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Estimating Induced Effects in IO Impact Analysis: Variation in the Methods for Calculating the Type II Leontief Multipliers

Tobias Emonts-Holley , Andrew Ross & Kim Swales

Abstract

Type II Input-Output (IO) multipliers are frequently used for impact analysis. Unfortunately, there is no standard way to calculate these. The fundamental issue is that these multiplier methods endogenise household consumption but all have drawbacks because the IO accounts are missing key information required to consistently link household income and consumption to domestic economic activity. Using compatible regional and national data sets, we evaluate the values for various IO Type II multipliers to a Benchmark value calculated with the aid of Social Accounting Matrix (SAM) data. The results suggest that the variation in Type II IO multiplier values generated by these alternative methods is an empirically non-trivial issue.

Key words: Input-Output, Social Accounting Matrix, impact analysis, multipliers.

JEL: E16, E17, R13, R15

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1. Introduction

Demand-driven Input-Output (IO) multipliers are extensively used by academics, policy makers and consultants. These multipliers claim to quantify the total demand-side consequences of actual or planned changes in exogenous expenditures. Whilst multipliers can be used to identify national impacts, they are more often employed at a regional or local spatial level. In the academic literature, examples occur in areas of interest which include: quantifying the local economic impact of major events (Chhabra et al., 2003; Collins et al., 2012; Gelan, 2003); institutions (Allan et al., 2007; Goldstein, 1990; Morgan et al., 2017); disasters (Hallegatte, 2008; Okuyama and Santos, 2014) and the introduction of new products/technologies (Allan, 2015).

Broadly, a Type I multiplier comprises both the initial exogenous direct demand disturbance and subsequent supply-chain impacts, known as the indirect effects; a Type II multiplier also includes the induced consequences of changes in household consumption. These induced effects reflect the adjustment to household income that accompanies the direct and indirect changes in economic activity. In this way, Type II multipliers combine the traditional Keynesian consumption and Type I IO multiplier models.[1]

The motivation for the present paper is straightforward. Although the construction and interpretation of Type I multipliers is very clearly understood, the same cannot be said for the Type II approach. This is despite the fact that in principle Type II multipliers give a more comprehensive, and therefore more useful, measure of the impact of exogenous expenditure on a local economy. Any neglect of household consumption is problematic as it is a major contributor to final demand, accounting for up to 70% of GDP (on the expenditure side) in some cases. It is important to recognise that numerous variants of the standard Type II multiplier exist and that their relative values differ significantly and systematically.

[1] The Keynesian consumption multiplier is attributed to Kahn (1931). Goodwin (1949) is an early attempt to combine IO and Keynesian multipliers. Examples of early theoretcal and applied use of Type II multipliers include Miernyk (1965) and Smith and Morrison (1974).

Practitioners require clear and consistent methods to calculate and comment on the economy-wide impacts of exogenous demand shocks experienced by individual sectors. In the present paper we outline a number of popular alternative approaches to constructing Type II multipliers. We then explore which formulation is likely to replicate most accurately the flow of incomes generated in production to factors of production and then subsequently to domestic households. It is this income stream that funds endogenous domestic consumption.

A central finding is that all the standard methods used to calculate Type II IO multipliers are necessarily defective. This is because the data required to track accurately the relevant income flows are missing from the IO accounts. However, such information is typically available in a Social Accounting Matrix (SAM). We therefore construct a Benchmark multiplier using consistent SAM data against which the standard IO multipliers can be compared.[2] The aim is principally to indicate the size and nature of the potential error in using different Type II IO multiplier formulations.

It is important to be aware from the start that we are not attempting to develop a more sophisticated model of household consumption so as to extend the standard Type II IO approach, in the manner of scholars such as Batey and Madden or Miyazawa; we are not criticising existing models for their lack of detailed household disaggregation.[3] Nor do we wish to discuss work that focuses on inter-reginal modelling and the role of income flows in such models in the manner of Rose and Stevens (1991) and Robison (1997). Further our analysis does not concern the use of eclectic IO/econometric approaches where consumption is modelled in a more flexible and data intensive manner (Klein, 1989). Rather, we want to advise users who are prepared to accept the standard assumptions and conventions of IO analysis as to which Type II multiplier option, each of which is supported by authoritative sources, should be incorporated in their own empirical

[2] As we note later, it is not necessary to construct the full SAM but extensive additional data not available in standard IO accounts are required (Robison, 1997; Rose and Stevens, 1991).

[3] For example, Miyazawa and Masegi (1963) attempt to incorporate household disaggregation within a Kaleckian version of the standard Keynesian model (Hewings et al. 1999). This is limited because it requires restrictive assumptions for reasons similar to those discussed here, though extensive household disaggregation is possible in a demand-driven SAM-type model, if the appropriate data are availible (Pyatt, 2001).

research. However, we also want to warn that endogenising household consumption involves very basic difficulties that conventional texts appear to ignore.

Section 2 introduces the IO Type I multiplier. This establishes the terminology and method in a non-controversial setting. Section 3 details how the generic Type II multiplier endogenises household expenditure. Section 4 outlines five alternative Type II multiplier formulations together with a Benchmark multiplier. Section 5 comprises an analytical comparison of the various multiplier approaches and Section 6 contrasts the multiplier values for individual sectors using the 2010 IO and SAM regional (Scotland) and national (UK) data. Section 7 is a short conclusion.

