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Detection of weak transient broadband signals: Subspace and likelihood ratio test approaches

Cornelius A.D. Pahalson, Louise H. Crockett, Stephan Weiss *

Dept. of Electronic & Electrical Eng, University of Strathclyde, Glasgow, Scotland, UK

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ABSTRACT

We investigate the detection of a weak transient broadband signal, and compare a polynomial subspace detection approach to a likelihood ratio test. The former is based on an analytic eigenvalue decomposition of the array data in order to derive a subspace projection away from stronger stationary sources that obscure the transient signal. An energy detection in the noise-only subspace has been demonstrated to work well in a number of broadband array applications. In this contribution, we aim to explore its comparison to a statistically optimum test, the likelihood ratio test (LRT). The LRT requires more information about the scenario than the subspace test — namely the data covariance due to the transient signal — but can still serve as a suitable benchmark. Somewhat surprisingly, simulation results show that the more dispersive the propagation environment and the weaker the transient signal is compared to any stationary sources, the better it is to base a test — either the LRT or even a simple energy criterion — on the data in the noise-only subspace. This is due to the reduced matrix dimensions and enhanced condition numbers of the involved space-time covariance matrices.

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Figures and tables

Fig. 1. Background in defence: in the presence of stationary users (in green) we would like to detect an emerging transient source (in red) which could represent a mobile phone signal triggering an improvised explosive device. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

* Corresponding author.
E-mail address: stephan.weiss@strath.ac.uk (S. Weiss).

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Fig. 2. Background cognitive radio: in the presence of secondary users (in red), the aim is to detect an emerging primary user (in green), such that secondary users can vacate the frequency band. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

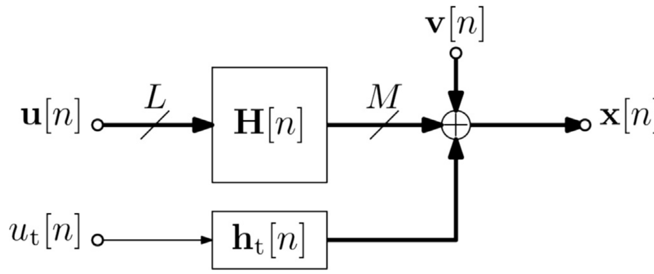

Fig. 3. Signal model. The measurement vector $x[n] \in C^M$ acquired by M sensors, which are illuminated by $L < M$ stationary sources through a convolutive mixing system $H[n]$. This system also models the source power spectral densities, such that the source signal vector $u[n] \in C^L$ contains temporally and spatially uncorrelated Gaussian excitations. The measurement vector is corrupted by additive Gaussian noise $v[n] \in C^M$; a transient signal may be present, modelled the source $u_t[n]$, which contributes to the sensors via a vector of impulse responses $h_t[n]$. Many applications aim to detect such a transient signal [1–16]. In the innovation filter model [17], the system $H[n]$ defines the space-time covariance matrix in the stationary case with the transient signal absent; its z -transform $R(z)$, a parahermitian matrix [18] such that $R^P(z) = (R(1/z^*))^H = R(z)$ admits an analytic eigenvalue decomposition $R(z) = Q(z)\Lambda(z)Q^P(z)$, with an analytic paraunitary matrix $Q(z)$ such that $Q^{-1}(z) = Q^P(z)$ [19–22]. The diagonal matrix $\Lambda(z) = diag(\lambda_1(z), \ldots, \lambda_M(z))$ contains the analytic eigenvalues $\lambda_m(z)$, $m = 1, \ldots, M$. Only L of these eigenvalues will be significant, with the remainder defining the noise-only subspace.

