

Fracture, Damage and Structural Health Monitoring

High Cycle Fatigue Analysis with induced Residual stress based on Fracture Mechanics

Xuran Xiao^{a*}, Volodymyr Okorokov^b, Donald Mackenzie^a

^aDepartment of Mechanical & Aerospace Engineering, University of Strathclyde, Glasgow, G1 1XQ, UK

^bWeir Minerals Netherlands, Venlo, NL

Abstract

A numerical method based on crack initiation and growth theory to predict the high cycle fatigue life with induced compressive residual stress is presented. Fatigue crack growth in a double-notched S355 low carbon steel specimen is considered. The total life of the specimen is divided into crack initiation life and crack propagation life, which can be calculated separately. To obtain the crack initiation life, an assumed crack initial length is employed to create an S-N equation for crack initiation. The process of crack propagation is simulated by finite element analysis (FEA) and the influence of compressive residual stress included by a stress superposition method, hence changing the stress intensity factor ratio. The total fatigue life with residual stress is calculated by corrected Paris law. Comparison of numerical fatigue results with previous experimental results showed good agreement, indicating that the proposed numerical fracture method is suitable for calculating high cycle fatigue life considering induced residual stress.

© 2023 The Authors. Published by Elsevier B.V.

This is an open access article under the CC BY-NC-ND license (<https://creativecommons.org/licenses/by-nc-nd/4.0>)

Peer-review under responsibility of Professor Ferri Aliabadi

Keywords: fracture; fatigue life; crack growth; residual stress

* Corresponding author. Tel.: +44-7940-715528.

E-mail address: xuran.xiao.2017@uni.strath.ac.uk

1. Introduction

Autofrettage is widely used to increase the fatigue life of components, which makes it important to find an accurate method to predict the fatigue life of components with compressive residual stress induced by autofrettage pressure. The stress-life method is one of the most widely applied methods in industry to calculate fatigue life and has been improved by several extended methods. For example, the theory of critical distance was proposed by Taylor (2008) to calculate an average stress to investigate the influence of notches on fatigue life calculation. However, if residual stress is induced, the stress distribution is more complex compared to when it is under single applied load, and this method is not applicable due to the variable stress ratio. Therefore, in recent studies, researchers have applied fracture mechanics to determine the fatigue life with induced residual stress. Among these studies, the main discussions focus on how to identify the stress intensity factor of the residual stress (K_{rs}) and how to employ the value of K_{rs} into crack propagation (Lee, Chung et al. 1998, LaRue and Daniewicz 2007, Ma, Staron et al. 2011).

In this paper, the K_{rs} is obtained by finite element analysis from two crack growth simulations with and without residual stress. The proposed numerical method can be employed simply by engineers.

Regarding the influence of K_{rs} on crack propagation, two approaches, the crack closure method and the superposition method, have been proposed. The crack closure method was first proposed by Elber (1971) where the effect of K_{rs} can be applied through an effective stress intensity factor proposed by Elber. In the superposition method, the total range of stress intensity factor (K_{tot}) is independent on the K_{rs} , but the ratio of stress intensity factor (R) is dependent on the K_{rs} , as:

$$R = \frac{K_{min} + K_{rs}}{K_{max} + K_{rs}} \quad (1)$$

where, K_{min} and K_{max} are the values of minimum and maximum stress intensity factors without induced residual stress.

Due to the influence of residual stress, the value of R is not zero. The Paris law based on zero stress ratio load therefore needs to be correlated to R , and the method used in this paper is that proposed by Dinda and Kujawski (2004).

The ΔK in the traditional $R=0$ Paris law is replaced by K^* as:

$$\Delta K = K^*(1 - R)^\alpha \quad \text{for } R > 0 \quad (2)$$

$$\Delta K = K^*(1 - R) \quad \text{for } R < 0 \quad (3)$$

The crack growth life with induced residual stress can then be calculated by corrected Paris law.

The crack initiation life in this paper is based on an assumed crack initial length, and an S-N equation of crack initiation is created by FEA. Procedures to determine the crack initial life with variable stress amplitude can be shown as:

- Determine the smooth S-N curve of total life (N_{tot}) with different stress amplitude.
- An initial crack is assumed, and the crack growth is simulated in a smooth specimen by FEA to obtain crack growth life (N_g).
- The crack initiation life can then be calculated by: $N_i = N_{tot} - N_g$.