2. IO Type I Multipliers

The standard IO model quantifies the total demand-side effects of changes in exogenous final demands.[4] The model adopts a set of conventional assumptions concerning production; fixed technical coefficients and no supply constraints. For an economy with n sectors, the Input-Output accounts can be represented in the standard way:

$$(1) \qquad [I - A]x = f_T$$

In equation (1), $\boldsymbol{A}$ is the nxn matrix of technical production coefficients of domestically produced intermediate inputs, where each element a_{ij} is the input from sector i needed to produce one unit of output in sector j. These coefficients are parameterised on the IO base-year data. The $\boldsymbol{x}$ and $\boldsymbol{f_T}$ are $nx1$ vectors of sectoral outputs and total final demands respectively. (Table 1 lists and defines all the variables and parameters used in the main text). Pre-multiplying both sides of (1) by $[I - A]^{-1}$ produces the familiar IO relationship that links endogenous outputs to exogenous final demands:

$$(2) \qquad x = [I - A]^{-1} f_T$$

[4] Models based on the IO accounts can be built and employed for other purposes but this is the standard demand-driven model and the one most frequently used (Miller and Blair, 2009, Ch. 12).

Table 1: Variables and parameters used in the main text

Variable	Definition
$\boldsymbol{A}$	Standard matrix of domestic industrial coefficients
M_j^I	The Leontief type I multiplier for industry *j*
M_j^{II}	The Leontief type II multiplier for industry *j*
$\boldsymbol{a_W}$	Row vector of industrial wage coefficients (wage input per unit of output)
$\boldsymbol{a_\Pi}$	Row vector of industrial other value added coefficients
c	Scalar total household consumption
$\boldsymbol{f_H}$	Column vector of household final demand
$\boldsymbol{f_T}$	Column vector of total final demand (= $\boldsymbol{f_X}+\boldsymbol{f_H}$)
$\boldsymbol{f_X}$	Column vector of exogenous non-household final demand (exports, investment, government)
$\boldsymbol{f_Y}$	Scalar exogenous household income (e.g. extra-regional, government and pension income transfers)
$\boldsymbol{h_C}$	Column vector of household coefficients in the B matrix (= $\boldsymbol{f_H}y^{-1}$)
r	Scalar share of other value added that is distributed directly or indirectly to domestic (regional) households
w	Scalar total wage payments
$\boldsymbol{x}$	Column vector of industrial outputs
y	Scalar total household income
α_{ij}^I	Entry in the *i*th row and the *j*th column of the Leontief Type I inverse
α_{ij}^{II}	Entry in the *i*th row and the *j*th column of the Leontief Type II inverse
π	Scalar total payments of other value added

In this case $[I-A]^{-1}$ is the Type I Leontief inverse where the representative element α_{ij}^I is the direct and indirect output in sector *i* required to produce one unit of exogenous final demand in sector *j*. Summing the elements of column *j* gives the Type I multiplier for sector *j*, M_j^I. This is the total output across all domestic sectors associated with a unit increase in exogenous demand for the output of sector *j*, given as:

$$(3) \quad M_j^I = \sum_{i=1}^{n} \alpha_{i,j}^I$$

If all the relevant assumptions are applicable, then equation (2) becomes a model in which exogenous final demand drives, in a linear and deterministic manner, endogenous production. Importantly, equation (2) can also be interpreted as an accounting identity. Any set of IO accounts can be presented in this way, so that the vector of actual outputs is attributed to the vector of actual final demands. The ability of the Leontief inverse to replicate the original output vector when applied to the original vector of final demands is an important practical check on the validity of the model.

The output multiplier values calculated through equation (3) can also be employed to identify the impact on any variable that is linearly linked to a sector's output. This can apply to economic or physical variables, such as employment, value added, greenhouse gas emissions or water use. A particular instance is the use of the Type I model in an accounting mode as the most rigorous way to calculate carbon footprints (Swales and Turner, 2017).

3. IO Type II Multiplier: General Issues

The previous section outlined what is known as the open IO model; all final demands are treated as exogenous. However, as Miller and Blair (2009, p. 34) state: "In the case of households, especially, this "exogenous" categorisation is something of a strain". If economic activity increases, employment and household incomes rise, generating an expected increase in household consumption. Endogenous consumption is the central element of basic Keynesian demand-driven models. As argued by Miyazawa (1960, p. 53) "… unlike the Keynesian model [Leontief's Type I approach] lacks an analysis of the multiplier process via the consumption function. Formally, the Leontief system can regard the household sector as an industry whose output is labour and whose inputs are consumption goods". The standard procedure is to "… move the household sector from the final demand column and labour-input row and place it inside the technically interrelated table [of intermediate demands] making it one of the *endogenous* sectors. This is known as closing the model with respect to households" (Miller and Blair, 2009, p.35).

This implies disaggregating the final demand vector used in equations (1) and (2) into household, $\boldsymbol{f_H}$, and non-household, $\boldsymbol{f_X}$, expenditures so that:

$$(4) \qquad f_T = f_H + f_X$$

Non-household final demand comprises exports, investment, and government expenditure and is taken to be exogenous; however, the Type II multiplier now treats household consumption as endogenous. [5] Following Batey (1985), with appropriate adjustments for notation, we represent the generic Type II equation as:

$$(5) \qquad \begin{bmatrix} 1-A & -h_C \\ -(a_W + ra_\Pi) & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f_X \\ f_Y \end{bmatrix}$$

In equation (5) y is total household income, $\boldsymbol{h_c}$ is the *nx1* vector of household consumption coefficients, $\boldsymbol{a_W}$ and $\boldsymbol{a_\Pi}$ are *1xn* vectors of wage and other value added coefficients, *r* is the share of other value added that is distributed, directly or indirectly to domestic (regional) households and f_Y is exogenous household income. [6] Exogenous household income is that part of household income sourced from exogenous transfers. These transfers generally include capital or rental income from outwith the local economy and welfare or pension transfers from the capital and government accounts. [7]

Again, through the conventional procedure of matrix inversion, the values in the vector of endogenous variables can be derived as a function of the exogenous variables, so that:

$$(6) \qquad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1-A & -h_C \\ -(a_W + ra_\Pi) & 1 \end{bmatrix}^{-1} \begin{bmatrix} f_X \\ f_Y \end{bmatrix}$$

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[5] It is also possible to endogenise investment and government expenditures, for example by linking investment to output or savings and government expenditure to implied population or tax receipts. However, these extensions are not included in the conventional Type II model.

