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# Vibration analysis of a circular disc backed by a cylindrical cavity

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**Abstract:** This paper describes the free vibration analysis of a thin disc vibrating and interacting with an acoustic medium contained in a cylindrical duct. The effects of structural–acoustic coupling are studied by means of an analytical–numerical method that is based upon classical theory and the Galerkin method. The coupling effects are discussed, and results obtained from the analysis are compared with corresponding values obtained both experimentally and from a finite element analysis. There is good agreement between the three sets of results.

**Keywords:** vibration, plates, acoustic, interaction

## NOTATION

$a$	peripheral radius of disc/acoustic cavity
$c_1$	speed of acoustic wave propagation
$E$	Young's modulus
$f$	natural frequency (Hz)
$J_m, I_m, Y_m$	Bessel functions, order $m$
$l$	depth of acoustic cavity
$m$	number of circular waves
$q$	number of radial waves in acoustic medium
$r$	radial coordinate
$s$	number of radial waves on disc
$w$	lateral vibratory deflection of disc
$x$	axial distance from base of acoustic cavity
$\Theta$	circumferential coordinate
$\nu$	Poisson's ratio
$\rho_d$	disc density
$\rho_f$	fluid density
$\Phi$	velocity potential functions
$\omega$	natural frequency (rad/s)

## 1 INTRODUCTION

The general analysis of acoustic/structural vibration interaction problems is presented in references [1] and [2], where infinite series solutions for the acoustic pressure and the displacement of the structure are derived from a fundamental solution of the uncoupled problems, namely vibration of the structure *in vacuo*, and acoustic resonance in a closed cavity with undeformable walls. These basic models were extended and applied to problems involving rectangular plates backed by rectangular cavities [3–6]. More recently, a study has been performed on the case of a *circular membrane* vibrating in contact with a compressible fluid contained in both a closed and open cylindrical cavity [7]. One of the authors of reference [7], Bhat, at an earlier stage considered the specific case of an Indian musical drum [8]. With respect to the case of vibro-acoustic effects involving a *circular plate*, Lee and Singh [9] analysed the characteristics of the acoustic radiation emitted from a vibrating circular plate in free space. More recently, Bauer and Chiba [10] considered the case of a circular plate backed by a cylindrical cavity; however the fluid contained within the cavity was assumed to be *viscous and incompressible*.

In the present paper the natural frequencies and mode shapes of a flexible disc mounted on top of a cylindrical cavity which has rigid walls and base will be examined. The cavity contains *inviscid and compressible* fluid. The static inplane pre-stress within the disc is not taken into account in this instance. The disc is modelled as a

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thin-walled elastic circular plate and it is assumed to be clamped around the edge of the top of the cavity.

A theoretical–numerical analysis (TNA) is developed, which is based upon the equations describing the vibration characteristics of the disc *in vacuo* and the potential flow theory describing the free vibration of the compressible fluid contained in the cylindrical cavity. Determination of the vibration characteristics describing the *coupled* motion of the structure and compressible fluid is then achieved by consideration of the boundary conditions on the surface of the disc and via the Galerkin method. The Galerkin method implies that the normal mode shapes of the coupled system are similar to those of the disc *in vacuo*. Results are then compared with corresponding values of natural frequency obtained using the finite element (FE) code ANSYS and laboratory experiments. Consequently, the study presents the opportunity to appraise the performance of a commercial finite element code, such as ANSYS, when dealing with a vibroacoustic problem in which the structural and fluid elements are deployed simultaneously.

## 2 THEORETICAL ANALYSIS

### 2.1 The acoustic cavity

The inviscid compressible fluid inside the cylindrical duct, schematically shown in Fig. 1, is described by the following equation in cylindrical coordinates for the perturbed velocity potential,  $\Phi$ :

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r} \frac{\partial \Phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Phi}{\partial \theta^2} + \frac{\partial^2 \Phi}{\partial x^2} = \frac{1}{c_1^2} \frac{\partial^2 \Phi}{\partial t^2} \quad (1)$$

where  $c_1$  is the speed of sound. The harmonic solution is assumed to be of the form

$$\Phi = H(x) \cdot Q(r)(\cos m\theta) e^{i\omega t} \quad (2)$$

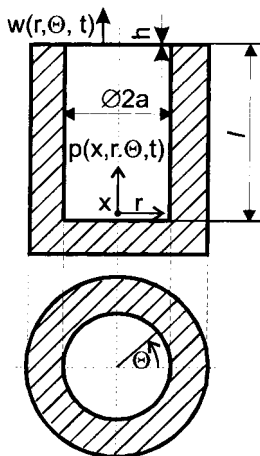


Fig. 1 Schematic of the system under investigation

where  $m = 0, 1, 2, \dots$ . Separation of variables therefore yields

$$\frac{H''}{H} = \left( \frac{m^2}{r^2} - \frac{\omega^2}{c_1^2} \right) - \left( \frac{Q''}{Q} + \frac{1}{r} \frac{Q'}{Q} \right) \equiv \pm k^2 \quad (3)$$

where

$$H'' = \frac{d^2 H}{dx^2}, \quad Q'' = \frac{d^2 Q}{dr^2} \quad \text{and} \quad Q' = \frac{dQ}{dr}$$

