

The effect of relative population growth on the Barnett squeeze

by J R Cuthbert

This note is concerned with the Barnett squeeze as narrowly defined, (that is leaving aside the impact of local sources of funding like non-domestic rates and council tax outwith the Scottish Executive's DEL) and considers the implications of relative population growth in England compared to Scotland. It appears intuitively plausible that, if the population of England is growing relative to Scotland, then this will tend to dampen the rate of convergence of per capita spending levels in Scotland and England. It is difficult, however, without going into the algebra, to assess whether this statement is a truism which is likely to have little practical impact, or whether it is likely to have a significant impact on convergence rates in the real world.

To investigate this question further, an idealised model of how Barnett works has been set up. As will become apparent this model represents a simplified description of the public expenditure planning process and the application of the Barnett formula. Nevertheless this model captures the key aspects of the system, and enables some useful, and perhaps surprising, conclusions to be drawn.

The government plans its future programme of public expenditure on the basis of a three year rolling programme. Thus, in the Spending Review 2000 exercise. (SR2000), the government set its programmes for the future years of 2001-02, 2002-03 and 2003-04. When the government comes to carry out its 2001 planning round, therefore, the first thing it has to do is to establish a starting, or baseline, figure for the new end year in the three year planning horizon. In this case this will be 2004-05. The way the government does this is of critical importance for the application of the Barnett formula. Since 1993, the government has set the new final year base line for each programme as being equal to the previous final year in cash: (source: personal communication from H.M.Treasury). Prior to 1993, the new final year base line was apparently set as the previous final year figure, uprated for inflation: (see H.M.Treasury (1997)). The government then considers what adjustments are required to the English baseline figures for each of the three future years. In the 2001 planning round it will be considering years 2002-03 to 2004-05. These adjustments to the English baseline figures will include any required uplift for inflation in the new final year. These adjustments then feed into the Barnett formula to give the required changes to the Scottish DEL for each of the three years. In applying the Barnett formula, Scotland is given its

population share of changes to English department base lines, based on the most recently available historic population estimates for Scotland and England. Typically, therefore, the population shares used will relate to a date two years before the year in which the spending review is being carried out. Hence for the 2001 SR, the population estimates would relate to the year 1999. (For details on the operation of Barnett, see H.M.Treasury (2000)).

It will be apparent from the description in the previous paragraph why modelling the operation of Barnett is not simple. In particular, the Scottish DEL for any given year will be modified by the operation of Barnett in three successive planning rounds, each time using population estimates relating to a different historic date, and with a different lag each time between the date of the population estimates and the year being adjusted. These lags will, typically, be five years, four years, and three years successively. The average lag, therefore, between the date of the population estimates used and the year being adjusted is likely to be of the order of four years.

The following simplified model of how Barnett works abstracts from these complications. In this model, the Scottish DEL for any given year is adjusted only once by Barnett, when the new baseline for that year is first established. Another simplification is the assumption that public expenditure in England is growing by a constant percentage each year.

Specifically, the following notation and assumptions are used:

Let E_t denote expenditure in England in year t , and E_t^S expenditure in Scotland.

Let p_t denote population in England in year t , and p_t^S population in Scotland.

Let R_t denote the ratio of per capita expenditures between Scotland and England at time t .

Let k denote lag, (in years).

Suppose that

- $E_{t+1} = \theta E_t$: (i.e., expenditure in England grows at a constant rate.)
- $\frac{p_{t+1}}{p_t} = \lambda \frac{p_{t+1}^S}{p_t^S}$ for all t , where $\lambda \geq 1$: (i.e., there is a constant relative rate of growth of population in England relative to Scotland).
- In the annual public expenditure planning round, the new final year baseline is determined as being equal to the previous end year figure: and Barnett applies only to that end year, with population shares determined at a lag k .

The above model can be solved, to show how the per capita spending relativity between Scotland and England, evolves through time from its initial starting value in year 0.

The relevant formula for R_t is as follows:

$$R_t = \left(\frac{\lambda}{\theta}\right)^t \left[R_0 - \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)} \right] + \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$$

for $\lambda \neq \theta$. (1)

The derivation of formula (1) is given in the Annex.

What formula (1) means is that, in the normal circumstance where

$$\frac{\lambda}{\theta} < 1$$

then the initial per capita spending relativity, R_0 , will decay geometrically to the limiting value

$$\lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$$

which is a function of the expenditure growth rate in England, the rate of relative population growth, and the lag.

If $\lambda = 1$, then the limiting value $\lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}$

collapses to 1, (as is expected), and the rate of convergence is governed by the term θ^t : that is, the larger θ is, the faster the rate of convergence, which, again, is exactly as expected.

