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EFFICIENT SUBBAND ADAPTIVE FILTERING WITH OVERSAMPLED GDFT FILTER BANKS

Stephan Weiß, Lutz Lampe[†], *and Robert W. Stewart*

Signal Processing Division, Dept. of Electronic and Electrical Eng.
University of Strathclyde, Glasgow G1 1XW, UK
{weiss,lutz,bob}@spd.eee.strath.ac.uk

Abstract. This paper addresses the numerical efficiency of adaptive filtering implemented in subbands. Our approach first focuses on oversampled GDFT filter banks and their potential benefits over other possible subband decompositions. Although the subband filters presented use complex arithmetic, the discussed method allows factorization into a real valued polyphase network, followed by a complex GDFT modulation, which can be mostly implemented via an FFT. Secondly we discuss the advantages and potential savings that can be gained by processing complex subband signals, with particular reference to adaptive system identification problems, for which we give demonstrations of the potential benefits of our GDFT approach compared to adaptive identification in both fullband and critically sampled DCT-IV based pseudo-QMF subbands.

1. INTRODUCTION

Adaptive identification of long impulse responses, as required for acoustic echo cancellation, is unlikely to be implemented as a fullband FIR system due to computational limitations [4]. Strategies to lower the computational complexity of adaptive DSP algorithms include the application of decimated subband structures [7, 3, 13], where both input and desired signal are split into a number of frequency bands, as depicted in Fig. 1. The reduced spectra allow a downsampling of the subband signals, which enables computational savings when applying adaptive filtering to the subbands. Via a synthesis bank operation, a fullband error signal can then subsequently be reconstructed.

The downsampling operation in filter banks causes aliasing, which usually is suppressed in the synthesis stage by careful design [14]. For processing that involves correlating between different subband signals, like adaptive filtering, aliasing is a disturbing non-linear distortion, which decreases performance. There exist several subband approaches to overcome this problem. Critically sampled systems require the use of cross-terms

[†] Visiting from Lehrstuhl für Nachrichtentechnik, Friedrich-Alexander Universität Erlangen-Nürnberg

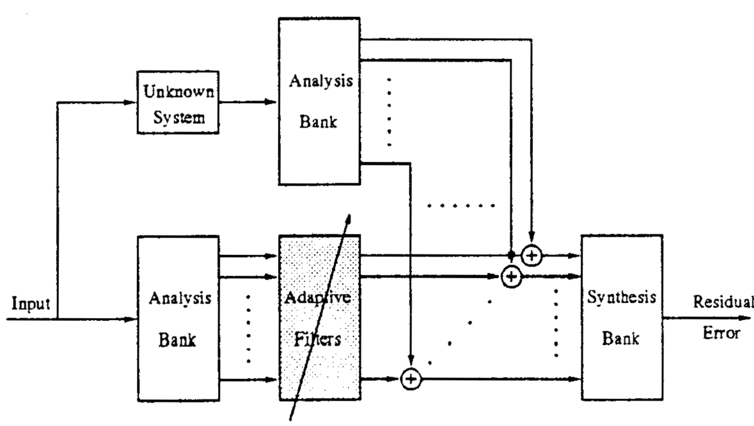

Fig. 1: Subband adaptive filter architecture to identify an unknown system.

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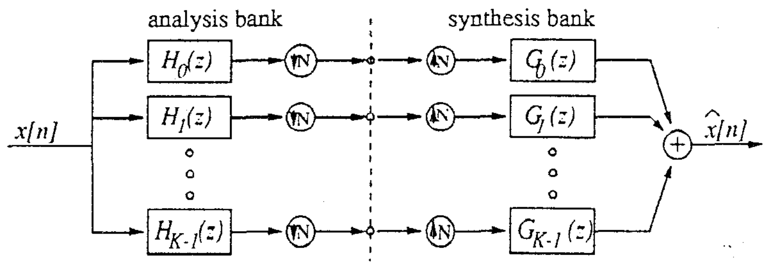


Fig. 2: Analysis and synthesis branch of a K-channel filter bank with subbands decimated by N.

between adjacent bands [3] to cover for lost information, or gap filter banks [17] which are non-perfectly reconstructing by definition. For real-valued bandpass signals, non-critical subsampling may violate the sampling theorem and requires signle sideband (SSB) modulation into the baseband prior to decimation [1] or the use of non-uniform filter banks with unequal decimation ratios [5], which require the handling of different sampling rates. On the contrary, uniform complex valued subbands are unproblematic to decimate [1, 2] and can usually be implemented at least as efficiently as real valued subband adaptive filter systems, as we will show in the sequel.