Considering  $H''/H = +k^2$ , where  $k^2 > 0$ , yields

$$H(x) = A \cosh kx + B \sinh kx \quad (4)$$

If, however,  $k^2$  were less than zero, equation (4) would be

$$H(x) = A \cos kx + B \sin kx$$

The physical boundary condition

$$\left. \frac{\partial \Phi}{\partial x} \right|_{x=0} = 0 \quad (5)$$

implies that  $B = 0$ ; therefore

$$H(x) = A \cosh kx \quad (6)$$

From the Bessel equation (3),

$$Q'' + \frac{1}{r} Q' + \left[ \left( \frac{\omega^2}{c_1^2} + k^2 \right) - \frac{m^2}{r^2} \right] Q = 0$$

It then follows that

$$Q(r) = C J_m(\lambda r) + D Y_m(\lambda r) \quad (7)$$

where

$$\lambda = \left( k^2 + \frac{\omega^2}{c_1^2} \right)^{1/2} \quad (8)$$

and  $D = 0$  since  $Q$  must be finite when  $r \rightarrow 0$ . Thus, the following can be written:

$$Q = C J_m(\alpha \bar{r}) \quad (9)$$

where  $\bar{r} = r/a$  and  $\alpha = a\lambda$ . Since there is no radial component of the fluid velocity at  $\bar{r} = 1$ , for a given value of  $m$  there is a set of roots  $\alpha = \alpha_{m,q}$  ( $q = 1, 2, 3, \dots$ ), where  $J'_m(\alpha) = 0$  [11]. Therefore, from equation (8),

$$k^2 = k_{m,q}^2 = \frac{1}{a^2} \left[ \alpha_{m,q}^2 - \left( \frac{\omega a}{c_1} \right)^2 \right] \quad (10)$$

and finally

$$\Phi = \sum_{m=0}^{+\infty} \sum_{q=1}^{+\infty} \Phi_{m,q}(x, \bar{r}, \theta, t) \quad (11)$$

where

$$\Phi_{m,q} = D_{m,q}(\cosh k_{m,q}x)[J_m(\alpha_{m,q}\bar{r})(\cos m\Theta) e^{i\omega t}] \quad (12)$$

and  $D_{m,q}$  are unknown constants given by the condition at  $x = l$ . The compressible fluid pressure distribution is finally given by

$$p = -\rho_f \frac{\partial \Phi}{\partial t} \quad (13)$$

**2.2 Vibration of a thin disc in vacuo**

The governing equation for the lateral vibration of a thin disc in vacuo is [12]:

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \Theta^2}\right)^2 w = -\left(\frac{\rho_d}{D_0}\right) \frac{\partial^2 w}{\partial t^2} \quad (14)$$

where  $D_0 = Eh^2/[12(1 - \nu^2)]$ ,  $D = D_0h$ . For a peripherally clamped circular plate,

$$w(a) = \frac{\partial w}{\partial r} \Big|_{\hat{r}=a} = 0 \quad (15)$$

and, for a given value of  $m$ , the radial mode shape of vibration is

$$W_m(r) = A_m J_m(\hat{\alpha}_{m,s}r) + C_m I_m(\hat{\alpha}_{m,s}r) \quad (16)$$

where  $\hat{\alpha}_{m,s}$  are roots ( $s$ ) of the equation

$$J_m(\hat{\alpha}_{m,s}a)J'_m(\hat{\alpha}_{m,s}a) - I_m(\hat{\alpha}_{m,s}a)J'(\hat{\alpha}_{m,s}a) = 0 \quad (17)$$

Introducing  $\xi_{m,s} = \hat{\alpha}_{m,s}a$ , the natural frequencies of the disc in vacuo are

$$\omega_{m,s} = \xi_{m,s}^2 \frac{1}{a^2} \sqrt{\frac{D_0}{\rho_d}} \quad (18)$$

and the modes of vibration are

$$W_{m,s}(r) = -\frac{I_m(\xi_{m,s})}{J_m(\xi_{m,s})} J_m\left(\xi_{m,s} \frac{r}{a}\right) + I_m\left(\xi_{m,s} \frac{r}{a}\right) \quad (19)$$

