This is a peer-reviewed, author's accepted manuscript of the following research article:
Devesse, V. A. P. A., Akartunali, K., Arantes, M. D. S., & Toledo, C. F. M. (Accepted/In press).
Linear approximations to improve lower bounds of a physician scheduling problem in emergency rooms.
Journal of the Operational Research Society. https://doi.org/10.1080/01605682.2022.2125841

Linear Approximations to Improve Lower Bounds of a Physician Scheduling Problem in Emergency Rooms

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ARTICLE HISTORY
Compiled September 20, 2022

ABSTRACT
The physician assignment process consists of coverage of shifts and duties allocated to physicians in a planning period, taking into account work regulations, individual preferences, and organizational rules, which mostly conflict with each other. In this work, we propose a reformulated mixed-integer programming model based on the literature to tackle fairness in physician scheduling in Emergency Rooms. In particular, we propose two mixed-integer quadratic programming formulations that consider quadratic costs and two models with linear costs. Our approaches provide balanced schedules concerning target hours and weekends in terms of fairness. Our models also provide a high degree of demand coverage, providing decision-makers a significant advantage.

KEYWORDS
Physician Scheduling Problem, Mixed-Integer Quadratic Programming, Fairness, Healthcare Management

1. Introduction

Emergency rooms are usually the main entry points to hospitals. They are divisions that operate around the clock every day of the year to provide adequate and immediate treatment. The visits to these divisions include a wide range of illnesses, often because ER usually provides the only medical treatment free of charge in some countries such as Spain, Brazil, and the USA (Cabrera et al., 2012). As a result, ERs often work 50% or even more over the capacity, despite services being provided 24 hours a day (Barish et al., 2012).

One of the most concerning situations in the ER is crowding associated with increased mortality and a more extended stay in the hospital, primarily due to uninsured patients waiting for approximately twice as long as recommended, which leads to overuse as mentioned by (Sun et al., 2013; NEHI, 2010). Some approaches to tackle this problem include the use of co-location of a primary care clinician, doctor triage, rapid assessment, and patient streaming (Edwin, 2016). Indeed, these measures lead

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to reduced patient time in emergency rooms. However, these measures do not suffice to create fair schedules between the physicians.

The recent work of Erhard et al. (2018) provides a comprehensive review of quantitative methods, relevant characteristics, modeling features, and fairness in nurse and physician scheduling. Physician scheduling consists of coverage of shifts and allocation of duties to physicians in a specified planning period, taking into account a set of work regulations, individual preferences, and organizational rules, which often conflict. As emphasized by Gendreau et al. (2007), fairness needs to be carefully observed to balance the distribution of different types of shifts among physicians, particularly with often extensive overtimes. This aspect becomes paramount in the ER setting.

This paper proposes an overarching mixed-integer programming model for Physician Scheduling Problem in Emergency Room (PSP-ER), a reformulation and expansion of previous models from the literature. In order to effectively tackle fairness, we suggest two objective functions with quadratic costs, resulting in mixed-integer quadratic formulations, and two objective functions with linear costs. One function estimates the target values of hours and weekends for the quadratic costs, and the other uses parameters from the instances. We also study some approaches to linearize the quadratic models and alternative distance functions. Our experiments using benchmark instances demonstrate that the proposed techniques result in fairer schedules without sacrificing the high degree satisfaction of demand coverage.

The remainder of this paper is organized as follows: in Section 2, we present a brief literature review on the physician scheduling problem in emergency rooms. Section 3 presents the problem, as well as the most important features to consider, and a discussion of the model reformulation. We report the computational experiments in Section 4, along with an extensive discussion.

2. Literature Review

PSP-ER has been studied in the literature since the mid-80s. To the best of our knowledge, the first approach was proposed by Vassilacopoulos (1985) with a dynamic program to determine the number of doctors to cover each shift every week at Saint Peter Hospital in England. Since then, operations research and computer science practitioners have proposed a range of methods to solve this NP-hard problem.

The general trend in physician scheduling problems consists of applying exact solution methods over heuristics, with the majority of the literature using modeling derived from mathematical programming (Erhard et al., 2018). The same review notes that around 53% of the literature about emergency room physician scheduling was produced before 2010, and 69% uses mathematical programming-based approaches.

2.1. Solution approaches for the PSP-ER

Earlier mathematical programming approaches include Beaulieu et al. (2000), solving the problem with an integer programming model, and Carter & Lapierre (2001), proposing a generic mathematical model. A combination of heuristics and mathematical programming has also been effective in tackling this highly constrained problem, e.g., Rousseau et al. (2002) proposed a general and hybrid method integrating constraint programming, local search, and genetic algorithms. Gendreau et al. (2007) gathered a series of generic constraints to describe emergency room physician scheduling based on the characteristics of five Canadian hospitals and reviewed alternative solution techniques, namely Tabu Search, column generation, mathematical programming, and constraint programming. In this paper, the authors explore techniques and features of the problem formerly explored by Beaulieu et al. (2000), Carter & Lapierre (2001) and Rousseau et al. (2002).

The computational limitation was one of the major impediments faced by OR practitioners in the early 2000s when solving mid-sized instances of the PSP-ER via mathematical programming approaches. A common measure was easing the problem's constraints to find a feasible solution. As an example, due to the size of the instances used in the PSP-ER, Beaulieu et al. (2000) noted in their earlier work that it was impractical to solve the entire problem. Instead, they approached the problem by removing all conflicting constraints and then adding the constraints iteratively when a partial branch-and-bound procedure found an acceptable solution.

An essential subset of ER physicians is residents, who are part of the graduate medical education seeking a full license in ER specialty. Topaloglu (2006) proposed a goal programming model and applied the analytical hierarchy process (AHP) in a university hospital to measure the relevance of each of the soft constraints of the problem when scheduling emergency medicine residents.

A derivation of PSP-ER is the Resident Scheduling Problem (RSP), where the residents are assigned to day and night shifts over a given planning horizon subject to numerous working regulations and staffing requirements. It is important to remark that residents have a unique position as learners and providers of services, working long duty hours.

With time, computational performance improved, and some techniques could be effectively implemented. Unlike Beaulieu et al. (2000), who removed conflicting constraints considering the challenging sizes of the instances and highly constrained structure, Topaloglu & Ozkarahan (2011) proposed a mixed-integer programming model for small instances of the RSP, while employing column generation (CG) to larger instances. As the reader can notice, the current paper and Topaloglu & Ozkarahan (2011) have benefited from computational evolution. A difference is in the application of MIP even for larger instances in our case, whereas Topaloglu & Ozkarahan (2011) used CG. Therefore, we can infer that computational resources can have an impact when solving problems.

The conflicts of work regulations, organizational requirements, and personnel preferences are particularly magnified in new hospitals. Therefore, Ferrand et al. (2011) proposed a mixed-integer programming model to create cyclic schedules for ER physicians, with an application in a children's hospital in Cincinnati. This work had a significant impact since it helped physicians identify trade-offs in formulating their requests and classifying what was critical instead of desirable.

Carter & Lapierre (2001) applied a Tabu Search (TS) to solve computational issues inherent in PSP-ER. TS procedure was in general satisfactory in generating feasible schedules, where fairness was nominally achieved even though not every physician could have their desired schedule.

Rousseau et al. (2002) proposed a general hybrid method merging Constraint Programming (CP), Local Search, and Genetic Algorithms. However, not all solutions were suitable, which led to an improvement process that combines the proper individual schedules from different solutions into a single complete roster. This method is part of two real-world scheduling problems for physicians in Montreal and is also included in the study of Gendreau et al. (2007).

Gendreau et al. (2007) proposed a series of generic constraints to describe PSP-ER based on the characteristic of five Canadian hospitals. The variety and specificity of constraints made the authors conclude that it is challenging to come up with general solution methods to solve the problem and to obtain fair comparisons of different techniques.

Puente et al. (2009) addressed another particular case using Genetic Algorithms (GA) to automate the creation of timetables in a Spanish hospital. The problem is less constrained, as hard constraints are limited and primarily soft constraints are used. The work proposed an encoding type that allows creating feasible solutions and a crossover operator that allows finding new feasible solutions evolving favorably according to fitness measures. (Frey et al., 2009) also applied GA in PSP-ER with data from a Swiss hospital, where only two sets of hard constraints were used among various soft constraints.

However, the solution quality of generated schedules leaves room for improvement. Despite the challenges faced by the authors to create benchmark instances, with the progress of computational resources, they seem to be a suitable tool to compare different approaches, as can be observed in recent attempts of Curtois & Qu (2014); Rahimian et al. (2017a,b).

We also note that over-staffing is not adequate to resolve overtime issues, as demonstrated by Al-Najjar & Ali (2011) simulation of patients' arrivals to generate schedules for two large public hospitals in Baghdad. As this paper demonstrates, PSP-ER requires an out-of-the-box solution approach, often building a hybrid method benefiting from the strengths of mathematical programming and meta-heuristics.

2.2. Fairness in the PSP-ER

Al Ghathbar et al. (2019) proposed an ILS(Iterated Local Search) procedure to tackle the physician scheduling problem in ERs for King Khalid University Hospital in Saudi Arabia. Fairness measures cover an equal number of weekends off for all physicians and night shifts for all physicians. Different from our setting, they consider weekends off in the definition of the problem, while we consider weekends on assignment. We can translate the measure as an even distribution of weekends, which we apply. As in Gharbi et al. (2017), these constraints may suffer from violations in case of priority rules such as seniority.

Savage et al. (2015) optimized the physician shift schedules by minimizing the difference between physician productivity and patient demand in the emergency room. The problem consists of four main constraints based on uncertainty in patient arrival. This problem and ours do not have similarities with regards to constraints but resemble in the solution technique, as both use mixed-integer programming models.

Vermuyten et al. (2018) combined VNDS(Variable Neighborhood Descent Search) and CG to tackle an integrated approach to staff scheduling on an Emergency Service Health Center in Portugal named INEM. Medical doctors' and psychologists' schedules are built by their institutions, whereas the INEM creates nurses' and technical personnel schedules. Fairness measures are: every person needs to work at least a predetermined number of night shifts, morning shifts, and afternoon shifts, and working hours should be met as much as possible. The similarity that we find here with ours lies in the workload. Both Vermuyten et al. (2018) and our proposal assume the desired number of hours as a target.