Here, we discuss a complex valued filter bank which is derived from a real prototype lowpass filter by generalized DFT (GDFT) modulation [1], such that the frequency range $\Omega = [0, \pi]$ is covered by $K/2$ subbands, which can be decimated by a factor $N \leq K$. In Sec. 2, we introduce the modulation procedure, specify design criteria, and propose an efficient polyphase structure for this filter bank. Polyphase structures are often believed to be restricted to critical decimation and integer oversampling ratios [1, 10]. Our formulation uses a factorization similar to [2] and allows any integer decimation $N \leq K$, thus including non-integer oversampling ratios. Based on our polyphase filter bank realization, we derive some results for the complexity of subband filtering in Sec. 3, depending on the order of the algorithm. Finally, Sec. 4 compares the GDFT method to a critically decimated subband adaptive filter approach [3] for the normalized least mean squares (NLMS) algorithm.

2. OVERSAMPLED GDFT FILTER BANK

2.1. GDFT Filter Banks

A general structure of a filter bank is shown in Fig. 2. The analysis bank decomposes a signal $x[n]$ into K subbands, each produced by a branch $H_k(z)$ of the analysis bank. After decimation and expansion by a factor N, the fullband signal is reconstructed from the subbands in the synthesis bank by filtering with filters $G_k(z)$ followed by summation. The analysis filters $h_k[n]$ are derived from a real valued lowpass prototype FIR filter $p[n]$ of even length L_p by a generalized discrete Fourier transform (GDFT),

$$h_k[n] = t_{k,n} \cdot p[n] \quad , \qquad t_{k,n} = e^{j\frac{2\pi}{K}(k+k_0)(n+n_0)}, \qquad k, n \in \mathbb{N}. \tag{1}$$

The term generalized DFT [1] stems from offsets k_0 and n_0 introduced into the frequency and time indices, which serve two purposes. Firstly, a frequency offset $k_0 = \frac{1}{2}$ shifts the bandpass characteristics of the filters $h_k[n]$ and yields the frequency range $[0; \pi]$ to be covered by exactly $K/2$ subbands for an even K. For real input signals $x[n]$, it is sufficient to keep only these $K/2$ subbands, from which the complete spectrum can be reconstructed by a real operation $\mathrm{Re}\{\cdot\}$. An example for the described GDFT modulation of an $\frac{1}{8}$-band lowpass filter with $k_0 = \frac{1}{2}$ in (1) and the spectral coverage can be seen in Fig. 3. This *reduced* filter bank with only $K/2$ subbands has serious implications on the computational efficiency of processing complex subbands, as will be further explained in Sec. 3. Secondly, we can ensure the linear phase property of the filters $h_k[n]$ by choosing a linear phase prototype filter $h_p[n]$ and a symmetric transform with respect to $(L_p - 1)/2$ by appropriately setting n_0.

For the synthesis filters, we employ time reversed copies of the analysis filters,

$$g_k[n] = \tilde{h}_k[n] = h_k[L_p - n + 1]. \tag{2}$$

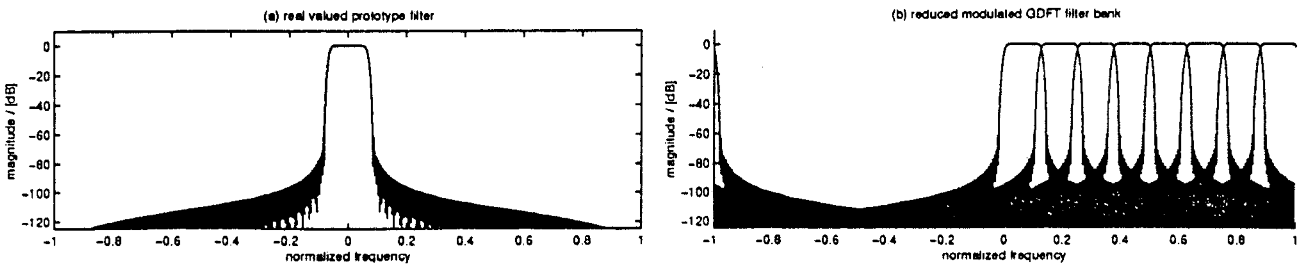


Fig. 3: (left real valued prototype filter; (right) reduced GDFT filter bank covering the frequency range $[0; \pi]$.