[6] In the Batey (1985) formulation shown as equation (5), household income from wages and other value added produce qualitatively similar consumption responses, which is in the spirit of standard IO and SAM multiplier practice. Behavioural work suggests that individuals might allocate income from different sources to different "pots" and treat consumption from these pots differently (Thaler, 2016). Pyatt (2001) also raises this issue in discussing the link between endogenous consumption, saving and the Kalecki macro-closure. These are interesting lines of enquiry and can be accommodated by further disaggregating the analysis but are not central to our account.

[7] They can also include wage income from commuters and tourists, if tourist expenditure is included in the household consumption accounts (Rose and Stevens, 1991; Hermannsson, 2016). In the Scottish and UK data used in the Section 6 tourism expenditure from non-residents is identified separately and treated as exports.

In equation (6), $\begin{bmatrix} 1-A & -h_C \\ -(a_W + ra_\Pi) & 1 \end{bmatrix}^{-1}$ is the Type II Leontief inverse, where α_{ij}^{II} is the *ith* element in the *jth* column which gives the direct, indirect and induced effects on sector *i* of a unit increase in final demand in sector *j*. Again, summing the first *n* elements of column *j* gives the Type II multiplier for sector *j*, M_j^{II}.[8]

$$(7) \qquad M_j^{II} = \sum_{i=1}^{n} \alpha_{ij}^{II}$$

Key practical issues raised in endogenising household expenditure for an impact model include the question of whether average and marginal consumption propensities are equal and whether different household types have different consumption propensities (Miyazawa, 1960, 1976; Miller and Blair, 2009; Kim et al. 2014, 2016).[9] These topics clearly warrant investigation and many economists have treated consumption in a more detailed way in extended IO systems, including incorporating endogenous demographic effects and welfare payments (Batey and Madden, 1983; Batey, 1985; Batey and Weeks, 1989). However, in this paper we focus on a prior and more basic problem. This is that even if one accepts the limitations of a unified household sector, a linear consumption function and exogenous government transfers, the information required to link economic activity to household income and consumption is unavailable in the IO accounts.

There are two problems posed by the Type II multiplier formulation outlined in equation (6). First, the IO accounts do not have information on income flows between domestic institutions. There are therefore no figures for the sources of household income so that the values of *r* and f_Y are not available. The second problem is that the vector of household consumption coefficients, $\boldsymbol{h_C}$, cannot be calculated from base-year IO data. This is because the IO accounts do not include a figure for total household income, *y*. The difference between the various Type II IO multipliers rests on the strategies that they adopt to deal with these informational shortfalls.

[8] The *(n+1)th* element is the impact on household income.
[9] Analogous concerns of homogeneity within sectors and the nature of marginal adjustments also apply to industrial sectors, though these issues might be thought to be more important in household consumption.

These issues can be considered in more detail by rearranging the final row of equation (5). We here, and subsequently, identify the different Type II multiplier formulations solely through a superscript, in this case the generic *K*, and apply the equation to the base-year data set, indicated by the subscript 0.

$$(8) \qquad y_0^K = (a_W + r^K a_\Pi)x_0 + f_{Y,0}^K$$

In equation (8), and in subsequent discussions, for each multiplier formulation this accounting identity must hold. This is central to our analysis. Recall that the two parameters identifying the factor coefficients, $\boldsymbol{a_W}$ and $\boldsymbol{a_\Pi}$, can be determined from the base-year IO accounts, but the parameters r^K and $f_{Y,0}^K$, and the base year household income, y_0^K, have to be imposed.

In order to calculate the Type II multiplier as shown in equation (6) we need the value of *r* and implicitly the value of y_0. This is because the vector of household consumption coefficients, $\boldsymbol{h_C}$, is derived by dividing the base-year vector of household expenditures, $\boldsymbol{f_{H,0}}$, by the base-year household income, so that:

$$(9) \qquad h_C^K = f_{H,0}(y_0^K)^{-1}$$

However, once two of the parameters/variables r^K, $f_{Y,0}^K$ and y_0^K are imposed, equation (8) determines the third. This means that whilst the level of exogenous household income does not appear explicitly in the Type II multiplier formulations, it can be thought of as being present implicitly via equation (8).

The value of r^K is problematic for the following reasons. The IO accounts do give the sectoral coefficients for the income distributed to the factors of production. These are the *1* x *n* vectors $\boldsymbol{a_W}$ and $\boldsymbol{a_\Pi}$,; in the same way as the sectoral domestic intermediate coefficients in the **A** matrix, these are parameterised on the IO base year data. However, the sum of the endogenous payments to locally-based factors does not equal the sum of the endogenous receipts of income by domestic households. To determine this, we need information of the subsequent income flows from factors to domestic households and the corporate sectors, and then subsequent income transfers between these accounts. In particular, in a regional context, the existence of cross boundary income flows is important.

