An IGA-BEM Method for the Open-Water Marine Propeller Flow Problem

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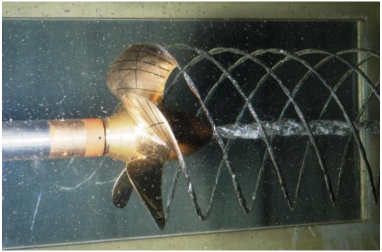
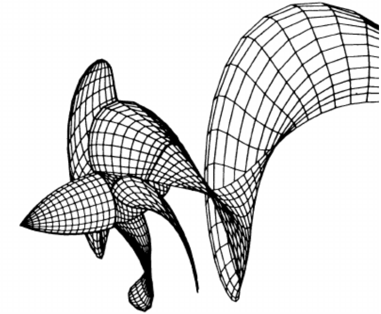
Overview

The Steady State Problem



PPTC (Heinke 2011)



$S = S_B \cup S_W \cup S_H$ (Hoshino 1989)

Perturbation Potential and Boundary Conditions

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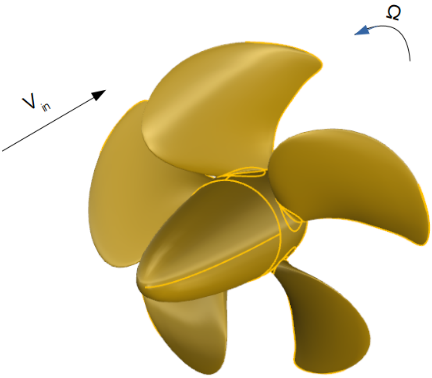
Velocity Potential

$$\mathbf{V}_{\infty} = \mathbf{V}_{in} + \mathbf{\Omega} \times \mathbf{r}$$
$$\phi_{total} = \phi + \phi_{\infty}$$
$$\nabla^2 \phi = 0$$

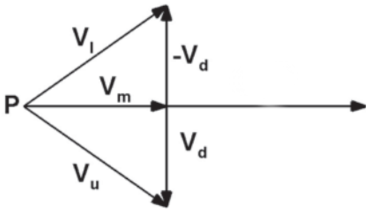
Boundary Conditions

$$\frac{\partial \phi}{\partial n} = -\mathbf{V}_{\infty} \cdot \mathbf{n}, \; S_B \cup S_H$$
$$p_u - p_l = 0, \; S_W$$
$$\frac{\partial \phi_u}{\partial n} - \frac{\partial \phi_l}{\partial n} = 0, \; S_W$$



Pressure Jump Condition

- $\mathbf{V_m} = \frac{1}{2}(\mathbf{V_u} + \mathbf{V_l}), \quad \mathbf{V_d} = \frac{1}{2}(\mathbf{V_u} - \mathbf{V_l}).$
- $\mathbf{V_u}$, $\mathbf{V_l}$ orthogonal
- Kutta condition
- Correct wake geometry

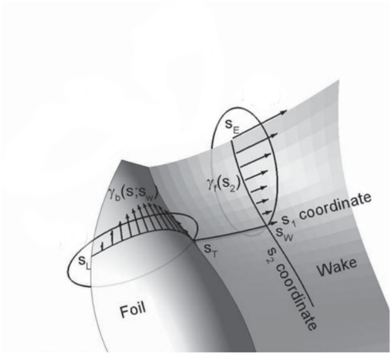


Boundary Integral Equation

$$2\pi\phi(\mathbf{P}) - \int_{S_H+S_B} \phi(\mathbf{Q}) \frac{\partial G(\mathbf{P},\mathbf{Q})}{\partial n} dS - \int_{S_W} \Delta\phi(\mathbf{Q}) \frac{\partial G(\mathbf{P},\mathbf{Q})}{\partial n} dS = \int_{S_H+S_B} (\mathbf{V}_\infty \cdot \mathbf{n}) G(\mathbf{P},\mathbf{Q}) dS$$

- $G(\mathbf{P},\mathbf{Q}) = \frac{1}{4\pi r(\mathbf{P},\mathbf{Q})}$
- r: distance between **P** and **Q**
- ϕ: potential on boundary surface
- $\Delta\phi = \phi_u - \phi_l$: potential jump on wake

Zero Pressure Jump - Kutta Condition



$s_1 \perp s_2$, $s_1 \parallel s_2$ (Kerwin & Hadler 2010)

Potential jump on wake

$$\Delta\phi(\mathbf{Q}) = \Delta\phi(s_2)$$

$$\Delta\phi(s_2) = \Delta\phi_{TE}(s_2)$$

Kutta Condition $\longrightarrow$ $\Delta\phi_{TE}(s_2)$

Zero Pressure Jump - Kutta Condition

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2D: Equal s_1 velocity components:

$$V^u_{s_1,TE} = V^l_{s_1,TE} \Rightarrow \frac{\partial \Phi_u}{\partial s_1} = \frac{\partial \Phi_l}{\partial s_1}$$