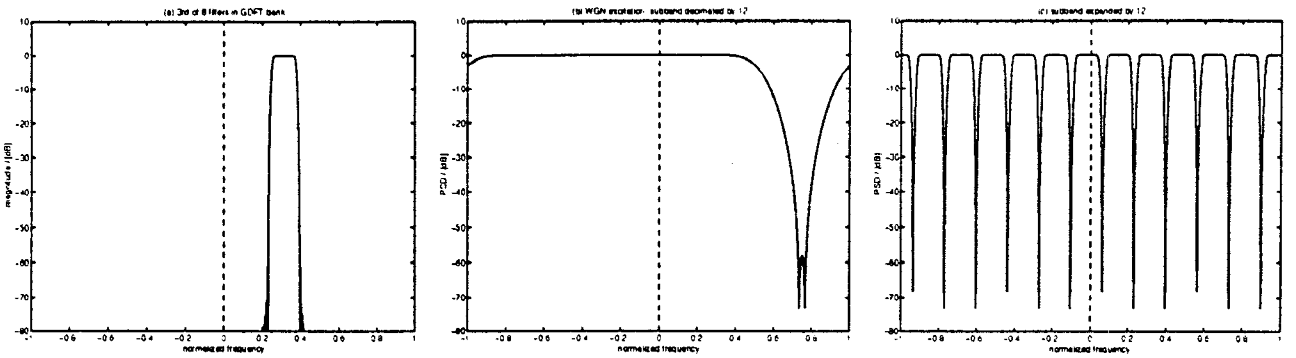


Fig. 4: Example for GDFT modulation, with (a) 3rd bandpass filter of a $K/2 = 8$ channel filter bank; (b) the PSD of the 3rd decimated channel for white Gaussian filter bank input; (c) the PSD of the expanded channel signal.

Thus, all filters can be derived from one single prototype filter $h_p[n]$. Conditions for near perfect reconstruction, i.e. $\hat{x}[n] \approx x[n-\tau]$ will be discussed in Sec. 2.3 solely based on the properties of $h_p[n]$.

The GDFT transform described by (1) can be linked to a DCT-IV modulation used for cosine modulated pseudo-QMF banks [14] by a real operation performed on the complex bandpass filters, apart from a phase shift by $\pm\pi/4$ in the DCT-IV for alias cancellation, which can be omitted here, as aliasing will be suppressed as best as possible in Sec. 2.3 due to oversampling.

Example. To further develop the example introduced in Fig. 3, Fig. 4(a) shows the 3rd complex bandpass filter. For white Gaussian input, the power spectral density of the resulting subband signal decimated by $N = 12$ is given in (b), while (c) depicts the PSD of the same, expanded signal. Note, that for filtering with the bandpass filter in (a), the relevant signal part can be easily isolated.

2.2. Efficient Filter Bank Implementation

For efficient implementation of the oversampled GDFT filter bank, we employ polyphase representation of the analysis and synthesis filters. Generally, savings due to a polyphase implementation are gained in two steps: firstly, filter output samples which are decimated are omitted from the calculation; secondly, computations common to different branches of the analysis or synthesis bank are combined.

2.2.1. Polyphase Representation

With the kth analysis filter written in terms of its N polyphase components $H_{k|j}(z)$, $j = 0(1)N-1$,

$$H_k(z) = \sum_{j=0}^{N-1} z^{-j} H_{k|j}(z^N) \tag{3}$$

a matrix $\mathbf{H}_r(z)$ with polynomial entries can be created for the analysis filter bank:

$$\mathbf{H}(z) = \begin{bmatrix} H_{0|0}(z) & H_{0|1}(z) & \cdots & H_{0|N-1}(z) \\ H_{1|0}(z) & H_{1|1}(z) & & H_{1|N-1}(z) \\ \vdots & & \ddots & \vdots \\ H_{K-1|0}(z) & H_{K-1|1}(z) & \cdots & H_{K-1|N-1}(z) \end{bmatrix} . \tag{4}$$

Together with a polyphase decomposition $X(z) = \sum_{j=0}^{N-1} z^{-j} X_j(z^N)$ of the input signal $x[n]$ analogue to (3),

$$\underline{X}(z) = \begin{bmatrix} X_0(z) \\ X_1(z) \\ \vdots \\ X_{N-1}(z) \end{bmatrix} , \qquad x_i[n] = x[nN+i], \quad i = 0(1)N-1. \tag{5}$$

the analysis bank operation denotes as

$$\underline{Y}(z) = \mathbf{H}(z) \cdot \underline{X}(z) \quad , \tag{6}$$

where $\underline{Y}(z) \in \mathbb{C}_{(z)}^{K\times 1}$ contains the K subband signals, with the notation $\mathbb{C}_{(z)}^{K\times 1}$ referring to the set of K-by-1 matrices with complex valued polynomials in z.

If the polyphase matrix $\mathbf{H}(z)$ is paraunitary, the synthesis of the subband signals may be performed by $\hat{\underline{X}}(z) = \tilde{\mathbf{H}}(z) \cdot \underline{Y}(z)$, where $\tilde{\mathbf{H}}(z)$ is the hermitian of $\mathbf{H}(z)$ with reversed polynomial entries. The reconstructed fullband signal $\hat{x}[n]$ is given in the polyphase representation by $\hat{\underline{X}}(z)$. If we combine analysis and synthesis, i.e. $\hat{\underline{X}}(z) = \tilde{\mathbf{H}}(z) \cdot \mathbf{H}(z) \cdot \underline{X}(z)$, perfect reconstructed is characterized by $\tilde{\mathbf{H}}(z) \cdot \mathbf{H}(z) = z^{-L_p+1} c\mathbf{I}$, $c \in \mathbb{C}/\{0\}$.

