Large-Eddy Simulation of Non-Homogeneous Turbulence Subjected to Sudden Distortion

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ABSTRACT

In this paper we investigate the distortion of an unsteady flow through a two-dimensional sudden area expansion using Large-Eddy Simulations. We perform a series of numerical experiments at various Mach numbers ($M_b = (0.1, 0.5)$) based on the inlet volume flow rate and fixed $\mathcal{O}(1)$ contraction ratio. At $M_b = 0.1$, we find that the transverse components of the auto-correlation of the two-point time-delayed velocity correlation tensor de-correlates slower in the downstream expansion compared to the channel section. We discuss this result in the context of classical and rapid-distortion theory.

1. Introduction

The study of the linear evolution of turbulence with small spatial scales began with Prandtl [1] and Taylor [2] who analysed the distortion of unsteady flow through a contracting stream. This work was primarily motivated by ensuring optimum wind tunnel performance; i.e. that the flow in the working section downstream of a contraction has very low intensity turbulence. It is based on the idea that the magnitude of the mean rate of strain (S^*) multiplied by the eddy turnover time, $T^*_{\text{eddy}} = D^*_{\text{inlet}}/U^*_{\text{inlet}}$, is large inasmuch as $S^*T^*_{\text{eddy}} \gg 1$ (asterisks are dimensional quantities). This work was continued by Batchelor & Proudman [3] a few decades later and much more comprehensively by Hunt [4] two decades after that. Batchelor & Proudman's study involving homogeneous turbulence distorting through a contraction was placed on a restrictive basis since the spatial scale of the turbulence (l^*_T) was taken to be much less than the scale of mean flow gradients (proportional to the entry diameter in the contracting stream such that $l^*_T/D^*_{\text{inlet}} \ll 1$) and moreover, that the upstream turbulence intensity was taken to be much less than this ratio (Goldstein & Durbin, [5]).

Hunt's analysis showed that the Batchelor-Proudman estimates for the viscous terms in the incompressible Navier Stokes equations were too large ([4], p.632), and the same rapid-distortion theory equations apply to large-scale (that is relatively non-homogeneous over an $\mathcal{O}(1)$ dimension such as the entry diameter D^*_{inlet}) turbulence interacting over short distances. That is, the time of turbulence interaction, τ_{int}, is small compared to the turnover time, i.e. $\tau_{\text{int}} \ll l^*_{\text{T}}/|\boldsymbol{v}'^*|$ of a typical turbulent eddy over which non-linear interaction (through advection term) and viscous dissipation take place, where l^*_{T} is the upstream turbulence spatial scale ([6], p. 264 & [7]). Hence the flow Reynolds number is large, $R = U^*_{\text{inlet}}D^*_{\text{inlet}}/\nu^* \gg \mathcal{O}(1)$ and the turbulence (interaction) Reynolds number is fixed at order 1, i.e. $R_T = \alpha R = \mathcal{O}(1)$ where (U^*_{inlet}, D^*_{inlet}) (the latter scaling also allows for $R_T \gg \mathcal{O}(1)$) are the upstream mean flow reference velocity and length scale, ν^* is the kinematic viscosity, $\alpha = |\boldsymbol{v}'^*|/U^*_{\text{inlet}} \ll \mathcal{O}(1)$ is the turbulence intensity of the flow and $|\boldsymbol{v}'^*|$ is the magnitude of the local rms turbulence velocity. RDT is then used to determine the distorted velocity field. Hunt's [4] analysis, was later advanced by Goldstein [8] to include compressibility effects and also to reduce the complexity of Hunt's formulation.

Our aim is to analyze the archetypal sudden expansion problem that involves the distortion of non-homogeneous turbulence in the novel scenario where the mean flow at the inlet possesses a finite degree of shear. While the structure of homogeneous turbulence with high rate of shear in a channel flow was determined using DNS in [9], in our case the shearing and distortion both modify the downstream development of the non-homogeneous turbulence field that is a function of spatial position y as well. We believe that this effect has not been adequately covered in the existing turbulence research. The problem statement is therefore to determine the space-time structure of correlation functions of the velocity fluctuation possessing tensorial rank 2 and 4, upstream and downstream of a sudden expansion of $\mathcal{O}(1)$ contraction ratio (σ_c) and at an upstream Mach number of $M_b = U^*_{\text{bulk}}/c^*_{\text{inlet}} = 0.1$.

