

Measuring Nothing

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Measurement is integral to quantum information processing and communication; it is how information encoded in the state of a system is transformed into classical signals for further use. In quantum optics, measurements are typically destructive, so that the state is not available afterwards for further steps. Here we show how to measure the presence or absence of the vacuum in a quantum optical field without destroying the state, implementing the ideal projections onto the respective subspaces. This not only enables sequential measurements, useful for quantum communication, but it can also be adapted to create novel states of light via bare raising and lowering operators.

I. INTRODUCTION

At first glance, measuring the vacuum is trivial, a perfect photodetector will reveal the vacuum state upon the non-occurrence of a click. However, the converse result, i.e. not measuring the vacuum, ideally should preserve the information in the non-vacuum sector for further interrogation - something which is difficult to achieve with direct photodetection. Formally, we would like to implement the following measurement projectors, $\{|0\rangle\langle 0|, \mathbb{I} - |0\rangle\langle 0|\}$, where the latter non-vacuum outcome removes the vacuum component without affecting the relative amplitudes or coherences of the other Fock states. This is crucial for sequential measurement schemes, such as in [1–3], and rules out other projective schemes such as quantum nondemolition measurements of photon number [6, 7]. The development of practical methods for non-destructive measurements on optical fields [4, 5] is therefore an important topic for future practical quantum information processing systems.

Measurement is also a key element in performing non-Gaussian operations, e.g. for entanglement purification of continuous variable states. Recent examples include the implementation of the quantum optical creation and annihilation operators, both of which rely on postselection [8–10]. Extending the type of possible operations is crucial for the production of tailored states in quantum information systems. Our method can be simply extended to provide a first realization of the bare photon addition and subtraction operators.

We consider a single mode of an optical cavity in an arbitrary quantum state, ρ , as our system to be measured. To perform the measurement, we introduce a probe which consists of a three level atom in the Λ -configuration (See Fig. 1a). The cavity mode can be controllably coupled to transition B whereas transition A interacts with an externally applied laser field [11, 12]. In these papers the general adiabatic mapping of atomic levels to cavities was

introduced. Our particularly simple configuration is insensitive to all field amplitudes other than the vacuum.

The Hamiltonian of the combined system can be written in the rotating wave approximation as

$$H^{RWA} = \hbar\Delta|e\rangle\langle e| + \hbar\gamma_A(t)(|e\rangle\langle g| + |g\rangle\langle e|) + \hbar\gamma_B(t)(|e\rangle\langle g'|a + |g'\rangle\langle e|a^\dagger), \quad (1)$$

where the coupling constants γ_A and γ_B between the atom and the two fields depend on the strength of the respective fields at the point where the atom is located. An optional detuning Δ can be applied to both fields in order to suppress single-photon resonance effects as long as we maintain the two-photon resonance condition,

$$E'_g - E_g = \hbar(\omega_B - \omega_A). \quad (2)$$

The situation is similar to the V-STIRAP scheme for producing single photons [13] where a cavity evolves from $|0\rangle \rightarrow |1\rangle$ through a dark state adiabatic evolution of an atom $|g\rangle \rightarrow |g'\rangle$.

In our measurement procedure, we run the V-STIRAP sequence in reverse: the initial state of the atom is $|g'\rangle$, and the order of the A and B couplings is switched (see Fig. 1b). If the cavity field initially contains at least one photon, at the end of the sequence the atom is left in $|g\rangle$ and the field has one photon subtracted. However, if the cavity was originally in the vacuum state, the atom stays in $|g'\rangle$ and the cavity is left unchanged. An initial superposition of the cavity evolves as

$$|g'\rangle \sum_{n=0}^{\infty} \alpha_n |n\rangle \rightarrow |g'\rangle \alpha_0 |0\rangle - |g\rangle \sum_{n=1}^{\infty} \alpha_n |n-1\rangle. \quad (3)$$

The state of the atom is now entangled with that of the cavity. By measuring the atomic state in either $|g\rangle$ or $|g'\rangle$, we have determined whether the initial cavity state had at least one photon or none. By coherent rotations of the ground states before a population measurement, projections onto more general subspaces are also possible.