For real input signals $x[n]$, an efficient implementation omits $K/2$ subbands

$$\hat{\underline{X}}(z) = \mathrm{Re}\left\{\tilde{\mathbf{H}}_r(z) \cdot \underline{Y}_r(z)\right\} = \mathrm{Re}\left\{\tilde{\mathbf{H}}_r(z) \cdot \mathbf{H}_r(z) \cdot \underline{X}(z)\right\} \tag{7}$$

where the subscript r refers to a reduced matrix representation including only the upper $K/2$ rows of $\mathbf{H}(z)$,

$$\mathbf{H}(z) = \begin{bmatrix} \mathbf{H}_r(z) \\ \times \end{bmatrix} . \tag{8}$$

2.2.2. Polyphase Factorization

Let M be the least common multiple (lcm) of the periodicity of the transform in (1), $2K$, and the decimation ratio N, $M = \mathrm{lcm}(2K, N)$, with $M = J \cdot 2K = L \cdot N$, $J, L \in \mathbb{Z}$. To exploit common calculations between filters, the polyphase components of the analysis filters $\mathbf{H}(z)$ can be written in terms of the M polyphase components of the prototype filter $P(z) = \sum_{m=0}^{M-1} z^{-m} P_m(z^M)$,

$$H_{k|n}(z) = \sum_{l=0}^{L-1} z^{-l} \cdot t_{k,lN+n} \cdot P_{lN+n}(z^L). \tag{9}$$

If the periodicity $2K$ of the transform coefficients $t_{k,n}$ is considered, it is possible to formulate a dense matrix notation

$$\mathbf{H}_r(z) = \mathbf{T}_{\mathrm{GDFT},r} \cdot \mathbf{P}(z) \tag{10}$$

analogue to [2], with the upper half of a GDFT matrix $\mathbf{T}_{\mathrm{GDFT},r} \in \mathbb{C}^{K/2\times 2K}$, fulfilling $\mathrm{Re}\{\mathbf{T}_{\mathrm{GDFT},r}^H \cdot \mathbf{T}_{\mathrm{GDFT},r}\} = 2K\mathbf{I}_{K/2}$, and a generally sparse matrix $\mathbf{P}(z) \in \mathbb{R}_{(z)}^{2K\times N}$ with M non-zero polynomial entries

$$\mathbf{P}(z) = [\mathbf{I}_{2K} \ldots \mathbf{I}_{2K}] \cdot \mathrm{diag}\{P_0(z^L), P_1(z^L), \ldots, P_{M-1}(z^L)\} \cdot \begin{bmatrix} \mathbf{I}_N \\ z^{-1}\mathbf{I}_N \\ \vdots \\ z^{-L+1}\mathbf{I}_N \end{bmatrix} . \tag{11}$$

Additionally, the GDFT transform matrix $\mathbf{T}_r \in \mathbb{C}^{K/2\times 2K}$ in (10) can be factorized according to

$$\mathbf{T}_{\text{GDFT},r} = \mathbf{D}_1 \cdot \mathbf{T}_{\text{DFT},r} \cdot [\mathbf{I}_K \; \mathbf{I}_K] \cdot \mathbf{D}_2, \tag{12}$$

where $\mathbf{D}_1 = e^{-j\frac{\pi}{K}n_0} \cdot \mathbf{I}_{K/2}$ applies a phase correction and $\mathbf{D}_2 \in \mathbb{C}^{2K\times 2K}$ is a diagonal matrix with elements $e^{j\frac{\pi}{K}(n-n_0)}, n = 0(1)2K-1$. The representation in (12) allows savings, as $\mathbf{T}_{\text{DFT},r} \in \mathbb{C}^{K/2\times K}$ consists of the upper $K/2$ rows of a K-point DFT matrix with entries $e^{j\frac{2\pi}{K}kn}$, which can be implemented using standard FFT algorithms. Although half the solution of this K-point FFT will be discarded, the calculation can present a major reduction in computations over the evaluation of the matrix multiplication in (10).

