

# Uncertainty Analysis of Two Gas Measurement DGA Ratios for Improved Diagnostics Applications

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**Abstract:** This paper formulates the exact analytical probability density function (PDF) for the ratio of two independent dissolved gas analysis (DGA) measurements that include individual gas measurement errors. It is demonstrated that for small DGA gas measurement errors, the correct two-gas ratio PDF approaches a conventional Gaussian distribution. As the measurement accuracy decreases, the ratio PDF becomes non-Gaussian with the maximum likelihood value of the PDF deviating from the true underlying value. For larger errors, the maximum likelihood estimate of the gas ratio deviates significantly from presumed Gaussian statistics. A method for debiasing measured gas ratio values is presented and a simple application is used to demonstrate the proposed approach.

increase up to 30% [8]. Duval and Dukarm highlight that gas measurement errors can range from 5% to 30%, and if measurement accuracies are unknown, they recommend the adoption of a default 15% measurement error [9]. Though many modern gas measurement instruments are capable of attaining 5% measurement errors or better, many other gas measurements still have large measurement uncertainties. Indeed, historical gas measurements collected by legacy measurement equipment will likely have larger or unknown measurement errors. It is therefore crucial to know how best to evaluate the accuracy of gas ratio measurements for these situations, and accordingly, interpret the outcome of any DGA diagnostic methods.

## I. INTRODUCTION

Transformers are key assets for the correct operation of the power grid [1], and accordingly, transformer health monitoring activities are crucial to ensure reliable grid operation [2]. Dissolved gas analysis (DGA) is an important industry-accepted method for assessing the health of the electrical insulation of oil-filled power transformers [3]-[6]. DGA involves extracting oil samples and making measurements of key combustible gases including hydrogen ( $H_2$ ), methane ( $CH_4$ ), acetylene ( $C_2H_2$ ), ethylene ( $C_2H_4$ ), ethane ( $C_2H_6$ ), carbon monoxide (CO) and carbon dioxide ( $CO_2$ ), measured in  $\mu$ litres/litre or parts-per-million (ppm). Diagnostic health assessment is often performed through determining the levels of these gases and then evaluating a set of key gas ratios, labelled from R1 to R5, as defined in Table I.

TABLE I  
KEY GAS RATIO DEFINITIONS

Ratio	R1	R2	R3	R4	R5
Gases	$CH_4/H_2$	$C_2H_2/C_2H_4$	$C_2H_2/CH_4$	$C_2H_6/C_2H_2$	$C_2H_4/C_2H_6$

The IEEE C57.104 and IEC 60599 standards outline methods for the measurement and analysis of key gas ratios such as Rogers ratios, Doernenburg ratios and the Duval triangle [7], [8]. These methods define boundary gas levels and gas-ratio threshold levels that correspond to insulation fault types.

When measuring gas levels, hardware equipment and laboratory measurement procedures induce gas measurement errors. For an analytical detection limit  $S$ , the IEC 60599 states that above  $10 \times S$  the uncertainty on gas measurements is 15% and for measurements below  $10 \times S$  the uncertainty can

In this context, a general understanding of the correct theoretical and analytical framework for determining a two-gas ratio error, based on individual gas measurement errors is required. Normally practitioners presume gas ratio values are Gaussian distributed around the true value (e.g. [10]). However, this is not the case as will be presented below.

The statistical distributions of the ratio of two independent Gaussian distributed measurement variables has previously been studied [11-14]. However, these papers do not provide an analysis of best estimate methods for establishing the maximum likelihood (ML) ratio probability for applications such as DGA ratio-based diagnostics. To this end, the main contributions of this paper are twofold. Firstly, the formulation of the correct probability density function (PDF) for two-gas ratio dissolved gas evaluations with measurement errors. Secondly, the conception of a methodology to elicit the best estimate of the true underlying gas ratio through ML estimation methods. In the evaluations, it is shown that the correct PDF for a two-gas ratio determination is non-Gaussian. Both the correct gas ratio PDF and a presumed standard Gaussian formulated PDF are compared. A simple application to DGA data demonstrates the underlying principles in relation to best-estimate gas-ratio values.