If the atom is found in $|g\rangle$, the field amplitudes have been shifted by one. The ideal projection ($\mathbb{I} - |0\rangle\langle 0|$) results if we replace the subtracted photon, this is simply

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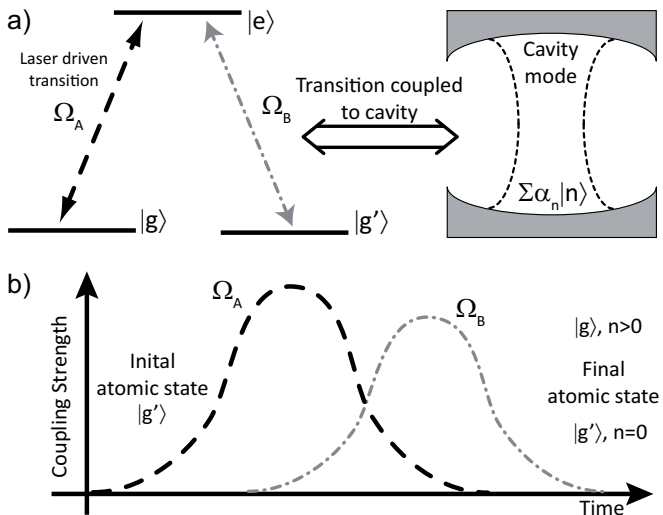


FIG. 1: a) Lambda atomic system coupled to cavity. A three level atom with two ground states ($|g\rangle$, $|g'\rangle$) and a single excited level ($|e\rangle$) can be used to probe for the vacuum component of an optical field with annihilation operator a . The A transition is driven by a STIRAP laser and the B transition can be controllably coupled to the cavity mode to be measured. Initially, the atom is in the state $|g'\rangle$. The final state of the atom $|g\rangle$ or $|g'\rangle$ depends on the presence or absence of photons in the field respectively. b) Counter-intuitive pulse sequence for the couplings. Consider the case where there are $n \geq 1$ photons in the field. With the initial state of the combined atom-mode system $|g', n\rangle$, coupling A is turned on first. As coupling B is slowly increased, the atom-cavity state adiabatically follows the dark-state manifold $\sin\theta|g, n-1\rangle - \cos\theta|g', n\rangle$, $\theta = 0 \rightarrow \pi/2$. Coupling A is now turned off, followed by coupling B, leaving the final state of the system as $|g, n-1\rangle$. If the cavity was originally vacuum ($n = 0$), the final state of the atom remains as $|g'\rangle$.

achieved by running the V-STIRAP procedure forwards. Note that this does not require the initial cavity state to be vacuum, we can add a photon to an arbitrary state of the field. As discussed later, the shifting property of the procedure can be exploited to perform novel operations and generate non-classical states of light.

The key aspect of the adiabatic process is that the evolution of the system does not rely on the dynamics of the Hamiltonian, provided that the conditions of adiabatic transition are satisfied. In this way, the state of the ancilla atom can be made asymptotically insensitive to the cavity photon number, except for the critical case of the vacuum. In the usual Jaynes-Cummings scenario, the dynamics in each of the combined Fock subspaces proceeds at a rate proportional to the square root of photon number, leading in general to different states of the atom. In our scheme, the atom does not distinguish between different photon numbers $n = 1, 2, 3, \dots$, which allows us to perform the ideal projection onto the complement of the vacuum, in contrast to previous proposals for quantum non-demolition measurements of the optical field [6, 7].