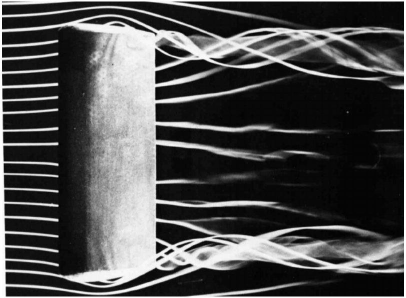
3D: Equal s_1 velocity components and opposite s_2 velocities:

$$V^u_{s_1,TE} = V^l_{s_1,TE} \Rightarrow \frac{\partial \Phi_u}{\partial s_1} = \frac{\partial \Phi_l}{\partial s_1}$$

$$V^u_{s_2,TE} = -V^l_{s_2,TE} \Rightarrow \frac{\partial \Phi_u}{\partial s_2} = -\frac{\partial \Phi_l}{\partial s_2}$$

Zero Pressure Jump - Wake Alignment

- $p_u - p_l = 0$ may not be satisfied yet
- $s_1 \parallel \mathbf{V_m}$ $\rightarrow$ requires solution
- Simple blade models $\rightarrow$wake surface is known
- Empirical methods
- Wake alignment iterative methods



Wake roll-up (Van Dyke 1982)

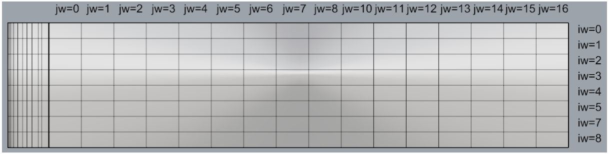
Isogeometric Boundary Element Method

Wing and wake in uniform flow represented by bicubic T-spline surfaces: $S_B = \cup_{e=1}^{n_B} S_{e,B}$, $S_W = \cup_{e=1}^{n_W} S_{e,W}$:

$$\tilde{\mathbf{x}}^{e,m}(\tilde{\xi}) = (\mathbf{d}^{e,m})^T \mathbf{R}_B^{e,m}(\tilde{\xi}) = \frac{(\mathbf{d}^{e,m})^T \mathbf{W}^{e,m} \mathbf{C}_{e,m} \mathbf{B}(\tilde{\xi})}{(\mathbf{w}^{e,m})^T \mathbf{C}_{e,m} \mathbf{B}(\tilde{\xi})} \quad m = B, W.$$

$$\phi(\mathbf{P}) = \sum_{i=1}^{n_A} \phi_i \tilde{R}_i(\mathbf{P}), \quad \mathbf{P} \in S_{e,B}$$

Discrete Kutta condition



Numbering of elements on the wake

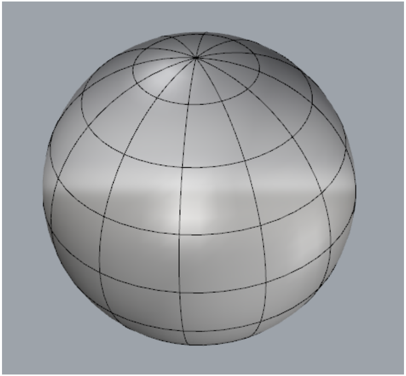
- $\Delta\phi(\mathbf{Q}) = \Delta\phi_{iw,jw} = \Delta\phi_{iw}$
- $\Delta\phi_{iw} = \sum_{i=1}^{n_A} \phi_i(\tilde{R}_i(\tilde{\xi}_{u,TE,iw}) - \tilde{R}_i(\tilde{\xi}_{l,TE,iw}))$
- $\tilde{\xi}_{u,TE,iw}, \tilde{\xi}_{l,TE,iw}$: Parent domain parametric values of upper and lower trailing edge blade elements respectively.

Discrete Form of BIE

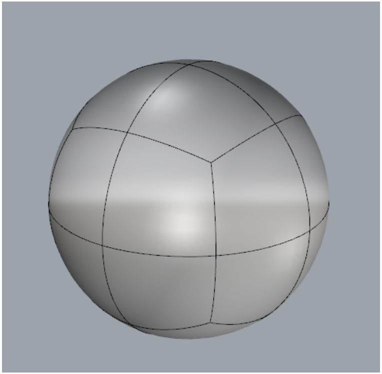
$$2\pi \sum_{i=1}^{n_A} \phi_i \tilde{R}_i(\mathbf{P}_j) - \sum_{i=1}^{n_A} \phi_i \sum_{e=1}^{n_B} \int_{S_{e,B}} \tilde{R}_i(\mathbf{Q}) \frac{\partial G(\mathbf{P}_j,\mathbf{Q})}{\partial n} dS$$
$$- \sum_{e=1}^{n_W} \phi_i \sum_{jw=0}^{jmax-1} \sum_{iw=1}^{imax-1} (\tilde{R}_i(\tilde{\xi}_{u,TE,iw}) - \tilde{R}_i(\tilde{\xi}_{l,TE,iw})) \int_{S_{e,iw,jw}} \frac{\partial G(\mathbf{P}_j,\mathbf{Q})}{\partial n} dS =$$
$$- \sum_{e=1}^{n_B} \int_{S_{e,B}} V_\infty \cdot n(\mathbf{Q}) \left(\frac{1}{r(\mathbf{P}_j,\mathbf{Q})} \right) dS(\mathbf{Q}), \quad j = 1, \ldots, n_A \quad (1)$$

- $\mathbf{P}_j$: Collocation points based on the generalisation of Greville abscissae (Scott et al. 2013).