**2.3 The coupled solution**

It follows from equations (11) and (12) that, for a fixed integer value  $m$  (the number of nodal diameters), the potential is

$$\Phi_m(x, \Theta, r, t) = \sum_{q=1}^{+\infty} D_{m,q} \cosh(k_{m,q}x) J_m\left(\alpha_{m,q} \frac{r}{a}\right) \times \cos(m\Theta) e^{i\omega t} \quad (20)$$

and for the forced vibrations of the disc it is possible to

write

$$w_m(r, \Theta, t) = W_m(r) \cos(m\Theta) e^{i\omega t} \quad (21)$$

where

$$W_m(r) = \sum_{s=1}^{+\infty} W_{0,m,s} W_{m,s}(r) \quad (22)$$

and  $W_{0,m,s}$  are unknown constants.

The impermeability condition

$$\frac{\partial \Phi_m}{\partial x} \Big|_{x=l} = \frac{\partial w_m}{\partial t} \quad (23)$$

yields

$$\sum_{q=1}^{+\infty} D_{m,q} k_{m,q} \sinh(k_{m,q}l) J_m\left(\alpha_{m,q} \frac{r}{a}\right) = i\omega W_m(r) \quad (24)$$

At this stage the orthogonality relationship [13] is also introduced:

$$\int_0^a r J_m\left(\alpha_{m,q_1} \frac{r}{a}\right) J_m\left(\alpha_{m,q_2} \frac{r}{a}\right) dr = 0$$

for  $q_1 \neq q_2$ . For  $q_1 = q_2 = q$ ,

$$\int_0^a r J_m^2\left(\alpha_{m,q} \frac{r}{a}\right) dr = \left(\frac{a^2}{2}\right) \left(1 - \frac{m^2}{\alpha_{m,q}^2}\right) J_m^2(\alpha_{m,q}) \quad (25)$$

Therefore, multiplication of equation (24), which describes the impermeability condition, by  $[r J_m(\alpha_{m,q} r/a)]$  and integration over the interval  $(0, a)$  gives

$$D_{m,q} = \frac{i\omega \int_0^a r W_m(r) J_m(\alpha_{m,q} r/a) dr}{k_{m,q} \sinh(k_{m,q}l) J_m^2(\alpha_{m,q} a^2/2) (1 - m^2/\alpha_{m,q}^2)} \quad (26)$$

Therefore

$$\begin{aligned} \Phi_m(x, \Theta, r, t) \Big|_{x=l} &= i\omega l \sum_{q=1}^{+\infty} \frac{\text{cotgh}(k_{m,q}l)}{(k_{m,q}l)} \frac{J_m(\alpha_{m,q} r/a)}{(a^2/2)(1 - m^2/\alpha_{m,q}^2) J_m^2(\alpha_{m,q})} \\ &\times \cos(m\Theta) e^{i\omega t} \int_0^a r W_m(r) J_m\left(\alpha_{m,q} \frac{r}{a}\right) dr \end{aligned} \quad (27)$$

and

$$\begin{aligned} p_m(r, \Theta, x, t) \Big|_{x=l} &= -\rho_f \frac{\partial \Phi_m}{\partial t} \Big|_{x=l} \\ &= \rho_f l \omega^2 \frac{2}{a^2} \cos(m\Theta) e^{i\omega t} \sum_{q=1}^{+\infty} p_{m,q}(r) \end{aligned} \quad (28)$$

where

$$p_{m,q}(r) = \frac{\text{cotgh}(k_{m,q}l)}{(k_{m,q}l)} \frac{J_m(\alpha_{m,q}r/a)}{(1 - m^2/\alpha_{m,q}^2)J_m^2(\alpha_{m,q})} \times \int_0^a r W_m(r) J_m(\alpha_{m,q} \frac{r}{a}) dr \tag{29}$$

Substituting  $W_m(r)$  from equation (20) into the integrals  $\int_0^a r W_m(r) J_m(\alpha_{m,q} r/a) dr$  gives

$$\int_0^a r W_{m,s}(r) J_m(\alpha_{m,q} \frac{r}{a}) dr = a^2 J_m(\alpha_{m,q}) I_m(\xi_{m,s}) G_{m,s,q} \tag{30}$$

where

$$G_{m,s,q} = \frac{\xi_{m,s}}{\alpha_{m,q}^2 + \xi_{m,s}^2} \frac{I'_m(\xi_{m,s})}{I_m(\xi_{m,s})} - \frac{\xi_{m,s}}{\alpha_{m,q}^2 - \xi_{m,s}^2} \frac{J'_m(\xi_{m,s})}{J_m(\xi_{m,s})} \tag{31}$$