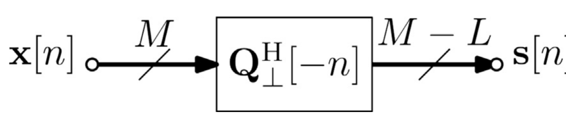

Fig. 4. Subspace projection and syndrome vector. Using algorithms for the analytic eigenvalue decomposition of $R(z)$ [23–27] or for its approximation [28–31], the eigenvectors that correspond to eigenvalues due to noise form the columns of $Q_\perp(z) \circ\!\!-\!\!\bullet Q_\perp[n]$. The latter can be used to project the measurement data $x[n]$ onto the noise-only subspace. The resulting projection is refered to as syndrome vector $s[n]$; it will ideally only contain noise, but if a transient source is present, some of its energy will fall into $s[n]$. Thus, the syndrome energy, which follows a generalised chi-square distribution, forms a surprisingly good discriminator for the presence of a transient source [9], and has been successfully applied to e.g. voice activity detection for weak speakers in the presence of background noise [10], other speakers [11], or for primary user detection [12] in cognitive radio scenarios [13]. This complements related findings that successfully exploit polynomial subspace decompositions in the area of audio processing [32,33].

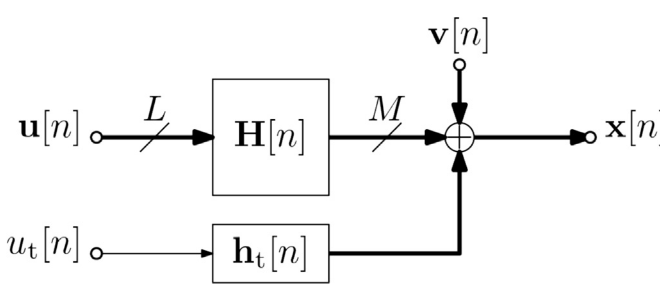

Fig. 5. Simulation parameters for example scenario. We operate with $L = 5$ sources and $M = 8$ sensors; the transient signal, if present, has the same power as the additive noise in $v[n]$. The SNR, the power level between the stationary sources and the noise, is set to either 10 dB or 20 dB. We also vary the order of the convolutive mixing system as $K = 8$ or $K = 16$. The simple syndrome energy detector will be compared to a likelihood ratio test [34,35], which assumes significantly more knowledge of the scenario, since the covariance of the transient signal contribution to $x[n]$ must be known.

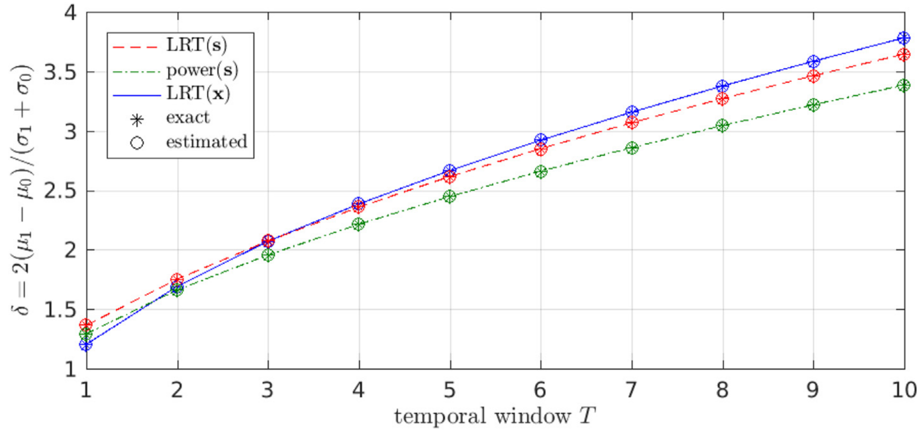

Fig. 6. Simulation results for separability of distributions for moderately strong stationary users and moderate temporal correlation. The stationary users are 10 dB above the noise floor and all signals are observed through convolutive mixing of order $K = 8$. As decision variable, we either use the energy of the syndrome vector, or an LRT test applied to either the measurement vector $x[n]$ or the syndrome vector $s[n]$, and measure a simple separability between the distributions for the decision variable with and without transient signal. The decision variables can be averaged over T subsequent snapshots. For small value of T, methods operating on the syndrome vector perform slightly better due to the temporal decorrelation effect of the subspace projection. As the window T gets large enough, the LRT applied to the measurement vector captures enough of this correlation to perform best. Results marked by circles refer to statistics estimated from 1e5 snapshots of data rather than from the signal model in Fig. 3 [34–36]. The covariance matrices are sufficiently accurate to yield no significant difference to the case where the ground truth is available (marked by asterisks).