2. Simulation Description

The numerical study employs the CFD code CNS3D (Compressible Navier-Stokes Solver in 3D), which solves the fully-compressible Navier–Stokes equations. The code uses the finite volume (upwind) Godunov method in conjunction with the HLLC Riemann solver [10] to estimate the numerical fluxes at the cell-faces. The primitive variables are reconstructed at the cell-faces using an 11^{th}-order WENO scheme [11]. For time integration, an explicit five-stage 4^{th}-order accurate strong stability preserving Runge–Kutta (SSP-RK) scheme is employed [12].

The size of the domain in the spanwise direction is $5h^*$, where $h^* = 3$ cm is the height of the inlet section. The inlet channel length is $4h^*$, whereas that of the expansion section is $27h^*$. The expansion ratio is $ER = 3$ and the aspect ratio is $AR = 5$. The grid resolution at the inlet in the streamwise, wall-normal and spanwise directions is $\Delta x^+ = 30$, $y^+ = 2$ (giving a $y^+ = 1$ at the cell-center) and $\Delta z^+ = 20$, respectively. Such grid resolution has been shown to suffice in order to yield accurate results [17].

A synthetic turbulent boundary condition based on the digital filter (DF) approach [13] is prescribed at the inflow, the implementation of which in CNS3D has

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been validated [14] and extensively used [15, 16]. The DF approach produces time and spatially varying velocity components which match first- and second-order statistical moments, length and time scales and correlations gained from experimental data or previous numerical simulations. No-slip wall condition is used for all domain boundaries in the wall-normal direction. At the outlet, a sponge layer is specified in order to avoid non-physical oscillations from contaminating the flow field. Since the flow is turbulent in the inlet region prior to the expansion, turbulent boundary layers grow on the upper and lower walls in the inlet region and the mean flow is non-uniform and sheared inasmuch as $U = U(y_2)$. The Mach and Reynolds numbers of the flow at the inlet are 0.1 and 10^5, respectively, based on the inlet bulk velocity U^*_{bulk}, and the contraction ratio is $\sigma_c = D^*_{\text{exit}}/D^*_{\text{inlet}} = 3$. The total simulation time was $300\,U^*_{\text{bulk}}/h^*$, while statistics were obtained over the last $100\,U^*_{\text{bulk}}/h^*$.

3. Effect of sudden expansion on the temporal de-correlation of $R_{i,j}$

There are several types of correlation functions that we will consider using the LES solution described in the previous section. Our prime concern in this paper is to analyze what the effect of $\mathcal{O}(1)$ values of σ_c have on the structure of the correlations at fixed subsonic inlet velocity $M_b = U^*_{\text{bulk}}/c^*_{\text{inlet}} = \mathcal{O}(1)$ where c^*_{inlet} is the upstream uniform speed of sound.

We concentrate on determining the amplitude and auto-correlation curves for $R_{i,j}(\boldsymbol{y},\boldsymbol{\eta},t) = \overline{\sqrt{\rho}v'_i(\boldsymbol{y},\tau)\sqrt{\rho}v'_j(\boldsymbol{z},\tau')}$ where $\boldsymbol{z} = \boldsymbol{y} + \boldsymbol{\eta}$ and $\tau' = \tau + t$ and the time average is interpreted in the usual manner where $\overline{\bullet}(\boldsymbol{y}) \equiv \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \bullet(\boldsymbol{y},\tau)\,d\tau$. The spatial field point $\boldsymbol{y}$ is positioned at, $(y_1, 0, 0)$ where the separation vector is $\boldsymbol{\eta} = (\eta_1, 0, 0)$ and y_1 is positioned in the region of maximum turbulence kinetic energy in the upper/lower shear layers to determine the maximum value of $|R_{i,j}(\boldsymbol{y},\boldsymbol{\eta},t)|$. The comma between suffixes in the correlation function refers to the spacetime location of the measurement probe, i.e. the suffix on the right side of the comma is at the shifted location $\boldsymbol{y} + \boldsymbol{\eta}$.