Therefore the pressure equation (28) becomes

$$p_m(r, \Theta, t)|_{x=l} = 2(\rho_f l) \omega^2 \cos(m\Theta) e^{i\omega t} \sum_{q=1}^{+\infty} \frac{\text{cotgh}(k_{m,q}l)}{(k_{m,q}l)} \times \frac{J_m(\alpha_{m,q}r/a)}{(1 - m^2/\alpha_{m,q}^2)J_m(\alpha_{m,q})} \sum_{s=1}^{+\infty} W_{0,m,s} I_m(\xi_{m,s}) G_{m,s,q} \tag{32}$$

For the disc *in vacuo*,

$$\nabla^4 W_{m,s}(r) = \frac{\rho_d}{D_0} \omega_{m,s}^2 W_{m,s}(r) \tag{33}$$

and the substitutions of

$$w_m(r, \Theta, t) = \cos(m\Theta) e^{i\omega t} \sum_{s=1}^{+\infty} W_{0,m,s} W_{m,s}(r) \tag{34}$$

and  $p_m|_{x=l}$  into the equation of motion describing the vibrating disc interacting with the fluid, i.e.

$$\nabla^4 w = -\frac{\rho_d}{D_0} \frac{\partial^2 w}{\partial t^2} + \frac{p}{D} \Big|_{x=l} \tag{35}$$

yields

$$\frac{\rho_d}{D_0} \sum_{s=1}^{+\infty} W_{0,m,s} (\omega_{m,s}^2 - \omega^2) W_{m,s}(r) = \frac{p_m}{D} \Big|_{x=l} \tag{36}$$

After substitutions of  $W_{m,s}$  and  $p_m$ ,

$$\sum_{s=1}^{+\infty} \left( \frac{\rho_d}{D_0} \right) (\omega_{m,s}^2 - \omega^2) W_{0,m,s} \times \left[ -\frac{I_m(\xi_{m,s})}{J_m(\xi_{m,s})} J_m\left(\xi_{m,s} \frac{r}{a}\right) + I_m\left(\xi_{m,s} \frac{r}{a}\right) \right] = -2 \left( \frac{\rho_f l}{D_0 h} \right) \omega^2 \sum_{s=1}^{+\infty} W_{0,m,s} I_m(\xi_{m,s}) \sum_{q=1}^{+\infty} C_{m,q}(\omega) G_{m,s,q} \times \frac{J_m(\alpha_{m,q}r/a)}{(1 - m^2/\alpha_{m,q}^2)J_m(\alpha_{m,q})} \tag{37}$$

where

$$C_{m,q}(\omega) = -\frac{\text{cotgh}(k_{m,q}l)}{(k_{m,q}l)} \tag{38}$$

Multiplying equation (37) by  $[r J_m(\alpha_{m,p}r/a)]$  and integrating over the interval  $(0, a)$ , the following equation is obtained:

$$\sum_{s=1}^{+\infty} (\omega_{m,s}^2 - \omega^2) W_{0,m,s} \int_0^a r J_m\left(\alpha_{m,p} \frac{r}{a}\right) \times \left[ -\frac{I_m(\xi_{m,s})}{J_m(\xi_{m,s})} J_m\left(\xi_{m,s} \frac{r}{a}\right) + I_m\left(\xi_{m,s} \frac{r}{a}\right) \right] dr = -2\rho\omega^2 \sum_{s=1}^{+\infty} W_{0,m,s} I_m(\xi_{m,s}) \sum_{q=1}^{+\infty} C_{m,q}(\omega) G_{m,s,q} \times \frac{1}{(1 - m^2/\alpha_{m,q}^2)J_m(\alpha_{m,q})}$$

$$\times \int_0^a r J_m\left(\alpha_{m,p} \frac{r}{a}\right) J_m\left(\alpha_{m,q} \frac{r}{a}\right) dr$$

where  $\rho = \rho_f l / \rho_d h$ . After the integration has been performed, the following equation is obtained:

$$\sum_{s=1}^{+\infty} (\omega_{m,s}^2 - \omega^2) W_{0,m,s} I_m(\xi_{m,s}) G_{m,s,q} J_m(\alpha_{m,q}) = -\rho\omega^2 \sum_{s=1}^{+\infty} W_{0,m,s} I_m(\xi_{m,s}) C_{m,q}(\omega) G_{m,s,q} J_m(\alpha_{m,q}) \tag{39}$$

which, upon rearranging, gives

$$\sum_{s=1}^{+\infty} W_{0,m,s} I_m(\xi_{m,s}) [(\omega_{m,s}^2 - \omega^2) G_{m,s,q} J_m(\alpha_{m,q}) + \rho\omega^2 C_{m,q}(\omega) G_{m,s,q} J_m(\alpha_{m,q})] = 0 \tag{40}$$

