A New Modulated Model Predictive Current Controller with Reduced Computational Burden

Euan T. Andrew  
Department of Electronic and Electrical Engineering,  
University of Strathclyde  
Glasgow, UK  
euan.andrew@strath.ac.uk

Khaled Ahmed  
Department of Electronic and Electrical Engineering,  
University of Strathclyde  
Glasgow, UK  
khaled.ahmed@strath.ac.uk

Derrick Holliday  
Department of Electronic and Electrical Engineering,  
University of Strathclyde  
Glasgow, UK  
derrick.holliday@strath.ac.uk

Abstract—Model Predictive Control (MPC) has been widely used for grid connected converters, due to its rapid dynamic response and easy inclusion of system constraints and non-linearities. Existing finite control set MPC approaches suffer from variable switching frequency, whilst existing modulated MPC implementations suffer from a high computational burden. This paper proposes a new implementation of modulated MPC which offers half the computational burden whilst retaining identical performance. The proposed method is applied to a two-level voltage source converter and its performance is compared to existing approaches in simulation. Experimental results are included to prove the reduced computational burden.

Index Terms—Current control, grid connected, model predictive control, modulated model predictive control, voltage source converter

I. INTRODUCTION

The installation of small-scale generators has received considerable attention recently, due to their ease of installation in remote areas and low capital cost [1]. Distributed energy resources (DER) are typically connected to low voltage distribution networks [2], [3], where the two-level voltage source converter (VSC) remains the most prevalent converter choice. The power injected into the grid must be of a high quality to satisfy grid regulations. In the grid-connected mode, the voltage is dictated by the grid, therefore the converter must inject high-quality current to achieve a high power quality [4], [5]. The challenge of controlling the current exchanged between the DER and the grid has received considerable attention in literature [6].

Model predictive control (MPC) is a powerful control strategy whereby a model of the system is used to predict future behavior of the controlled variables for a given input. These predictions are evaluated against a desired reference based on a cost function, then, the control sequence that minimizes the cost function is chosen [7]. MPC has been widely proposed as an alternative current controller in VSCs, due to its rapid dynamic response, ability to control non-linear systems (or those with constraints) and ease of implementation on digital signal processor (DSP) platforms [8]. MPC has been used in different power converter circuit topologies, including uninterruptable power supplies (UPS) [9], grid-interface [10], [11] and machine drive applications [12], [13].

Finite control set MPC (FCS-MPC) exploits the discrete nature of power converters and has been widely used in literature due to its low computational burden. FCS-MPC has the advantage of intuitive implementation, favorable dynamic performance, and relative ease of inclusion of multiple control objectives. FCS-MPC applies one switching state or output voltage vector for the whole sample period, therefore, no modulator is adopted. This results in a large ripple of the output current and hence lower power quality. There is no restriction in the switching sequence unless it is explicitly featured in the cost function and so the switching frequency is variable. This leads to a large proportion of low order harmonics in the output current, which are difficult to filter out and reduce the power quality.

Modulated MPC (MMPC) has been proposed [14] to overcome the variable switching frequency limitation of FCS-MPC. MMPC selects two active vectors and two zero vectors and switches between them during the sample period. This approach satisfies the objectives of fixing the switching frequency and improving the steady-state performance. The complexity of existing MMPC implementations is of concern [15], [16]. In particular, the computational burden required to evaluate all available voltage vectors when only two active vectors are actually required has been criticized [17], [18]. Ideally, the DSP implementation of the current controller should not take up too much of the control period, otherwise there may not be enough time left for other functions.

In [18], an attempt is made to reduce the calculation time of MMPC by first calculating a reference voltage vector, thereby reducing the number of candidate vectors which must be evaluated during the modulation stage. This technique reduces the number of sectors involved in the calculations from 6 to 1, however, an additional deadbeat controller is required to synthesize the reference voltage vector.