As is apparent from equation (5), all the multiplier models that we consider assume that wage income generated within the region is fully retained within the region as household income.[10] However, this is not typically the case for the income created from other value added, which comprises payments to inputs such as capital, property and land. These are not necessarily owned by local residents and in so far as this true, a proportion will not be received by domestic households. This flow of income is particularly difficult to track and certainly is not identified in standard IO accounts. However, this income stream can be derived from data available in an appropriate Social Accounting Matrix (SAM).[11]

In Section 4 we compare five IO-based Type II multiplier formulations found in the literature. These formulations use only data available in the standard IO accounts to derive the multiplier values.[12] In the absence of available data, alternative Type II approaches impose different values for the parameters r^K, h_C^K and $f_{Y,0}^K$. A benchmark multiplier is also outlined where the values for the missing parameters are derived from a Social Accounting Matrix (SAM).[13]

4. IO Type II Multiplier: Specific Formulations

[10] Where this assumption fails to hold, for example if inter-regional commuting occurs, then the multiplier can be adjusted. Rose and Stevens (1991) and Hermannsson (2016) outline how such spatial wage/household/expenditure flows can be accommodated. They are important in multiplier effects for cities and need a more sophisticated treatment than the standard Type II IO..

[11] Again we stress that the availability of an appropriate SAM is sufficienct, but not necessary, for deriving this figure (Robison, 1997).

[12] This does not strictly apply to the formulation which we call Batey1 where an externally-derived value for household income is used.

[13] Accepting the standard assumptions of demand-driven IO/SAM modelling and taking the SAM data as accurate, in previous versions we have referred to this as the “true” Type II multiplier value. However, we have adopted the more neutral label “Benchmark value” partially to reflect the fact that its calculation does not require the construction of the whole SAM framework (see, for example, Robison, 1997).

The specific multiplier formulations that we consider in this paper are summarised in Table 2. Early conceptual attempts to combine Leontief (1936, 1966) and Keynesian (1936) multiplier models endogenously linked consumption to total value added (Goodwin, 1949; Miyazawa, 1960). This approach, which we allocate the superscript *G*, is consistent with a basic Keynesian model where all expenditure on domestic output becomes domestic income. In terms of equation (8) this implies that base-year exogenous household income, f_Y^G , is zero and share of other value added going to household income (r^G) equals one. Given that the vector of base year sectoral outputs, $\mathbf{x_0}$, and the wage and other value added coefficients can be derived from the IO accounts, this approach uses equation (8) to calibrate base-year household income, y_0^G , as the total base-period wages w_0 (= $\boldsymbol{a_W x_0}$) nd other value added π_0 (= $\boldsymbol{a_\Pi x_0}$) .

However, in practice, income transfers typically prove more complex, particularly at a regional level. One primary concern is cross-boundary income flows, with endogenous factor income earned locally leaving the domestic economy but with exogenous income transfers generated elsewhere adding to domestic household income. However, it is not only geographical income flows that add complications. At both the national and regional level there are also flows across generations, in terms of private and public pensions, plus government funded welfare payments.

Table 2: Alternative formulations for the Type II multiplier using equations (5), (6) and (8).

	Label	$f_{Y,0}^K$	r^K	h_C^K	*Estimate*
Goodwin (1949)	M^G	0	1	$\frac{f_{H,0}}{w_0 + \pi_0}$	$M^G > M^B$

Rose and Stevens (1991)	M^{R+S}	0	$\frac{c_0 - w_0}{\pi_0}$	$\frac{f_{H,0}}{c_0}$	$M^{R+S} > M^B$
Miller and Blair (1985, 2009)	M^{M+B}	0	0	$\frac{f_{H,0}}{w_0}$	$M^{M+B} > M^B$
Batey (1985)	M^{B1}	$y_0^B - w_0$	0	$\frac{f_{H,0}}{y_0^B}$	$M^{B1} < M^B$
Batey (1985)	M^{B2}	$c_0 - w_0$	0	$\frac{f_{H,0}}{c_0}$	$M^{B2} > or < M^B$
Benchmark	M^B	$f_{Y,0}^B$	$\frac{y_0^B - f_{Y,0}^B - w}{\pi_0}$	$\frac{f_{H,0}}{y_0^B}$	

For the determination of endogenous income the problem lies primarily, though not entirely, with other value added. Usually some elements of profits and rents accruing to capital and land located in a specific region will be distributed to households and corporations located outwith the region. Rose and Stevens (1991, p. 256) reports a formulation which it refers to as a "typical approach" where the total household consumption expenditure is used as the control total for deriving the household consumption vector h_C^{R+S}. If we define total base-year household consumption as $c_0 = \sum f_{H,0}$, then this implies imposing $y_0^{R+S} = c_0$. As in the Goodwin model, household income is assumed to be derived solely from domestic production, so that again $f_{Y,0}^{R+S}$ is zero.[14] All wage income is allocated to households plus a share, r^{R+S}, of other value added that is calibrated to the base-year values. Using equation (8), $r^{R+S} = \frac{c_0 - w_0}{\pi_0}$.