Expressing equation (40) in matrix form for fixed  $m$  and  $q, s = 1, 2, \dots, N$  gives

$$\begin{bmatrix} a_{11}(\omega) & a_{12}(\omega) & \dots & a_{1N}(\omega) \\ a_{21}(\omega) & a_{22}(\omega) & \dots & a_{2N}(\omega) \\ \vdots & \vdots & & \vdots \\ \dots & a_{qs}(\omega) & \dots & \\ \vdots & \vdots & & \vdots \\ a_{N1}(\omega) & a_{N2}(\omega) & \dots & a_{NN}(\omega) \end{bmatrix} \begin{bmatrix} \bar{W}_{m,1} \\ \bar{W}_{m,2} \\ \vdots \\ \bar{W}_{m,s} \\ \vdots \\ \bar{W}_{m,N} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad (41)$$

where

$$a_{qs}(\omega) = J_m(\alpha_{m,q})G_{m,s,q}\{\omega_{m,s}^2 - \omega^2[1 - \rho C_{m,q}(\omega)]\}$$

$$\bar{W}_{m,s} = W_{0,m,s}I_m(\xi_{m,s}) \quad (42)$$

### 3 NUMERICAL AND EXPERIMENTAL ANALYSIS

The finite element system ANSYS 5.4 [14] was used for modal analysis of the model. The plate was modelled by  $8 \times 32$  SHELL 63 finite elements. The acoustic volume was modelled by  $8 \times 32 \times 9$  or 30 FLUID 30 space acoustic finite elements. Both types of elements were coupled on the disc surface.

The experimental layout for the validation of the results of the TNA and FE analysis is shown in Fig. 2. The structural–acoustic system was excited by means of a standard miniature electromagnetic shaker applied to the disc. Natural frequencies, and associated mode shapes, of the plate were obtained by the Chladni sand technique, particularly when the mode of vibration was *predominantly structural*. Similarly, acoustic resonance of the air inside of the cylindrical container was detected, particularly when the mode of vibration was *predominantly acoustic*, by microphones that were inserted and fixed carefully into the base and side wall of the container. In cases of strong structural–acoustic interaction modes, both the sand and the microphones responded and, depending upon the relative levels of each component of response, the authors were required to make