Like Carter & Lapierre (2001) that compiled characteristics of a hospital and modeled using MIP, Tan et al. (2019) gathered a set of management rules and physicians' preferences between other features of a Chinese hospital to build a mixed-integer programming model for physician schedules in ERs. Fairness in this problem is very similar to Topaloglu (2006), Gendreau et al. (2007), and Topaloglu & Ozkarahan (2011) since all of them aim to schedule fairly and reasonably the number of night shifts.

After being a subject of study in the early 2000s, Hospital Sacré-Cœur was a case of study in the late 2010s, when Camiat et al. (2019) proposed productivity-driven schedules to emergency physicians to align physician productivity with demand without losing fairness between physician. Like Beaulieu et al. (2000), fairness here is related to the distribution of shifts. Although our proposal and Camiat et al. (2019) aim to increase physician welfare, we address two different measures of fairness.

Fairness is a matter of concern when Damcı-Kurt et al. (2019) deals with the physician scheduling problem faced by Flagstaff Medical Center Hospitals in the United States. In the work, authors leverage workload to obtain schedules with fewer violations through a set of equalization constraints. Particularly they try to balance the number of night work, weekend assignments, and even distribution of other shift types among the personnel. This type of fairness is similar to our approach as we tackle the least absolute deviation to leverage the workload between the personnel. The authors use mixed-integer programming model as in, e.g., Beaulieu et al. (2000), Ferrand et al. (2011). When modeling this highly constrained problem, they apply relaxation variables to control violation, which is later penalized in the objective function. We use a similar approach as will be described later.

Wickert et al. (2020) tackled the physician scheduling problem in multiple locations (including ERs) at Hospital de Clínicas de Porto Alegre (HCPA), Brazil. The authors tackle workload fairness in this work and formulate an extended model that obtains high-quality results. We tackle the workload balance in our proposal by changing how workload constraints are defined. We add the concept of the least absolute deviations. Moreover, we reformulated the cited work by adapting some constraints and objective functions to achieve a balance in the workload. To solve this problem, Wickert et al. (2020) proposed a mixed-integer programming model for small-sized instances and a fix-and-optimize math-heuristic for large instances, with a constructive heuristic based on the challenging constraints of the problem. We also apply a similar procedure when solving a variant of our problem, but we use a relax-and-fix heuristic to construct the initial solution.

Another discussion we raise in this section is related to fairness. Some of the works, e.g., Carter & Lapierre (2001), Rousseau et al. (2002), Topaloglu (2006) consider fairness by simply creating bounds to certain variables like weekends, workload, and night shifts. On the other hand, Beaulieu et al. (2000) aims to balance weekly hours, nights, and conflicting shifts. With this measure, we can infer that fairness is not only related to balance between the workforce but can be defined in the approximation to maximum or minimum values a variable can assume. Practitioners can be deceived if they observe just one side of the coin. Indeed, fairness has been a topic of interest among healthcare practitioners since the early days of Operations Research in ERs (see, e.g., Beaulieu et al. (2000) and Carter & Lapierre (2001)), as well as in the general context of staff scheduling (Zhong et al., 2017). A comprehensive literature review on personnel scheduling considering fairness was presented recently by Wolbeck (2019). Each problem has its specificity when it comes to deciding what to consider for fairness. Table 1 summarizes the comparison between our paper and existing fairness aspects in the literature, where column "Fairness Aspect" provides specific details and column "Objective Function" notes when personal preferences are used, denoted by PP. Our proposal is the only one considering target as a parameter to be estimated, aiming for a fair workload among physicians.

Table 1. Overview of PSP-ER literature including on fairness, objective functions and methods used

Author	Fairness	Country	Fairness Aspect	Objective Function	Method
This Paper	✓	BR	Target number of weekends Target number of week hours	PP+Cost	MIP
Beaulieu et al. (2000)	✓	CA	Workload balance; Balanced night shifts and duties; Balance on the assignment of antagonist shift.	PP	MIP
Carter & Lapierre (2001)	✓	CA	Cyclic schedule; Distribution of weekend shifts; Distribution of night shifts; Overall distribution of shifts.	PP	MIP
Rousseau et al. (2002)	✓	CA	Distribution constraints	PP	CP, LS, GA
Topaloglu (2006)	✓	TR	Distribution of night shifts in certain days of the week; Distribution of supervising resident positions.	PP	GP
Gendreau et al. (2007)	✓	CA	Equal distribution of types of shifts	PP	TS
Puente et al. (2009)	✓	ES	Balanced number of different types of shifts; Work the same number of Saturday and Sundays per month.	PP	GA
Frey et al. (2009)	✓	CH	Same amount of shifts; Same amount of Sunday shifts; Same amount of Saturday shifts; Same amount of Thursday shifts.	PP	GA
Topaloglu & Ozkarahan (2011)	✓	TR	Balance in the number of on-call night; Balance between on-call nights and night during the weekends	PP	MIP, CG
Ferrand et al. (2011)	✓	US	Cyclic schedule	PP	MIP
Al-Najjar & Ali (2011)	✓	IQ	Team size	Cost	SIM
Lo & Lin (2011)	×	TW		PP	PSO
Savage et al. (2015)	×	CA		PP	MIP
Gharbi et al. (2017)	✓	SA	Distribution of weekend shifts; Distribution of night shifts.	PP	GP
Devesse et al. (2017)	✓	BR	Workload balance; Balanced night shifts and duties; Balance on the assignment of antagonist shift.	PP	MIP, RFFO
Vermuyten et al. (2018)	✓	PT	Even distribution of shifts; Even distribution working hours.	Cost	IP, CG, VNDS
Damcı-Kurt et al. (2019)	✓	US	Distribution of night shifts; Distribution of weekend; Even distribution of shifts.	PP	MIP
Al Ghathbar et al. (2019)	✓	CN	Weekends off balance; Balanced night shifts.	PP	ILS
Tan et al. (2019)	✓	CN	Distribution of night shifts.	PP	MIP
Camiat et al. (2019)	✓	CA	Distribution of shifts	Cost	MIP
Wickert et al. (2020)	✓	BR	Balanced distribution of shifts	PP+Cost	MIP, FOh
Marchesi et al. (2020)	×	BR	PP+Cost	SP, DES	

CG- Column Generation; CP - Constraint Programming; DES - Discrete Event Simulation; FOh - Fix-and-Optimize heuristic; GA - Genetic Algorithms; GP - Goal Programming; ILS - Iterated Local Search; LS - Local Search; MIP - Mixed-Integer Programming; PS - Particle Swarm Optimization; SIM - Simulation Model; SP - Stochastic Programming; TS - Tabu Search; VNDS - Variable Neighborhood Descent Search; VNS - Variable Neighborhood Search; RFFO - Relax and Fix and Fix-and-Optimize heuristic

3. Problem and Model Descriptions

PSP-ER aims to schedule available physicians P to cover the demand of shifts S, within the days of the planning horizon D. The problem can be established by the sequence of day-on and day-off duties in the planning horizon. The constraints are often classified as hard and soft, where hard constraints must be satisfied under any circumstances, while soft constraints are preferably met, with tolerance to violations.

The objective of physician scheduling is the maximization of staff satisfaction, as physician retention is the most critical issue faced by hospital administrators, see, e.g., Beaulieu et al. (2000); Bruni & Detti (2014). However, it is also necessary to minimize demand violations and planned overtime. Thus, we describe next the Physician Scheduling Problem (PSP) through its objective functions and hard and soft constraints. Table 2 shows the notation employed.

Table 2. Sets, parameters and variables

Sets	
P	Set of physicians.
D	Set of days.
S	Set of shifts.
Υ_s	Set of shifts conflicting with s.
M	Set of months.
W	Set of weeks.
U_p	Set of unavailable shifts for the physician p.
D_m	Set of days of the month m.
Parameters	
DEM_{ds}	Number of physicians to assign to shift s on day d.
R_p^+	Maximum number of consecutive work days for the physician p.
V_p^+	Maximum number of weekends physician p can work.
H_s	Length of shift $s \in S$, in hours.
LS_{ps}	Maximum number of shifts of the type s worked by physician p.
A_{pm}	Maximum number of days physician p can be assigned on month m.
c_p^-	Minimum number of consecutive days physician p can work.
o_p^-	Minimum number of consecutive vacation days by physician p.
q_{pds}^+	Weights for shift on requests by physician p on day d and shift s.
q_{pds}^-	Weights for shift off requests by physician p on day d and shift s.
v_{ds}^+	Weights for over-coverage on day d and shift s
v_{ds}^-	Weights for under-coverage on day d and shift s.
h_{pm}^{max}	Maximum number of month hours for physician p in month m.
h_{pm}^{min}	Minimum number of month hours for physician p in month m.
Variables	
x_{pds}	1 if physician p is assigned to shift s on day d, 0 otherwise.
y_{pw}	1 if physician p works weekend w, 0 otherwise.
σ_{pm}^H	Deviation between the number of hours of physician p during month m and the target number $\mathcal{T}_{pm}^H$.
σ_p^W	Deviation between the number of weekends of a physician p during the planning horizon and the target number of weekends.
w_{ds}	Negative deviation from the demand constraint on shift s on day d.
z_{ds}	Positive deviation from the demand constraint on shift s on day d.
$\mathcal{T}_{pm}^H$	Target number of monthly hours for physician p in month m.
$\mathcal{T}_{\mathcal{P}}^{\mathcal{W}}$	Target number of weekends for physician p in the planning horizon.

3.1. Objective functions

We introduce two objective functions that differ in how the quadratic costs are handled. The objective function (1) maximizes the allocation of an employee for shift requests and minimizes under- and over-staffing, as well as deviations in the number of hours and weekends.

$$\begin{aligned}\mathcal{G}(x_{pds}, \sigma^H_{pm}, \sigma^W_p) = &\sum_{p\in P}\sum_{d\in D}\sum_{s\in S}\left(q^+_{pds}(1-x_{pds}) + q^-_{pds}x_{pds}\right)\\ &+\sum_{d\in D}\sum_{s\in S}\left(v^-_{ds}w_{ds} + v^+_{ds}z_{ds}\right) + \sum_{p\in P}\sum_{m\in M}\left(\sigma^H_{pm}\right)^2 + \sum_{p\in P}\left(\sigma^W_p\right)^2 \end{aligned} \tag{1}$$

The parameters q^+_{pds} and q^-_{pds} represent the weights for the shift-on and shift-off requests, respectively. The higher the weight, the more relevant the request is to the employee. If there is no request, then the parameter is set to zero. The variables w_{ds} and z_{ds} are the total numbers of staff below and above the preferred cover level for each shift, s on the day d, and the parameters v^-_{ds} and v^+_{ds} are weights indicating the importance of under- and over-coverage. The last term of the objective function is the summation deviation from target hours and target number of weekends. The inclusion of these terms is one of the key differences between our formulation and the formulation of Curtois & Qu (2014).