2.2.3. Computational Complexity

Using the above polyphase factorization for the expansion-reconstruction operation in (5)

$$\begin{aligned} \hat{\underline{X}}(z) &= \tilde{\mathbf{H}}(z)\cdot\mathbf{H}(z)\cdot\underline{X}(z) = \text{Re}\left\{\tilde{\mathbf{H}}_r(z)\cdot\mathbf{H}_r(z)\cdot\underline{X}(z)\right\} & (13)\\ &= \tilde{\mathbf{P}}(z)\cdot\text{Re}\{\mathbf{T}_r^T\cdot\mathbf{T}_r\cdot\mathbf{P}(z)\cdot\underline{X}(z)\}, & (14)\end{aligned}$$

we see that complex multiplications are confined to the evaluation of the transform, while the actual polyphase filtering consists of real operation in both analysis and synthesis stage. Together with (10) and (12) the formalism of a filter bank operation becomes apparent. By evaluating matrix operations of the analysis side from right to left, and performing efficient numerical operations on diagonal matrices and an FFT for $\mathbf{T}_{\text{DFT},r}$

$$\underline{Y}(z) = \underbrace{\mathbf{D}_1}_{2K}\cdot\underbrace{\mathbf{T}_{\text{DFT},r}}_{4K\log_2 K}\cdot\underbrace{[\mathbf{I}_K\mathbf{I}_K]\cdot\mathbf{D}_2}_{4K}\cdot\underbrace{\mathbf{P}(z)}_{L_p}\cdot\underline{X}(z) \tag{15}$$

the complexity per fullband sample results in

$$C_{\text{bank}} = \frac{1}{N}(4K\log_2 K + 6K + L_p) \tag{16}$$

real multiplications, which is similarly performed for a synthesis operation at identical expense.

2.3. Relaxation of the Perfect Reconstruction Condition

In-band aliasing for white Gaussian input may be defined in terms of an SNR measure

$$\text{SNR} = \frac{\int_0^{\pi/N} P(e^{j\Omega})P^*(e^{j\Omega})d\Omega}{\int_{\pi/N}^{\pi} P(e^{j\Omega})P^*(e^{j\Omega})d\Omega}, \tag{17}$$

which completely depends on the frequency response of the prototype filter $p[n]$. Therefore, a suitable filter is required to have a stopband edge at $\Omega_s = \frac{\pi}{N}$, as shown in Fig. 5. Thus if aliasing is approximately suppressed, perfect reconstruction reduces to the power complementary condition [14]

$$\sum_{k=0}^{K-1} |H_k(e^{j\Omega})|^2 \stackrel{!}{=} 1. \tag{18}$$

Deviations from (18) manifest as filter bank distortion. As the SNR in (17) limits the minimum mean squared error (MMSE) in the adaptive identification, and the bank distortion creates a lower limit for the reconstruction error of the fullband model reconstructed from the adapted subband responses [15], both distortion terms should be balanced, while exact perfect reconstruction is not required. However for some application such as acoustic echo cancellation, the power complementary condition may be further relaxed to allow several dB ripple in the bank distortion function to achieve low-delay filter banks with good alias suppression and yet sufficient quality for speech signals.

With an expression of (18) in terms of the polyphase components of the prototype filter [14] and an error criterion based on (17), an iterative least-squares method has been applied to design a prototype filter [5]. A design example for $K/2 = 8$ subband channels, a decimation ratio of $N = 12$, and filter length $L_p = 288$ is given in Figs. 3 and 4, with SNR = 74.51dB and a reconstruction error of -73.42dB.

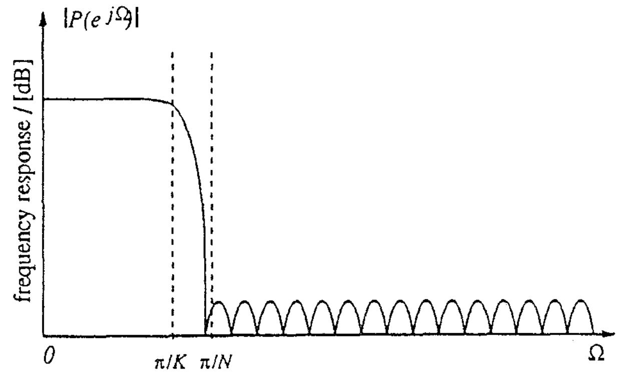


Fig. 5: Frequency response of prototype filter: all signal parts in the stopband above $\Omega_s = \frac{\pi}{N}$ will be aliased in the decimation stage and cause a non-linear distortion.

3. PROCESSING OF COMPLEX SUBBAND SIGNALS

As the previous section has shown, it is sufficient for real valued input signals to keep only half the GDFT subband channels, as the other half is complex conjugate and carries no additional information. Considering that the DCT-IV filter bank mentioned in Sec. 2.1 for the same prototype filter yields an equal number of subband signals, the decimation ratio appears effectively doubled for complex valued subband signals. This facts allows us to process complex subbands competitively compared to real valued subband signals, as a complex filter tap represents about twice the sampling period of a real valued tap, even though commonly known algorithms like the LMS [16] and RLS [6], or the affine projection algorithm often used in AEC [12, 11] require a 4 times higher computational load for complex inputs. The ratio of computational complexity between filtering real and complex valued subband signals for order $\mathcal{O}(L_a^i)$ algorithms can be derived as

