A computational study of the local cuts from two-period convex hull closures for big-bucket lot-sizing problems

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Abstract

We study the big-bucket capacitated lot sizing problem with setup times. We use the novel methodology of Akartunali et al. (2014) that exploits two-period relaxations of the formulation in order to generate inequalities that cut-off the optimal solution of the linear programming relaxation. Our approach applies column generation in an unconventional way, with the master problem being a distance minimizing formulation and the subproblems being combinatorial two-period relaxations of the original problem. We identify a lower bound of the dimensionality of the generated cuts and provide extensive computational experiments that show how the generated bounds compare with other state-of-the-art approaches. Our results show that, for certain classes of problems, the bound improvement is considerable.

1 Introduction

The capacitated lot-sizing problem with setup times (CLSP) is a well-studied combinatorial mixed-integer problem of high practical and theoretical importance [7, 6]. The problem is defined over a given discrete and finite time horizon. The goal is to schedule the production amounts of a set of items that has known demands in each time period so that all item demands are covered. Items are produced in a single machine, and a production is possible only after a setup takes place. Each discrete period has finite time capacity, which is consumed by the setup and production processes that take place. Setup operations and production quantities help in inventory carry given, item-specific, fixed and unit costs respectively. The mathematical programming formulation seeks to find a feasible production schedule that minimizes the joint cost of inventory holding and setups. We first define our notation.

Indices and Sets:

NT	Number of periods
NI	Number of items

Variables:

x_t^i	Production quantity of item i in period t
y_t^i	Setup of item i in period t (1 if production occurs, 0 otherwise)
s_t^i	Inventory held of item i at the end of period t

Parameters:

f_t^i	Fixed cost per setup of item i in period t
h_t^i	Holding cost per unit of item i from period t to period $t+1$
d_t^i	Demand for item i in period t
$d_{t,t'}^i$	Total demand from period t to t', i.e., $d_{t,t'}^i = \sum_{\bar{t}=t}^{t'} d_{\bar{t}}^i$
a^i, ST^i	Per-unit processing/setup time for item i

Then, the CLSP formulation can be written as follows:

$$\min \sum_{t=1}^{NT}\sum_{i=1}^{NI} f_t^i y_t^i + \sum_{t=1}^{NT}\sum_{i=1}^{NI} h_t^i s_t^i \tag{1}$$

$$\text{s.t. } x_t^i + s_{t-1}^i - s_t^i = d_t^i \qquad t \in [1,\ldots,NT], i \in [1,\ldots,NI] \tag{2}$$

$$\sum_{i=1}^{NI}(a^i x_t^i + ST^i y_t^i) \leq C_t \qquad t \in [1,\ldots,NT] \tag{3}$$

$$x_t^i \leq M_t^i y_t^i \qquad t \in [1,NT], i \in [1,\ldots,NI] \tag{4}$$

$$y \in \{0,1\}^{NT\text{x}NI}, x \geq 0, s \geq 0 \tag{5}$$

2 Two-period closures and valid inequalities

We introduced in [1] a polytope that can be used to describe subspace relaxation of the CLSP. The structure of the two-period polytope defined over periods $(t, t+1)$ and parametrized by period $k \geq t+1$, hereby referred to as $X_{t,k}^{2PL}$, is as follows:

$$x_{t'}^i \leq M_{t'}^i y_{t'}^i \qquad i = [1,...,NI], t' = t, t+1 \tag{6}$$

$$x_{t'}^i \leq d_{t',k}^i y_{t'}^i + s_k^i \qquad i = [1,...,NI], t' = t, t+1 \tag{7}$$

$$x_t^i + x_{t+1}^i \leq d_{t,k}^i y_t^i + d_{t+1,k}^i y_{t+1}^i + s_k^i \qquad i = [1,...,NI] \tag{8}$$

$$x_{t,k}^i + x_{t+1,k}^i \leq d_{t,k}^i + s_k^i \qquad i = [1,...,NI] \tag{9}$$

$$\sum_{i=1}^{NI}(a^i x_{t'}^i + ST^i y_{t'}^i) \leq C_{t'} \qquad t' = t, t+1 \tag{10}$$