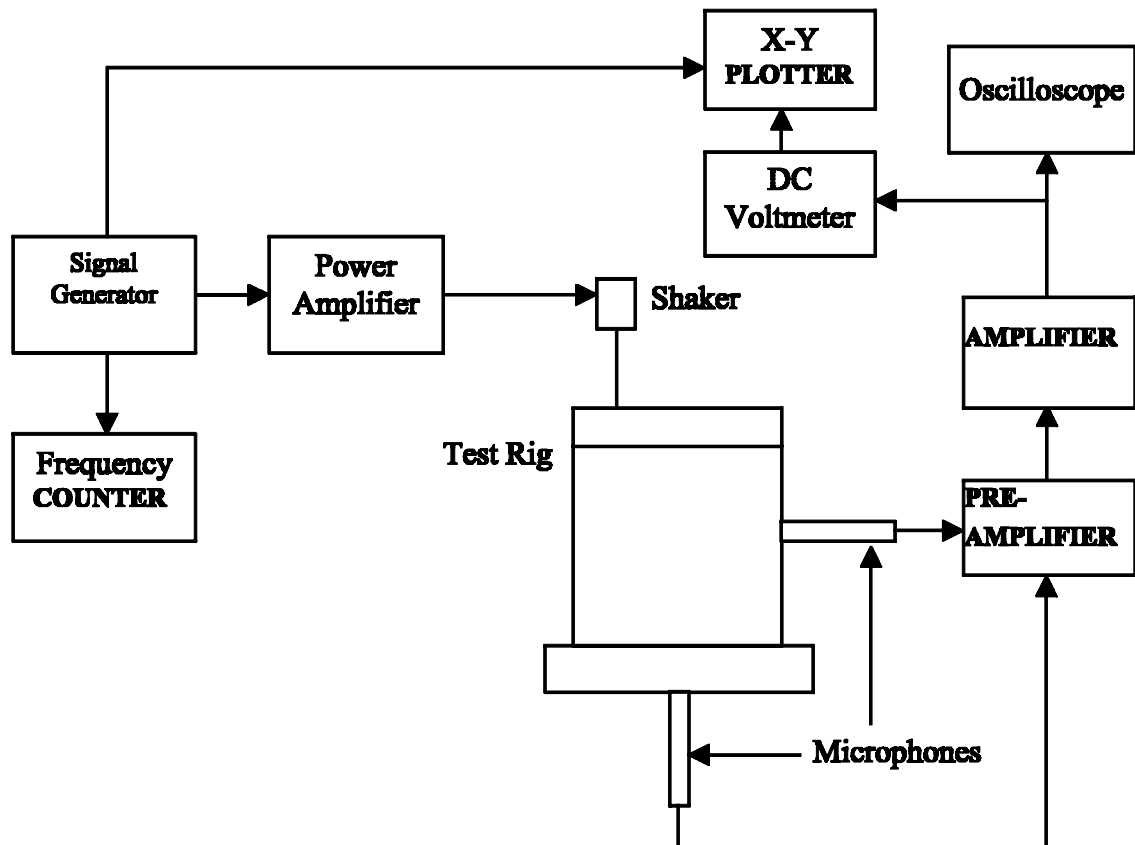


Fig. 2 Layout of instrumentation and transducer

**Table 1** Characteristics of the uncoupled systems

Number	Acoustics in rigid duct					
	Clamped plate <i>in vacuo</i>		<i>l</i> = 255 mm			
	<i>f</i> (Hz)	<i>m, s</i>	<i>f</i> (Hz)	<i>m, s, q</i>	<i>f</i> (Hz)	<i>m, s, q</i>
1	671.8	0, 0	672.5	0, 0, 1	2117	0, 0, 1
2	1398	1, 0	1345	0, 0, 2	2645	1, 0, 0
3	2293	2, 0	2018	0, 0, 3	3388	1, 0, 1
4	2615	0, 1	2645	1, 0, 0	4234	0, 0, 2
5	3356	3, 0	2690	0, 0, 4	4387	2, 0, 0
6	4000	1, 1	4387	2, 0, 0	4871	2, 0, 1

a judgement as to whether the coupled mode was acoustically (ac/st) or structurally (st/ac) dominant. These straightforward experiments were conducted for the purpose of detecting resonance (assuming negligible damping) conditions only.

**4 NUMERICAL EXAMPLES**

The parameters considered were as follows:

1. *Circular plate*

- Radius *a* = 0.038 m
- Thickness *h* = 0.00038 m
- Young’s modulus *E* = 2.1 × 10<sup>11</sup> Pa
- Density  $\rho_d$  = 7800 kg/m<sup>3</sup>
- Poisson’s ratio  $\nu$  = 0.3

2. *Acoustic medium*

- Radius *a* = 0.038 m
- Depth of cylinder *l* = 0.081 m and 0.255 m
- Density of air  $\rho_f$  = 1.2 kg/m<sup>3</sup>
- Sound velocity *c*<sub>1</sub> = 343.0 m/s

The lower natural frequencies of the uncoupled systems calculated for the plate *in vacuo* and for the compressible fluid in a rigid cylindrical duct are listed in Table 1. The

calculated natural frequencies for the coupled systems are presented in Table 2 for *l* = 81 mm and in Table 3 for *l* = 255 mm. The results calculated according to the developed theory (TNA) are compared with the natural frequencies calculated by the ANSYS finite element package and those obtained from the experimental investigation.

In the TNA solution, only the range up to *N* = 3 in matrix (41) was considered, i.e. the number of nodal diameters was *m* = 0, 1 and 2 as well as the number of nodal inner circles *s* = 0, 1, 2 except the clamped edge at *r* = *a*. The number of the inner nodal circular planes for the pressure inside the cylinder is denoted by *q* = 0, 1. The percentage differences,  $\epsilon$  and  $\kappa$ , between the natural frequencies computed according to the two methods are defined by

$$\epsilon = \left( \frac{f_{TNA} - f_{EXP}}{f_{EXP}} \right) \times 100\%$$

and

$$\kappa = \left( \frac{f_{TNA} - f_{ANSYS}}{f_{ANSYS}} \right) \times 100\%$$

and the predominantly structural (st), acoustic (ac) or coupled (ac/st and st/ac) modes of vibration are indicated in Tables 2 and 3.