We can extend the method to project onto the joint

n -mode vacuum state or complement, as required in the decoding scheme of [2]. This requires a probe atom with $n + 2$ levels in an $(n + 1)$ -pod configuration. Let $|g_0\rangle$ denote the initial state of the atom, the remaining ground states be denoted $|g_n\rangle$ for $n = 1, \dots, n$, and $|e\rangle$ be the excited level. The $|e\rangle - |g_n\rangle$ ($n > 0$) transitions are driven by lasers with strength Γ_j and the $|e\rangle - |g_0\rangle$ transition is selectively coupled to each of the n -modes in turn with strength γ_j . In real atoms, this may be difficult but it may be simpler in engineered systems, e.g. superconducting qubits coupled to transmission lines. We apply in turn the same procedure as for the single mode measurement by sequential pairwise adiabatic variation of $\{\Gamma_j, \gamma_j\}$, $j = 1, \dots, n$, after which the population of $|g_0\rangle$ is determined. If the atom is detected in $|g_0\rangle$, then the n -modes are projected onto the joint vacuum state $|00\dots 0\rangle$, otherwise the atom and n -modes are left in a (generally entangled) state where the $|g_0\rangle|00\dots 0\rangle$ state has been truncated. To disentangle the atom and add back subtracted photons, running the sequence of couplings backwards and in reverse order returns the atom to $|g_0\rangle$ which erases any information of the photon number distribution of the n -modes.

II. IMPLEMENTATION

The experimental setup for V-STIRAP [13] can be adapted to perform our vacuum measurement (Fig.2). We use a cavity with a long storage time to reduce leakage and decoherence. We also introduce preparation and readout zones for the atom. The motion of the atom is reversed in the case of measuring the atom in $|g\rangle$ in order to replace a subtracted photon. There are several experimental challenges, mainly the lifetime of the field compared to the time required to implement the measurement. The cavity field must last long enough for the atom to be adiabatically transported, measured, and returned. The damping rate of the cavity and atom-cavity coupling are both highly dependent upon the effective mode volume of the cavity and balancing these factors will be system dependent. To give an indication of the performance of the protocol under non-ideal condition, we have simulated the measurement of a lossy cavity with finite sweep times, the results displayed in Fig.3.

III. APPLICATIONS

A straightforward application of this measurement is in sequential decoder schemes as discussed in [3]. In the protocol of [2], the state of a n -mode system has to be identified. The state is taken from an ensemble of products of coherent states, $\{|\alpha_1^k, \alpha_2^k, \dots, \alpha_n^k\rangle\}$. A sequence of displacements and projections onto the n -mode vacuum or its complement has been shown to decode the message successfully in the $n \rightarrow \infty$ limit as long as the rate of transmission is below the Holevo bound.

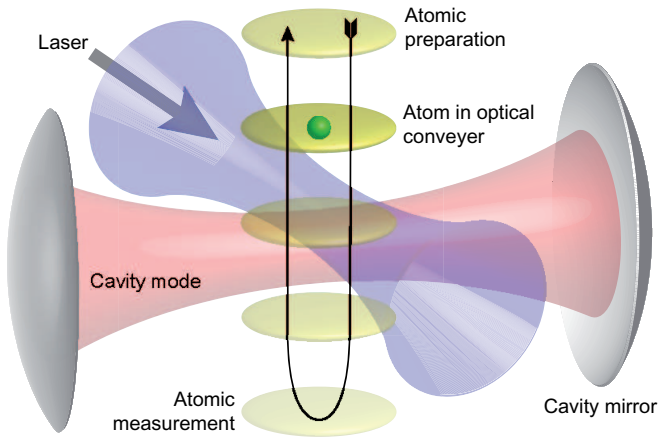


FIG. 2: Cavity QED vacuum measurement. The configuration of the cavity mode, driving laser and trajectory of the atom are similar to the V-STIRAP scheme, except that the laser beam is encountered before the cavity mode. An optical lattice traps and controls the position, hence coupling, of the atom with both the driving laser and cavity mode. State preparation in $|g'\rangle$ is performed before the atom is transported into the cavity. After the atom has crossed the cavity and adiabatically interacted with laser and mode, it is measured, e.g. by fluorescence shelving [14] or cavity enhanced detection [15], to discover which ground state it is in. To perform the ideal $(\mathbb{I} - |0\rangle\langle 0|)$ operation in the case of the $|g\rangle$ result, the motion of the atom is reversed in order to replace the photon extracted from the cavity.

