

The same analysis holds true also for infinite cylinders.

We consider now an incident field generated by external source with $f_n \neq 0, f_{n\perp} = 0$. i_n, s_n, f_n are exact solutions of the Maxwell's equations that, for particles with continuously varying surface tangents (particles of class C^1), have Stratton-Chu representation [18]

$$\int [i\omega\mu(n \times S_n^H) - \nabla \times (n \times S_n^E) - \nabla(n \cdot S_n^E)]g(x,y)d\sigma = 0 \quad (13)$$

$$\int [i\omega\mu(n \times F_n^H) - \nabla \times (n \times F_n^E) - \nabla(n \cdot F_n^E)]g(x,y)d\sigma = -F_n^E(x), \quad (14)$$

with μ the magnetic permittivity of the external medium, ω the frequency of light, the outgoing normal, n , the fields F, S and the infinitesimal area $d\sigma$ depend on a point y on the surface of the particle, x is a point inside the particle, ∇ acts on x and $g(x,y,k)$ is the Green function of the scalar Helmholtz equation for the external medium [28]. For x infinitesimally close to the surface, using Eq. (13) and the boundary conditions in Eq. (14) and the conservation of energy we get

$$\begin{aligned} -n(x) \times \left(n(x) \times \int [i\omega\mu(n \times I_n^H) - \nabla \times (n \times I_n^E) - \frac{\varepsilon_i}{\varepsilon} \nabla(n \cdot I_n^E)] g(x,y)d\sigma \right) = \\ \frac{a_n^s}{a_n^i} s_n(x) - i_n(x) \end{aligned} \quad (15)$$

$$W_n = |a_n^i|^2 W_n^i - |a_n^s|^2 W_n^s + \frac{1}{2} \text{Re} \left[\int n \cdot (a_n^i a_n^{s*} i_n^E \times s_n^{H*} + a_n^{i*} a_n^s s_n^E \times i_n^{H*}) d\sigma \right] = 0, \quad (16)$$

where $\varepsilon_i, \varepsilon$ are the dielectric constant of the internal and external media and W_n, W_n^i, W_n^s are the power fluxes of f_n, i_n, s_n [28]. Eq. (15) and its magnetic equivalent are independent complex equations that must be solved up to a factor by a_n^i, a_n^s ; because a_n^i, a_n^s do not depend on x , these equations can be solved only if the dependence on x in all the terms in Eq. (14) can be factored out. When this happens, as for spheres and cylinders, Eq. (16) shows that the condition in Eq. (3) cannot be satisfied by incident fields generated by sources outside the particle.

Finally, the procedure in Eq. (4) can be generalized fixing, for instance, the amplitude of one mode, say $a_1^i = c$, and setting to zero the amplitudes of m internal modes and l scattering modes using $m+l+2$ incident fields that satisfy the following equation

$$\begin{bmatrix} \frac{i_1^i \cdot f^1}{\sin(\xi_1)} & \cdots & \frac{i_1^i \cdot f^{m+l+1}}{\sin(\xi_1)} \\ \vdots & & \vdots \\ \frac{s_l^i \cdot f^1}{\sin(\xi_l)} & \cdots & \frac{s_l^i \cdot f^{m+l+1}}{\sin(\xi_l)} \end{bmatrix} \begin{bmatrix} A_1 \\ \vdots \\ A_{m+l+1} \end{bmatrix} = - \begin{bmatrix} A \frac{i_1^i \cdot f}{\sin(\xi_1)} - c \\ A \frac{i_2^i \cdot f}{\sin(\xi_2)} \\ \vdots \\ A \frac{s_l^i \cdot f}{\sin(\xi_l)} \end{bmatrix}. \quad (17)$$

where f^1, \dots, f^{m+l+1} are arbitrary. The solution for the amplitudes A^1, \dots, A^{m+l+1} is unique if the determinant of the matrix is not null.