The objective function (2) differs only by applying a linear approximation to describe the quadratic costs. We will later explain the piecewise linear function, $\mathcal{N}(\cdot)$, and its constraints.

$$\begin{aligned}\mathcal{H}\Big(x_{pds}, \mathcal{N}(\sigma^H_{pm}), \mathcal{N}(\sigma^W_p)\Big) = &\sum_{p\in P}\sum_{d\in D}\sum_{s\in S}\left(q^+_{pds}(1-x_{pds}) + q^-_{pds}x_{pds}\right)\\ &+\sum_{d\in D}\sum_{s\in S}\left(v^-_{ds}w_{ds} + v^+_{ds}z_{ds}\right) + \sum_{p\in P}\sum_{m\in M}\left(\mathcal{N}(\sigma^H_{pm})\right) + \sum_{p\in P}\mathcal{N}(\sigma^W_p) \end{aligned} \tag{2}$$

3.2. Constraints

We introduce hard constraints in equations (3)-(11), and soft constraints in equations (12)-(18).

3.2.1. Set of hard constraints

H1*: Single shift assignment*

An employee cannot be assigned to more than one shift in a day. This constraint was also considered by Curtois & Qu (2014), Beaulieu et al. (2000) and Topaloglu (2006) as a hard constraint. Some hospitals require that physicians cannot be assigned to more than a shift a day so they can be assigned to a longer stretch, while many hospitals are required to follow this legally due to work regulations.

$$\sum_{s\in S} x_{pds} \leq 1 \qquad \forall p \in P,\ d \in D, \tag{3}$$

***H2**: Antagonist shift control*

A minimum amount of rest is required after each shift, and hence, some shifts cannot follow others, e.g., a morning shift cannot follow a night shift. Differently from Beaulieu et al. (2000), Topaloglu (2006), Ferrand et al. (2011) and Topaloglu & Ozkarahan (2011) who modelled this constraint as a hard constraint for specific shifts, we impose it as a general constraint to tackle the antagonism between shifts, i.e., rather than only restricting one specific pair, we define for each shift a set of conflicting shifts.

$$x_{pds} + x_{p(d+1)i} \leq 1 \qquad \forall p \in P,\ d \in \{1, \ldots, |D| - 1\}, s \in S, i \in \Upsilon_s, \tag{4}$$

***H3**: Maximum consecutive work days*

This ensures that the maximum number of days an employee can work without a day off is not violated. From the current day to the last day in the stretch, a physician cannot exceed the maximum established in R_p^+. This constraint was described as a soft constraint by Topaloglu (2006), whereas Beaulieu et al. (2000), Carter & Lapierre (2001), Ferrand et al. (2011) and Topaloglu & Ozkarahan (2011) considered it as a hard constraint.

$$\sum_{z=d}^{d+R_p^+} \sum_{s \in S} x_{pzs} \leq R_p^+ \qquad \forall p \in P,\ d \in \{1, \ldots, |D| - R_p^+\}, \tag{5}$$

***H4**: Minimum consecutive work days and vacation days*

Minimum consecutive workdays and vacation days are modeled in constraints (6) and (7), respectively. Carter & Lapierre (2001) was the first to model this constraint, which was also considered by Topaloglu & Ozkarahan (2011). These constraints may stem from workplace regulations but also allow managers to design schedules with more organized and standardized assignments (workdays) and employees having proper recovery periods (vacation days).

$$\sum_{s \in S} x_{pds} + c - \sum_{j=d+1}^{d+c} \sum_{s \in S} x_{pjs} + \sum_{s \in S} x_{p(d+c+1)s} \geq 1$$
$$\forall p \in P,\ c \in \{1 \ldots c_p^- - 1\},\ d \in \{1 \ldots |D| - (c+1)\}, \tag{6}$$

$$1 - \sum_{s \in S} x_{pds} + \sum_{j=d+1}^{d+o} \sum_{s \in S} x_{pjs} + \sum_{s \in S} x_{p(d+o+1)s} \geq 0$$
$$\forall p \in P,\ o \in \{1 \ldots o_p^- - 1\},\ d \in \{1 \ldots |D| - (o+1)\}, \tag{7}$$

***H5**: Weekend assignment*

A weekend is assumed to be worked if the employee has a shift on a Saturday or Sunday, as ensured by (8), while the maximum number of work weekends is presented in equation (9). Carter & Lapierre (2001) also considers weekend constraints with a maximum number of consecutive work weekends. A weekend is regarded as being worked if the employee has a shift on a Saturday or Sunday, in the constraints expressed by (7*w-1) and (7*w), respectively.

$$\sum_{s\in S}(x_{p(7*w-1)s} + x_{p(7*w)s}) \leq 2y_{pw} \qquad \forall p \in P,\ w \in W, \tag{8}$$

$$\sum_{w\in W} y_{pw} \leq V_p^+ \qquad \forall\ p \in P, \tag{9}$$

***H6**: Holidays*

The constraint (10) indicates days that employees cannot work, e.g., due to holidays or training. This is a hard constraint, and can also be found in Beaulieu et al. (2000), Topaloglu (2006) and Puente et al. (2009).

$$x_{pds} = 0 \qquad \forall p \in P,\ (d,s) \in U_p, \tag{10}$$

***H7**: Maximum number of shifts*

The constraint (11) ensures a maximum number of shifts of each type that can be assigned to a physician, e.g., due to contractual requirements allowing only a maximum number of night shifts. Beaulieu et al. (2000) and Topaloglu & Ozkarahan (2011) consider this as a hard constraint.

$$\sum_{d\in D} x_{pds} \leq LS_{ps} \qquad \forall p \in P,\ s \in S, \tag{11}$$

3.2.2. Set of soft constraints

***S1**: Demand*

The demand should be satisfied, but deviations for over- and under-staffing are allowed, which are penalized in the objective function. This constraint is originally found in Curtois & Qu (2014).

$$\sum_{p\in P} x_{pds} - z_{ds} + w_{ds} = DEM_{ds} \qquad \forall d \in D,\ s \in S, \tag{12}$$

***S2**: Hour balance*

These constraints control fairness for hours worked by two means. The first, with hour parameters, and the second estimating the values through a variable. We note that the formulation of Curtois & Qu (2014) defined hour constraints as in (13), where parameters b_p^{min} and b_p^{max} are, respectively, minimum and maximum hours a physician has to complete in the planning horizon.

$$b_p^{min} \leq \sum_{d \in D} \sum_{s \in S} H_s x_{pds} \leq b_p^{max} \qquad \forall p \in P. \tag{13}$$

We introduce the constraints in equations (17)-(20) aiming for a fair hour balance. For this purpose, we make the necessary definitions in equations (14)-(16), where $length(m)$ indicates the length of the month m. In equation (14), $\mathcal{A}_{pm}$ is rounded down if not integer, to indicate the number of cycles in a month when a maximum number of consecutive work days and a minimum number of consecutive vacation days are used. $\overline{\mathcal{A}}_{pm}$ in equation (15) indicates the number of remaining days if such cycles are used throughout the month. Finally, we calculate the total number of hours using equation (16).

$$\mathcal{A}_{pm} = \frac{length(m)}{R_p^+ + o_p^-} \qquad \forall m \in M,\ p \in P, \tag{14}$$

$$\overline{\mathcal{A}}_{pm} = length(m) \bmod (R_p^+ + o_p^-) \quad \forall m \in M,\ p \in P, \tag{15}$$

$$\mathcal{T}_{pm}^H \leq (\mathcal{A}_{pm} \times R_p^+) + \overline{\mathcal{A}}_{pm} \qquad \forall p \in P,\ m \in M. \tag{16}$$

The estimation of the best target value for the number of hours a physician can satisfy within a month is denoted by $\mathcal{T}_{pm}^H$. However, this estimation needs attention to ensure a reasonable value for a given problem instance. Using a parameterized approach, the target is defined by the equation $\mathcal{LH}_{pm} = \frac{h_{pm}^{max} + h_{pm}^{min}}{2}$.

$$\sigma_{pm}^H \geq \sum_{d \in D_m} \sum_{s \in S} H_s x_{pds} - \mathcal{LH}_{pm} \qquad \forall p \in P,\ m \in M, \tag{17}$$

$$\sigma_{pm}^H \geq \mathcal{LH}_{pm} - \sum_{d \in D_m} \sum_{s \in S} H_s x_{pds} \qquad \forall p \in P,\ m \in M, \tag{18}$$

$$\sigma_{pm}^H \geq \sum_{d \in D_m} \sum_{s \in S} H_s x_{pds} - \mathcal{T}_{pm}^H \qquad \forall p \in P,\ m \in M, \tag{19}$$

$$\sigma_{pm}^H \geq \mathcal{T}_{pm}^H - \sum_{d \in D_m} \sum_{s \in S} H_s x_{pds} \qquad \forall p \in P,\ m \in M, \tag{20}$$

S3: *Weekend balance*

The formerly defined weekend constraint in (9) only establishes a maximum limit on the number of work weekends. We propose the weekend constraints as flexible constraints defined by (21) and (22). Thus, we have parameters for maximum weekend assignments, $\mathcal{V}_{\mathcal{P}}^{+}$, and deviation variables, σ_p^W, to control the deviations of weekend assignments in the so-called parameterized approach (constraints (23) and (24)). On the other hand, we employ the decision variable T_P^W in the estimated approach to find the best assignment for weekends (constraints (21) and (22)).

$$\sigma_p^W \geq \sum_{w \in W} y_{pw} - \mathcal{T}_{\mathcal{P}}^{\mathcal{W}} \qquad \forall p \in P, \tag{21}$$

$$\sigma_p^W \geq \mathcal{T}_{\mathcal{P}}^{\mathcal{W}} - \sum_{w \in W} y_{pw} \qquad \forall p \in P, \tag{22}$$

$$\sigma_p^W \geq \sum_{w \in W} y_{pw} - TV_P^{+} \qquad \forall p \in P, \tag{23}$$

$$\sigma_p^W \geq TV_P^{+} - \sum_{w \in W} y_{pw} \qquad \forall p \in P, \tag{24}$$

Originally, constraints (21) and (22) were defined as (9) by Curtois & Qu (2014). Parameter V_p^{+} is the maximum number of weekends a physician can be assigned. A similar procedure was applied to hour balance constraints.

