Vehicle Density Estimation of Freeway Traffic with Unknown Boundary Demand-Supply: An IMM Approach

Liguo Zhang and Xuerong Mao

Abstract

As distributed parameter systems, dynamics of freeway traffic are dominated by the current traffic parameter and boundary fluxes from upstream/downstream sections or on/off ramps. The difference between traffic demand-supply and boundary fluxes actually reflects the congestion level of freeway travel. This paper investigates simultaneous traffic density and boundary flux estimation with data extracted from on-road detectors. The existing studies for traffic estimation mainly focus on the traffic parameters (density, velocity, etc.) of mainline traffic and ignore flux fluctuations at boundary sections of the freeway. We propose a stochastic hybrid traffic flow model by extending the cell transmission model (CTM) with Markovian multi-mode switching. A novel interacting multiple model (IMM) filtering for simultaneous input and state estimation is developed for discrete-time Markovian switching systems with unknown input. A freeway segment of Interstate 80 East (I-80E) in Berkeley, Northern California, is chosen to investigate the performance of the developed approach. Traffic data is obtained from the Performance Measurement System (PeMS).

Index Terms

Lighthill-Whitham and Richards (LWR), stochastic hybrid traffic model, simultaneous input and state estimation, IMM filter, boundary flow.

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I. Introduction

Real-time knowledge about the traffic conditions of freeway transportation systems is critical for traffic management and control. There are many well-established technologies for collecting vehicle speed and traffic flux, including loop detectors or automatic vehicle identification (AVI) systems. Recently, the novel ubiquitous sensing technologies, such as dedicated probe vehicles or smartphone-based applications, have the potential of unprecedented data collection at any positions of large-scale traffic networks. However, the equipped measurement is typically low and not representative of the urban network as a whole, which leaves the traffic conditions in most of the network unknown.

Estimate unknown states of freeway traffic with filtering have been extensively studied in the past two decades. In the literature [1], Wang etc. proposed the extend Kalman filter (EKF) based on the stochastic version of macroscopic traffic flow model, METANET. A particle filter (PF) is developed in [2] using the extended stochastic cell transmission model (CTM) [3]-[5]. Sun etc. propose a solution to the traffic estimation by a sequential Monte Carlo algorithm [6], the so-called mixture Kalman filter. A switching mode model (SMM) is developed in [7] to represent freeway traffic with two distinct traffic phenomena, as a free-flow mode and a full congestion mode respectively. A fuzzy observer is designed in [8] based on SMM model to estimate vehicle densities of a freeway link. Most recently, the PF method is developed in [9] to real-time estimate traffic states with a jumped Markov traffic model coming from a variant of CTM. While, all of the above methods assume that the boundary flux of the estimated freeway traffic is known or available (as system measurement model) in advance.

In practise, however, when using mobile sensing technologies to perform measurement, it is reasonable difficult to collect the driving-in or the driving-out flux at boundaries of freeway traffic since the detectors are moving with the traffic flow. These situations are also arising when dealing with traffic big-data by using the distributed data-fusing methods for the large-scale traffic networks. In this case, the whole network is usually dynamically divided into several sub-regions for the computation reasons, without consider whether or not exist fixed detectors between the region boundaries. Moreover, the real-time flux information is sometimes absent as the communication of the traffic network is limited or the computing burden for system analysis and control is heavy.

At the same time, as a typical distributed parameter system, the traffic dynamics (system state models) are dominated not only by the traffic parameters on road, but by the driving-in or driving-out flux at the network boundaries. In the discrete-time state-space realization of traffic flow model, such as METANET model, the boundary fluxes are usually formulated as the system input outside. Therefore, develop a real-time filtering method without known of the boundary flux information is an important theoretical supplement to the state estimation of freeway traffic.

In this paper, we estimate the vehicle densities of a freeway link with unknown traffic demand-supply relationship at the network boundaries. The traffic flow model used for the state predictive is the Markov switched stochastic cell transmission model (SCTM) [11], with the boundary traffic fluxes as system inputs. For the state update stage, a novel interacting multiple model (IMM) filtering [14] is developed to estimate state and input variable, i.e. vehicle density and boundary flux, simultaneously.