$$C_{\text{real}} \propto L_a^i \qquad , \qquad C_{\text{complex}} \propto 4 \cdot \left(\frac{1}{2} \cdot \left(\frac{L_a}{2}\right)^i\right) \tag{19}$$

$$\longrightarrow \quad \frac{C_{\text{complex}}}{C_{\text{real}}} = \frac{1}{2^{i-1}}. \tag{20}$$

with L_a being the length of an adaptive filter in the real valued band, which for the complex valued case can be halved and is updated at only half the rate. Thus in terms of processing load, order $\mathcal{O}(L_a)$ algorithms like LMS and NLMS have same computational complexity for complex and real implementations, while for quadratic dependencies like the RLS the computational burden can be halved by going complex.

Complex subband processing also doubles the range of possible decimation ratios to choose from $\mathbb{N} \ni N \leq K$ over real valued methods like SSB [1] or non-uniform filter banks [5].

4. ADAPTIVE FILTERING IN GDFT MODULATED SUBBANDS

The overall computational complexity of a subband adaptive system as introduced in Sec. 1 consists of two analysis and one synthesis bank operation plus the complexity spent for adaptive filtering, i.e. $C_{\text{total}} = C_{\text{adapt}} + 3 \cdot C_{\text{bank}}$. To demonstrate the benefit of the subband filtering approach for the computational complexity, we look at the length of the fullband equivalent model for different methods and a given number of real multiplications, which is often imposed as a benchmark when implementing an adaptive system on a DSP. Fig. 6 shows the length of the equivalent fullband model for subband adaptive filtering over a different number of subbands, relative to a fullband NLMS adaptive filter. A number of curves is given for different benchmark numbers. Fig. 6(a) is based on the comlpexity of a critically sampled, polyphase implemented DCT-IV filter bank with real valued subband signals and cross-terms between adjacent bands with 1/3 length of the length of the main adaptive filters as suggested by [3]. The same curves are given in Fig. 6(b) for a GDFT modulated complex valued filter bank implemented in polyphase representation and an FFT transformation. Fig. 6(b). Clearly, the