3.3. Parameterized and Estimated Models

The objective functions and constraints previously stated lead us to introduce four modeling approaches to describe the PSP with considerations about fairness. The so-called parameterized approach deals with fairness using a target parameter for work hours and weekend assignments, while the estimated approach uses a decision variable for fairness assignments. $PSP - MIQP_p$ is the parameterized model, whose objective function (1) handles quadratic costs, as follows:

$$\begin{aligned} PSP - MIQP_p\text{: Minimize} \quad & \mathcal{G}\Big(x_{pds}, \sigma_{pm}^{H}, \sigma_p^W\Big) \\ & \text{s.t. } (3) - (12), (17) - (18) \end{aligned}$$

$PSP - MIP$ formulation applies a linear approximation for the quadratic costs. Thus, we use the objective function (2) and include constraints (26) - (29):

$$
\begin{aligned}
PSP - MIP_p\text{: Minimize} \quad & \mathcal{H}\Big(x_{pds}, \mathcal{N}(\sigma^H_{pm}), \mathcal{N}(\sigma^W_p)\Big) \\
\textbf{s.t.} \quad & (3) - (12) \\
& (17) - (18) \\
& (26) - (29)
\end{aligned}
$$

The estimated approach works with variables to adjust work hours and weekend assignments; therefore, we also have two formulations, which differ by the objective functions (1) and (2).

$$
\begin{aligned}
PSP - MIQP_s\text{: Minimize} \quad & \mathcal{G}\Big(x_{pds}, \sigma^H_{pm}, \sigma^W_p\Big) \\
\text{s.t.} \quad & (3) - (8) \\
& (10) - (12) \\
& (19) - (22)
\end{aligned}
$$

$$
\begin{aligned}
PSP - MIP_s\text{: Minimize} \quad & \mathcal{H}\Big(x_{pds}, \mathcal{N}(\sigma^H_{pm}), \mathcal{N}(\sigma^W_p)\Big) \\
\text{s.t.} \quad & (3) - (8) \\
& (10) - (12) \\
& (19) - (22) \\
& (26) - (29)
\end{aligned}
$$

3.4. Linear approximations

To tackle the inherent complexity of the Mixed-Integer Quadratic Programming (MIQP) in the PSP-ER, we propose a linear approximation of the quadratic objective function using a piece-wise linear function. The domain of the function is divided into N linear problems as formalized in equation (25), where λ_{pmn} is an auxiliary variable and B_n is the growth coefficient of the straight segment that models the curve, with $n = 0 \ldots N-1$. σ^{max} represents the maximum value σ_{pm} deviation variable can take.

$$
\mathcal{N}(\sigma^H_{pm}) = \sum_{p \in P} \sum_{m \in M} \Big((\sigma^H_{pm})^2 \Big) \tag{25}
$$

Through linear approximation, the quadratic objective function term in equation (25) can be expressed by constraints (26) - (29).

$$\mathcal{N}(\sigma_{pm}^{H}) = \sum_{p}\sum_{m}\sum_{n} B_n \lambda_{pmn} \tag{26}$$

$$\sigma_{pm}^{H} = \sum_{n} \lambda_{pmn} \tag{27}$$

$$0 \leq \lambda_{pmn} \leq \frac{\sigma^{max}}{N} \tag{28}$$

$$B_n = \frac{(2n+1)\sigma^{max}}{N} \tag{29}$$

The linear approximation defines values of λ_{pmn} depending on N linear segments and in terms of σ^{max}, i.e., the maximum deviation found. Constraints (26) - (29) are later included in the model as cuts.

4. Computational results

We evaluate the parameterized ($PSP - MIP_p$, $PSP - MIQP_p$) and estimated ($PSP - MIP_s$, $PSP - MIQP_s$) models using the benchmark instances of Curtois (2014) for the Nurse Rostering Problem. These instances are challenging for state-of-the-art algorithms, see, e.g., Rahimian et al. (2017a,b). It is worth mentioning that the benchmark set can be used in physician scheduling problems, such as ours, given the similar characteristics. Table 3 summarizes the instances' parameters. This benchmark set has 24 instances, and each instance was solved 10 times, corresponding to once for each objective function, once for each of the five intervals of linear approximation, and once for the nurse scheduling problem by Curtois & Qu (2014).

The experiments were run on a PC with Intel Xeon E5-2680v2 2.8 GHz processor and 128 GB of RAM. The four proposed models employ the L^2 norm in the objective functions for all reported results, and the linear approximation sets an 8-interval for segments. The computational results that lead us to choose these parameter settings are discussed in Appendix A and B. We took the model in Curtois & Qu (2014) for comparison because this model and ours are solving a similar problem of scheduling for medical staff (physicians in our case, nurses in Curtois & Qu (2014)), sharing similar constraints as pointed out in Section 3. The main point for comparison against the model of Curtois & Qu (2014) is the impact of finding fairness solutions for physicians, the time performance to reach such solutions, and the number of benchmark instances solved under the proposed fairness constraints. We have implemented all models, including the one in Curtois & Qu (2014), and run under the same computer platform and solver parameters. We used the Java version of callable libraries from IBM ILOG CPLEX 12.8 optimization solver.

Table 3. The characteristics of the set of benchmark instances proposed by Curtois (2014)

Instances	Days	Nurses	Shift types	Day off requests	Shift on/off requests
Instance01	14	8	1	8	26
Instance02	14	14	2	14	62
Instance03	14	20	3	20	64
Instance04	28	10	2	20	71
Instance05	28	16	2	32	106
Instance06	28	18	3	36	135
Instance07	28	20	3	40	168
Instance08	28	30	4	60	225
Instance09	28	36	4	72	232
Instance10	28	40	5	80	284
Instance11	28	50	6	100	336
Instance12	28	60	10	120	422
Instance13	28	120	18	240	841
Instance14	42	32	4	128	359
Instance15	42	45	6	180	490
Instance16	56	20	3	120	280
Instance17	56	32	4	160	480
Instance18	84	22	3	176	414
Instance19	84	40	5	320	834
Instance20	182	50	6	900	2318
Instance21	182	100	8	1800	4702
Instance22	364	50	10	1800	4638
Instance23	364	100	16	3600	9410
Instance24	364	150	32	5400	13809

4.1. Parameterized vs Estimated approaches

This section presents the performance associated with each model considering fairness. We summarize the comparison between the proposed models in terms of demand compliance as shown in Figure 1, measuring demand satisfaction with equations (30)-(31).

$$demViol_{ds} = \begin{cases} 1 & \text{If} \quad \sum_{p\in P}\sum_{p\in P} x_{pds} - z_{ds} + w_{ds} < DEM_{ds} \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

$$\Delta_{dem} = \frac{\sum_{d,s} demViol_{ds}}{TotalDemConstr} \times 100 \tag{31}$$

The parameterized and estimated approaches did not find a feasible solution within 1 hour for instances 21 to 24. For other instances, $PSP - MIP_s$ and $PSP - MIQP_s$ have mostly results below 25% in terms of demand satisfaction when compared to $PSP - MIP_p$ and $PSP - MIQP_p$. Indeed, $PSP - MIP_s$ outperformed the other three models for 45% of the instances. Thus, if demand compliance is critical, the estimated approaches are more beneficial than the parameterized ones for the set of benchmark instances.

Another fairness aspect is the violation of weekend constraints given by equations (32) and (33). We have $X_P^W = TV_P^+$ and $X_P^W = \mathcal{T}_\mathcal{P}^\mathcal{W}$, respectively, for parameterized

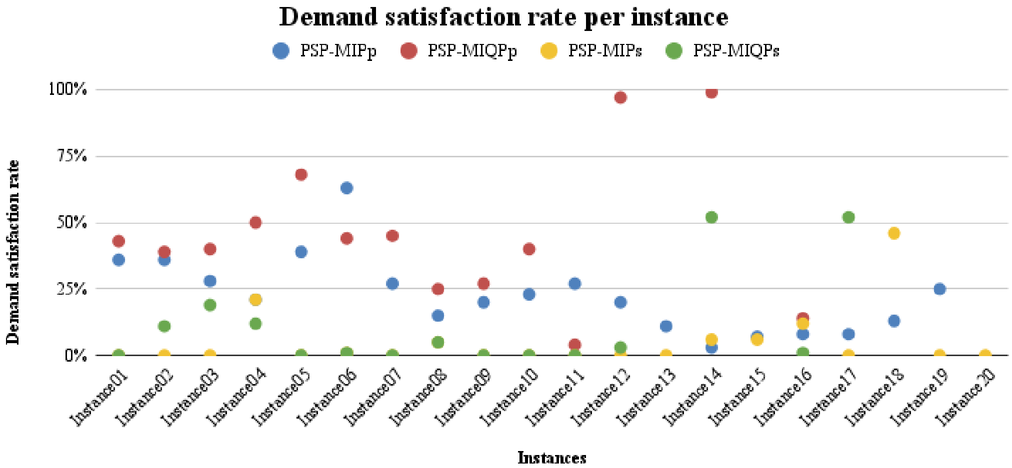

Figure 1. The demand satisfaction rate per instance for $PSP-MIP_p$, $PSP-MIQP_p$, $PSP-MIP_s$ and $PSP-MIQP_s$

and estimated approaches

$$wkndViol_p = \begin{cases} 1 & \text{If} \quad |\sum_{w \in W} y_{pw} - X_P^W| > 0 \quad p \in P, \\ 0 & \text{otherwise} \end{cases} \tag{32}$$

$$\Delta_W = \frac{\sum_{p,m} wkndViol_p}{TotalWknConstr} \times 100 \tag{33}$$

Figure 2 summarizes weekend violation rates and Figure 3 illustrates weekend deviations for Instance07. $PSP-MIQP_p$ violates weekend constraints in 50% of the solved instances, and this rate decreases to 16% using $PSP-MIP_p$. Instance 15 is the only one where $PSP-MIP_s$ violated weekend constraints, while the constraint is fully satisfied for instances solved by $PSP-MIQP_s$. Again, the possibility of deciding the target values for hours and weekends improves the chance of satisfying such constraints.