However, if $1 < \lambda < \theta$, then the limiting value will be greater than 1: so the per capita spending ratio will never converge to unity.

In order to establish whether formula (1) just represents a curiosity, or whether it has any practical importance, it is necessary to know what range of values of λ and θ is likely to be encountered in practice:

- a) The table below shows values of λ back to 1961, for individual years, or averaged over groups of years, with projected values forward to 2021. The projected figures relate to the 1998 based official population projections: the figures on which the table is based are taken from the *Annual Abstract of Statistics, 2000*.

For present purposes, it is relevant to note that, in the 1990s, λ increased from a value of 1.0013 in 1994, to 1.0056 in 1996, and is projected to remain at a value around 1.005 until 2001.

- b) As regards θ , through the mid 1990s growth in government expenditure was typically in low single figures: for example, the annual growth in the aggregate control total for the expenditure of government departments was between 1.6% and 4% between 1993-94 and 1997-98. This contrasts with the current

situation: for example, in SR2000, total DELs are projected to rise by 10.4%, 8.7%, 8.1%, and 7.1% over the next four years respectively.

Population Growth in England Relative to Scotland

Year	λ	Year	λ
1961		1996	1.0056
1971	1.0055	1997	1.0050
1981	1.0020	1998	1.0049
1991	1.0044	2001	1.0054
1992	1.0027	2006	1.0039
1993	1.0014	2011	1.0038
1994	1.0013	2021	1.0040
1995	1.0030		

For illustrative purposes, therefore, a λ value of 1.005, and θ values of 1.03 and 1.09 are considered, (as being typical of values in the mid 1990s, and currently). An illustrative lag of 4 years is considered, as is a starting value of 1.2 for the level of public expenditure per head in Scotland relative to England, (close to the current value). The relevant results are illustrated in Charts 1 to 3.

Chart 1 shows the path of the per capita expenditure relativity, when $\theta = 1.03$, and $\lambda = 1$: in this case, of course, the limiting ratio is 1, and, because of the relatively small growth rate in expenditure in England, convergence is fairly slow.

Chart 2 shows what happens when $\lambda = 1.005$: (note that this is the only change from the Chart 1 case.) This time, a radically different picture emerges. The limiting value is now 1.22, (that is, above the starting value), so that what is observed is slow growth of R_t to this level.

Finally, Chart 3 shows what happens when θ is increased to 1.09, but still retaining a λ value of 1.005. In this case, the limiting value is 1.08. Although this value is higher than the limiting value of 1 in Chart 1, the values of R_t in Chart 3 are actually below those in Chart 1 for about the first 20 years, because the rate of convergence to the limiting value is faster in Chart 3.

These results illustrate how levels of relative population growth similar to those currently being experienced can have a very marked effect indeed on the behaviour of relative per capita spending, when expenditure growth rates are in low single figures. However, this effect is significantly reduced when expenditure growth rates approach double figures. These findings are very relevant to the debate about the likely historical impact of the Barnett squeeze. At times during the 1990s, given the low growth rate in overall public expenditure, and the high relative growth rate of population in England which applied latterly, the model indicates that we would not have expected convergence in per capita spending levels. The situation now, with the high

current levels of public expenditure growth, is markedly different.

Finally, it is worth noting that the illustrative results quoted here are not very sensitive to variations in the lag parameter, (which, it will be recalled, has been set at a value of 4 years, representing the likely average lag in practice.) This is illustrated by the following table, which shows the limiting value of the per capita expenditure ratio, given different choices of the lag parameter, and θ , (and with $\lambda = 1.005$ in each case).

Limiting Values of Per Capita Expenditure Ratio

Lag (years)	Expenditure growth factor		
	1.03	1.06	1.09
1	1.206	1.096	1.064
2	1.212	1.102	1.069
3	1.218	1.107	1.075
4	1.224	1.113	1.08

In other words, the key factor driving the results illustrated in this note is the relative values of θ and λ .

References

HM Treasury (1997) Supplementary Memorandum in the Second Report of the Treasury Committee, "The Barnett Formula", (London: The Stationery Office).

HM Treasury (2000) "Funding the Scottish Parliament, National Assembly for Wales and the Northern Ireland Assembly".

Annex: Proof of Formula 1.

Notation and assumptions are as above.