The paper structure is as follows. Section 2 derives the PDF for two measurement gas ratios with individual gas measurement uncertainties and compares the correct PDF with a Gaussian PDF formulation as a function of measurement errors. Section 3 outlines a method for determining the best estimate of the true underlying ratio based on the correct PDF and measured ratio value. Section 4 provides some examples to demonstrate the principles outlined in the paper. Finally, Section 5 summarizes the main conclusions.

## II. UNCERTAINTY OF DGA DIAGNOSTICS: DERIVATION OF GAS RATIO PROBABILITY DENSITY FUNCTIONS

Let any measured gas ratio measurement be defined as  $R = x/y$ , where  $x$  and  $y$  are the numerator and denominator gas measurements respectively. Also, let  $R_0 = x_0/y_0$  represent the true underlying gas ratio, where  $x_0$  and  $y_0$  are the true gas levels with no measurement errors. If  $x$  and  $y$  are independent Gaussian (Normal) distributed variables, then the resulting individual PDFs are expressed as:

$$P_z(z, z_0) = \frac{1}{\sqrt{2\pi}\sigma_z} \exp\left[-\frac{(z-z_0)^2}{2\sigma_z^2}\right] \quad (1)$$

where  $z = \{x, y\}$ , is the gas measurement,  $z_0$  is the underlying true gas level and  $\sigma_z$  is the standard measurement error on  $z$ .

### A. Gas Ratio PDF with Measurement Errors

The PDF for the ratio  $R$ ,  $P_R(R, R_0)$ , is determined from the following formulation:

$$P_R(R, R_0)dR = P_x(x, x_0)dx P_y(y, y_0)dy \quad (2)$$

which results in

$$P_R(R, R_0) = \int_{-\infty}^{+\infty} P_x(x|R, x_0) \frac{dx}{dR} P_y(y, y_0) dy. \quad (3)$$

Substituting the relevant terms from (1) and (2) into (3), then

$$P_R(R, R_0) = \frac{1}{2\pi\sigma_x\sigma_y} \int_{-\infty}^{+\infty} y \exp\left[-\left(\frac{(yR-x_0)^2}{2\sigma_x^2} + \frac{(y-y_0)^2}{2\sigma_y^2}\right)\right] dy. \quad (4)$$

Expanding terms and performing the integration results in:

$$P_R(R, R_0) = \frac{(y_0\sigma_x^2 + x_0\sigma_y^2 R)}{\sqrt{2\pi}(\sigma_x^2 + \sigma_y^2 R^2)^{3/2}} \exp\left[-\frac{(x_0 - y_0 R)^2}{2(\sigma_x^2 + \sigma_y^2 R^2)}\right]. \quad (5)$$

Substituting  $R_0 = x_0/y_0$  and defining  $a = \sigma_x/x_0$  and  $b = \sigma_y/y_0$  as the fractional errors on  $x_0$  and  $y_0$ , then (5) can be expressed as:

$$P_R(R, R_0) = \frac{(a^2 R_0^2 + b^2 R_0 R)}{\sqrt{2\pi}(a^2 R_0^2 + b^2 R^2)^{3/2}} \exp\left[-\frac{(R_0 - R)^2}{2(a^2 R_0^2 + b^2 R^2)}\right]. \quad (6)$$

It can be seen that  $P_R(R, R_0)$  is a function of  $R_0$ ,  $a$  and  $b$ . Equation (6) can be normalized in relation to  $R_0$  by defining a scaling factor  $\alpha = R/R_0$ . The corresponding PDF distribution of  $\alpha$ , denoted  $P_\alpha(\alpha)$ , is defined as follows:

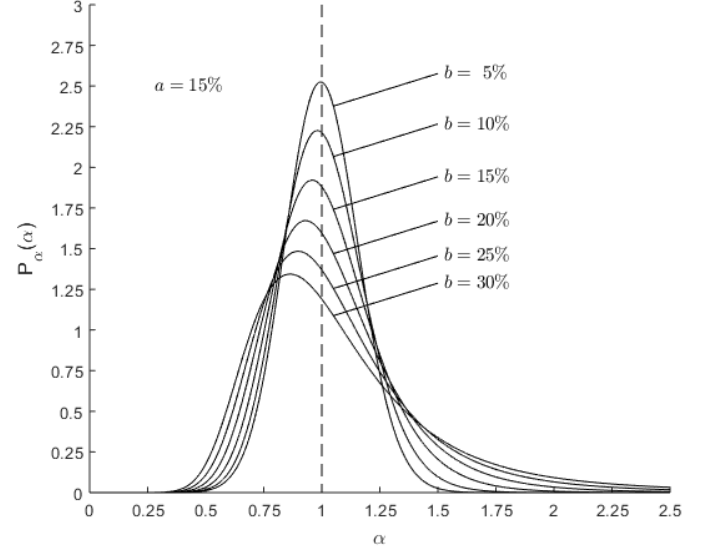
$$P_\alpha(\alpha) = P_R(R|\alpha, R_0) \frac{dR}{d\alpha}. \quad (7)$$

Formulating (7) leads to the resulting PDF for  $\alpha$ , i.e.

$$P_\alpha(\alpha) = \frac{(a^2 + \alpha b^2)}{\sqrt{2\pi}(a^2 + \alpha^2 b^2)^{3/2}} \exp\left[-\frac{(1-\alpha)^2}{2(a^2 + \alpha^2 b^2)}\right]. \quad (8)$$

Equation (8) depends solely on  $a$  and  $b$ , and permits the evaluation of gas ratio probabilities for all values of  $R_0$  and all fractional errors on gas measurements through the scaling process. That is, the relationship  $R = \alpha R_0$  permits scaling of all  $\alpha$  values to  $R$  and  $R_0$  through Equation (8).

Figure 2 shows some examples of the PDF  $P_\alpha(\alpha)$  for different measurement error values of  $a$  and  $b$ , and demonstrates that  $P_\alpha(\alpha)$  is generally non-Gaussian in nature.



**Figure 1.** Example  $P_\alpha(\alpha)$  values as a function of  $\alpha$  for  $a = 15\%$  and  $b = \{5\%; 10\%; 15\%; 20\%; 25\%; 30\%\}$ .

### B. Standard Gaussian Gas Ratio PDF

For simplicity, the usual practice when analyzing a gas ratio measurement,  $R$ , is to adopt a Gaussian PDF [10]. For comparison purposes, the PDF for a presumed Gaussian distribution,  $P_G(R, R_0)$ , is described by:

$$P_G(R, R_0) = \frac{1}{\sqrt{2\pi}\sigma_G} \exp\left[-\frac{(R-R_0)^2}{2\sigma_G^2}\right] \quad (9)$$

where  $\sigma_G$  is determined through:

$$\sigma_G = \sqrt{\left(\frac{\partial R}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial R}{\partial y}\right)^2 \sigma_y^2} \Bigg|_{x=x_0, y=y_0} \quad (10)$$

which results in:

$$\sigma_G = \sqrt{x_0^2 \sigma_y^2 + y_0^2 \sigma_x^2} / \sigma_y = R_0 \sqrt{a^2 + b^2}. \quad (11)$$

Substituting  $\alpha = R/R_0$  and noting that  $dR/d\alpha = R_0$ , then (9) can be transformed into the following Gaussian form for variable  $\alpha$ :

$$P_{\alpha G}(\alpha) = \frac{1}{\sqrt{2\pi}\sigma_{\alpha G}} \exp\left[-\frac{(1-\alpha)^2}{2\sigma_{\alpha G}^2}\right] \quad (12)$$

where  $\sigma_{\alpha G} = \sigma_G/R_0 = \sqrt{a^2 + b^2}$ . It can be seen in (12) that  $P_{\alpha G}(\alpha)$  is independent of  $R_0$  and is an explicit function of  $a$  and

b. Scaling to  $R_0$  is again performed through  $R = \alpha R_0$  noting that  $P_G(R, R_0) = P_{\alpha G}(\alpha|R)/R_0$ .