Next, the fairness is evaluated by measuring work hours deviation, where $\overline{H}$ stands for average hours accomplished and $\overline{TH}$ for average target hours as follows:

$$\overline{H} = \frac{\sum_m \sum_{d \in D_m} \sum_{s \in S} H_s x_{pds}}{|P||M|} \tag{34}$$

$$\overline{TH} = \begin{cases} \sum_{m,p} \frac{\mathcal{LH}_{pm}}{|P||M|} & \text{For parameterized approach} \\ \sum_{m,p} \frac{\mathcal{T}^H_{pm}}{|P||M|} & \text{For estimated approach} \end{cases} \tag{35}$$

Table 4 shows $\overline{H}$ and $\overline{TH}$ results for the parameterized and estimated approaches. $PSP-MIP_p$ and $PSP-MIQP_p$ return $\overline{H}$ values that reach exactly $\overline{TH}$ for 14 out of

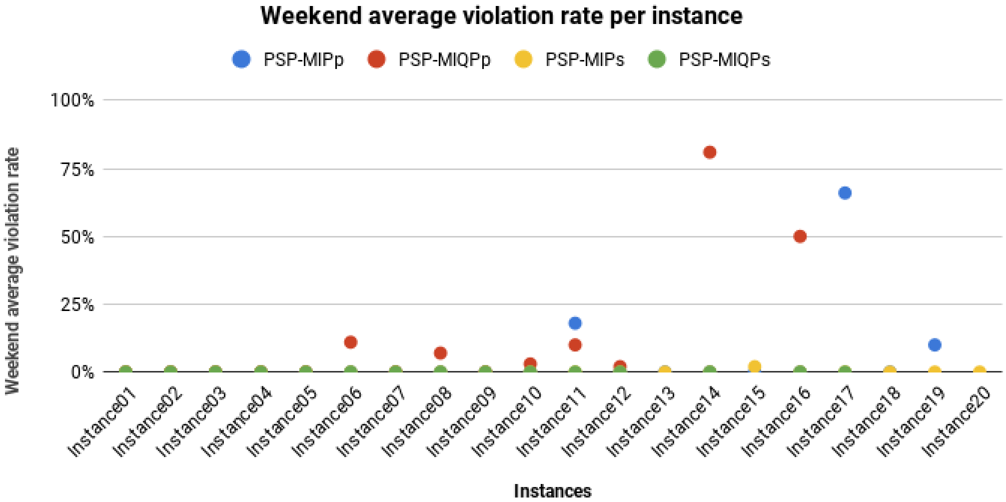

Figure 2. The weekend average violation rate for all models.

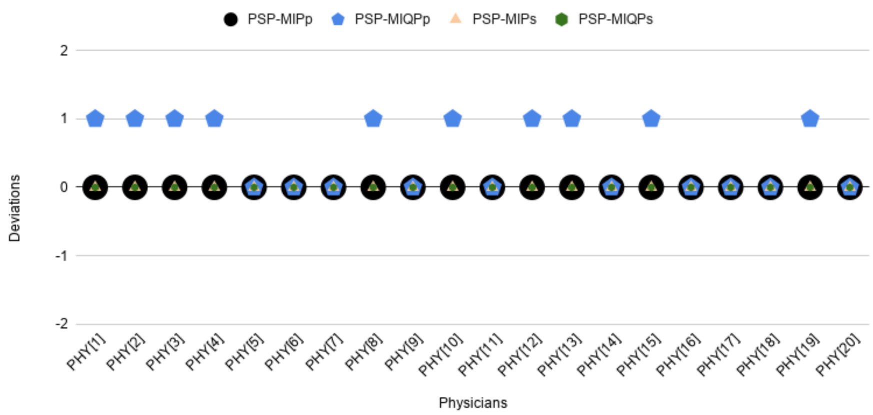

Figure 3. Weekend deviations between four models in instance 07.

20 and 7 out of 20 instances, respectively. $PSP-MIP_s$ and $PSP-MIQP_s$ manage the target variable $(\mathcal{T}^{H}_{pm})$ to exactly reach the work hours $(H_s x_{pds})$ for physicians in all instances. The variables set did not increase or decrease $\overline{TH}$ by a significant amount when compared with the same average figures in the parameterized case. Thus, the estimated models can suggest a reasonable adjustment for target hours to decision-makers. In terms of the objective function, the models guided by the proposed quadratic objective function seem to have more problems finding solutions for some instances. In contrast, the linear approximation employed by MIP models returned the same optimal solutions for some instances as the quadratic ones and found solutions for more instances.

The depicted figures do not present a standard deviation from the average for the majority of instances applying both approaches (estimated and parameterized) in $\overline{H}$. It means that the physicians were assigned to the target number of working hours, which is relevant for fairness considerations. There are some exceptions in the parameterized models. $PSP - MIP_p$ has $\overline{H} = 121.6 \pm 11.8$ for instance 04, $\overline{H} = 83.38 \pm 3.58$ for instance 09, $\overline{H} = 142 \pm 3.61$ for instance 16, and $\overline{H} = 141 \pm 4.5$ for instance 17. The $PSP - MIQP_p$ has also $\overline{H} = 121.6 \pm 11.8$ for instance 04 and $\overline{H} = 84.72 \pm 3.58$ for instance 09, and presents $\overline{H} = 123.67 \pm 5.12$ for instance 11, $\overline{H} = 22.53 \pm 26.98$ for instance 12, $\overline{H} = 110.25 \pm 16.87$ for instance 14, and $\overline{H} = 129.40 \pm 7.93$ for instance 16. $PSP - MIP_s$ and $PSP - MIQP_s$ were able to adjust the target values to avoid deviations in all instances, except for instance 13 by $PSP - MIP_s$ with a deviation of ± 0.92. Surely, this deviation leads $PSP - MIP_s$ to satisfy demand (see Figure 1) as well as weekend assignment (see Figure 2).

Table 4. The hour compliance for $PSP - MIP_p$, $PSP - MIQP_p$, $PSP - MIP_s$ and $PSP - MIQP_s$. $\overline{H}$ stands for average hours and $\overline{TH}$ for average target hours.

Instances	$PSP - MIP_p$		$PSP - MIQP_p$		$PSP - MIP_s$		$PSP - MIQP_s$	
	$\overline{H}$	$\overline{TH}$	$\overline{H}$	$\overline{TH}$	$\overline{H}$	$\overline{TH}$	$\overline{H}$	$\overline{TH}$
Instance01	72	64	72	64	71	71	71	71
Instance02	52.57	52.57	52.57	52.57	61.70	61.70	61.70	61.70
Instance03	54	54	54	54	61.60	61.60	61.60	61.60
Instance04	121.60	128	121.60	128	136.80	136.80	136.80	136.80
Instance05	128	128	128	128	135	135	135	135
Instance06	124	124	124	124	135.10	135.10	135.10	135.10
Instance07	110	110	110	110	128.80	128.80	128.80	128.80
Instance08	120.80	120.80	120.80	120.80	125.60	125.60	125.60	125.60
Instance09	83.38	84.72	83.38	84.72	95.10	95.10	95.10	95.10
Instance10	128.15	128.15	128.15	128.15	141.75	141.75	141.75	141.75
Instance11	128	128	123.67	128	129.92	129.92	129.92	129.92
Instance12	118.67	118.67	22.53	118.67	135.07	135.07	134.80	134.80
Instance13	118.10	118.10	0	0	131.43	131.52	0	0
Instance14	124.30	124.30	110.25	124.30	124	124	124	124
Instance15	147.20	147.20	0	0	149.82	149.82	0	0
Instance16	142	138.50	129.40	138.50	134.8	134,8	134.80	134.80
Instance17	141	141.43	0	0	136.25	136.25	136.25	136.25
Instance18	130.42	130.42	0	0	129.45	129.45	0	0
Instance19	130.68	130.68	0	0	127.68	127.68	0	0
Instance20					138.52	138.52		

The results show that the estimated model with the linear approximation performs better than the parameterized models. The linear approximation is an effective alternative to the original quadratic formulation, providing reasonable quality solutions. As we will report in the following sections, the linear approximation is also a better approach when considering execution time.

4.2. Analyzing $PSP - MIP_p$ *and* $PSP - MIQP_p$ *models*

In this section, we compare the results achieved by $PSP - MIP_p$ and $PSP - MIQP_p$ against the benchmark model of Curtois & Qu (2014). There are 19 out of 24 instances solved using $PSP - MIP_p$, whereas $PSP - MIQP_p$ and Curtois & Qu (2014) could solve only 14 out of 24 instances (corresponding to 79% and 58% success rates, respectively). The different performance rates between the linear and quadratic approaches are not surprising, considering that MIQPs, despite many significant computational improvements in recent decades, often remain more challenging to solve in practice than their MILP counterparts. Figures 4 and 5 show computational time and gap, respectively. All instances are included, and, for cases when a model did not solve an instance, we assume $> 100\%$ gap and $> 3,600$ seconds. We consider the gap as the optimality gap after 3,600 seconds.

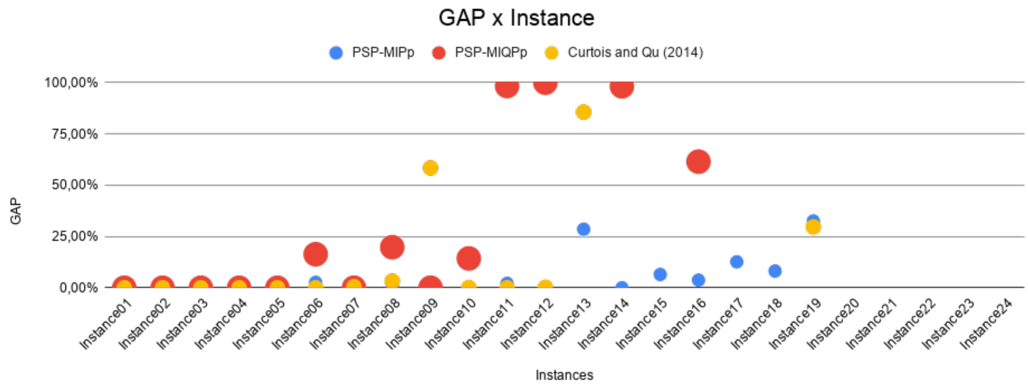

Figure 4. Gap comparison between parameterized approaches of PSP and Curtois & Qu (2014).

