

# Quantum structures in nonlinear optics and atomic physics: a background overview

Luigi A. Lugiato

*INFN Dipartimento di Fisica dell'Università di Milano,  
Via Celoria 16, 20133 Milano, Italy*

*luigi.lugiato@mi.infn.it*

Gian-Luca Oppo

*Department of Physics and Applied Physics, University of Strathclyde,  
107 Rottenrow, Glasgow, G4 0NG, Scotland, U.K.*

*gianluca@phys.strath.ac.uk*

**Abstract:** A brief overview of quantum effects in spatial structures such as nonlinear optical patterns, chains of trapped ions and atoms in optical lattices is presented. Some of the main results of the contributions to this Focus Issue are also briefly described.

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## References and links

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## Introduction

Spontaneous formation of spatial structures and patterns is ubiquitous in several branches of science such as, for example, Physics, Chemistry and Biology. Crystals, metals, spotted animal coats, liquid crystals and sand dunes are just few common examples. In nonlinear physics, the combination of nonlinear dynamics with a spatial cross-talk mechanism, e.g. diffusion or diffraction, gives rise to self-organization processes in spatially extended systems. These phenomena, which arise in macroscopic systems, are typically studied on a purely classical basis. This Focus Issue of Optics Express is devoted, however, to two outstanding cases of spatial coherence in which such effects can exhibit relevant quantum aspects.

### *Quantum Structures in Optical Pattern Formation*

The first case is provided by Optical Pattern Formation [1]. Radiation beams with large section, diffracting and interacting with nonlinear media, form naturally a wide variety of spatial structures. Such patterns, which range from the simple to the very complex, arise typically from the interference of a number of waves in the near field, while in the far field the spatial structure consists in a set of spots, each corresponding to one of the waves in play. Nonlinear optical processes such as, for example, parametric down-conversion or four-wave mixing, give rise to the simultaneous absorption and emission of photons, a situation which creates quantum correlations among the waves in play. Quantum correlations are a manifestation of one of the most intriguing concepts of Quantum Physics, i.e. entanglement, which is ubiquitous in Quantum Optics and plays a crucial role, for instance, in Quantum Information, Cavity Quantum Electrodynamics, Quantum Nondemolition Measurements, and Squeezing (twin photon beams). It is well known that quantum correlations display surprising aspects, as those related to the famous Einstein-Podolski-Rosen "paradox". In the case of nonlinear optical patterns, the various waves are entangled or, equivalently, the spots in the far field are entangled, i.e. quantum correlated. Hence the quantum aspects are not a cosmetic addition to the classical picture but lie, rather, at the very root of optical pattern formation. In this context entanglement phenomena, with their Einstein-Podolski-Rosen aspects, can be studied for macroscopic and spatially extended radiation beams instead of single photon pairs, as usually happens in parametric down-conversion. Even below the threshold of pattern formation, quantum fluctuations are capable of generating radiation fields that display a noteworthy level of spatial order; the term "quantum images" was coined to designate such noise-generated patterns [2]. The field of spatial quantum optical structures allows also us to investigate the spatial aspects of squeezing and to exploit them, for example, for the observation of faint images [3,4].

This Focus Issue includes four articles on quantum aspects of spatial structures. Santagiustina et al. [5] show numerical evidence of structures sustained by quantum noise, with an intensity comparable with that of above-threshold patterns. In a series of breathtaking videoclips, they show that in convective instability regions noise-sustained structures can arise in Kerr resonators and optical parametric oscillators in the presence of both diffraction and advection (i.e. pump tilting and walk-off, respectively) [5]. Marte et al. [6] discuss classical and quantum aspects of spatial patterns in optical parametric oscillators with spherical mirrors. They not only present a colorful videoclip of the formation of a spatial structure above threshold but also determine the dependence of the quantum image below threshold on the width of the input pump, an important feature for future experimental observations [6]. Jost et al. [7] describe, both theoretically and experimentally, spatial correlations in the quantum process of spontaneous parametric down-conversion. By making use of a single-photon-sensitive intensified charge-coupled-

device they simultaneously detect photon-pairs over a broad spatial area to measure quantum spatial correlations [7]. The information retrieved from the entangled photon measurements can be used to significantly reduce background, detector and quantum noise in optical images.

#### *Quantum Structures in Atomic Physics*

In Atomic Physics one can also find fascinating spatial structures. In the late eighties, H. Walther and his group in Garching (Germany) realized microcrystals formed by a small number of ions in a trap and observed via their fluorescent emission. When their kinetic energy is increased, the crystal melts and one observes chaotic motion. They also obtained linear arrangements of ions in storage rings. The groups of H. Walther and of R. Blatt (Innsbruck, Austria) are presently realizing linear configurations of trapped ions tailor-made for applications to quantum computing. In this Focus Issue on Quantum Structures the reader can find an experimental article with fascinating images of an ion chain: Nägerl et al. [8] have prepared crystal structures of up to 15 Calcium ions in a linear Paul trap and observed their normal modes of oscillation via resonant rf-fields. In particular the eigenmodes of the oscillations can be selectively excited in sequence [8]. Experimental observation of the collective motion of the ion string has been done by recording CCD images directly visualized in [8].

Another exploding area of research in the field of Quantum Structures is related to atoms trapped in Optical Lattices. They constitute a new form of matter which combines the crystalline order of a solid with the density of a high vacuum. They are created by the interference pattern of a number of intersecting laser beams; according to the number and the arrangement of the beams, optical lattices can have spatially periodic (or even quasi-periodic [9]!) 1D, or 2D, or 3D structure. Each well of the optical lattice works as a trap for cold neutral atoms; the atomic lattice is formed by a collection of atoms in different sites of the optical lattice. In contrast to solid state physics, the atoms in an optical lattice can be controlled and modulated from the outside. Such patterns have been realized, for example, in the laboratories of G. Grynberg in Paris (France), Th. Hänsch in Garching (Germany) and W.D. Phillips in Gaithersburg (U.S.A.). The trend towards lower and lower temperatures allows now for exploring the quantum aspects of such atomic patterns such as, for example, tunneling [10], Bloch oscillations [11] or Wannier ladders [12]. Far detuned lattices where dissipation is almost absent are being developed. Hänsch and his group realized recently a configuration in which the microtraps of the optical lattice are highly non dissipative and larger than usual [13], so that many atoms can be loaded into each site, achieving phase space densities less than three orders of magnitude away from Bose-Einstein condensation [14]. In this Focus Issue Schlipf et al. [15] of the Max-Planck Institut für Quantenoptik in Garching (Germany) discuss diffusion of a single ion in a one-dimensional optical lattice. A Mg<sup>+</sup> ion was radially confined in a 2D rf-trap while an optical lattice was superimposed along the free axis [15]. A breathtaking videoclip of the experimental loading of the trap from four to just one ion is presented in [15]. Numerical Monte-Carlo simulations are successfully compared with the experimental results which allow to measure the spatial diffusion coefficient [15].

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