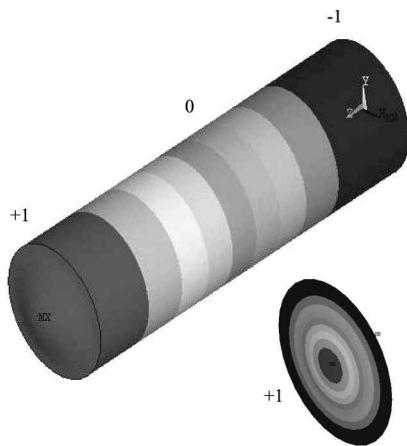
**Table 2** Characteristics of the coupled system for *l* = 81 mm (ac, acoustic; st, structural)

Number	Mode ( <i>m, s, q</i> )	TNA (Hz)	ANSYS (Hz)	Experimental frequency (Hz)	$\epsilon$ (%)	$\kappa$ (%)
1	0, 0, 0 (st)	675	673.6	675	0.0	0.25
2	1, 0, 0 (st)	1394	1387	1430	-2.50	0.5
3	0, 0, 1 (ac)	2119	2130	2167	-2.2	-0.5
4	2, 0, 0 (st)	2289	2273	2216	3.3	0.7
5	0, 1, 0 (st)	2615	2595	2565	1.9	0.77
6	1, 0, 0 (ac)	2646	2659	—	—	-0.49
7	1, 0, 1 (ac)	3388	3405	3488	-2.9	-0.5
8	1, 1, 0 (st)	3997	3953	3630	10	1.1
9	0, 0, 2 (ac)	4237	4324	4236	0.02	-2.0
10	2, 0, 0 (ac)	4388	4425	—	—	-0.84

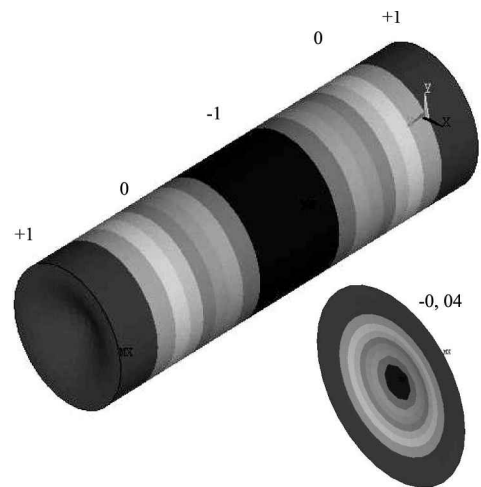


**Table 3** Characteristics of the coupled system for  $l = 255$  mm

Number	Mode ( $m, s, q$ )	TNA (Hz)	ANSYS (Hz)	Experimental frequency (Hz)	$\epsilon$ (%)	$\kappa$ (%)
1	0, 0, 1 (st/ac)	636.9	636.5	630	1.1	0.06
2	0, 0, 1 (ac/st)	707.7	707.5	685	3.3	0.03
3	0, 0, 2 (ac)	1347	1350	1348	-0.74	-0.22
4	1, 0, 0 (st)	1394	1388	1376	1.3	0.43
5	0, 0, 3 (ac)	2018	2027	2040	-1.1	-0.44
6	2, 0, 0 (st)	2289	2274	2170	5.5	0.66
7	0, 1, 4 (st/ac)	2607	2589	2596	0.42	0.7
8	1, 0, 0 (ac)	2645	2658	—	—	-0.49
9	1, 0, 1 (ac)	2730	2742	2756	-0.94	-0.44
10	0, 1, 4 (ac/st)	2697	2714	2689	0.3	-0.63
11	1, 0, 2 (ac)	2968	2980	2971	-0.1	-0.40

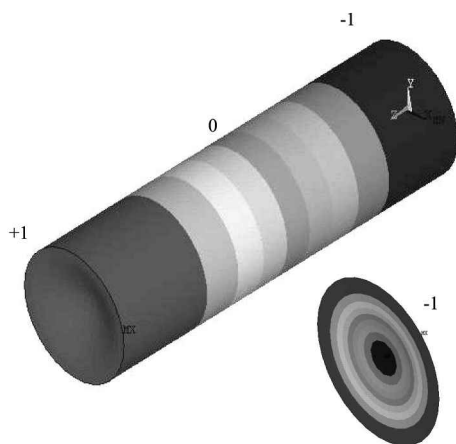


**Fig. 3**  $f = 636.5$  Hz ( $m = 0, s = 0, q = 1$ )



**Fig. 5**  $f = 1350$  Hz ( $m = 0, s = 0, q = 2$ )

There was a relatively weak acoustic–structural coupling for the cavity with depth  $l = 81$  mm. All modes of vibration are predominantly structural or acoustic and the natural frequencies in Table 2 are close to the values for the uncoupled systems shown in