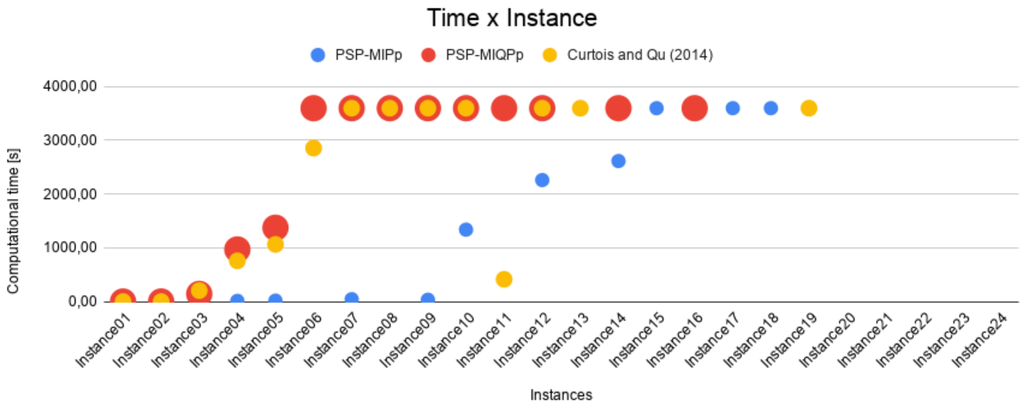

Figure 5. Computational time comparison between parameterized approaches of PSP and Curtois & Qu (2014)

Excluding the ties, the model $PSP - MIP_p$ outperforms the others in 11 of the instances in terms of the gap, and in 5 out of these 11 instances, it has reached the optimal. It is also the only model that found solutions for four instances. The behavior of $PSP-MIP_p$ is consistent throughout the instances, including significantly difficult instances such as Instance13, Instance14, and Instance16. This indicates that the cuts we have applied are not too limited to the size of the instances, and with their inclusion, it is slightly easier for the solver to reach the feasibility region. This kind of cut can then be useful for problems in which we aim to find a feasible solution in a faster and reasonable amount of time, especially for MILP. In terms of computational time, $PSP - MIP_p$ is faster in 12 of the solved instances (three times faster than Curtois & Qu (2014)), while $PSP - MIQP_p$ has often exhausted the time limit. From the results, we can observe that $PSP - MIP_p$ performs better than $PSP - MIQP_p$, in particular for medium and large-sized instances.

4.3. Analyzing the $PSP - MIP_s$ and $PSP - MIQP_s$ models

Figures 6 and 7 compare the results from estimated models against the benchmark model of Curtois & Qu (2014). The results indicate that 20 instances are solved by $PSP - MIP_s$, in comparison to 14 solved by $PSP - MIQP_s$ and Curtois & Qu (2014) with 79% and 58% of the instances, respectively. The behavior of $PSP - MIP_s$ is consistent throughout the instances, including significantly difficult instances such as Instance16 and Instance17. In the set of instances solved by $PSP - MIP_s$, only one instance has a higher value of gap in comparison to its counterparts, albeit with a negligible difference.

$PSP - MIP_s$ provides the overall best performance with respect to gaps and computational times. For instance, the model was solved to optimality without any cost for Instance03, which is the most desired scenario in the realm of physician scheduling. This can be explained by the less restricted values for hours and weekends and also a fair trade-off between demand compliance and workload disturbances.

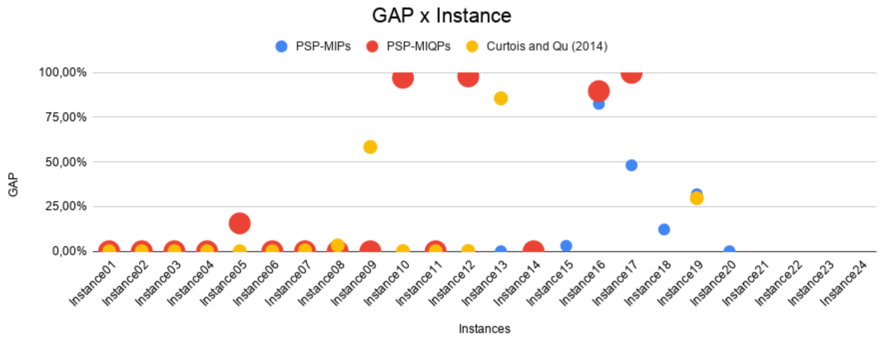

Figure 6. Gap comparison between estimated approaches of PSP and Curtois & Qu (2014).

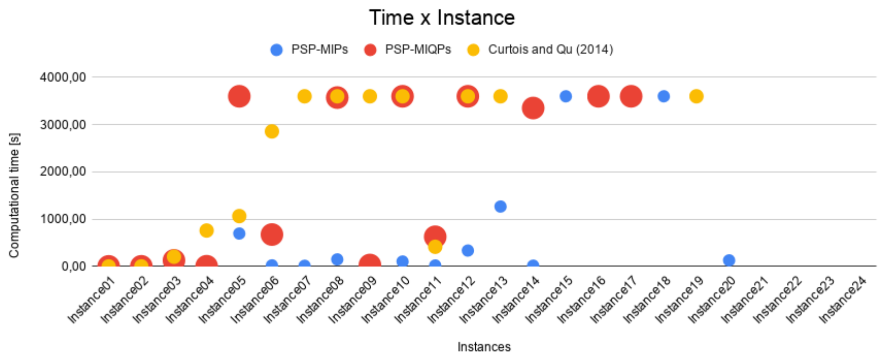

Figure 7. Computational time comparison between estimated approaches of PSP and Curtois & Qu (2014)

Regarding linear approximation, it acts as a cut leading to satisfactory upper and lower bounds improvements. An example of this behavior is illustrated in Figures 8 and 9. For the $PSP - MIQP_s$, the initial upper bound is around 400 while the lower bound is below 50. It takes nearly 1,000 seconds to converge the upper bound towards the lower bound, while the lower bound barely changes from its initial value. On the other hand, $PSP - MIP_s$ starts with a shallow value for the upper bound, and the convergence of both bounds is steady. In general, the $PSP - MIP_s$ surpasses the $PSP - MIQP_s$ and the parameterized approaches. Firstly, since $MIQP$ naturally contains $MILP$ as a special case, we note that $PSP - MIQP_s$ faces computational challenges for more complex instances. Another impact on this performance is considering the target number of hours as a variable. With such a measure, there is a smaller set of constraints to compute, which allows the B&B algorithm used by the solver to reach a more significant set of feasible solutions, even for large-sized instances.

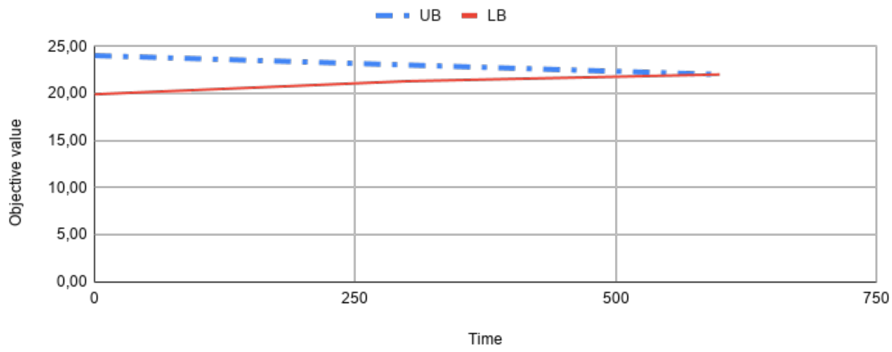

Figure 8. The evaluation of the lower and upper bound for $PSP - MIP_s$

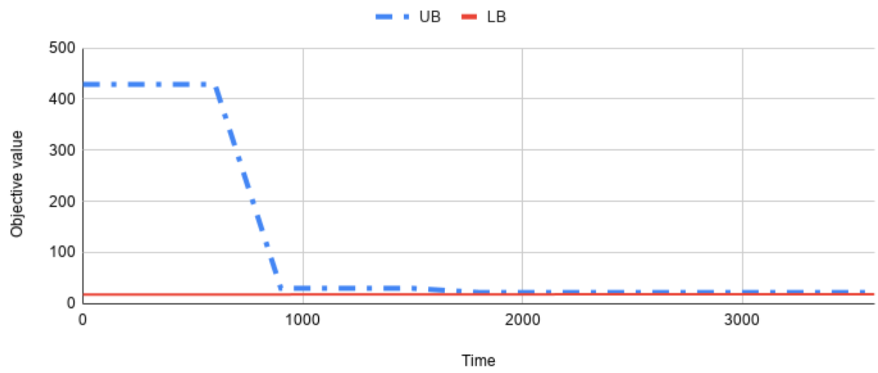

Figure 9. The evaluation of the lower and upper bound for $PSP - MIQP_s$

5. Conclusion

The Physician Scheduling Problem in Emergency Rooms, with a particular focus on fairness in the number of worked hours and worked weekends, is investigated in this paper. The use of L^1, L^2, and L^∞ norms as well as linear approximations of quadratic models were evaluated. We estimated values of target hours and weekends to achieve a better level of satisfaction for fairness in hours and weekends, and for demand coverage. Our formulations are closely linked to MIP models applied to classical Nurse Rostering Problems, albeit with various extensions, particularly for handling fairness.

Using benchmark instances and a model from the literature, computational experiments were conducted to gain critical insights into the behavior of different approaches.

The quality of linear approximations was tested with different values of numbers of intervals to approximate the curve, and different norms to define fairness aspects were also evaluated. Computational results were analyzed mainly regarding fairness and solutions found in the limited time. In both criteria, our approaches demonstrated relevant results, where applying $PSP - MIP_p$ model 79% of instances were solved, against 58% by the model of Curtois & Qu (2014). Real-world settings can benefit from our findings, in particular, ERs where the aim is to balance the workload among physicians. Currently, we are in the process of applying our approach in a local hospital.