### C. Comparison of $P_\alpha(\alpha)$ and $P_{\alpha G}(\alpha)$

As examples, Figure 2 shows the differences in the PDFs and the ML values between  $P_\alpha(\alpha)$  and  $P_{\alpha G}(\alpha)$  for: (a)  $a=5\%$ ,  $b=15\%$  and (b)  $a=10\%$ ,  $b=30\%$ .  $P_\alpha(\alpha)$  is seen to be asymmetric compared to  $P_{\alpha G}(\alpha)$  and also possesses a wider PDF skirt. It is also noted that the ML  $\alpha$  value,  $\alpha_{ML}$ , of  $P_\alpha(\alpha)$  is always  $< 1$ , while for  $P_{\alpha G}(\alpha)$ , which is symmetric,  $\alpha_{ML} = 1$ . It is also noted that when  $b$  is small, or when both  $a$  and  $b$  are small, then  $P_\alpha(\alpha)$  tends towards the Gaussian form  $P_{\alpha G}(\alpha)$ .

For diagnostics purposes the ML estimate of a PDF is often adopted as the most likely measured value. As a consequence, the measured value,  $R$ , requires to be “de-biased” to the best estimate of  $R_0$  based on the ML value from  $P_\alpha(\alpha)$  (see below).

### III. INFERENCE OF TRUE GAS RATIO $R_0$

As the ML value of  $P_\alpha(\alpha) < 1$ , a bias correction process is required to estimate the true underlying value of  $R_0$ , i.e.  $\hat{R}_{R0}$ . The measured value  $R$  is always an underestimate of the best estimate of the true value  $R_0$ , i.e. the underlying value of  $R_0$  must be larger to ensure  $R$  is the peak (ML value) of the correct underlying PDF  $P_R(R, R_0)$ . The correction process involves calculating  $\alpha_{ML}$  using  $P_\alpha(\alpha)$  and then adjusting the measured value  $R$  by dividing it by  $\alpha_{ML}$  to obtain the corrected or unbiased estimate  $\hat{R}_{R0}$ . The method for inferring  $\hat{R}_{R0}$  is as follows. Using the values of  $a$  and  $b$ , determine analytically the ML value of  $\alpha$  from  $P_\alpha(\alpha)$  through:

$$\left. \frac{\partial P_\alpha(\alpha)}{\partial \alpha} \right|_{\alpha_{ML}} = 0 \quad (13)$$

Differentiating (13) and solving for  $\alpha_{ML}$ , results, after simplification, in the following quadratic equation:

$$A_4 \alpha_{ML}^4 + A_3 \alpha_{ML}^3 + A_2 \alpha_{ML}^2 + A_1 \alpha_{ML} + A_0 = 0 \quad (14)$$

where  $A_4 = 2b^6$ ,  $A_3 = b^4(1+3a^2)$ ,  $A_2 = b^2(a^2b^2+2a^2-b^2)$ ,  $A_1 = a^2(a^2 + 3a^2b^2 - 2b^2)$ , and  $A_0 = -a^4(1+b^2)$ .

Equation (14) may be solved through numerical iteration methods, e.g. Newton-Raphson iteration. However, there are some special cases. If  $b \ll a$  or  $b = 0$ , then (14) reduces to  $a^4 \alpha_{ML} - a^4 = 0$  resulting in  $\alpha_{ML} = 1$ . If  $a \ll b$  or  $a = 0$ , then (14) reduces to  $2b^2 \alpha_{ML}^2 + \alpha_{ML} - 1 = 0$ . Solving directly and taking the positive root gives  $\alpha_{ML} = (-1 + \sqrt{1 + 8b^2})/4b^2$ . Once  $\alpha_{ML}$  has been evaluated and with the measured value  $R = x/y$  presumed to be the ML value of  $P_R(R, R_0)$ , then the true underlying  $R_0$  (which is always greater than  $R$ ) is determined through the relationship:

$$\hat{R}_{R0} = R/\alpha_{ML} \quad (18)$$

Example calculations of  $\alpha_{ML}$  as a function of gas measurement errors  $a$  and  $b$  are shown in Figure 3. As can be seen, when errors increase,  $\alpha_{ML}$  reduces. In addition,  $\alpha_{ML}$  is more susceptible to variations in  $a$  when  $b$  is constant.