The calculated S- N_i equation can then be employed to calculate the crack initiation life with induced residual stress for a given stress amplitude.

2. Material properties

The material considered in this study is S355 low carbon steel. The parameters of Paris law are determined based on a numerical model by Mlikota (2017). The Chaboche kinematic hardening model is applied to describe the material

cyclic stress-strain curve and the loop data, with several strain amplitudes used in calculation of the parameters in the Chaboche model, is collected from pervious experiments by Okorokov (2018).

The material properties are summarized as:

Table 1 Material properties of low carbon steel

Material	S355 low carbon steel
Young's Modulus /Pa	2×10^{11}
Poisson's ratio	0.3
Yield stress/ MPa	255
Paris Law parameter C (reference unit m)	1.43×10^{-11}
Paris Law parameter m	2.75
Material constant C_1	30489
Material constant γ_1	135.41

Figure 1 shows the high cycle fatigue S-N curve for S355 low carbon steel under room temperature with corresponding stress ratios of 0 and -1 respectively.

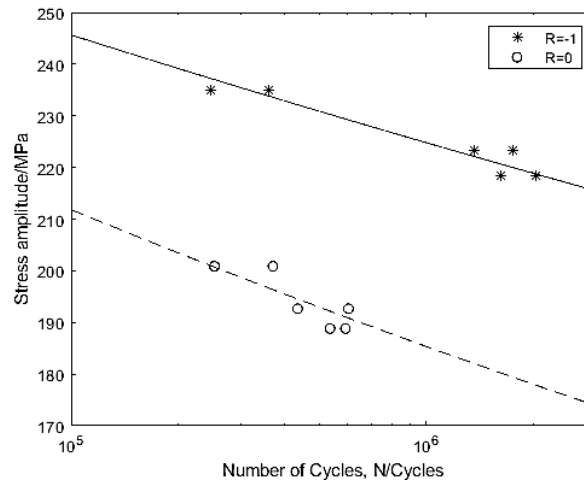


Fig.1. S-N curve of low carbon steel

The S-N curve can be approximated by Basquin's power law (Dowling, Prasad et al. 2013):

$$\sigma_a = \sigma_f' (2N_f)^b \tag{4}$$

Table 2 Fatigue parameters of low carbon steel

Stress Ratio (R)	σ_f'	b
R=-1	393	-0.0385
R=0	430	-0.0580

3. Crack initiation Stress-Life curve

In this paper, an initial crack length is assumed as 0.2mm and, to create a standard $S-N_i$ equation, the crack growth life is obtained first. Therefore, the process of crack propagation in a standard smooth specimen must be investigated to calculate the crack growth life. The ANSYS Separating Morphing and Adaptive Remeshing Technology (SMART)

was employed to simulate the crack growth of specimens. In the SMART crack growth method, the crack front is re-meshed during crack propagation. This method has previously been applied successfully by Ignatijev (2022), Doğan and Yuce et al. (2021), Lee and Lu (2022). In the SMART crack growth method, a local coordinate system is located at the crack surface, to determine the crack front, top, and bottom surfaces. The elements at the front of the crack tip are re-meshed during crack growth and the stress intensity factor with crack extension for each step can be determined. The dimensions of the standard smooth specimen are obtained according to ASTM E466 (2006), which are shown in Figure 2.

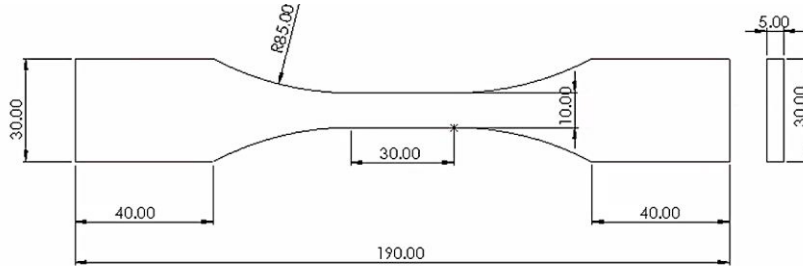


Fig.2. Standard smooth fatigue test specimen

The model was subject by fully reversed forces from 10.5kN to 11.4kN. The relationship of range of stress intensity factor K and crack growth length a can be obtained by SMART. The stress intensity factor (SIF) for one step is shown in Figure 3.