Since the CTM model is computationally efficient and preserves many traffic phenomena such as queue build-up and dissipation and propagation of congestion waves, it has been extensively used for designing traffic estimators and controllers. The modified cell transmission model (MCTM) [7] and its simplification, SMM, avoid the triangular or trapezoidal flow-density relationship in CTM by switching among a set of linear equations which represent different traffic status of the freeway. SCTM extends CTM by explicitly defining parameters governing the sending and receiving functions between upstream and downstream segments as Markovian random variables. We take advantage of both the SMM model and SCTM model, and propose a modified stochastic hybrid dynamic model with boundary input for a freeway link. Only two distinct traffic phenomena, free-flow mode and full congestion mode, are formulated in the freeway dynamics as discrete-time subsystems. A mode-based switching rule that is formulated as a stochastic Markovian process is added to characterize the probability of occurrence of each mode.

Interacting Multiple Model (IMM) filtering [13],[14] is a computationally efficient and well performing suboptimal estimation algorithm for Markovian switching systems, in which the unknown system structure is estimated from a set of candidate models. In this paper, we firstly extend the classical IMM filtering to simultaneous input and state estimate of discrete-time Markovian switching systems, by designing a two-stages Kalman estimators in the filtering step. State estimation under unknown inputs has a wide range of applications and received considerable attention in recent decades [17]-[21]. In these applications, inputs and state variables are often unmeasurable or inaccessible. To the best of our knowledge, simultaneous input and state estimation for Markovian switching systems has not been studied in the literature.

The outline of the paper is as follows: Section II gives a brief review on the macroscopic LWR and CTM traffic flow models, and formulates the switched stochastic hybrid traffic flow model. IMM simultaneous input and state estimation for Markovian switching systems is presented in Section III. Numerical simulations and comparison results between CTM model and the proposed stochastic hybrid traffic flow model are provided in Section IV, respectively. Lastly, conclusions and future research issues are highlighted in Section V.

II. STOCHASTIC HYBRID TRAFFIC FLOW MODEL

Among the macroscopic traffic flow models, Lighthill-Whitham-Richards (LWR) [22], [23] would be the most popular and most-cited one. The traffic dynamics of a freeway link modeled by the LWR model is governed by the following two fluid dynamical equations.

One is the principle of conservation for vehicles,

$$\frac{\partial \rho(\xi,t)}{\partial t} + \frac{\partial q(\xi,t)}{\partial \xi} \quad = \quad 0 \tag{1}$$

where $\rho(\xi,t)$ and $q(\xi,t)$ denote the traffic density and the traffic flow (as a function of location ξ and time t, respectively), the other is from the flow-density relationship also known as the fundamental diagram,

$$q(\xi,t) \quad = \quad v(\xi,t)\rho(\xi,t) \tag{2}$$

where $v(\xi,t)$ is the traffic speed.

Numerical solutions of equations (1)-(2) could be found using the finite volume method, whereby each segment in the freeway link is discretized into cells with length Δl_i, $i = 1, \cdots, N$, and time is discretized into intervals with length Δt, (see Fig. 1). The length of cells Δl_i could be chosen based on the Courant-Friedrichs-Lewy condition [24], such that the numerical solution is stable, i.e.,

$$\Delta l_i \quad \leq \quad v\Delta t \tag{3}$$

where v is the free flow speed of this freeway segment.

Given this discretization, the conservation equation can be rewritten in spacestate form

$$\rho_{i,k+1} \quad = \quad \rho_{i,k} + \frac{\Delta t}{\Delta l_i}(q_{i,k}^{in} - q_{i,k}^{out}) \tag{4}$$

where $\rho_{i,k}$ is the vehicle density of cell i at time index k, and $q_{i,k}^{in}$ and $q_{i,k}^{out}$ are vehicle flows entering and leaving cell i during the time interval $[k\Delta t, (k+1)\Delta t]$, respectively.