[14] Rose and Stevens (1991) gives no references to the use of this approach but is critical of this method as a basis for

More frequently, applied Type II multipliers only track labour income, which then drives endogenous household consumption. In all such approaches, $r^K = 0$. Miller and Blair (1985, 2009) adopt this procedure and also assumed that all income is endogenous. This implies that that $r^{M+B}, f_{Y,0}^{M+B} = 0$, so that $y_0^{M+B} = w_0$. In a regional context this means that all wage income goes to local households but that all other value added either leaves the region or in other ways fails to reach domestic households (maybe saved by the corporate sector or taken in taxes). The primary problem for this method is that in fact typically only around 60% of all household income comes from wages (Emonts-Holley et al., 2015). Moreover, perhaps more critically, recall that certain elements of household income, such as pensions and some government transfers, are independent of income generated in current production.[15]

The Type II multiplier approach outlined in Batey (1985), which we label as Batey1, acknowledges the existence of exogenous household expenditure.[16] But as with Miller and Blair, endogenous household income is linked to wage income alone, so that $r^{B1} = 0$. Base-year Benchmark household income is used to generate the household consumption coefficients, so that $y_0^{B1} = y_0^B$. However, as stressed already, the Benchmark household income is not given in the IO accounts, so that it has to be derived from some other source. From equation (8), $f_{Y,0}^{B1}$ is determined through calibration as $y_0^B - w_0$.

A variant of the Batey approach, that we call Batey2, uses total household consumption data from the IO accounts to generate the vector of household consumption coefficients, h_C^{B2}. As under the method identified in Rose and Stevens (1991) $y_0^{B2} = c_0$ but in this case $r^{B2} = 0$. Again, calibrating using equation (8) produces ` $f_{Y,0}^{B2} = c_0 - w_0$.

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generating sound Type II multiplier values.

[15] This issue is fudged in the numerical examples given in Miller and Blair (1985, p. 28; 2009, p. 38) where the sum of household consumption is given as arbitrarily equal to the total wage payment, so that in those cases $f_{Y,0}^{M+B} = 0$.

[16] This is Model 2 shown in Figure 1 in Batey (1985).

Finally, a Benchmark multiplier is constructed on the basis that the modeller has access to a set of base-year accounts which provide direct information on variables required to parameterise equation (6). These accounts would require an IO base and compatible information on the flow of income and expenditure between domestic factor incomes, and domestic businesses and households. These figures can be calculated using the data available in a standard Social Accounting Matrix (SAM). This derivation is shown in detail in Appendix A. We refer to this as the Benchmark multiplier because it is constructed on the basis of data not available to the other Type II IO formulations.

5. Comparing the Type II Multiplier Formulations

It is of interest to compare analytically the different Type II IO and the Benchmark approaches. (This is done more formally in Appendix B). From Table 2 note that M^G, M^{R+S} and M^{M+B} all embody the assumption that there is no exogenous household income, so that $f_{Y,0}^K = 0$. But the three measures have different r^K values, varying from 0 to 1, with $1 = r^G > r^{R+S} > r^{M+B} = 0$.[17] Recall that the r^K value shows the share of locally generated other value added that is passed on to household income. Other things being equal, one would expect the average multiplier values to be ranked in line with the r^K figure. However, note that the vector of household coefficients is calculated by dividing the values in the vector of household consumption by the imposed or calibrated initial household income.

For an individual sector, the multiplier values derived by these three models, M^G, M^{R+S} and M^{M+B}, will differ. But if the multipliers for all sectors are weighted by the initial vector of exogenous final demands, $\boldsymbol{f_{X,0}}$, the average value for each of these measures will be the same. This is because in each of the three cases the Type II multipliers must be able to replicate equation (6) with the base-year data for the final demand vector $\boldsymbol{f_{X,0}}$ and zero

[17] This is assuming that base-year household consumption is greater than total wages.

$f_{Y,0}^{K}$

h_{C}^{K}.

Moreover, this weighted average will be greater than the weighted Benchmark multiplier value, because in each of these three Type II IO cases the multiplier model links the whole of household consumption to income generated in domestic production, whereas in the Benchmark model some will be funded by exogenous household income. The accuracy of the multiplier value for individual sectors, as compared to the Benchmark figure will depend on the breakdown of the sector's value added between wages and other value added.

There are also three Type II multiplier approaches that have endogenous household income sourced solely from domestic wages, M^{M+B}, M^{B1} and M^{B2}, so that in all these cases, $r^{K} = 0$.[18] However, they differ in the control total (implied household income) used to calculate the vector of household consumption coefficients, so that the multiplier values will differ. From equation (6), under these conditions, the Type II multiplier takes the form $\begin{bmatrix} 1-A & -\frac{f_{H,0}}{y_0^K} \\ -a_W & 1 \end{bmatrix}^{-1}$. The multiplier value will be positively related to the value of the household consumption coefficients, which implies that it will be negatively related to the value of y_0^K. On the assumption that base-period wage income is less than consumption, which is, in turn, less than Benchmark income, then for any sector j, $M_j^{M+B} > M_j^{B2} > M_j^{B1}$. This result also corresponds to ranking the multipliers inversely related to their implied exogenous income, given that in this case exogenous household income equals total income minus wage income.

[18] Note that in the Miller and Blair approach both $f_{Y,0}^{M+B}$ and r^{M+B} equal zero.