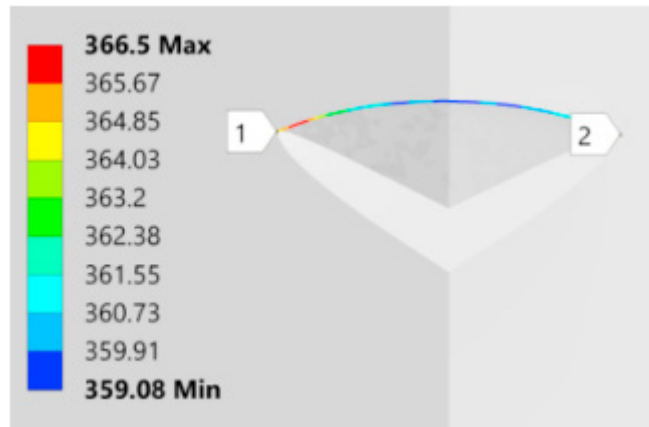


Fig.3. An example of stress intensity factor ($\text{MPa}\sqrt{\text{mm}}$) on SMART crack growth

The crack tip SIFs and their associated crack extensions are shown in Figure 4. Continuous curves describing the relationship are defined by 5th order polynomial interpolation for each load:

$$\Delta K = f(a) = A_0 + A_1a^1 + A_2a^2 + A_3a^3 + A_4a^4 + A_5a^5 \tag{5}$$

where, A_0 to A_5 are constant polynomial coefficients. The crack growth life can then be calculated from integral function:

$$N_g = \int_{a_0}^{a_f} \frac{1}{C(f(a))^m} \tag{6}$$

where, a_0 and a_f are crack initial length and final crack length.

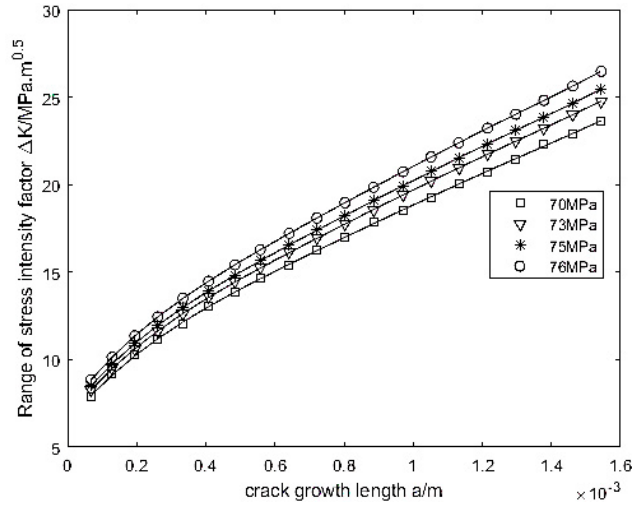


Fig.4. Range of stress intensity factor with crack growth length in smooth specimen

As the Paris law parameters are dependent on $R = 0$, the Walker Equation (Dowling, Calhoun et al. 2009) is applied to calculate the equivalent stress intensity factor (ΔK_{eqv}) from the range of SIFs shown in Figure 4, for fully reversed load $R = -1$.

$$\Delta K_{eqv} = \frac{\Delta K}{(1 - R)^{1-\gamma}} \tag{7}$$

The results for ΔK_{eqv} are shown in Figure 5. The crack growth life of standard smooth specimens is shown in Figure 6, where the results are also approximated by exponential function as:

$$\log(N_g) = 7.39 - 0.00824S_a \tag{8}$$

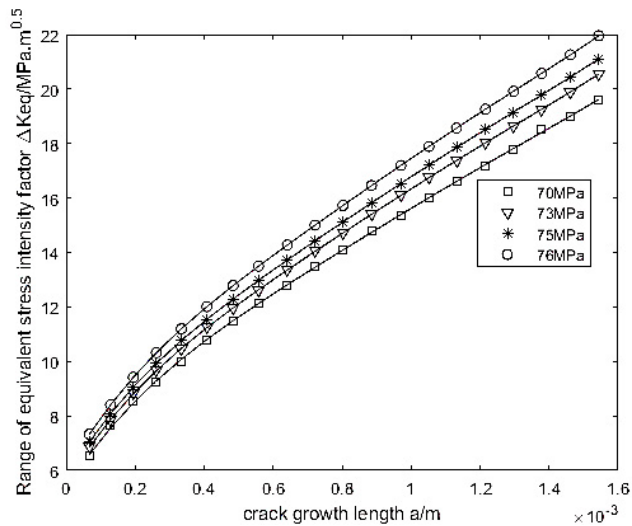


Fig.5. ΔK_{eqv} with crack growth length

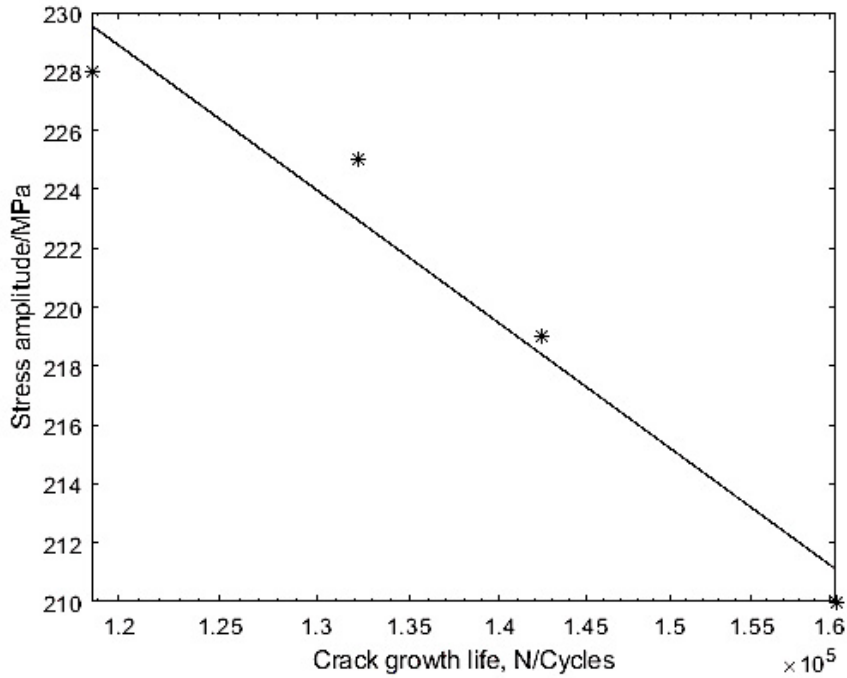


Fig.6. Crack growth life of smooth specimen

The high cycles crack initiation life for the standard model with stress amplitude can then be calculated as:

$$N_i = N_{tot} - N_g = 10^{16.9-0.0497S_a} - 10^{6.63-0.00678S_a} \tag{9}$$

4. Calculation of crack initiation life with induced residual stress

Inducing compressive residual stress in structures through processes such as autofrettage, deep rolling, shot peening, etc. is a commonly used method of increasing fatigue life. These processes are applied to a structure before it is subject to working loads. The induced compressive residual stress increases the fatigue life by reducing the mean stress. Therefore, to calculate the crack initiation life by (9) with induced residual stress, the distributions of stress amplitude, mean stress and stress ratio must be obtained.

The double-notched specimens designed by Okorokov (2018) shown in Figure 7 was used to determine the influence of residual stress. The specimens are subjected to a 75kN single overload and 21kN, 22kN, 23kN and 25kN working force amplitudes. A one-eighth of the specimen with appropriate symmetry boundary conditions is modelled in ANSYS, with a refined mesh at the notch root. The stress amplitude with 21kN applied force amplitude and the stress amplitude along a path starting from the notch root are shown in Figure 8.