Finally, of the four models proposed in the study, $PSP - MIP_s$ yielded better demand coverage and fairer assignments for 45% instances in the tests.

For future work, one research direction is to investigate the effectiveness of decomposition techniques for very large-scale problems, where a straightforward application of a model is not viable. Moreover, integrating constraint programming techniques with classical integer programming is also a promising approach for such large-scale problems. Finally, we foresee that studying the theoretical properties of the problem is crucial to obtain stronger lower bounds, e.g., via valid inequalities.

Disclosure statement

No potential conflict of interest was reported by the authors.

Acknowledgments

The second author's work is supported by the Air Force Office of Scientific Research under award number FA9550-18-1-7003. CEPID/FAPESP project (process 2013/073750) granted the computational resources used for computational experiments. We are also grateful to three anonymous reviewers, whose comments improved the presentation and content of the paper.

Notes on contributor(s)

Valdemar A.P.A. Devesse: Conceptualization, Methodology, Software, Formal Analysis, Data curation, Writing- Original draft preparation. **Kerem Akartunalı:** Methodology, Formal Analysis, Writing - Review & Editing, Supervision, Funding acquisition. **Márcio da Silva Arantes:** Methodology, Software & Validation. **Claudio F.M. Toledo:** Methodology, Writing - Review & Editing, Supervision, Funding acquisition.

Appendix A. Evaluating granularity of linear approximations

Tables A1 and A2 report results for 8, 16, 32, 64, and 128 intervals, where we remove those instances without a solution within the time limit. The tables show upper (UB) and lower bounds (LB), related $GAP = \frac{UB-LB}{UB}$, and execution time (in seconds) of CPLEX at termination.

Table A1. Linear approximation with 8, 16 and 32 intervals. Time limit = 3,600 seconds

Instance	8 intervals				16 intervals				32 intervals			
	UB	**LB**	**GAP**	**Time**	**UB**	**LB**	**GAP**	**Time**	**UB**	**LB**	**GAP**	**Time**
Instance01	716	716	0.00%	0.94	716	716	0.00%	0.41	716	716	0.00%	0.59
Instance02	1620	1620	0.00%	0.36	1620	1620	0.00%	0.23	1620	1620	0.00%	0.27
Instance03	1900	1900	0.00%	2.07	1900	1900	0.00%	1.92	1900	1900	0.00%	2.51
Instance04	2204	2203.78	0.01%	9.08	2204	2203.78	0.01%	8.33	2204	2203.78	0.01%	11.36
Instance05	3211	3210.75	0.01%	9.85	3211	3210.75	0.01%	8.93	3211	3210.75	0.01%	19.49
Instance06	2246	2186.39	2.65%	3600	2246	2187.73	2.59%	3600	2245	2187.7	2.55%	3600
Instance07	4030	4030	0.00%	37.14	4030	4030	0.00%	35.73	4030	4030	0.00%	39.18
Instance08	2966	2946.19	0.67%	3600	2967	2946.91	0.68%	3600	2968	2946.87	0.71%	3600
Instance09	3866	3866	0.00%	23.67	3866	3866	0.00%	24.34	3866	3866	0.00%	24.98
Instance10	6329	6328.4	0.01%	1014.41	6329	6328.4	0.01%	1779.78	6329	6328.4	0.01%	1053.62
Instance11	2310	2260.44	2.15%	3600	2310	2260.44	2.15%	3600	2310	2260.44	2.15%	3600
Instance12	11721	11721	0.00%	3268.92	11722	11721	0.01%	1749.2	11721	11721	0.00%	3312.74
Instance13	9181	6558.29	28.57%	3600	9067	6558.29	27.67%	3600	8699	6558.27	24.61%	3600
Instance14	1235	1234.95	0.00%	1871.39	1235	1234.95	0.00%	1900.61	1235	1234.95	0.00%	1874.13
Instance15	4019	3758.32	6.49%	3600	4130	3758.32	9.00%	3600	4185	3758.3	10.20%	3600
Instance16	2945	2834.98	3.74%	3600	2945	2835	3.74%	3600	2943	2834.98	3.67%	3600
Instance17	4886	4267.61	12.66%	3600	5181	4267.61	17.63%	3600	4781	4267.61	10.74%	3600
Instance18	4646.13	4266.95	8.16%	3600	4728.25	4266.75	9.76%	3600	4633	4266.95	7.90%	3600
Instance19	5116.38	3450.16	32.57%	3600	5078.63	3450.16	32.07%	3600	5760.38	3450.16	40.11%	3600

Table A2. Linear approximation with 64 and 128 intervals. Time limit = 3,600 seconds

Instance	64 intervals				128 intervals			
	UB	**LB**	**GAP**	Time	**UB**	**LB**	**GAP**	**Time**
Instance01	716	716	0.00%	0.92	716	716	0.00%	0.57
Instance02	1620	1620	0.00%	0.19	1620	1620	0.00%	0.27
Instance03	1900	1900	0.00%	2.1	1900	1900	0.00%	4.28
Instance04	2204	2203.78	0.01%	17.27	2204	2203.78	0.01%	8.58
Instance05	3211	3210.75	0.01%	8.69	3211	3210.75	0.01%	8.76
Instance06	2246	2187.76	2.59%	3600	2246	2187.78	2.59%	3600
Instance07	4030	4030	0.00%	36.56	4030	4030	0.00%	37.46
Instance08	2966	2946.94	0.64%	3600	2967	2946.95	0.68%	3600
Instance09	3866	3866	0.00%	23.06	3866	3866	0.00%	23.763
Instance10	6329	6328.4	0.01%	1054.23	6329	6328.4	0.01%	1013.84
Instance11	2310	2260.44	2.15%	3600.44	2310	2260.44	2.15%	3600
Instance12	11722	11721	0.01%	1758.01	11722	11721	0.01%	2020.61
Instance13	8069	6558.29	18.72%	3600	9464	6558.27	30.70%	3600
Instance14	1235	1234.95	0.00%	3315.61	1235	1234.95	0.00%	1917.12
Instance15	4027	3758.32	6.67%	3600	4055	3758.32	7.32%	3600
Instance16	2947	2835	3.80%	3600	2948	2835	3.83%	3600
Instance17	4813	4267.61	11.33%	3600	4903	4267.61	12.96%	3600
Instance18	4640.13	4266.95	8.04%	3600	4528.13	4266.95	5.77%	3600
Instance19	8063.13	3450.16	57.21%	3600	5760.38	3450.16	40.11%	3600

Appendix B. Computing L^1 and L^∞ norms

A question arises on whether using L^1 or L^∞ norms would be useful in order to achieve a linear model or avoid convergence issues, as investigated for other problems, e.g., Cadoux (2010); Akartunalı et al. (2016). The L^1 norm (or Manhattan distance) is simply the sum of the magnitudes of the vector in the space, which is the magnitude

Table A3. Deviation from UB of 8 intervals approach

Instances	Intervals				Instances	Intervals			
	16	**32**	**64**	**128**		**16**	**32**	**64**	**128**
Instance01	0,00	0,00	0,00	0,00	Instance11	0,00	0,00	0,00	0,00
Instance02	0,00	0,00	0,00	0,00	Instance12	0,01	0,00	0,01	0,01
Instance03	0,00	0,00	0,00	0,00	Instance13	-1,26	-5,54	-13,78	2,99
Instance04	0,00	0,00	0,00	0,00	Instance14	0,00	0,00	0,00	0,00
Instance05	0,00	0,00	0,00	0,00	Instance15	2,69	3,97	0,20	0,89
Instance06	0,00	-0,04	0,00	0,00	Instance16	0,00	-0,07	0,07	0,10
Instance07	0,00	0,00	0,00	0,00	Instance17	5,69	-2,20	-1,52	0,35
Instance08	0,03	0,07	0,00	0,03	Instance18	1,74	-0,28	-0,13	-2,61
Instance09	0,00	0,00	0,00	0,00	Instance19	-0,74	11,18	36,55	11,18
Instance10	0,00	0,00	0,00	0,00					

of the deviation variables' vector in our case. For L^1 norm, we have the following constraints:

$$\mathcal{N}_1(\sigma^H_{pm}) = \sum_p \sum_m \Delta^H_{pm} \tag{B1}$$

$$\Delta^H_{pm} \geq \sigma^H_{pm} \qquad \forall p \in P,\ m \in M, \tag{B2}$$

$$\Delta^H_{pm} \geq -\sigma^H_{pm} \qquad \forall p \in P,\ m \in M, \tag{B3}$$

$$\mathcal{N}_1(\sigma^W_p) = \sum_p \Delta^W_p \tag{B4}$$

$$\Delta^W_p \geq \sigma^W_p \qquad \forall p \in P, \tag{B5}$$

$$\Delta^W_p \geq -\sigma^W_p \qquad \forall p \in P \tag{B6}$$

Then, the objective function will be as follows:

$$\begin{aligned}\mathcal{H}\Big(x_{pds}, \mathcal{N}_1(\sigma^H_{pm}), \mathcal{N}_1(\sigma^W_p)\Big) &= \sum_{p\in P}\sum_{d\in D}\sum_{s\in S}\Big(q^+_{pds}(1-x_{pds}) + q^-_{pds}x_{pds}\Big) \\ &+ \sum_{d\in D}\sum_{s\in S}\Big(v^-_{ds}w_{ds} + v^+_{ds}z_{ds}\Big) + \Big(\mathcal{N}_1(\sigma^H_{pm}) + \mathcal{N}_1(\sigma^W_p)\Big)\end{aligned} \tag{B7}$$

Then, the formulation with the L^1 norm is:

$$\begin{aligned} \text{Minimize} \quad & \mathcal{H}\Big(x_{pds}, \mathcal{N}_1(\sigma^H_{pm}), \mathcal{N}_1(\sigma^W_p)\Big) \\ \text{s.t} \quad & (3) - (12), \\ & (17) - (18) \\ & (B1) - (B6) \end{aligned}$$