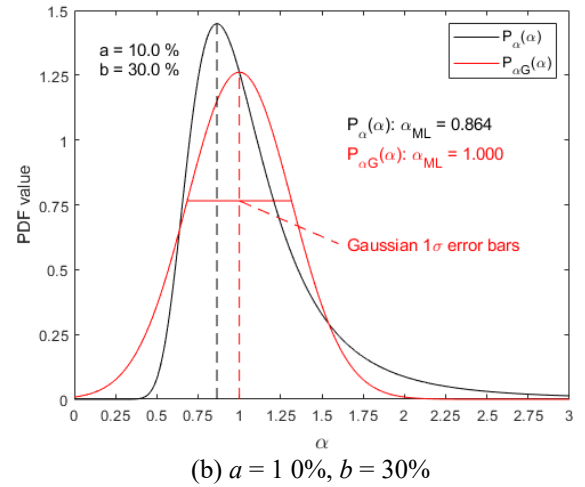
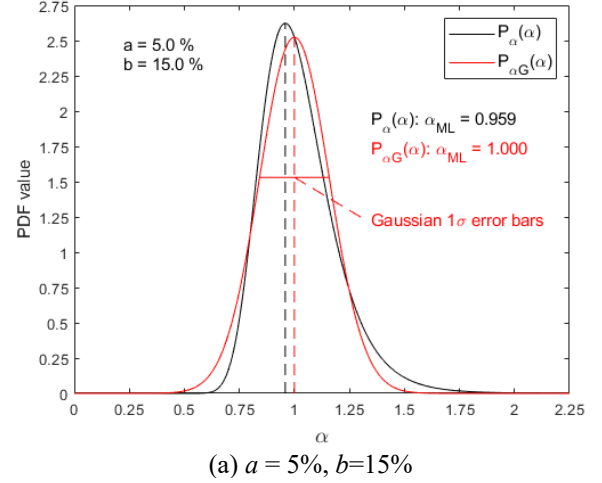


Figure 2. Example plots of  $P_\alpha(\alpha)$  and  $P_{\alpha G}(\alpha)$

### IV. NUMERICAL EXAMPLE

Different gas ratio determinations exist for DGA-based power transformer diagnostics [7, 8]. To demonstrate the principles of the proposed ML correction method, this section focuses on the application of Rogers ratios that classify faults according to the ratio values shown in Table II. As an example, consider the following key gas measurements taken from a power transformer:  $\text{CH}_4 = 156$  ppm,  $\text{H}_2 = 164$  ppm,  $\text{C}_2\text{H}_2 = 15$  ppm,  $\text{C}_2\text{H}_4 = 167$  ppm and  $\text{C}_2\text{H}_6 = 58$  ppm. With the selected gas values, the corresponding ratio values are  $R1 = 0.9512$ ,  $R2 = 0.0898$  and  $R5 = 2.8793$ . Assuming gas measurements without errors, the transformer health is classified as a low temperature thermal fault according to Table II. Assuming a Gaussian PDF, the ML estimates of the true underlying values  $R1_0$ ,  $R2_0$  and  $R5_0$  are the measured values, i.e.  $\hat{R}_{1G0} = R1$ ,  $\hat{R}_{2G0} = R2$  and  $\hat{R}_{5G0} = R5$ . For the correct PDF,  $P_R(R, \hat{R}_{R0})$ , the ML estimates, i.e.  $\hat{R}_{1R0}$ ,  $\hat{R}_{2R0}$  and  $\hat{R}_{5R0}$ , are obtained through solving (14), then applying (18). Corrected  $R_0$  value estimates as a function of  $a = b$  from 0 to 30% are shown in Figure 4. Depending on the gas errors, the diagnostic estimates cross over Rogers ratio boundaries, potentially changing the fault diagnosis outcome.