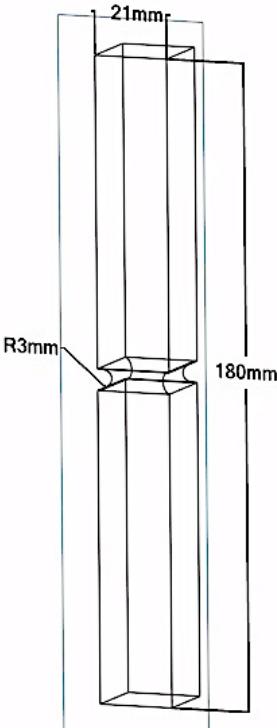


Fig.7. Double-notched specimen

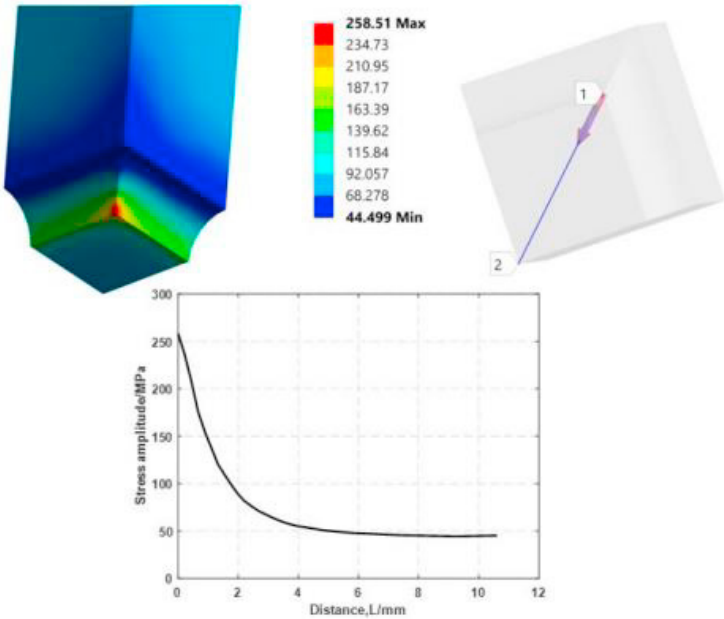


Fig.8. Stress amplitude (MPa) result with 21kN force amplitude

In Okorokov’s work, the modified critical distance method was applied to calculate a nominal or average stress amplitude for the load cycle. The average stress amplitudes with different working force amplitudes summarized in Table 3 are substituted into (9) to obtain crack initial life.

Table 3 Predicted crack initiation life

Force amplitude/kN	21	22	25
Equivalent Stress amplitude/MPa	219.8	220.1	223.2
Crack initiation life/Cycles	8.081×10^5	7.769×10^5	5.103×10^5

5. Calculation of crack growth life with induced residual stress

The residual stress distribution without the crack shown in Figure 9 was obtained by FEA and substituted into fracture simulation to calculate the stress intensity factor for the residual stress.

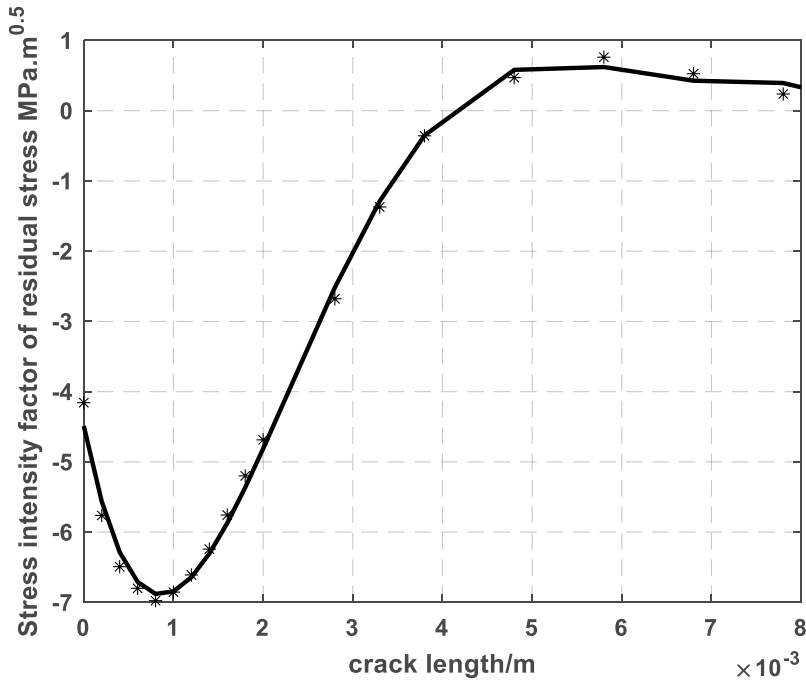


Fig.9. Krs with crack growth length

The range of applied stress intensity factor was obtained by the SMART crack growth method. An elliptical initial crack was located at the high stress concentration area as shown in Figure 10. After simulation, the process of crack growth was investigated and the results of K_{app} corresponding with crack extension length are shown in Figure 11.