This paper simplifies the selection of the active voltage vectors in an MMPC system by narrowing down the available vectors in the current domain rather than the voltage domain. The predicted current hexagon is analyzed geometrically, and then trigonometric identities are exploited to identify the two closest active vectors using the angle of the reference current vector. The new implementation of the MMPC algorithm...
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Fig. 1. Two-level grid-connected VSC.

achieves the same performance as existing methods while incurring only half of the computational cost.

The remainder of this paper is organized as follows. Section II derives a discrete mathematical model of a VSC. Section III describes how the active vectors are selected in a conventional MMPC scheme and proposes an alternative method. Section IV describes how the reference currents are generated and how calculation delay is compensated. Section V studies the computational burdens of the conventional and proposed technique.

II. Dynamic System Model

A circuit diagram of a grid-connected VSC is provided in Fig. 1. The converter terminal voltage can be described by the following system of differential equations in the alpha-beta frame:

\[
\begin{align*}
\frac{dv_{\alpha}}{dt} &= v_{\alpha} \alpha + R i_{\alpha} + L \frac{di_{\alpha}}{dt} \\
\frac{dv_{\beta}}{dt} &= v_{\beta} \beta + R i_{\beta} + L \frac{di_{\beta}}{dt}
\end{align*}
\]  

(1)

where \(v_{\alpha}\) and \(v_{\beta}\) are the alpha and beta components of the grid voltage, \(i_{\alpha}\) and \(i_{\beta}\) are the alpha and beta components of the grid current and \(R\) and \(L\) are the resistance and inductance of the interfacing filter respectively. The system can be approximately discretized using the forward Euler approximation for the current derivative:

\[
di_{\alpha} \approx \frac{i_{\alpha}(k+1) - i_{\alpha}(k)}{T_s}
\]

(2)

where \(i_{\alpha}(k+1)\) is the grid current sampled at instant \(k+1\), \(i_{\alpha}(k)\) is the grid current sampled at instant \(k\) and \(T_s\) is the sampling time. A discrete predictive model can then be obtained describing the future grid current as a function of the proposed terminal voltage:

\[
\begin{align*}
i_{\alpha}(k+1) &= \left(1 - R \frac{T_s}{L}\right) i_{\alpha}(k) \\
i_{\beta}(k+1) &= \frac{T_s}{L} \left(v_{\alpha}(k) - v_{\beta}(k)\right)
\end{align*}
\]

(3)

The model predictive current controller must then synthesize the required \(v_{\alpha\beta}(k)\) to minimize the current tracking error. This is achieved by selecting an optimal output voltage vector \(v_1\) and second-best voltage vector \(v_2\) and modulating between them and the zero vectors. According to the principle of deadbeat control, the current error should be reduced to zero by the end of the sampling period. Therefore, the duty factor for each of the selected active vectors and the zero vectors is obtained by solving the following system of equations:

\[
\begin{align*}
i_{\alpha}^1 d_1 + i_{\alpha}^2 d_2 + i_{\alpha}^0 d_0 &= i_{\alpha}^* \\
i_{\beta}^1 d_1 + i_{\beta}^2 d_2 + i_{\beta}^0 d_0 &= i_{\beta}^*
\end{align*}
\]

(4)

(5)

\[d_1 + d_2 + d_0 = 1\]

(6)

where \(i_{\alpha}^*\) is the current which would result from applying vector \(v_1\) for one whole sampling period, \(i_{\alpha}^2\) is the current which would result from applying vector \(v_2\) for one whole sampling period, \(i_{\alpha}^0\) is the current which would result from applying either of the zero vectors for one whole sampling period and \(d_1\), \(d_2\) and \(d_0\) are the duty factors for the active and zero vectors respectively.