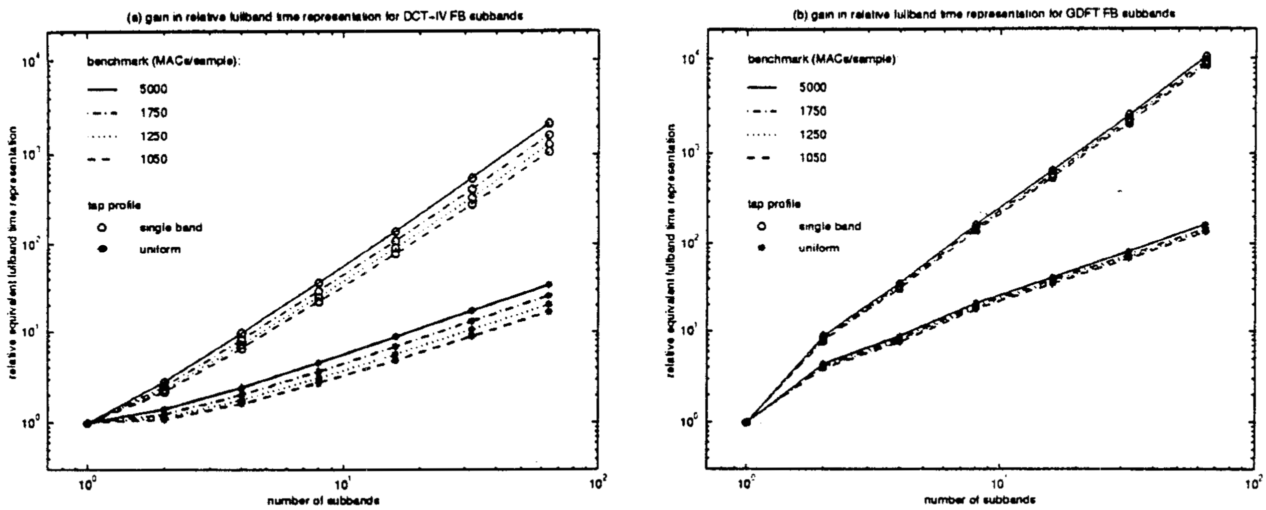


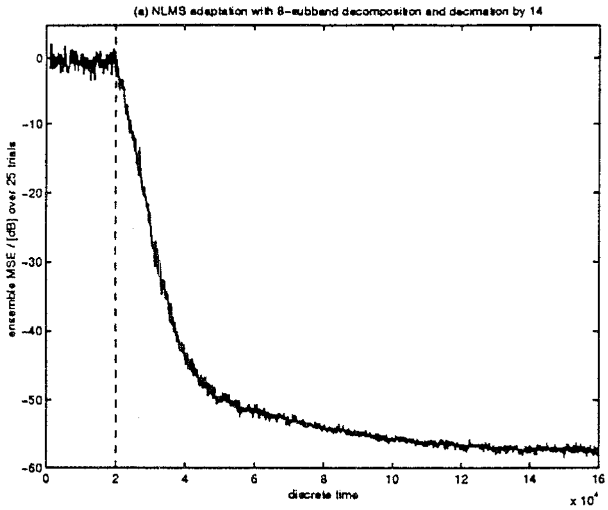
Fig. 6: Relative equivalent fullband model lengths for different number of subbands with (a) DCT-IV critically decimated bands according to [3,15] and (b) GDFT oversampled filter banks. The two groups of curves represent extreme tap profiles: uniform tap-distribution (all subband filter have equal length) and concentrated tap-profile (all filter taps/ computations are dedicated to one single band).

application of subbands can considerably increase the possible model representation of the adaptive system, with a clear advantage of the GDFT over the DCT-IV. Furthermore, the subband approach allows to vary the filter lengths to different subbands while keeping the overall system complexity constant; this may be achieved using an adaptive scheme [15]. For general filter tap profiles across the subbands, the groups of curves in Fig. 6 refer to two extreme cases of filter tap distributions, forming a lower and upper limit of possible equivalent fullband model lengths.

An example for convergence characteristics in an adaptive system identification setup is stated in Fig. 7(a) for $K/2 = 8$ subband signals and decimation by $N = 14$. Although the signals are non-white, the achieved final reduction in noise power of about -57.22dB is roughly limited by the SNR measure in (17) due to aliasing (57.42dB). Fig. 7(b) describes the power spectral densities (PSD) of desired and final error signal. The PSD of the final error exhibits peaks which are caused by aliasing of the system's dominant poles visible in the PSD of the desired signal. A PSD of the alias terms in the filter bank is shown in Fig. 7(c) and matches well with the PSD of final error signal. However note, that there are residuals of the peak at $\Omega = 0.9\pi$ and an insufficient adaptation at around $\Omega = 0.5\pi$ due to slow convergence caused by the position of a dominant pole at a band edge, which is also responsible for the slowly converging mode observed in Fig. 7(a)[9]. The equivalent fullband model can be calculated by sending an impulse through analysis bank, adapted subband filter, and the synthesis side [15]. As the accuracy of this model is limited by aliasing and the reconstruction by the filter bank distortion, the achieved model error of -53.12dB is close to its lower limit of -55.23dB filter bank distortion. If the task of adaptive filtering is to best identify the fullband equivalent model, this motivates a filter design that balances aliasing and bank distortion, as indicated in Sec. 2.3.

5. CONCLUSIONS

We have introduced a GDFT filter bank for subband adaptive filtering, and discussed some of its advantages over other subband approaches. An implementation using a factorization of the polyphase representation has been proposed. Contrary to our initial expectations, processing of complex subbands does not lead to an increased computational load in terms of real multiplications, due to a lower sampling rate and a reduced number of subbands for real input signals to the complex valued filter bank. For algorithms greater than $\mathcal{O}(L_a)$, savings can be gained in processing complex valued subbands. The computational efficiency of a subband adaptive filter using the GDFT modulation has been demonstrated in comparison to fullband and DCT-IV subband adaptive



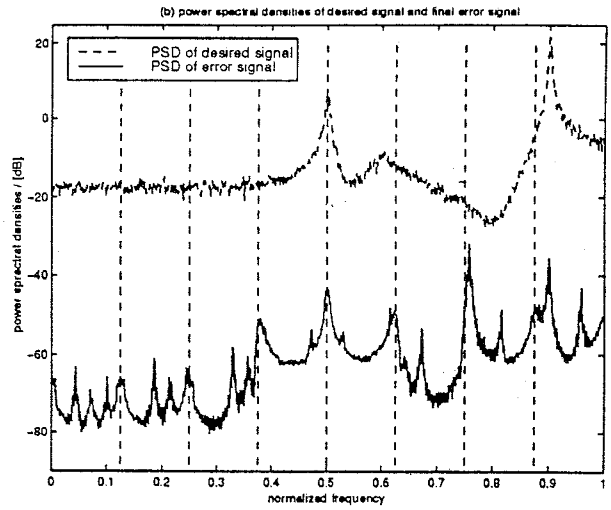
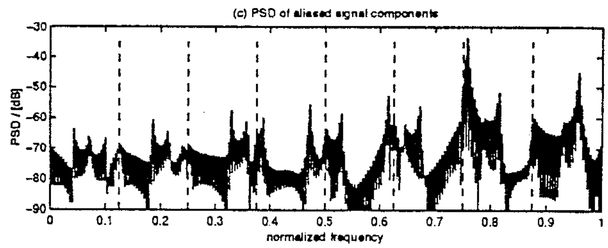


Fig. 7: (a) Ensemble mean square value of reconstructed error signal for a system characterized by 5 complex conjugate poles; (b) PSD of desired signal and final error signal; dashed vertical lines indicate the band edges; (c) PSD of alias components in the reconstructed signal.