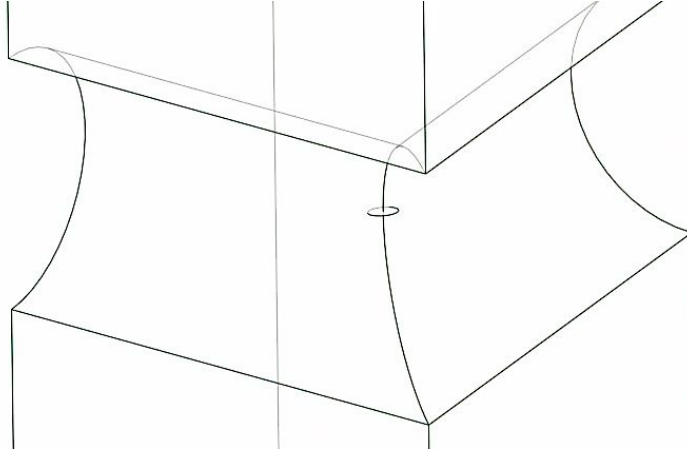


Fig.10. Crack initiation of double-notched specimen

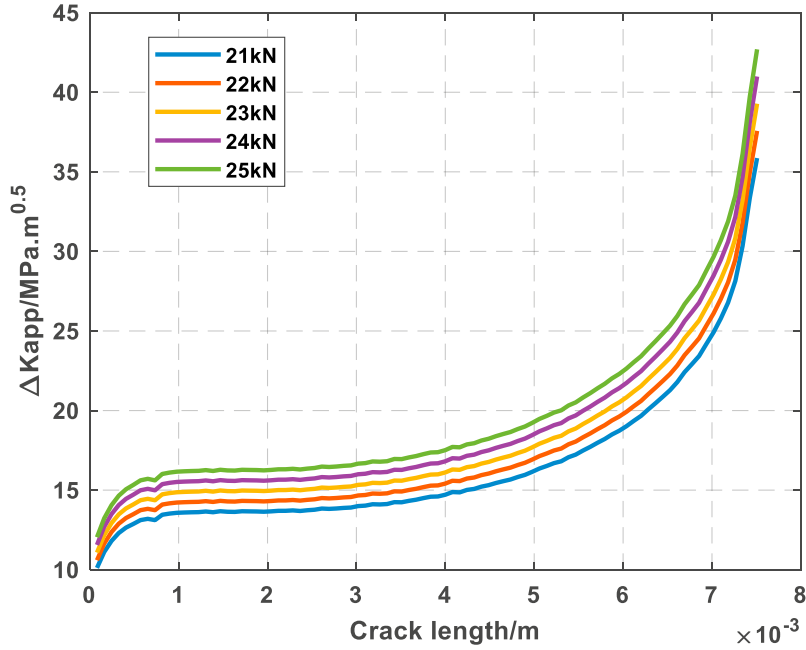


Fig.11. Kapp with crack growth length

Based on superposition method, equation (10) with induced residual stress can be derived from (6) as:

$$N = \int_{a_0}^{a_f} \frac{1}{C \left(\frac{\Delta K}{(1-R)} \right)^m} = \int_{a_0}^{a_f} \frac{1}{C \left(\frac{K_{app}}{\left(1 - \frac{K_{rs}}{K_{app} + K_{rs}} \right)} \right)^m} \quad (10)$$

The crack growth life with induced residual stress is calculated from (10). The total life with induced residual stress is then determined by adding the crack growth life and crack initiation life. The calculated total predicted life for the three working loads is shown in Table 4.

Table 4 Predicted crack growth life and total life

Force amplitude	21kN	22kN	25kN
Crack growth life	7.092×10^5	5.600×10^5	3.059×10^5
Total life	1.517×10^6	1.337×10^6	8.162×10^5

The fatigue results obtained by fracture method are compared with the experimental results in Figure 12.