## V. CONCLUSIONS

This paper has presented the correct PDF for DGA two-gas ratios under measurement uncertainty. The analysis has shown that for accurate or low level gas measurement errors, the exact ratio PDF is very close to a Gaussian PDF. However, with less accurate gas measurements, the ratio PDF differs from a Gaussian distribution. The correct ratio PDF is a skewed distribution, where the measured or calculated ratio value is not representative of the best ML estimate of the true gas ratio value, as it would be for a Gaussian PDF. An analytical correction method was presented based on ML PDF criteria, permitting determination of the most-likely estimate of the true gas ratio value  $R_0$ . It was also shown that individual gas measurement errors may influence potential gas ratio boundary decisions for diagnostic decision-making.

Based on the non-Gaussian nature of the correct PDF, future work will present: (i) the construction and determination of precise confidence intervals for two-gas ratio errors, and (ii) the development of suitable analytical formulae for fault classification probabilities based on specified or defined two-gas ratio fault-boundary levels.

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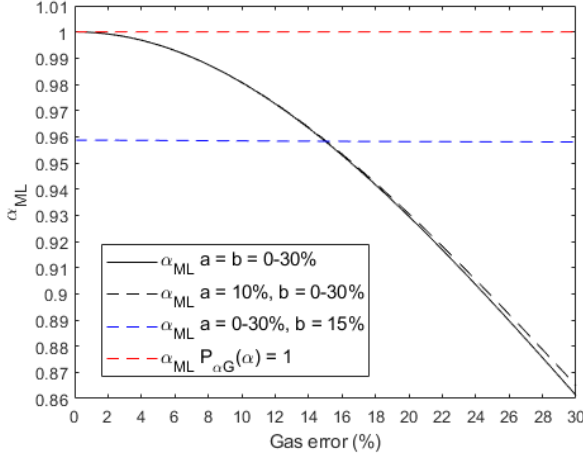


Figure 3. Example  $\alpha_{ML}$  plots as a function of gas errors  $a$  and  $b$

TABLE II  
ROGERS RATIO FAULT DIAGNOSTICS [7], [8]

Case	R2	R1	R5	Fault Diagnosis
0	< 0.1	0.1-1	< 1	Unit normal
1	< 0.1	< 0.1	< 1	Low energy arcing - PD
2	0.1-3	0.1-1	> 3	Arcing – high energy discharge
3	< 0.1	0.1-1	1-3	Low temperature thermal
4	< 0.1	> 1	1-3	Thermal < 700 °C
5	< 0.1	> 1	> 3	Thermal > 700 °C

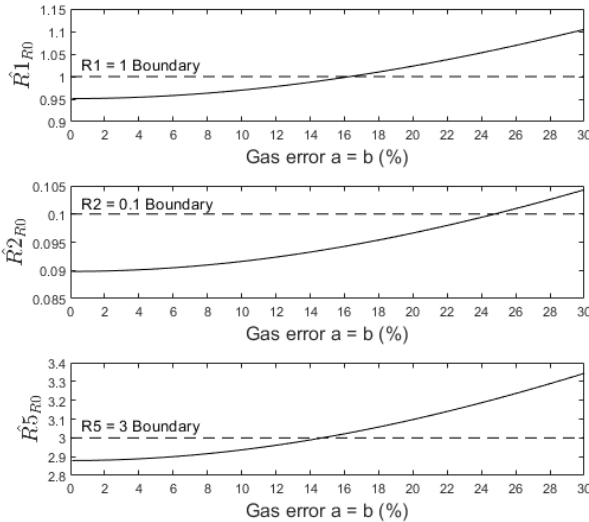


Figure 4. Unbiased estimates of the true Rogers ratios  $R1_0$ ,  $R2_0$  and  $R5_0$  as a function of gas measurement errors  $a = b$

For example, in Figure 4,  $\hat{R1}_0$  crosses the 1.0 boundary at 16.3% error,  $\hat{R2}_0$  crosses the 0.1 boundary at 24.7% error. and  $\hat{R5}_0$  crosses the 3.0 boundary at 14.7% error. At larger errors, the fault appears to translate into a Thermal fault < 700 °C. Obviously, the closer the  $R$  value to the boundary the higher the likelihood of the underlying true value crossing the boundary level when individual gas measurement errors are large.