III. Selection of Active Vectors

In the conventional MMPC [15], (3) is evaluated exhaustively for all available output voltage vectors and compared with the reference based on a cost function. Typically, a quadratic cost function is used to ensure good regulation of both the alpha and beta components as follows:

\[G_x = \left[i_{\alpha\beta}(k+2) - i_{\alpha\beta}(k+2)\right]^2, x \in [0,7]
\]

(7)

where \(i_{\alpha\beta}(k+2)\) is the current predicted two steps in advance as though each of the eight voltage vectors were applied for one whole sampling period.

The best and second-best vectors are selected by sorting the calculated costs \(G_x\) and selecting the lowest and second-lowest costs respectively. This is computationally expensive since the variables in (3) and (7) are complex valued and the expression must be evaluated repeatedly even though only three of the predicted currents are ultimately useful.

Fig. 2 shows a detailed view of the available output voltage vectors for the VSC and the currents which would result from applying each of them for one whole sampling period as calculated by (3). An arbitrary current reference vector is also shown. Clearly, the optimum voltage \(v_1\) and second-best \(v_2\) will be those which are closest to the reference when mapped to the current domain by (3). In the example shown, \(i^2\) is optimal and \(i^1\) is second-best, therefore, the controller should modulate between \(v^2\) and \(v^1\) and the zero vectors.

In fact, if the angle of the reference vector is known, the optimum vectors can be identified without having to repeatedly evaluate the cost function. Since the predicted current hexagon is not centered at zero, a direction vector, \(i_{\alpha\beta}^{dir}\) is obtained by subtracting \(i_{\alpha\beta}^*\) from \(i_{\alpha\beta}^0\). As shown in Fig. 3, the quadrant to which this direction vector points can then be efficiently obtained using compare-to-zero checks. For example, if Re \(i_{\alpha\beta}^{dir}\) > 0 and Im \(i_{\alpha\beta}^{dir}\) > 0, then the direction vector points to the first quadrant. From there, this quadrant is split into three subsectors. In subsector ‘A’, \(v^2\) is optimal and \(v^5\) is second best, in subsector ‘B’ \(v^2\) is optimal and \(v^1\) is second best and in subsector ‘C’ \(v^1\) is optimal and \(v^2\) is second best.
The subsector required can be easily identified by exploiting the exact values of the tangent function. For example, if
\[ \frac{\text{Im}\left(i_{\text{dir}}^{\alpha\beta}\right)}{\text{Re}\left(i_{\text{dir}}^{\alpha\beta}\right)} > \sqrt{3} \] (8)
then the direction vector lies in subsector ‘A’ and \( v^2 \) and \( v^3 \) should be selected as the optimum and second-best vectors respectively. Using a similar procedure, the active vectors can be identified rapidly using simple compare-to-zero and compare-to-constant operations for all quadrants and subsectors and the selected vectors are the same as if the vectors had been exhaustively evaluated.

IV. CURRENT REFERENCE GENERATION

To export a required active and reactive power to the grid, reference currents must be calculated according to the grid voltage. The current references are found by solving the following equation:
\[
\begin{bmatrix}
P^* \\
Q^*
\end{bmatrix} = \frac{3}{2} \begin{bmatrix}
v_{g\alpha}(k) \\ v_{g\beta}(k)
\end{bmatrix} \begin{bmatrix}
v_{g\beta}(k) \\ -v_{g\alpha}(k)
\end{bmatrix} \begin{bmatrix}
i^*_\alpha(k) \\
i^*_\beta(k)
\end{bmatrix}
\] (9)

As with any digital control scheme, MPC samples the input variables at regular instants and then performs calculations based on these measurements. If the system is sampled at instant k, by the time the necessary calculations have been performed, the control decision may be out of date when it is applied. To compensate this delay, the controller may extrapolate future quantities to be used to solve for the optimum control action to be applied at instant k + 1 to minimize the cost function at k + 2. The future current references and grid voltages can be extrapolated using a second-order Lagrange quadratic formula as follows:
\[
i^*_k = 3i^*_{k+1} - 3i^*_{k-1} + i^*_{k-2}
\] (10)
\[
i^*_k = 3i^*_{k+1} - 3i^*_{k-1} + i^*_{k-2}
\] (11)
where, \( i^*_{k+2} \) is the current reference extrapolated for instant k+2, \( i^*_{k+1} \) is the current reference extrapolated for instant k+1, \( i^*_{k} \) is the known current reference at instant k, \( i^*_{k-1} \) is the previous current reference at k–1 and \( i^*_{k-2} \) is the previous current reference at instant k–2. The same technique is used to extrapolate the future grid voltages.