**Fig. 4**  $f = 707.5$  Hz ( $m = 0, s = 0, q = 1$ )

Table 1. In contrast, there is a strong acoustic–structural coupling in the case of the longer cylindrical cavity,  $l = 255$  mm, especially for the first, second and seventh modes of vibration (see Table 3). Several of the lowest mode shapes of vibration calculated by ANSYS are shown in Figs 3 to 8. In all cases the normalized acoustic pressure amplitudes inside the cylindrical cavity and normalized vibration amplitudes of the elastic disc are shown by isolines. The main difference between the first two modes is in the fact that the plate vibrates with the pressure in phase (see Fig. 3) or in antiphase (see Fig. 4). The higher modes are typically predominantly acoustic (Figs 5 and 7) or structural (Figs 6 and 8). The predominantly structural modes, shown in Figs 6 and 8, are characterized by the observation that the normalized pressure amplitudes inside the cylindrical cavity decrease quickly with distance from the vibrating plate. In general there is good agreement between the TNA and experimental values, showing the general capability of the method described in this paper.



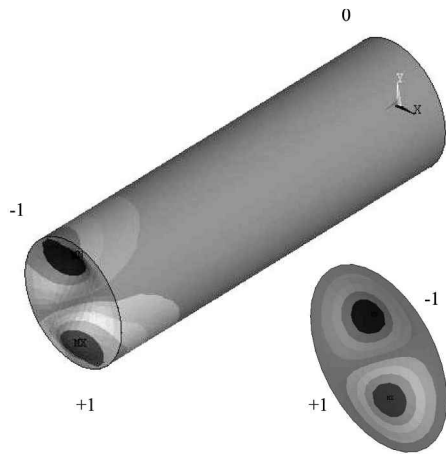


Fig. 6  $f = 1388$  Hz ( $m = 1, s = 0, q = 0$ )

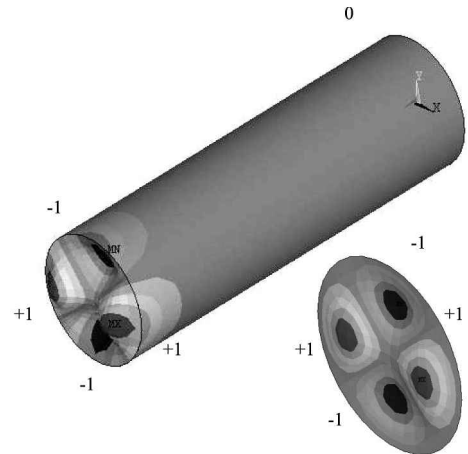


Fig. 8  $f = 2274$  Hz ( $m = 2, s = 0, q = 0$ )

## 5 CONCLUSIONS

A new theoretical–numerical analysis, based upon a combination of classical analysis and the Galerkin approximation, has been developed for the calculation of natural frequencies of a vibrating circular plate in interaction with a closed volume of compressible fluid. The analysis was validated by means of comparison with experimental results and those from an ANSYS finite element model. Accurate values of natural frequency were calculated by a simple iterative solution of a  $3 \times 3$  matrix for modes of vibration containing up to two nodal diametric lines on the circular plate.

This study has shown that in fluid–elastic systems strong acoustic–structural coupling can exist if the natural frequencies of the acoustic and structural subsystems are close and if the appropriate mode shapes are affined (cf. the values for the first uncoupled structural and acoustic modes in Table 1 and the first two modes listed in Table 3 for the coupled system). In this

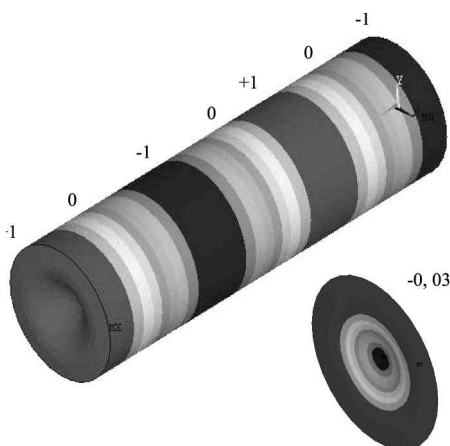


Fig. 7  $f = 2027$  Hz ( $m = 0, s = 0, q = 3$ )

case the coupled fluid–structural system has two natural frequencies substantially different from the frequencies of the uncoupled acoustic and structural subsystems. Corresponding mode shapes are neither acoustic nor structural and the dynamics of such a system cannot be studied by reference to the uncoupled subsystems.

Generally, even a light medium can significantly alter the spectrum of the natural frequencies of the structure, and vice versa, i.e. a flexible wall of an acoustic volume can significantly alter the spectrum of acoustic resonance.

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