Following the CTM model (Daganzo [4], [5]) and MCTM model (Muñoz, et al. [7]), the leaving flow between cell borders is determined by taking the minimum of three quantities:

$$q_{i,k}^{out} \quad = \quad min\{v\rho_{i,k}, q_{max}, \omega(\rho_J - \rho_{i+1,k})\} \tag{5}$$

where ω is the backward congestion wave speed, q_{max} is the maximum allowable flow, ρ_J is the jam density of the freeway link. For the adjacent cells, the flow-out of the upstream cell is equal to the flow-in of the downstream cell, i.e., $q_{i,k}^{out} = q_{i+1,k}^{in}$, for $i = 1, \cdots, N-1$.

Actual boundary flows of vehicles driving into or out of the freeway segment (boundary conditions of LWR model (1)-(2)), could also be described with this density-based formulation (5).

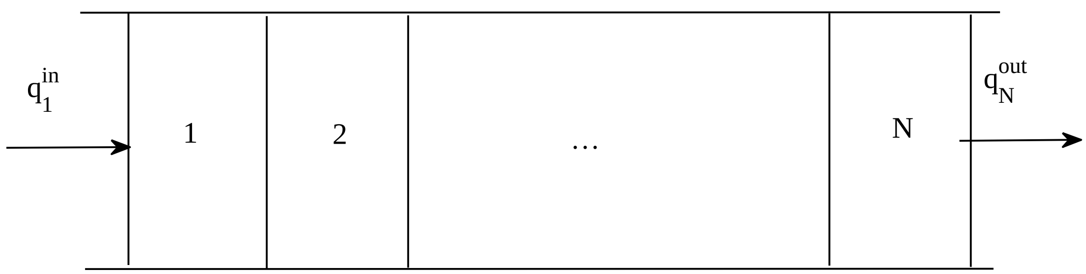

Fig. 1. A section of freeway segment divided into cells with uncertain boundary flows.

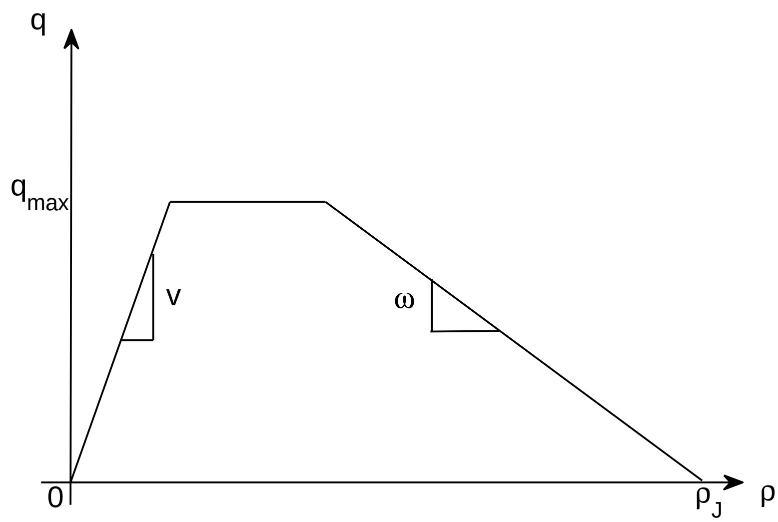

Fig. 2. A trapezoidal fundamental diagram for the cell transmission model.

For the upstream boundary cell, limited by the maximum allowable flow q_{max}, the actual entering flow is determined by comparing the prevailing traffic demand D_k to the maximum flow received by the first cell of the freeway segment under the congested condition, that is

$$q_{1,k}^{in} = min\{D_k, q_{max}, \omega(\rho_J - \rho_{1,k})\}. \tag{6}$$

For the downstream boundary cell, also limited by q_{max}, the actual leaving flow is determined by comparing the available traffic supply S_k to the maximum flow supplied by the end cell of the freeway segment under the free-flow condition, that is

$$q_{N,k}^{out} = min\{v\rho_{N,k}, q_{max}, S_k\}. \tag{7}$$

It is important to point out that the actual boundary flows $q_{1,k}^{in}$, $q_{N,k}^{out}$, are not only dependant on the available traffic demand-supply D_k, S_k, but on the traffic conditions of the freeway segment, and are changing dramatically. Therefore, different from the traditional traffic-state based filtering methods [1], [2], our research objective is trying to estimate vehicle densities and actual boundary flows of the freeway segment, simultaneously.