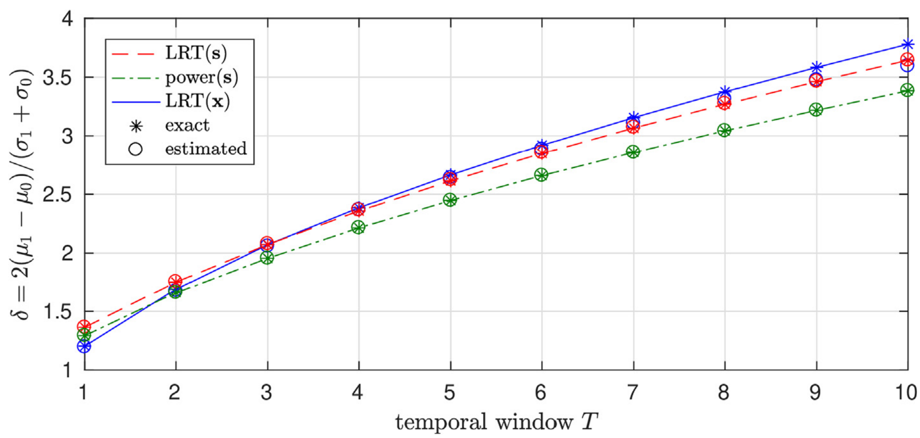

Fig. 7. An experiment which repeats the system parameters from Fig. 6 but here estimated quantities are based on only 1e3 snapshots. The experimental results for the LRT operating on the measurement vector are more sensitive to perturbations by the estimation errors [36–42] particularly as the data window T increases. This is due to the large dimensions and the increasingly poor conditioning of the involved covariance matrices, and as a result, the experimental values (blue circles) start to drop below the theoretical ones (blue asterisks). The experiments based on the subspace projection do not share this sensitive and remain largely unperturbed by the reduced sample size. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

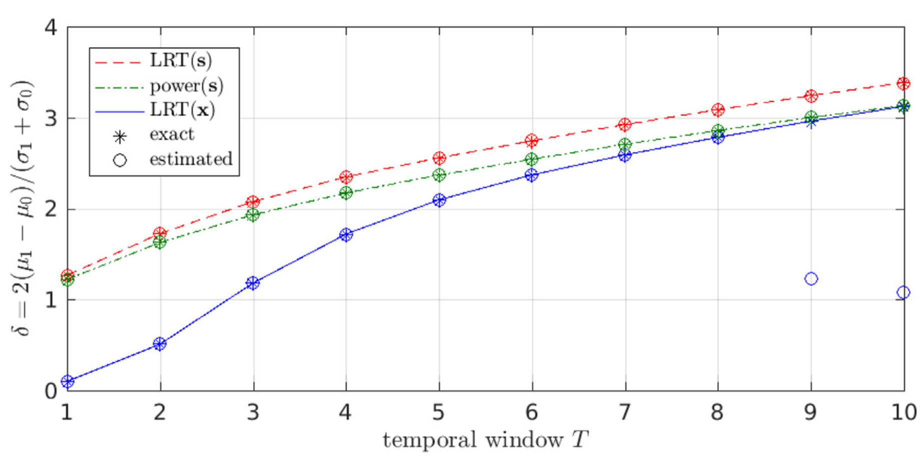

Fig. 8. Simulation results for separation of distributions with stronger stationary users and a more dispersive propagation environment. The stationary users are now 20 dB above the noise and the transient signal, and the order of the convolutive mixing matrices is or $K = 16$, and as in Fig. 6, the graph shows the separability of the distributions over the window T over which the decision variable is averaged. The temporal whitening of the projection in Fig. 4 now leads to an exaggerated advantage for operarting on the syndrome vector for lower values of T. For high values of T, where the LRT requires the inversion of covariance matrices of dimension $MT \times MT$, numerical problems start to surface, and the application to estimated space-time covariances, i.e. a generalised LRT, shows a catastrophic decline in performance.