From 6(a), it follows that $E_t = \theta^t E_0$. (A1)

From 6(b), it follows that $\frac{p_t^s}{p_t} = \frac{p_0^s}{p_0} \lambda^t$ (A2)

From 6(c), it follows that

$$\begin{aligned}
 E_t^s &= E_{t-1}^s + \frac{p_{t-k}^s}{p_{t-k}} (E_t - E_{t-1}) \\
 &= E_{t-1}^s + \frac{p_{t-k}^s}{p_{t-k}} \theta^{t-1} (\theta - 1) E_0, \text{ by (A1),} \\
 &= E_{t-1}^s + \lambda^{k-t} \theta^{t-1} (\theta - 1) E_0 \frac{p_0^s}{p_0}, \text{ by (A2),} \\
 &= E_{t-1}^s + \lambda^{k-1} \left(\frac{\theta}{\lambda}\right)^{t-1} (\theta - 1) E_0 \frac{p_0^s}{p_0}.
 \end{aligned}$$

It follows, on summing the resulting geometric series in $\frac{\theta}{\lambda}$, that

$$E_t^s = E_0^s + \lambda^{k-1} (\theta - 1) E_0 \frac{p_0^s (1 - (\frac{\theta}{\lambda})^t)}{p_0 (1 - \frac{\theta}{\lambda})}, \text{ when } \theta \neq \lambda, \tag{A3}$$

and

$$E_t^s = E_0^s + t \lambda^{k-1} (\theta - 1) E_0 \frac{p_0^s}{p_0}, \text{ when } \theta = \lambda. \tag{A4}$$

It follows from (A3), when $\theta \neq \lambda$, that

$$R_t = \frac{E_t^s p_t}{p_t^s E_t} = [E_0^s + \lambda^{k-1} (\theta - 1) E_0 \frac{p_0^s (1 - (\frac{\theta}{\lambda})^t)}{p_0 (1 - \frac{\theta}{\lambda})}] \frac{\lambda^t p_0}{\theta^t E_0 p_0^s}$$

which simplifies to

$$R_t = \left(\frac{\lambda}{\theta}\right)^t [R_0 - \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)}] + \lambda^k \frac{(\theta - 1)}{(\theta - \lambda)},$$

which is the required expression.

It follows from (A4), when $\theta = \lambda$, that,

$$\begin{aligned}
 R_t &= [E_0^s + t \lambda^{k-1} (\theta - 1) E_0 \frac{p_0^s}{p_0}] \frac{p_0}{E_0 p_0^s} \\
 &= R_0 + t \lambda^{k-1} (\theta - 1),
 \end{aligned}$$

so in this special, (and unlikely) case, R_t grows linearly.

Chart 1 Movement of Per Capita Expenditure Relative Towards Limit: $\Theta = 1.03$, $\Lambda = 1$, lag = 4

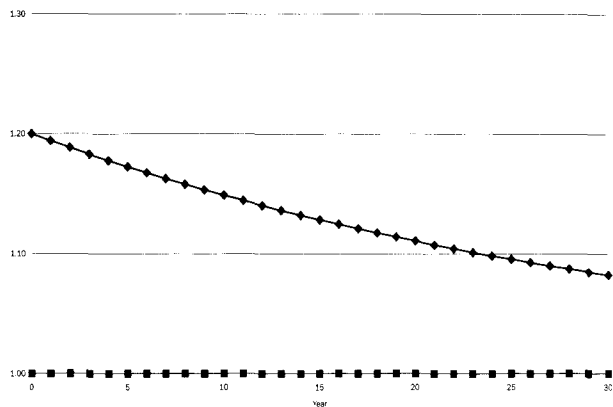


Chart 2 Movement of Per Capita Expenditure Relative Towards Limit: $\Theta = 1.03$, $\Lambda = 1.005$, lag = 4

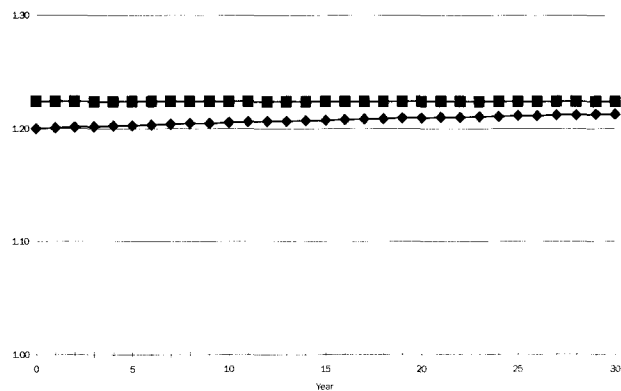


Chart 3 Movement of Per Capita Expenditure Relative Towards Limit: $\Theta = 1.09$, $\Lambda = 1.005$, lag = 4