In order to further simplify the nonlinear flow-density relationship (5) of the CTM model for filter design, we propose a stochastic hybrid model (SHM) for a local section of freeway traffic, which combines the advantages of both the SMM and SCTM models.

When the freeway segment is in free-flow mode, the first term in (5) dominates, and the discrete-time subsystem is

$$\begin{bmatrix} \rho_{1,k+1} \\ \rho_{2,k+1} \\ \rho_{3,k+1} \\ \vdots \\ \rho_{N,k+1} \end{bmatrix} = \begin{bmatrix} 1-\dfrac{\Delta t}{\Delta l_1}v & 0 & 0 & \cdots & 0 \\ \dfrac{\Delta t}{\Delta l_1}v & 1-\dfrac{\Delta t}{\Delta l_2}v & 0 & \cdots & 0 \\ 0 & \dfrac{\Delta t}{\Delta l_2}v & 1-\dfrac{\Delta t}{\Delta l_3}v & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \end{bmatrix} \begin{bmatrix} \rho_{1,k} \\ \rho_{2,k} \\ \rho_{3,k} \\ \vdots \\ \rho_{N,k} \end{bmatrix}$$
$$+ \begin{bmatrix} \dfrac{\Delta t}{\Delta l_1} & 0 \\ 0 & 0 \\ 0 & 0 \\ \vdots & \vdots \\ 0 & -\dfrac{\Delta t}{\Delta l_N} \end{bmatrix} \begin{bmatrix} q_{1,k}^{in} \\ q_{N,k}^{out} \end{bmatrix} \tag{8}$$

When the freeway segment is in full congestion mode, the third term in (5) dominates, and the discrete-time subsystem is

$$\begin{bmatrix}\rho_{1,k+1}\\ \rho_{2,k+1}\\ \rho_{3,k+1}\\ \vdots\\ \rho_{N,k+1}\end{bmatrix} = \begin{bmatrix}1 & \frac{\Delta t}{\Delta l_1}\omega & 0 & \cdots & 0\\ 0 & 1-\frac{\Delta t}{\Delta l_2}\omega & \frac{\Delta t}{\Delta l_2}\omega & \cdots & 0\\ 0 & 0 & 1-\frac{\Delta t}{\Delta l_3}\omega & \cdots & 0\\ \vdots & \vdots & \vdots & \vdots & \vdots\\ 0 & 0 & 0 & \cdots & 1-\frac{\Delta t}{\Delta l_N}\omega\end{bmatrix}\begin{bmatrix}\rho_{1,k}\\ \rho_{2,k}\\ \rho_{3,k}\\ \vdots\\ \rho_{N,k}\end{bmatrix} + \begin{bmatrix}\frac{\Delta t}{\Delta l_1} & 0\\ 0 & 0\\ 0 & 0\\ \vdots & \vdots\\ 0 & -\frac{\Delta t}{\Delta l_N}\end{bmatrix}\begin{bmatrix}q^{in}_{1,k}\\ q^{out}_{N,k}\end{bmatrix} + \begin{bmatrix}-\frac{\Delta t}{\Delta l_1}\omega\rho_J\\ 0\\ 0\\ \vdots\\ \frac{\Delta t}{\Delta l_N}\omega\rho_J\end{bmatrix} \tag{9}$$

Occurrence probabilities of two modes in the freeway segment could be estimated using the probability distribution of uncertainties of stochastic traffic demand-supply [12], or empirically assumed which satisfy Binomial or Gaussian distributions [10]. To this end, the overall traffic dynamics of the freeway segment are represented as the joint of the free-flow mode and the full congestion mode under the conjunction of occurrence probabilities of each operational model. Empirical studies using data from the California Performance Measurement System (PeMS) [26]-[28], have confirmed that the stochastic modeling method performs well for all traffic conditions ranging from light to very heavy traffic conditions [12].