Application - Spheres

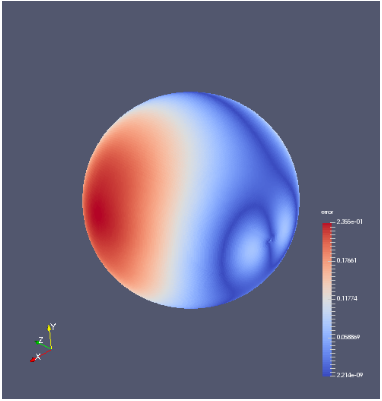


Sphere with poles - 132 Control Points, 96 Elements

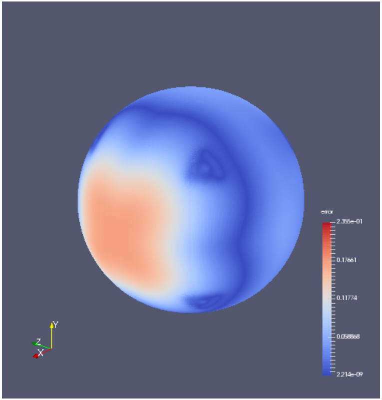


Quadball - 26 Control Points, 336 Elements

Application - Spheres

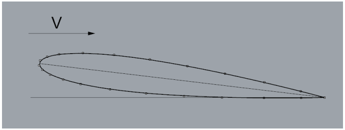


$l^2 = 0.10278, \; L^2 = 4.29251$

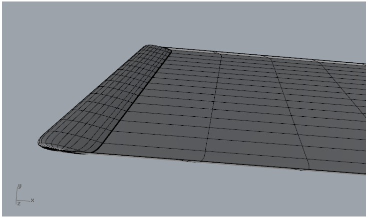


$l^2 = 0.04909, \; L^2 = 2.47136$

Application - Rectangular Wing



NACA 0012 profile - 6.75°



Rectangular wing and wake

Application - Rectangular Wing

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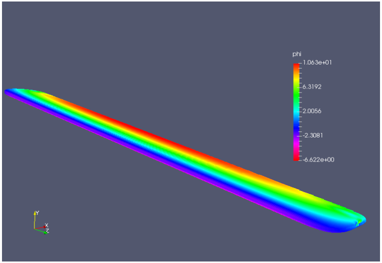
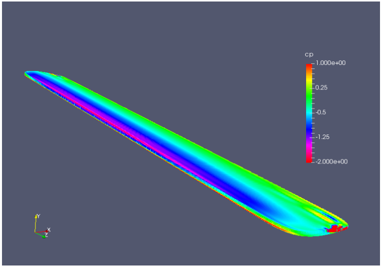
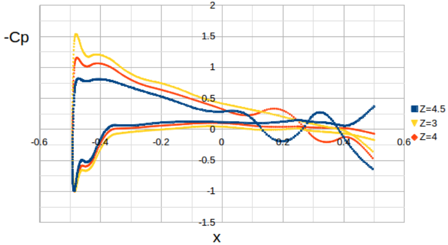


ϕ distribution on the wing

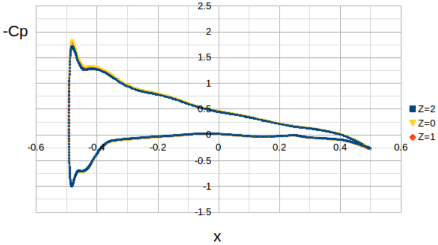


C_P distribution on the wing

Application - Rectangular Wing



C_P distribution for outter sections



C_P distribution for inner sections

Conclusions

- Quadball representation of the sphere is faster and more accurate than pole representation.
- 2D Kutta condition only works for sections far from the tip.
- Cross flow effects near the tip require alignment of spanwise velocities (3D Kutta condition).
- Singular behaviour on the tip region of the wake for this specific parametrisation.

Future Work

- Application of 3D Kutta condition → non-linear → Iterative Pressure Kutta
- Development of wake alignment iterative algorithms
- Investigation of aligned wake geometry (roll-up core etc.)

Wake roll-up section (Van Dyke 1982)

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- **A**lgebraic **R**epresentations in **C**omputer-**A**ided **D**esign for compl**E**x **S**hapes
- Coordinator: ATHENA Research and Innovation Center - Ioannis Emiris
- Duration: 1/2016-12/2019
- Aim: Bridge the gap between CAD industry and breakthroughs in mathematics.

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