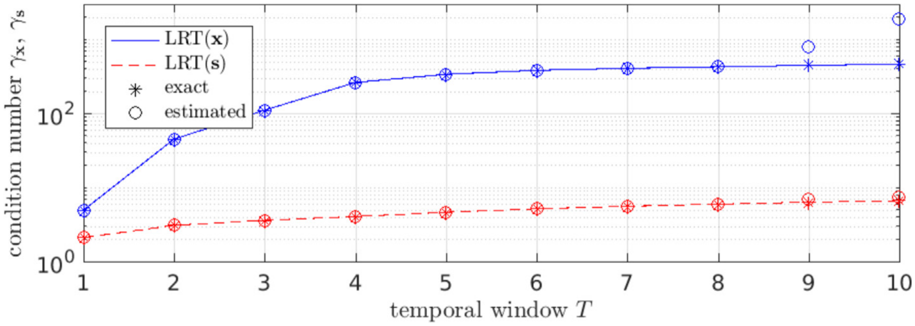

Fig. 9. Condition number for covariance matrices vs temporal window T. Over the window of snapshots over which the decision variable is averaged, we observe the condition number of the $MT \times MT$ covariance matrix required for the LRT and GLRT, as well as the condition number for the LRT/GLRT-internal $(M - LT \times (M - L)T$ covariance matrix when applied to the syndrome vector. Because the covariance matrix of the measurement vector contains the eigenvalues pertaining to the strong stationary sources, the resulting condition number is large. The resulting numerical instability is further emphasised when estimated covariance matrices (circles instead of asterisks) are utilised, justifying the performance drop in Fig. 8 for the LRT applied to the measurement vector for large values of T [43]. In case of applying the LRT/GLRT to the syndrome vector, the dominant eigenvalues relating to the stationary sources are removed.

CRediT authorship contribution statement

Cornelius A.D. Pahalson: Writing – review & editing, Software, Formal analysis. **Louise H. Crockett:** Writing – review & editing, Conceptualization. **Stephan Weiss:** Writing – review & editing, Writing – original draft, Software, Formal analysis, Conceptualization.

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Declaration of interests

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

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Cornelius A.D. Pahalson received his M.Eng degree in Communications Engineering from the Federal University of Agriculture Makurdi, Benue State, Nigeria, 2019. He is a member of the Nigeria Society of Engineers (NSE) and the Institute of Physics (MNIP). He is currently pursuing a Ph.D. degree at the University of Strathclyde Glasgow, United Kingdom. His research interests include signal processing and communications, emphasizing energy detection and communications. For his Ph. D. studies, he receives a scholarship from the Tertiary Education Trust Fund in Nigeria.

Louise H. Crockett was awarded MEng (distinction) and PhD degrees in Electronic and Electrical Engineering, both from the University of Strathclyde, in 2003 and 2008, respectively. She is currently a Senior Teaching Fellow and senior member of the StrathSDR (Strathclyde Software Defined Radio) research team, where she supervises / manages researchers and key sponsored projects. Her core research interests are in the implementation of DSP systems, FPGAs and SoCs, wireless communications, and SDR. Louise has previously co-authored three books on Xilinx/AMD technology. Her teaching focuses on digital systems design targeting FPGAs and SoCs, and builds practical skills to equip graduates for roles in industry.

Stephan Weiss received a Dipl.-Ing. degree from the University of Erlangen-Nürnberg, Erlangen, Germany, in 1995, and a Ph.D. degree from the University of Strathclyde, Glasgow, Scotland, in 1998, both in electronic and electrical engineering. He is professor for signal processing at the University of Strathclyde, Glasgow, following previous academic appointments at both the Universities of Strathclyde and Southampton. His research interests include adaptive, multirate, and array signal processing with applications in acoustics, communications, audio, and biomedical signal processing. Stephan is a member of EURASIP and a senior member of the IEEE. He was the technical co-chair for EUSIPCO 2009 and general chair of IEEE ISPLC 2014, both organised in Glasgow, a special session co-chair for ICASSP 2019, and part of the organising committee for IEEE SSP 2025. He is a subject editor for Elsevier Signal Processing.