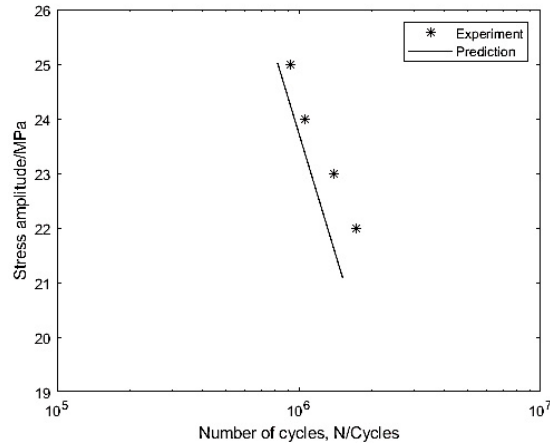


Fig.12. Experimental fatigue life results with predictions of numerical fracture method

These results show good agreement between experiment and prediction, indicating that the numerical fracture method based on ANSYS SMART crack growth simulation is suitable for calculating high cycle fatigue life with induced residual stress.

6. Conclusions

A $S-N_i$ equation was proposed based on an initial crack length of 0.2mm. The equivalent stress amplitude with induced residual stress can be substituted into this equation to determine the crack initiation life. The process of crack propagation was simulated by ANSYS SMART crack growth method and the relationship of applied stress intensity factors with crack extension length was described by polynomial equation. Through the superposition method and numerical method, the influence of compressive residual stress on K_{app} was represented by the ratio of SIFs, which can be correlated to the Paris law to calculate the crack propagation life. The results of total life can be determined by adding crack initiation life and growth life. Through comparing these results with previous experimental results, the numerical fracture method employed in this paper was shown have good accuracy on predicting the high cycle fatigue life with induced residual stress.

Reference

Dinda, S. and D. Kujawski (2004). "Correlation and prediction of fatigue crack growth for different R-ratios using K_{max} and $\Delta K+$ parameters." *Engineering Fracture Mechanics* 71(12): 1779-1790.

Doğan, O., et al. (2021). "Effects of rim thickness and drive side pressure angle on gear tooth root stress and fatigue crack propagation life." *Engineering Failure Analysis* 122: 105260.

- Dowling, N., et al. (2009). "Mean stress effects in stress - life fatigue and the Walker equation." Fatigue & Fracture of Engineering Materials & Structures **32**(3): 163-179.
- Dowling, N. E., et al. (2013). Mechanical Behavior of Materials: Engineering Methods for Deformation, Fracture, and Fatigue, Pearson.
- Elber, W. (1971). "The significance of fatigue crack closure."
- Huang, J.-Y., et al. (2006). "High-cycle fatigue behavior of type 316L stainless steel." Materials transactions **47**(2): 409-417.
- Ignatijev, A., et al. (2022). "Fatigue crack initiation and propagation in a PM-gear tooth root." Engineering Failure Analysis **138**: 106355.
- LaRue, J. and S. Daniewicz (2007). "Predicting the effect of residual stress on fatigue crack growth." International Journal of Fatigue **29**(3): 508-515.
- Lee, Y.-B., et al. (1998). "Effects of redistributing residual stress on the fatigue behavior of SS330 weldment." International Journal of Fatigue **20**(8): 565-573.
- Lee, Y. F. and Y. Lu (2022). "Advanced numerical simulations considering crack orientation for fatigue damage quantification using nonlinear guided waves." Ultrasonics **124**: 106738.
- Ma, Y. E., et al. (2011). "Size effects on residual stress and fatigue crack growth in friction stir welded 2195-T8 aluminium–Part II: Modelling." International Journal of Fatigue **33**(11): 1426-1434.
- Mlikota, M., et al. (2017). Numerical determination of Paris law constants for carbon steel using a two-scale model. Journal of Physics: Conference Series, IOP Publishing.
- Okorokov, V., et al. (2018). "High cycle fatigue analysis in the presence of autofrettage compressive residual stress." Fatigue & Fracture of Engineering Materials & Structures **41**(11): 2305-2320.
- Taylor, D. (2008). "The theory of critical distances." Engineering Fracture Mechanics **75**(7): 1696-1705.