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6. REFERENCES

[1] R. E. Crochiere and L. R. Rabiner. *Multirate Digital Signal Processing*. Signal Processing series. Prentice Hall, Englewood Cliffs, NJ, 1983.

[2] Z. Cvetković and M. Vetterli. "Oversampled Filter Banks". *To appear in IEEE Trans. on Signal Processing*, December 1994.

[3] A. Gilloire and M. Vetterli. "Adaptive Filtering in Subbands with Critical Sampling: Analysis, Experiments and Applications to Acoustic Echo Cancelation". *IEEE Trans. on Signal Processing.*, Vol.SP-40(No.8):pp.1862–1875, Aug. 1992.

[4] E. Hänsler. "From Algorithms to Systems — It's a Rocky Road". In *Proc. International Workshop on Acoustic Echo and Noise Control*, pages K1–K4, London, September 1997.

[5] M. Harteneck, J. Paez-Borallo, and R. W. Stewart. "A Filterbank Design for Oversampled Filter Banks without Aliasing in the Subbands". In *2nd IEEE UK Symposium on Applications of Time-Frequency and Time-Scale Methods*, Coventry, UK, August 1997.

[6] S. Haykin. *Adaptive Filter Theory*. Information and System Science Series. Prentice Hall, Englewood Cliffs, 2nd edition, 1991.

[7] W. Kellermann. "Analysis and Design of Multirate Systems for Cancellation of Acoustical Echoes". In *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, volume 5, pages pp.2570–2573, New York, 1988.

[8] L. Lampe. Adaptive filtering in oversampled subbands. Studienarbeit, University of Strathclyde, Glasgow, UK, August 1997.

[9] P. L. de León II and D. M. Etter. " Experimental Results with Increased Bandwidth Analysis Filters in Oversampled Subband Acoustic Echo Cancelers". *IEEE Signal Processing Letters*, Vol.2(No.1),pp.1-3, January, 1995.

[10] Q.-G. Liu and B. Champagne. "Simple Design of Filter Banks for Subband Adaptive Filtering". In *Proc. International Workshop on Acoustic Echo and Noise Control*, pages 132-135, Imperial College, London, September 1997.

[11] A. Nakagawa, Y. Haneda, and S. Makino. "Subband Acoustic Echo Canceller Using Two Different Analysis Filters and 8th Order Projection Algorithm". In *Proc. International Workshop on Acoustic Echo and Noise Control*, pages 140-143, Imperial College, London, September 1997.

[12] K. Ozeki and T. Umeda. "An Adaptive Filtering Algorithm Using an Orthogonal Projection to an Affine Subspace and Its Properties". *Electronics and Communications in Japan*, Vol.67-A(No.5):pp.19-27, February 1984.

[13] J. J. Shynk. "Frequency-Domain and Multirate Adaptive Filtering". *IEEE Signal Processing Magazine*, Vol.9:pp.14-37, Jan. 1992.

[14] P. P. Vaidyanathan. *Multirate Systems and Filter Banks*. Signal Processing series. Prentice Hall, Englewood Cliffs, 1993.

[15] S. Weiß, U. Sörgel, and R. W. Stewart. "Computationally Efficient Adaptive System Identification in Subbands with Intersubband Tap Assignment for Undermodelled Problems". In *Asilomar Conference on Signals, Systems, and Computers*, Monterey,CA, November 1996.

[16] B. Widrow, J. McCool, and M. Ball. "The Complex LMS Algorithm". *Proceedings of the IEEE*, Vol.63:pp.719-720, August 1975.

[17] Y. Yamada, H. Ochi, and H. Kiya. "A Subband Adaptive Filter Allowing Maximally Decimation". *IEEE Journal on Selected Areas in Communications*, Vol.12(No.9):pp.1548-1552, September 1994.

[18] H. Yasukawa. "Performance Degradation of a Subband Adaptive Digital Filter with Critical Sampling". *IEICE Transactions on Fundamentals of Electronics, Communications, and Computer Science*, Vol.E77A(No.9):pp.1497-1501, September 1994.

[19] H. Yasukawa, S. Shimada, and I. Furukawa. "Acoustic Echo Canceller with High Speech Quality". In *Proc. IEEE International Conference on Acoustics, Speech, and Signal Processing*, volume IV, pages 2125-2128, Dallas, April 1987.

[20] P. Yip and K. R. Rao. Fast discrete transforms. In D. F. Elliott, editor, *Handbook of Digital Signal Processing*, chapter 6. Academic Press, San Diego, CA, 1987.