Finally we compare the M^{M+B}, M^{B1} and M^{B2} multiplier values to the Benchmark value, M^{B}. For all sectors, the Benchmark value is greater than Batey1 because whilst both employ the same household coefficients, the Benchmark model incorporates a share other value added as endogenous household income. The weighted sum of the multiplier values for the Miller and Blair formulation are higher than the weighted sum of the Benchmark figures. This is because Miller and Blair endogenise all household income and the Benchmark case only part. The comparison between the Batey2 and the Benchmark multiplier values is less clear cut. Compared to the Benchmark, the Batey2 household coefficients are higher but the share of other value added income that is passed on to households is less. The relationship between M^{B} and M^{B2} therefore remains an empirical matter

6. Empirical Comparison of Multiplier Values

In this section, we report IO Type I, various IO Type II and the Benchmark multiplier values for 34 Scottish and UK industry sectors. The primary aim is to investigate how important the disparities between these multiplier values are in practice. The findings show, in the specific case of Scotland and the UK, which Type II methods give multiplier values closest to the Benchmark values. These data are derived from compatible Scottish and UK IO and SAM accounts for 2010, which are available in Ross (2019).[19] Appendix C gives a description of the abbreviated sectoral labels used in Table 3 and Figures 1 and 2.

6.1 Scotland

[19] These compatible data sets were developed through the ESRC Future of the UK and Scotland Initiative (Ref: ES/L003325/1). The computation of the SAM accounts is outlined in Emonts-Holley et al. (2014) with details of the data sources and the identification of income flows given in Ross (2019). Similar methods are used in the construction of the UK SAM.

The left-hand side of Table 3 shows the IO Type I, the five IO Type II and the Benchmark multiplier values for all Scottish production, disaggregated to individual sectors. For each sector, the deviations of the IO Type II multipliers from the corresponding Benchmark values are shown in the two-part Figure 1. In this Figure the horizontal axes represent the Benchmark values. This means that where the entry is above (below) this axis, the relevant IO Type II multiplier value is above (below) the corresponding Benchmark figure. Entries closer to this axis are therefore better approximations to the Benchmark multiplier value. Summary and error statistics for these results are given in the top half of Table 4.[20]

It is clear that including household expenditure into the calculation has a marked impact on the resulting multiplier values. Batey and Weeks (1989) uses IO data from the Greater Cork region of the Republic of Ireland to quantify the impact of incorporating first wage income and then increasingly detailed assumptions about the operation of the local labour market. At one level, our results allow a comparison along similar lines; in our case we can measure the cumulative impact on IO multiplier values of including other value added as well as wages as a source of endogenous household income.

The Type I IO multiplier comprises the direct exogenous output shock and the indirect effects on endogenous intermediate demand. Table 4 shows its unweighted average sectoral value for Scotland as 1.37. In all the multiplier calculations the direct effect - that is, the original stimulus - is unity. The multiplier effect is the remainder, so that on average for these Scottish data the indirect, supply-chain, effects increase the impact of a demand stimulus by 37%. We can make a comparison of the effects of incorporating additional components by comparing the Type 1 multiplier values with the Batey1 and Benchmark formulations.

Both the Batey1 and Benchmark multipliers use the Benchmark base period household income figure as the denominator in the household consumption coefficient calculations.

[20] Corresponding multiplier values disaggregated to 104 Scottish sectors are reported in Emonts-Holley et al. (2015) using a 2009 Scottish IO table and SAM.

The Batey1 formulation only includes wage payments as endogenous household income and this produces an average Type II multiplier value of 1.74. Simply incorporating the household consumption funded by wage payments therefore doubles the multiplier effect. [21] The Benchmark multiplier employs the best available estimate for the link between locally generated other value added and household income. When this is incorporated in the multiplier calculation the average (Benchmark) Type II multiplier value increases to 1.88. With these calculations, incorporating the endogenous wage income in the Type II IO multiplier increases the estimated impact by 37% of the direct effect and incorporating household consumption funded by other value added generates an additional 14%.

However, it is important to stress that information on total household income and the flow of payments going directly and indirectly from other value added to households is not given in the standard IO accounts. Therefore, analysts working only with the standard IO accounts cannot use the Batey1 or Benchmark methods to calculate the Type II multiplier values. As outlined in Section 4, there are a variety of Type II multiplier formulations given in the literature and Table 4 shows that it is extremely important which measure is used; the average Type II multiplier values range between 1.74 and 2.16.

The upper part of Figure 1 shows the relative results for the three Type II methods which embody the assumption that there is no exogenous source of household income. These are the M^G, M^{R+S} and M^{M+B} approaches. As argued in Section 5, this means that they systematically produce figures that are on average very similar and greater than the Benchmark value. However, for individual sectors the multiplier results can vary quite radically across the different methods.

With the Scottish data, in every single sector the M^G and M^{R+S} value is above the axis – that is, above the Benchmark value - but with the Miller and Blair formulation, M^{M+B}, there are a number of industrial sectors where the result is close to, and in three sectors - Agriculture, forestry and fishing (1, AGR), Coke and refined petroleum products (8,

[21] That is: £74million/£37 million = 2.

COK) and Electricity, transmission and distribution (14, ELE) - where it is marginally below, the axis. Also, in the Real Estate Activities (25, REA) sector the M^{M+B} value is 0.19 below the Benchmark. This reflects the high "other value added" intensity of this sector. Where the M^{M+B} value is relatively low, the M^{G} figure is relatively high with the M^{R+S} value somewhere in between.

The lower part of Figure 1 compares the values given by the three multiplier formulations where endogenous income comprises solely domestic wages. It is apparent that the way in which the relative values vary across sectors is qualitatively similar but these values are at different levels and do not move in parallel. This representation shows four clear results. First, there is a consistent ordering across the multiplier values that applies in each sector. As argued in Section 5: $M^{M+B} > M^{B2} > M^{B1}$. Second, in every sector the Batey1 formulation is always less than the Benchmark. Third, whilst the weighted average of the Miller and Blair multiplier must be above the Benchmark, there are a small number of sectors where this is not the case. Finally, the Batey2 is more evenly distributed across positive and negative relative values.