We can also use the photon number altering properties of our procedure to enact bare raising and lowering operations, in contrast to the creation a^\dagger and annihilation operators a as usually considered. The non-Hermitian a^\dagger and a operators represent non-Gaussian operations and have been realized probabilistically in experiments [8–10]. “Subtracting” a photon from squeezed light can produce an approximate Schrödinger cat state [16–19], and both processes have been used in super-optimal optical amplification protocols [20–23].

However, the a^\dagger and a operators do not simply add and subtract photons, but also Bose condition the state. Pure addition and subtraction of photons are represented by bare raising and lowering operators [24] (sometimes known as photon number shifting operators [25])

$$E^+ = \sum_{n=0}^{\infty} |n+1\rangle\langle n|, \quad E^- = \sum_{n=1}^{\infty} |n-1\rangle\langle n|. \quad (4)$$

These can produce nonclassical states of light; for example any state which has E^+ applied to it must violate the Klyshko criterion [25]. Applying E^+ to a coherent state produces a state with subpoissonian statistics, whereas applying E^- makes it superpoissonian.

There has been little study of the bare operators and their effects, mainly because they have not been realized experimentally [26]. Implementing E^+ and E^- requires cancellation of the \sqrt{n} Bose enhancement factors inherent in \hat{a} and \hat{a}^\dagger . The nature of the $(\mathbb{I} - |0\rangle\langle 0|)$ projection

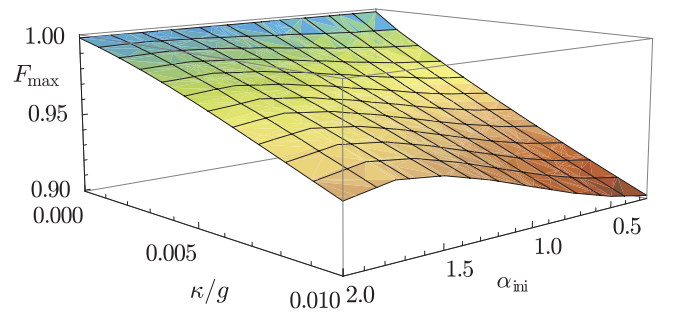


FIG. 3: Fidelity of the full measurement procedure, conditioned on the detection of the atom in $|g\rangle$. The initial cavity field is in a coherent state of amplitude α_{ini} . The coupling constants γ_A and γ_B were modulated as $\cos^2 t$ and $\sin^2 t$, respectively, and their amplitudes were in a 2 : 1 ratio. The detuning Δ was zero and the decay rate of the $|e\rangle$ state was 1% of the maximum cavity coupling $g = \max \gamma_B$. The cavity loss parameter κ is also plotted relative to g . Assuming the knowledge of the cavity properties, the fidelity optimizing time of transition was chosen in each data point. We note that assuming an educated guess for the initial mean photon number of the state, the point of maximum fidelity at a constant loss may be shifted along the α axis by altering the intensity of the laser field.

and the adiabatic process that we have described does not alter the relative weights of the amplitudes corresponding to different photon numbers, in contrast to other schemes which rely on a^\dagger . The V-STIRAP process implements E^+ and the reverse process realises E^- . In addressing the problem of quantum optical phase the measurement of moments of bare operators was proposed using a basic scheme similar to that considered here, without a detailed analysis of the effect of reachable experimental parameters [27]. We will explore the detailed consequences of this elsewhere.

We can further exploit the conditional dynamics preserving the relative amplitudes for all non-zero number states to implement a reverse quantum scissors [28]. In the original quantum scissors scheme a general superposition of photon numbers has photon numbers higher than 1 removed, without altering the zero and one photon amplitudes. The scheme has been theoretically extended to make the cut at higher photon numbers [29, 30]. By successive application of the measurement without photon replacement n times, we can truncate the first n amplitudes of a state, conditioned on not observing the vacuum. By adding n photons, we return the state to its original form but without the first n terms (Fig.4). The probability that this will occur is $1 - \sum_{k=0}^{n-1} P_k$ where P_k is the probability of observing k photons. Similarly, we also can simply use the protocol to perform photon number resolving measurements.