$$x, s \geq 0, y \in \{0,1\}^{2\times NI} \tag{11}$$

We also make a note of the single period relaxations of [3, 4] due to their relevance. We assume that an optimal solution of a linear programming relaxation of CLSP is at hand. For fixed t and k, it is possible to use that solution to construct a point $\overline{p} := (\overline{x}, \overline{y}, \overline{s}) \in \mathbf{R}_{+}^{5\times NI}$, formed by the production and setup variables of periods $(t, t+1)$ and the inventory variables of period k. Our methodology uses an extreme point representation of $X_{t,k}^{2PL}$ to generate valid inequalities that cut-off $\overline{p}$. First, we formulate the problem of finding the point of $X_{t,k}^{2PL}$ that has the minimum distance from $\overline{p}$ as a linear program, using the linearizable $\mathcal{L}_{\infty}$ norm. If we denote by p the index of the extreme points of $X_{t,k}^{2PL}$, where we dropped the subscripts (t, k) for ease of notation, then the minimum distance linear program can be cast as follows (associated dual variables of each constraint are given in brackets):

$$\min\ z_{\infty} \tag{12}$$

$$\text{s.t.}\ \sum_{p} \lambda_p (x_p)_{t'}^{i} - z_{\infty} \le \overline{x}_{t'}^{i} \qquad \forall i, t' = t, t+1 \qquad (\alpha^{-}{}_{t'}^{i}) \tag{13}$$

$$\overline{x}_{t'}^{i} \le \sum_{k} \lambda_p (x_p)_{t'}^{i} + z_{\infty} \qquad \forall i, t' = t, t+1 \qquad (\alpha^{+}{}_{t'}^{i}) \tag{14}$$

$$\sum_{p} \lambda_p (y_p)_{t'}^{i} - z_{\infty} \le \overline{y}_{t'}^{i} \qquad \forall i, t' = t, t+1 \qquad (\beta^{-}{}_{t'}^{i}) \tag{15}$$

$$\overline{y}_{t'}^{i} \le \sum_{p} \lambda_p (y_p)_{t'}^{i} + z_{\infty} \qquad \forall i, t' = t, t+1 \qquad (\beta^{+}{}_{t'}^{i}) \tag{16}$$

$$\sum_{p} \lambda_p (s_p)^{i} - z_{\infty} \le \overline{s}_{k}^{i} \qquad \forall i \qquad (\gamma^{i}) \tag{17}$$

$$\sum_{p} \lambda_p \le 1 \qquad (\eta) \tag{18}$$

$$\lambda_p \ge 0,\ z_{\infty} \ge 0 \tag{19}$$

Upon termination of column generation, and if $z_{\infty} > 0$, a valid inequality that cuts off $\overline{p} := (\overline{x}, \overline{y}, \overline{s})$ can be generated in the form:

$$\sum_{i=1}^{NI} \sum_{t'=t}^{t+1} [(\alpha^{*+}{}_{t'}^{i} + \alpha^{*-}{}_{t'}^{i}) x_{t'}^{i} + (\beta^{*+}{}_{t'}^{i} + \beta^{*-}{}_{t'}^{i}) y_{t'}^{i}] + \sum_{i=1}^{NI} \gamma^{*i} s_{k}^{i} + \eta^{*} \le 0 \tag{20}$$

We refer the interested reader for details (including validity proofs and other technical details) to [1]. It is interesting to note that a valid inequality can be generated before completing the column generation process, but it might not be as strong as the one generated upon column generation termination. Also, we investigate the strength of the generated inequalities by establishing tight lower bounds on their dimension.

3 Computational Results

Table 1 shows preliminary computational results that were run on the Trigeiro X dataset [7], and compares with the recent algorithms of [5, 2]. For the sake of brevity, we show results for the 10 X sets for which our algorithm had the larger integrality gaps, but report the average of all 34 X sets. Each X set consists of five instances.

Instance Group	Pimentel	PD	X2PL
X11429	9.6	4.99	4.74
X12429	8	3.92	3.56
X11419	10.37	7.07	3.33
X12419	5.97	4.25	2.13
X11229	3.05	2.07	2.01
X12119	1.73	3.46	1.9
X12229	3.24	1.92	1.65
X11119	1.54	3.13	1.62
X11129	2.53	2.87	1.46
X11219	2.38	2.51	1.41
Average	2.09	1.41	0.97

Table 1: Average integrality gaps for the decomposition approaches of [5, 2] and X2PL for the Trigeiro X dataset.

The talk will give an overview of the developed methodology, and will primarily elaborate on the computational results and implementation challenges that the separation algorithm poses. The interested reader is referred to [1] for extensive details of results and discussions.

References

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