The upper half of Table 4 shows the summary statistics for the Scottish results. The mean value for the Batey2 measure is closest to the benchmark and also has the lowest Root Square Mean Error (RMSE) and Mean Absolute Error (MAE) when compared to the Benchmark values. In the absence of the additional data required to fully specify the Type II multiplier, Batey2 would give results closest to the Benchmark and can be calculated solely from data available in the IO accounts. However, it has to be stressed that this multiplier value will tend to overestimate the induced effects and for some sectors will be especially inaccurate. From Figure 1 it is clear that the M^{B2} value is at least 0.20 above the Benchmark in the labour-intensive sectors: Scientific R & D (27, RND), Public Administration and Defence (29, PUB), Education (30, EDU), Human Health (31, HUM) and Care and Social Work Services (32, CAR). Recall that this is an error of 20% of the initial demand shock. If one generally prefers a more conservative impact estimate, then the Batey1 measure should be adopted if a separate and consistent estimate of the base year household income is available.

6.2 UK

The reason for computing the sets of multipliers for both regional and national economies is that the size of the multiplier is linked to the level of income and expenditure withdrawals. Essentially, the higher is the rate of withdrawal, the lower the multiplier value. The tax and savings rates will not vary systematically with the size of the geographic area under consideration. However, the level of imports and external transfers will typically increase as the scale of the area studied falls.[22] The sector-specific multiplier values for the UK are given on the right hand side of Table 3, the IO Type II multiplier values, relative to the Benchmark multiplier, are shown in Figure 2 and the summary and error statistics in the bottom half of Table 4.

Whilst the UK multiplier values for all formulations are higher than for Scotland, the figures are qualitatively comparable. The multipliers which assume no exogenous household income are the highest with very similar mean values and high RMSE and MAE results when compared to the Benchmark figures. With UK data, when other approaches are included, the Batey2 measure is more emphatically the closest estimate to the Benchmark though again the mean sectoral value remains higher (2.56 as against 2.47) and there are still some large deviations in labour intensive sectors. Again, Batey1 is a conservative alternative which for all sectors consistently produces a result below the Benchmark figure.

[22] By imports and external transfers we mean any purchases from, or transfers to, institutions or individuals located outwith the area under consideration.

Table 3: IO multipliers for Scotland and the UK, 2010

	Scotland							United Kingdom						
		Type II							Type II					
	Type I	M^{G}	M^{R+S}	M^{M+B}	M^{B1}	M^{B2}	Benchmark	Type I	M^{G}	M^{R+S}	M^{M+B}	M^{B1}	M^{B2}	Benchmark
1.AGR	1.58	2.37	2.14	2.00	1.78	1.92	2.01	1.83	3.31	3.01	2.81	2.25	2.57	2.66
2.COA	1.66	2.25	2.23	2.22	1.92	2.11	2.04	1.79	2.93	2.97	3.00	2.31	2.70	2.54
3.CRU	1.47	2.03	1.97	1.93	1.69	1.84	1.81	1.41	2.99	2.32	1.87	1.61	1.76	2.15
4.OMI	1.47	2.26	2.04	1.91	1.67	1.82	1.90	1.55	2.95	2.61	2.39	1.91	2.18	2.31
5.FOD	1.70	2.30	2.31	2.32	1.99	2.20	2.10	2.04	3.29	3.31	3.33	2.60	3.01	2.85
6.DRI	1.32	2.09	1.92	1.82	1.55	1.72	1.76	1.95	3.28	3.15	3.07	2.43	2.79	2.75
7.TEX	1.58	2.21	2.22	2.22	1.88	2.10	2.00	1.61	2.74	2.75	2.76	2.10	2.47	2.34
8.COK	1.14	1.33	1.27	1.23	1.18	1.21	1.24	1.42	1.97	1.92	1.89	1.62	1.77	1.75
9.CHE	1.31	2.00	2.07	2.11	1.68	1.95	1.79	1.70	2.70	2.70	2.70	2.13	2.45	2.34
10.PHA	1.20	2.04	1.84	1.72	1.44	1.62	1.67	1.45	2.78	2.52	2.35	1.84	2.13	2.19
11.RUB	1.49	2.11	2.16	2.18	1.81	2.04	1.91	1.62	2.65	2.71	2.75	2.11	2.47	2.30
12.ELM	1.39	1.96	2.00	2.03	1.68	1.90	1.78	1.65	2.83	2.84	2.85	2.16	2.55	2.41
13.OME	1.51	2.14	2.18	2.21	1.83	2.07	1.94	1.80	2.95	3.00	3.04	2.34	2.73	2.56
14.ELE	2.12	2.71	2.54	2.43	2.26	2.37	2.44	2.29	3.44	3.16	2.96	2.58	2.80	2.91
15.GAS	1.26	1.70	1.63	1.58	1.41	1.52	1.52	1.98	3.17	2.86	2.65	2.27	2.48	2.61
16.WAT	1.32	2.08	1.93	1.83	1.56	1.73	1.76	1.70	3.13	2.90	2.75	2.15	2.49	2.52
17.CON	1.70	2.45	2.45	2.44	2.04	2.30	2.19	1.83	3.21	3.08	2.99	2.33	2.70	2.66
18.WHO	1.42	2.17	2.17	2.17	1.77	2.02	1.91	1.66	3.09	3.08	3.07	2.27	2.72	2.57
19.TRA	1.47	2.14	2.19	2.22	1.82	2.07	1.93	1.73	3.10	3.15	3.19	2.36	2.82	2.63
20.ACS	1.36	2.06	2.06	2.06	1.68	1.92	1.82	1.63	2.95	2.90	2.87	2.16	2.56	2.45
21.ITC	1.33	2.12	2.14	2.16	1.72	1.99	1.86	1.45	2.84	2.80	2.77	2.02	2.44	2.32
22.FIN	1.26	2.08	1.95	1.86	1.54	1.74	1.74	1.50	2.97	2.76	2.62	1.98	2.34	2.35
23.INS	1.67	2.33	2.22	2.16	1.89	2.06	2.06	1.86	3.19	2.95	2.80	2.26	2.56	2.61
24.FIS	1.27	2.03	2.13	2.19	1.69	2.00	1.81	1.54	3.06	3.16	3.22	2.26	2.80	2.55
25.REA	1.30	2.13	1.73	1.49	1.39	1.45	1.68	1.54	3.16	2.49	2.05	1.76	1.92	2.31
26.PRO	1.38	2.18	2.15	2.14	1.73	1.99	1.89	1.56	3.09	3.02	2.98	2.17	2.63	2.51
27.RND	1.39	2.22	2.37	2.46	1.88	2.25	2.00	1.57	3.14	3.43	3.62	2.45	3.11	2.69
28.ADM	1.36	2.14	2.17	2.18	1.74	2.02	1.88	1.55	3.03	2.93	2.87	2.12	2.54	2.46
29.PUB	1.37	2.06	2.14	2.19	1.75	2.03	1.86	1.50	2.80	2.91	2.99	2.14	2.62	2.38
30.EDU	1.18	2.11	2.32	2.44	1.77	2.19	1.88	1.34	2.92	3.22	3.42	2.24	2.90	2.47
31.HUM	1.32	2.10	2.21	2.27	1.76	2.08	1.88	1.41	2.71	2.86	2.96	2.08	2.58	2.30
32.CAR	1.55	2.42	2.52	2.58	2.03	2.37	2.16	1.84	3.25	3.42	3.53	2.56	3.11	2.81
33.ART	1.32	2.14	2.04	1.98	1.62	1.85	1.82	1.51	2.95	2.87	2.82	2.08	2.50	2.40
34.OTR	1.26	2.11	2.12	2.12	1.66	1.95	1.82	1.34	2.86	2.81	2.77	1.96	2.42	2.29