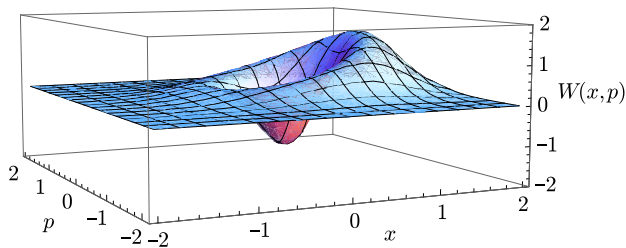


FIG. 4: Wigner function of a vacuum-stripped $|\alpha = 1\rangle$ coherent state. In the protocol a loss rate of $\kappa = 0.005g$ and other parameters as specified in Fig. 3 were used. It shares all the features of the idealised output state, including negativity. This state is obtained from $|\alpha = 1\rangle$ with a probability of 0.61. The probability of measuring the vacuum $|\langle 1|0\rangle|^2$ is 0.37, and a 2% error is caused by photon loss during the process. The fidelity is 96% compared with $(\mathbb{I} - |0\rangle\langle 0|)|\alpha = 1\rangle$.

IV. CONCLUSION

The ability to implement ideal projections on a field opens up new possibilities for quantum communication and computation. Adiabatic evolution in our method avoids the \sqrt{n} factors in dynamical schemes and achieves the unusual nonlinearity required. Though it will be challenging to engineer systems with the requisite long storage times with strong coupling, recent advances in microcavities as well as microwave and nanomechanical systems give grounds for optimism. The simplicity and utility of the system described here for implementing several quantum optical information protocols should be significant drivers towards this goal.

Appendix A: Technical Details

To see why the method works, we examine the form of the rotating-wave Hamiltonian for the atom-field system,

$$H^{RWA} = \hbar\Delta|e\rangle\langle e| + \hbar\gamma_A(t)(|e\rangle\langle g| + |g\rangle\langle e|) + \hbar\gamma_B(t)(|e\rangle\langle g'| + |g'\rangle\langle e|) \quad (A1)$$

Firstly, we note that for any $n \in \mathbb{N}$, the subspace $\{|g, n-1\rangle, |e, n-1\rangle, |g', n\rangle\}$ is coupled together by H^{RWA} . Other states are not coupled to this triplet, and this simplifies the dynamics significantly. Together with the linear span of the vector $|g', 0\rangle$, which itself is an eigenspace corresponding to the eigenvalue 0, these subspaces allow for a decomposition of the whole Hilbert space describing the coupled system of the atom and the cavity. Within each of the three-dimensional subspaces, we can identify three nondegenerate eigenenergies: 0 and $\frac{\hbar}{2}(\Delta \pm \sqrt{\Delta^2 + 4\gamma_A^2(t) + 4n\gamma_B^2(t)})$. The eigenstates corresponding to zero energy are, up to normalisation and phase factors, $\sqrt{n}\gamma_B(t)|g, n-1\rangle - \gamma_A(t)|g', n\rangle$, along with the special case of $|g', 0\rangle$, noted above.

In order to study the deviations from an ideal adiabatic transition, we solve the time-dependent Schrödinger equation. For a given n , we denote

$$\theta(t) = \arctan \frac{\sqrt{n}\gamma_B(t)}{\gamma_A(t)} \quad (A2)$$

and

$$\nu(t) = \sqrt{\gamma_A^2(t) + n\gamma_B^2(t)}. \quad (A3)$$

In this notation, the dark state can be expressed as

$$|a(t)\rangle = \sin\theta(t)|g, n-1\rangle - \cos\theta(t)|g', n\rangle. \quad (A4)$$

We rewrite the equation of motion in an orthonormal basis consisting of the vectors $|a(t)\rangle$,

$$|b(t)\rangle = \cos\theta(t)|g, n-1\rangle + \sin\theta(t)|g', n\rangle, \quad (A5)$$

and $|e\rangle$.

