P}^{[k]}$ is required to determine the weight vector $\mathbf{p}^{[k]}$. After that, the orientation of the vROA photodetector of each user is calculated according to (22) and (24), whose computational complexity can be considered negligible. The computational complexity of the heuristic method is $C_{\text{heu}} \approx C_{\text{x}}K$. It can seen that the proposed methodology avoids the search over the N_ν orientations of the vROA photodetector of each user.

V. Simulation Results

We consider $L = 4$ optical transmitters at the positions $(1.5, 1.5, 3)$, $(1.5, 3.5, 3)$, $(3.5, 1.5, 3)$, $(3.5, 3.5, 3)$ m and the users are uniformly distributed on a plane 2.15 m away from the ceiling. The ZF precoding vectors are obtained through the methods based on CVX proposed in [5]. In addition, the noise and all other parameters follow the values specified in [5].

The average sum-rate as the number of possible orientations increases is shown in Fig. 2. Notice that $N_\nu = 1$ corresponds to the traditional receiver composed of a single PD pointing to the ceiling. Besides, the values $N_\nu = \{8, 32, 50\}$ are highlighted, which correspond to users equipped with an ADR composed of the same number of PDs in a hemispherical arrangement [8]. The sum-rate increases as the vROA photodetector provides more possible orientations. For $N_\nu = 50$ the sum-rate converges to a constant value. However, ADRs comprising a large number of PDs such as this value may result impractical. Focussing on the devised algorithms, the proposed user ordering based on the channel strength provides a sum-rate close to, although below, the optimal ordering. Moreover, the heuristic algorithm also provides a solution close to the algorithm that evaluates all the possible orientations.

The sum-rate is shown as a function of the optical power at each transmitter in Fig. 3. For $K = 2$, the heuristic algorithm provides a sum-rate similar to the SRS algorithm since the error due to the approximation is only propagated for the second user (see (15)). For $K = 4$, notice that the slope of the sum-rate is almost twice in comparison to the obtained for $K = 2$. This result is expected since the degrees of freedom (DoF) are doubled, which are $\min(L, K)$. However, the sum-rate is considerably penalized assuming a traditional receiver. It can be seen that the performance of the heuristic algorithm is worse than the results obtained by the SRS algorithm due to the considered approximation (see (17) and (18)). In this sense, the penalty is also greater for a FoV of 60° than of 40°.

VI. Conclusions

In this letter, we analyze the impact of the ROA for precoding schemes in VLC. Based on the concept of vROA photodetector, we derive an algorithm for providing semi-orthogonal channels among users by evaluating the set of possible ROAs. An alternative algorithm is proposed to avoid this exhaustive evaluation reducing the complexity. Focussing on ZF precoding, it is shown that the capacity increases considerably by selecting the proper ROA at each user.

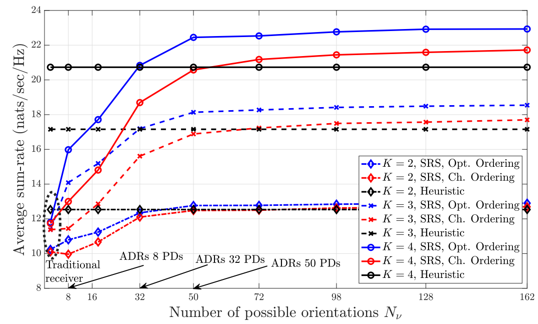

Fig. 2. Evolution of the average sum-rate versus accuracy of the vROA photodetector. $P_{\text{LED}} = 40$ dBm, $\Psi_c = 60°$.

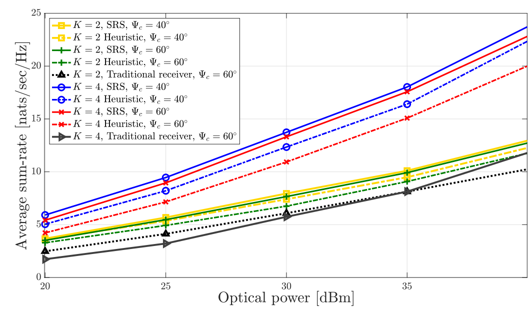

Fig. 3. Average sum-rate versus optical power per transmitter.

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