Table 4: IO multiplier summary and error statistics for Scotland and the UK

			Type II					
		Type I	M^G	M^{R+S}	M^{M+B}	M^{B1}	M^{B2}	Benchmark
Scotland	Mean	1.37	2.12	2.14	2.16	1.74	2.00	1.88
	Min	1.14	1.33	1.27	1.23	1.18	1.21	1.24
	Max	2.12	2.71	2.54	2.58	2.26	2.37	2.44
	RMSE	0.47	0.27	0.24	0.26	0.16	0.13	-
	MAE	0.45	0.26	0.23	0.22	0.15	0.11	-
United Kingdom	Mean	1.61	2.98	2.91	2.86	2.16	2.56	2.47
	Min	1.34	1.97	1.92	1.87	1.61	1.76	1.75
	Max	2.29	3.44	3.43	3.62	2.60	3.11	2.91
	RMSE	0.83	0.53	0.45	0.46	0.32	0.20	-
	MAE	0.82	0.51	0.43	0.41	0.30	0.16	-

Figure 1: Deviations of the Scottish IO Type II multipliers from the corresponding Benchmark values

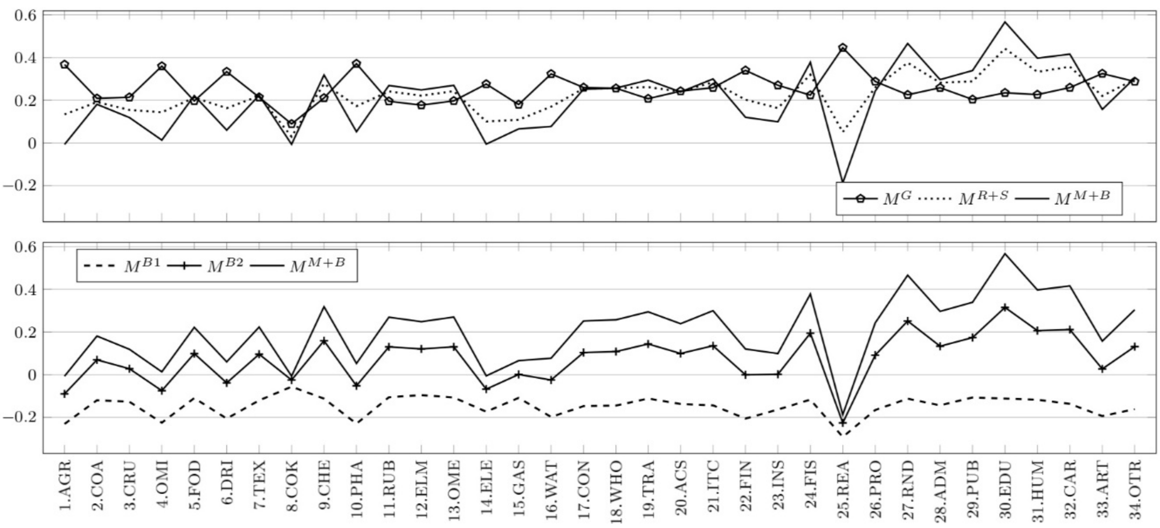

Figure 2: Deviations of the UK IO Type II multipliers from the corresponding Benchmark values