Resolving the instantaneous state as

$$|\psi(t)\rangle = \alpha_a(t)|a(t)\rangle + \alpha_b(t)|b(t)\rangle + \alpha_e(t)|e\rangle, \quad (A6)$$

we substitute this into the Schrödinger equation to find the equations of motion for $\alpha_i(t)$,

$$\begin{aligned} \dot{\alpha}_a &= \dot{\theta}\alpha_b, \\ \dot{\alpha}_b &= -\dot{\theta}\alpha_a - i\nu\alpha_e, \\ \dot{\alpha}_e &= -i\nu\alpha_b - i\Delta\alpha_e. \end{aligned} \quad (A7)$$

If the system begins in the dark state the initial conditions are $\alpha_a(0) = 1$, $\alpha_b(0) = \alpha_e(0) = 0$. The equations are best solved in terms of the projective space coordinates $\kappa_b = \alpha_b/\alpha_a$ and $\kappa_e = \alpha_e/\alpha_a$, where they become

$$\begin{aligned} \dot{\kappa}_b &= -\dot{\theta} - i\nu\kappa_e - \dot{\theta}\kappa_b^2, \\ \dot{\kappa}_e &= -i\nu\kappa_b - i\Delta\kappa_e - \dot{\theta}\kappa_b\kappa_e. \end{aligned} \quad (A8)$$

The last pair of equations can be solved asymptotically. For this purpose, we denote the complete time of transition T , such that all of $\dot{\theta}$, $\dot{\theta}/\theta$ and $\dot{\nu}/\nu$ are upper bounded by a constant multiple of T^{-1} . Also, we denote ν_0 the minimum modulus of the lower of the nonzero eigenvalues of H^{RWA} (restricted to the given subspace) reached during the transition. This value is also a lower bound for both of $\nu(t)$ and $\nu^2(t)/\Delta$. We use these inequalities to find that

$$\begin{aligned} \kappa_b(t) &= -i\frac{\dot{\theta}(t)\Delta}{\nu^2(t)} + O((\nu_0 T)^{-2}), \\ \kappa_e(t) &= i\frac{\dot{\theta}(t)}{\nu(t)} + O((\nu_0 T)^{-2}). \end{aligned} \quad (A9)$$

These formulas allow us to express

$$\alpha_a(t) = \exp\left(-i\Delta\int_0^t \frac{\dot{\theta}^2(t)}{\nu^2(t)} dt\right) + O((\nu_0 T)^{-2}). \quad (A10)$$

If the spatial distributions of intensity of the two light modes are smooth functions of coordinates, the time derivative of $\theta(t)$ for both $t = 0$ and $T = 0$ is zero. It immediately follows that the relative contributions of the states orthogonal to the desired final state, as given by Eq. (A9), vanish at the end of the transition, leaving only the asymptotic term. The probability of the diabatic transition is in turn $O((\nu_0 T)^{-4})$ and can be pushed arbitrarily close to zero by choosing a sufficiently long time T . The resulting state obtains a phase shift of

$$\phi = -\Delta \int_0^T \frac{\dot{\theta}^2(t)}{\nu^2(t)} dt + O((\nu_0 T)^{-2}), \quad (\text{A11})$$

which itself scales as $O((\nu_0 T)^{-1})$ and also has a limit of zero for $T \rightarrow \infty$.

The dependence of the results on the photon number n manifests itself through the constant

$$\nu_0 = \min_{t \in (0, T)} \left(\sqrt{\left(\frac{\Delta}{2}\right)^2 + \gamma_A^2(t) + n\gamma_B^2(t)} - \frac{\Delta}{2} \right). \quad (\text{A12})$$

This formula suggests that the scalings of both the probability of error and the phase ϕ are actually the more favourable the higher n one operates in. Studying the worst case, i.e., $n = 1$, we find that the transition is optimal if the two light modes overlap in such a way that the effective beginning or end of either one coincides with the point of maximal intensity of the other one.

We finish the analysis by noting that the state $|g', 0\rangle$ remains unchanged during the whole transition and obtains no phase shift due to the fact that it corresponds to exactly zero energy.