The other linear alternative, L^∞ norm, gives us the largest magnitude among all elements of a vector, which is equivalent to the largest deviations of hours and weekend assignments in our context. For L^∞, we have the following constraints:

$$\mathcal{N}_\infty(\sigma^H_{pm}) = \kappa \tag{B8}$$

$$\kappa \geq \sigma^H_{pm} \qquad \forall p \in P,\ m \in M, \tag{B9}$$

$$\kappa \geq -\sigma^H_{pm} \qquad \forall p \in P,\ m \in M, \tag{B10}$$

$$\mathcal{N}_\infty(\sigma^W_p) = \epsilon \tag{B11}$$

$$\epsilon \geq \sigma^W_p \qquad \forall p \in P \tag{B12}$$

$$\epsilon \geq -\sigma^W_p \qquad \forall p \in P \tag{B13}$$

Then, the objective function of the problem is as follows:

$$\begin{aligned} \mathcal{H}\Big(x_{pds}, \mathcal{N}_\infty(\sigma^H_{pm}), \mathcal{N}_\infty(\sigma^W_p)\Big) = & \sum_{p\in P}\sum_{d\in D}\sum_{s\in S}\Big(q^+_{pds}(1 - x_{pds}) + q^-_{pds}x_{pds}\Big) \\ & + \sum_{d\in D}\sum_{s\in S}\Big(v^-_{ds}w_{ds} + v^+_{ds}z_{ds}\Big) + \Big(\mathcal{N}_\infty(\sigma^H_{pm}) + \mathcal{N}_\infty(\sigma^W_p)\Big) \end{aligned} \tag{B14}$$

The formulation with the L^∞ norm then follows:

$$
\begin{aligned}
\text{Minimize} \quad & \mathcal{H}\Big(x_{pds}, \mathcal{N}_\infty(\sigma^H_{pm}), \mathcal{N}_\infty(\sigma^W_p)\Big) \\
& \text{s.t} \quad (3) - (12) \\
& (17) - (18) \\
& (\text{B8}) - (\text{B13})
\end{aligned}
$$

The results are detailed in Table B1, where instances not solved with any approach are not included. The L^1 norm takes a longer time than the other two norms, even for small instances, although it has competitive performance in terms of the gap for some instances with a planning horizon of 4 and 8 weeks. In terms of gaps, L^2 performs best for four instances, while L^1 and L^∞ perform best for 2 and 1 instances, respectively (the remaining instances have ties). Although L^2 outperforms the other two approaches in terms of average execution times, the difference is not significant in comparison to L^∞ and also not conclusive as times vary from instance to instance. We decided to use L^2 norm in section 4, based on its better performance in terms of gaps and fast execution times.

Table B1. Benchmark results for parameterized approach, testing L^1, L^2 and L^∞. Time limit = 3,600 seconds

Instance	**L^1 norm**				**L^2 norm**				**L^∞ norm**			
	UB	**LB**	**GAP**	**Time**	**UB**	**LB**	**GAP**	**Time**	**UB**	**LB**	**GAP**	**Time**
Instance01	716	716	0.00%	505.05	716	716	0.00%	0.94	716	716	0.00%	1.13
Instance02	1620	1620	0.00%	272.10	1620	1620	0.00%	0.36	1620	1620	0.00%	0.29
Instance03	1900	1900	0.00%	239.11	1900	1900	0.00%	2.07	1900	1900	0.00%	2.01
Instance04	2204	2203.78	0.01%	868.31	2204	2203.78	0.01%	9.08	2204	2203.78	0.01%	8.48
Instance05	3211	3210.75	0.01%	920.29	3211	3210.75	0.01%	9.85	3211	3210.75	0.01%	18.03
Instance06	2246	2187.77	2.59%	3600	2246	2186.39	2.65%	3600	2246	2187.77	2.59%	3600
Instance07	4030	4030	0.00%	365.07	4030	4030	0.00%	37.14	4030	4030	0.00%	35.54
Instance08	2967	2946.93	0.68%	3600	2966	2946.19	0.67%	3600	2967	2946.95	0.68%	3600
Instance09	3866	3866	0.00%	243.53	3866	3866	0.00%	23.67	3866	3866	0.00%	24.45
Instance10	6329	6328.4	0.01%	1509.26	6329	6328.4	0.01%	1014.41	6329	6328.4	0.01%	1034.88
Instance11	2310	2260.44	2.15%	3600	2310	2260.44	2.15%	3600	2310	2260.44	2.15%	3600
Instance12	11722	11721	0.01%	1666.92	11721	11721	0.00%	3268.92	11722	11721	0.01%	1541.38
Instance13	9075	6558.29	27.73%	3600	9181	6558.29	28.57%	3600	9181	6558.29	28.57%	3600
Instance14	1235	1234.95	0.00%	1832.49	1235	1234.95	0.00%	1871.39	1235	1234.95	0.00%	1772.02
Instance15	4404	3758.32	14.66%	3600	4019	3758.32	6.49%	3600	4136	3758.32	9.13%	3600
Instance16	2947	2835	3.80%	3600	2945	2834.98	3.74%	3600	2948	2835	3.83%	3600
Instance17	4806	4267.61	11.20%	3600	4886	4267.61	12.66%	3600	4813	4267.61	11.33%	3600
Instance18	4646.13	4266.95	8.16%	3600	4646.13	4266.95	8.16%	3600	4536.13	4266.95	5.93%	3600
Instance19	8365.25	3450.16	58.76%	3600	5116.38	3450.16	32.57%	3600	8265.25	3450.16	58.26%	3600

Appendix C. Detailed models evaluation

Tables C1 and C2 provides detailed results comparing the four proposed models, using fairness constraints, and the literature model in Curtois & Qu (2014) that does not handle fairness explicitly.

Table C1. Benchmark results for $PSP - MIP_p$ and $PSP - MIQP_p$, in comparison to the Curtois & Qu (2014) model. Time limit = 3,600 seconds

Instances	$PSP - MIP_p$				$PSP - MIQP_p$				Curtois & Qu (2014)			
	UB	LB	GAP	TIME	UB	LB	GAP	TIME	UB	LB	GAP	TIME
Instance01	716	716	0.00%	1.51	716	716	0.00%	1.77	607	607	0.00%	0.62
Instance02	1620	1620	0.00%	1.82	1620	1620	0.00%	2.59	828	828	0.00%	1.55
Instance03	1900	1900	0.00%	2.49	1900	1900	0.00%	143.31	1001	1001	0.00%	205.18
Instance04	2204	2203.78	0.01%	13.76	2204	2204.00	0.00%	970.29	1716	1716	0.00%	759.55
Instance05	3211	3210.75	0.01%	18.75	3211	3211.00	0.00%	1374.96	1143	1143	0.00%	1064.94
Instance06	2246	2186.39	2.65%	3600	2446	2045.93	16.36%	3600	1950	1950	0.00%	2857.30
Instance07	4030	4030	0.00%	45.26	4031	4029.85	0.03%	3600	1056	1050.52	0.52%	3600
Instance08	2966	2946.19	0.67%	3600	3667	2941.51	19.78%	3600	1315	1272.38	3.24%	3600
Instance09	3866	3866	0.00%	34.48	3867	3865.41	0.04%	3600	439	182.5	58.43%	3600
Instance10	6329	6328.4	0.01%	1339.48	7381	6324.74	14.31%	3600	4631	4627.02	0.09%	3600
Instance11	2310	2260.44	2.15%	3600	85775	1387.44	98.38%	3600	3443	3443	0.00%	415.73
Instance12	11721.0	11721	0.00%	2262.58	1.35E+08	11719.20	99.99%	3600	4043	4035.37	0.19%	3600
Instance13	9181	6558.29	28.57%	3600					9398	1347	85.67%	3600
Instance14	1235	1234.95	0.00%	2616.4	69472	1232.76	98.23%	3600				
Instance15	4019	3758.32	6.49%	3600								
Instance16	2945	2834.98	3.74%	3600	6077	2340.06	61.49%	3600				
Instance17	4886	4267.61	12.66%	3600								
Instance18	4646.13	4266.95	8.16%	3600								
Instance19	5116.38	3450.16	32.57%	3600					4186	2942.11	29.72%	3600

Table C2. The benchmark results for $PSPER - (HW)_s$ and $PSPER - (HW)_s^2$ in comparison with the original model run in CPLEX solver for 3600 seconds

Instance	$PSP - MIP_s$				$PSP - MIQP_s$				Curtois & Qu (2014)			
	UB	LB	GAP	TIME	UB	LB	GAP	TIME	UB	LB	GAP	TIME
Instance01	13	13	0.00%	0.21	13	13	0.00%	0.77	607	607	0.00%	0.89
Instance02	5	5	0.00%	0.25	5	5	0.00%	2.06	828	828	0.00%	152.55
Instance03	0	0	0.00%	0.66	0	0	0.00%	127.45	1001	1001	0.00%	120.45
Instance04	1112	1112	0.00%	1.20	1112	1112	0.00%	3.98	1716	1716	0.00%	1620.95
Instance05	22	22	0.00%	695.91	22	18.56	15.63%	3600	1150	874.81	23.93%	3600
Instance06	117	117	0.00%	22.16	117	117	0.00%	674.76	1950	1950	0.00%	2938.24
Instance07	14	14	0.00%	13.48	14	14	0.00%	1124.75	1056	1050.08	0.56%	3600
Instance08	1225	1225	0.00%	148.57	1225	1225	0.00%	3572.58	1642	1268.42	22.75%	3600
Instance09	32	32	0.00%	1.64	32	32	0.00%	29.76	444	185	58.33%	3600
Instance10	7	7	0.00%	103.61	248	7	97.18%	3600	4631	4626.28	0.10%	3600
Instance11	1	1	0.00%	20.18	1	1	0.00%	627.47	3443	3443	0.00%	92.13
Instance12	15	15	0.00%	335.99	730	15	97.95%	3600	4168	4035.92	3.17%	3600
Instance13	224	224	0.00%	1267.20								
Instance14	1217	1217	0.00%	15.80	1217	1217	0.00%	3351.52				
Instance15	3865	3747.82	3.03%	3600								
Instance16	119	20.76	82.55%	3600	219	22.56	89.70%	3600				
Instance17	27	14	48.15%	3600	21847	14	99.94%	3600				
Instance18	1097	963.11	12.21%	3600								
Instance19	47	32	31.91%	3600					6813	2942.81	56.81%	3600
Instance20	240	240	0.00%	128.27								

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