V. SIMULATION RESULTS

The proposed MMPC algorithm is studied using the Matlab/Simulink package to verify its effectiveness. The parameters of the simulation are shown in Table I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal Grid Phase Voltage (RMS)</td>
<td>( V_g )</td>
<td>100</td>
<td>V</td>
</tr>
<tr>
<td>Grid Fundamental Frequency</td>
<td>( f_{\text{grid}} )</td>
<td>50</td>
<td>Hz</td>
</tr>
<tr>
<td>Switching Frequency</td>
<td>( f_{\text{sw}} )</td>
<td>10</td>
<td>kHz</td>
</tr>
<tr>
<td>DC Link Voltage</td>
<td>( V_{dc} )</td>
<td>400</td>
<td>V</td>
</tr>
<tr>
<td>Filter Inductance</td>
<td>( L )</td>
<td>10</td>
<td>mH</td>
</tr>
<tr>
<td>Filter Resistance</td>
<td>( R )</td>
<td>0.1</td>
<td>Ω</td>
</tr>
<tr>
<td>Rated Power</td>
<td>( P_{\text{rated}} )</td>
<td>2</td>
<td>kW</td>
</tr>
<tr>
<td>Rated Current (Peak)</td>
<td>( i_{\text{rated}} )</td>
<td>9.428</td>
<td>A</td>
</tr>
</tbody>
</table>

The steady state current at full rated power for the proposed MMPC and the conventional MMPC is shown in Fig. 4. The proposed MMPC offers identical performance to the conventional MMPC. The total harmonic distortion (THD) and steady state error (SSE) are the same in both cases. The output currents of both controllers have little ripple and the THD is low, therefore, a high power quality is achieved in both cases.

The transient behavior of the proposed controller is also studied during a step change in active power from zero to...
rated power at unity power factor, as shown in Fig. 5. The fast transient response associated with MPC is evident for both controllers and a steady state is reached in 1.5 ms.

Fig. 4 and Fig. 5 combined prove that the proposed implementation of MMPC achieves identical performance to the established technique during both steady state and transient operation.

**VI. COMPUTATIONAL BURDEN**

The proposed MMPC algorithm was implemented on a Texas Instruments TMS320F28379D microcontroller forming part of a two-level grid-connected VSC system as shown in Fig. 6 A digital output pin is set high when the analog-to-digital (ADC) results are ready and the current control tasks begin, then it is set low again when the calculations are complete to verify the reduction in computation time. Fig. 7a shows the computation time for the conventional MMPC and Fig. 7b shows the computation time for the proposed new implementation.

As shown in Fig. 7 the execution time of the conventional MMPC algorithm is 20.5 µs while the proposed algorithm executes in only 11.1 µs. This represents a reduction in computation time of 46% for no reduction in performance. The reduced execution time of the proposed algorithm means that the sampling frequency could be increased to a maximum of 90 kHz, compared with 49 kHz for the existing technique. Alternatively, where faster sampling is not required, a lower cost and less capable microcontroller could be used, or time could be committed to performing other tasks.

**VII. CONCLUSION**

This paper has proposed an efficient implementation of the MMPC current control algorithm. Simulation results have been shown which prove that the proposed current controller achieves identical performance in steady state and transient conditions as the established technique. Experimental data has been included to verify the reduction in computational burden. The proposed method achieves identical performance to the well-known algorithm while requiring 46% less computation time.
REFERENCES