The contour plot in Figure 1 shows that the flow emerging into the expansion shifts into the upper part of the downstream enclosure. This is a chaotic feature (i.e. a bifurcation) of the flow in the sense that the shift of the main stream into the lower region could also be possible in another realization. When comparing the structure of the auto-correlations of $R_{1,1}$, $R_{2,2}$ & $R_{3,3}$ (Figure 1) before and after the sudden expansion (i.e. in the upstream channel compared to the downstream expansion), we found that only at $t > 5$ does a difference emerge between the correlation functions before and after the expansion. Most affected by the rapid-distortion is $R_{2,2}$ which displays a slower decay in its auto-correlation at $t > 5$ compared to its value within the upstream channel region. $R_{3,3}$ displays a similar trend albeit with smaller net difference in the normalized correlation between the two regions. The auto-correlation of $R_{1,1}$, interestingly, appears relatively unaffected for almost all values of time delay, t.

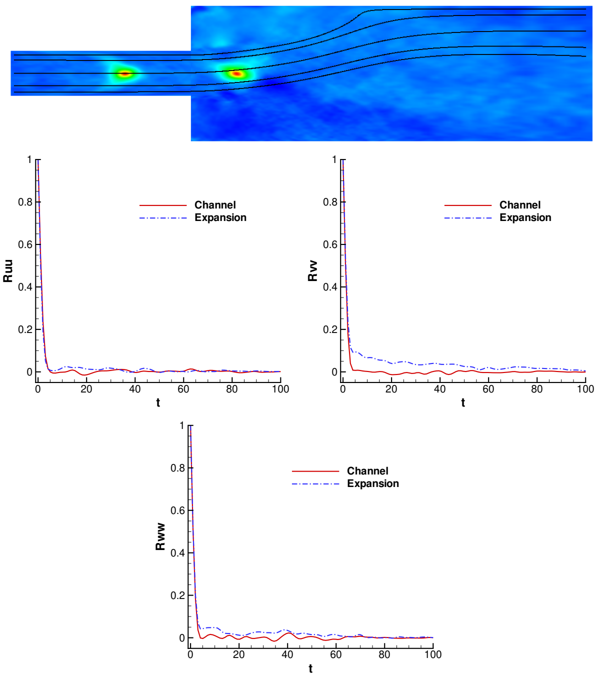

Fig. 1 Diagonal components of the correlation function, $R_{i,j}(\boldsymbol{y},\boldsymbol{\eta},t)$, against t: comparison before and after expansion.

References

[1] Prandtl, *N.A.C.A. Tech. Memo.* 726, (1933).

[2] Taylor, *Z.angew. Math. Mech.* 15, 91, (1935),

[3] Batchelor & Proudman , *Quart. J . Mech. Appl. Math.* 7, 83, (1954).

[4] Hunt, *J. Fluid Mech.* 61,(1973) 625–706.

[5] Goldstein & Durbin, *J. Fluid Mech.* 98, (1980), 473–508.

[6] Goldstein, *Annu. Rev. Fluid Mech.* 16, (1984), 263—286.

[7] Moffatt, *J. Fluid Mech.* 106, (1981), 27–47.

[8] Goldstein, *J. Fluid Mech.* 89, (1978), 433–468.

[9] Lee, Kim & Moin, *J. Fluid Mech.* 216, (1990), 561-583.

[10] Toro, Spruce & Speares, *Shock Waves* 4, (1994), 25–34.

[11] Balsara & Shu, *JCP* 160, (2000) 405–452.

[12] Spiteri & Ruuth, *SIAM JNA* 40, (2002) 469–491.

[13] Touber & Sandham, *TCFD*, 23, (2009) 79–107.

[14] Rana, Thornber & Drikakis, *PoF*, 23, 046103, (2011).

[15] Rana, Thornber & Drikakis, *Eng. Appl. CFD* 17, (2013), 13–39.

[16] Ritos, Kokkinakis, Drikakis, & Spottswood, *PoF*, 29, 046101, (2017).

[17] Kokkinakis & Drikakis, *J. Sound and Vib.*, 287, (2015) 229–261.