As a result, any linear superposition

$$\sum_{n=0}^{\infty} \alpha_n |g, n\rangle \quad (\text{A13})$$

evolves under ‘‘forward’’ STIRAP to

$$\sum_{n=0}^{\infty} \alpha_n |g', n+1\rangle, \quad (\text{A14})$$

in a deterministic manner, up to correction terms of order $(\nu_0^{(min)} T)^{-1}$. Similarly, letting the atom encounter the laser field A first, any state of the form

$$\sum_{n=0}^{\infty} \alpha_n |g', n\rangle \quad (\text{A15})$$

evolves into

$$\alpha_0 |g', 0\rangle + \sum_{n=1}^{\infty} \alpha_n |g, n-1\rangle. \quad (\text{A16})$$

In the case of a cavity in a mixed state, the corresponding operation is performed on each element of its Schmidt decomposition.

Appendix B: Experimental Considerations

For the implementation of the measurement, this requires that the system being measured does not significantly evolve over the timescale required to perform the adiabatic evolutions and atomic measurement. This will be a considerable experimental challenge, though advances in cavity QED have led to high coupling strengths compared to loss rates which are the dominant sources of decoherence.

1. Cavity lifetime

Very high Q-factors have been achieved in optical resonators, of the order of 3.5×10^{12} for large supermirror cavities [31], and approaching 10^{11} for Fabry-Perot cavities [32]. In addition, the atom must couple strongly with the cavity mode as this will limit the speed of the atomic transport at which adiabaticity can be maintained. Together with the low damping rate, we require that the atom-cavity system is within the strong coupling regime. Large coupling will be favoured by small mode volumes but this may conflict with the required storage time. Ringdown measurements for QED experiments have shown a decay time of 12.4 microseconds for a 1cm long cavity [33].

a. Cavity-atom coupling

The coupling γ_B limits the rate at which the atom can traverse the cavity. Coupling rates of the order of 10 MHz have been achieved in cavity QED [34], which leads to an adiabatic-safe interaction time on the order of ~ 1 microsecond. For a proof of principle experiment, the measurement of the atom can take far longer than the decay time of the cavity if one is not interested in restoring the subtracted photon. Much higher couplings have been reported for atoms interacting with evanescent field of a toroidal microresonator, of the order of 40 MHz [35].

b. Detection

A standard method of state detection is shelving fluorescence. A long cavity storage time is essential for the measurement as shelving fluorescence takes a finite time, of the order of 100 microseconds for 99.9% fidelity [14] due to the need for the atom to absorb and spontaneously emit photons and for these to be detected. This time could be reduced by increasing the collection solid angle and the probe power.

Faster atomic state detection could be achieved by using another cavity [36]. If the atom is strongly coupled to this detection cavity, this can affect the transmission of the cavity which can be interrogated by a probe beam. In practice, a different mode of the same cavity could

be used for detection. A detection time of 10 microseconds at 97% efficiency has been reported for fibre-coupled cavity detection [37]. Even faster detection has been achieved by combine both fluorescence and cavity coupling, 99.7% in less than 1 microsecond [15]. Detection of atoms coupled to cavities in 250 nanoseconds has also been reported [35].

c. Other physical systems

Photonic cavities may also be a possibility to tune the mode volumes, coupling strengths and system geometry. Whispering gallery modes in microspheres or microtoroids are also possibilities though the required geometry is more complex. Controlling the atomic position and shining the STIRAP laser may be issues. Projected $\gamma_B =$ several 100 MHz, $\kappa < 1$ MHz [38] for such microtoroid cavities. Coupled ion-cavity QED experiments may also be a viable system. The progress on segmented traps and in shuttling [39] allows controlled ion transport

through a cavity field [40].

We note that the same technique can be applied to superconducting systems of artificial three-level atoms [41] and stripline resonators using adjustable couplings [42, 43]. In these systems, very large couplings have been observed (200 MHz [44]). It is calculated that couplings of up to 360 MHz could be achieved in optimized geometries [45]. Cavity loss of $\kappa = 250$ kHz has been achieved in other experiments [46]. Tunable couplings between superconducting elements have been developed ((0–100) MHz) [42, 43]. Measurement of three level systems have been reported in [41].

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