In general, a discrete-time stochastic hybrid model (SHM) for a local freeway traffic could be described by

$$x_{k+1} = A_k^{\sigma(k)} x_k + B_k^{\sigma(k)} u_k + a_k^{\sigma(k)} + \varepsilon_k^{\sigma(k)} \tag{10}$$

$$z_k = C_k^{\sigma(k)} x_k + \epsilon_k^{\sigma(k)} \tag{11}$$

where $x_k = [\rho_{1,k}, \rho_{2,k}, \cdots, \rho_{N,k}]^T$ is the state vector, $u_k = [q^{in}_{1,k}, q^{out}_{N,k}]^T$ is an unknown input, z_k are the system measurements, respectively. Mode switching signal $\sigma(k) \in \{1, 2\}$ represents the freeway segment in the free-flow mode or in the full congestion mode, respectively. A^i_k, B^i_k, and a^i_k are corresponding subsystem matrices (8), (9). Observation matrix C^i_k is particular to the spatial location of loop detectors installed locally on the freeway segment. ε^i_k, ϵ^i_k are system modeling error and observation noise, respectively, assumed to be zero-mean Gussian signals with known covariance matrix $Q^i_k = E[\varepsilon^i_k(\varepsilon^i_k)^T]$ and $R^i_k = E[\epsilon^i_k(\epsilon^i_k)^T]$, for $i = 1, 2$.

The mode evolution $\sigma(k)$ is a Markov chain described by a mode transition matrix

$$\Pi = \{p_{ij}\}_{i,j=1,2} \tag{12}$$

where p_{ij} is the probability of a mode transition from mode i to mode j

$$p_{ij} := Pr\{\sigma(k+1) = j|\sigma(k) = i\}. \tag{13}$$

We denote $Pr\{\cdot|\cdot\}$ a conditional probability density function. The transition probability matrix Π is thus a second order square matrix, with elements satisfying $p_{ij} > 0$ and $\sum_{j=1}^{2} p_{ij} = 1$, for each $i \in \{1, 2\}$. Denote the prior probability distribution for each mode j as $\mu_0^j = Pr\{\sigma(0) = j\}$.

III. IMM FILTER OF SIMULTANEOUS STATE AND INPUT ESTIMATE

Basically IMM filter consists of three major steps: interaction (mixing), filtering and combination. In each time step we obtain the initial conditions for certain model-matched filter by mixing the state estimates produced by all filters from the previous time step under the assumption that this particular model is the right model at current time step. Then we perform standard Kalman filtering for each model, and after that we compute a weighted combination of updated state estimates produced by all the filters yielding a final estimate for the state and covariance of the Gaussian density in that particular time step. The weights are chosen according to the probabilities of the models, which are computed in filtering step of the algorithm.

In this section, we propose a novel simultaneous state and input estimate of IMM filters for the discrete-time Markovian switching systems (10)-(11) with unknown input.

The equations for each step are as follows:

1) Interaction:

In each time step k, initial conditions are obtained by mixing the state estimates produced at previous steps by all filters from a set of n models under the assumption that this particular model is the right model at the current time step.

The mixing probabilities $\mu_k^{i|j}$ for each model i and j are calculated as

$$\bar{c}_j = \sum_{i=1}^{n} p_{ij}\mu_{k-1}^i, \tag{14}$$

$$\mu_k^{i|j} = \frac{1}{\bar{c}_j} p_{ij}\mu_{k-1}^i, \tag{15}$$

where μ_{k-1}^i is the probability of model i in the step $k-1$ and $\bar{c}_j$ a normalization factor.

The mixing initial condition for each filter as

$$x_{k-1}^{0j} = \sum_{i=1}^{n} \mu_k^{i|j} x_{k-1}^i, \tag{16}$$

$$P_{k-1}^{0j} = \sum_{i=1}^{n} \mu_k^{i|j} \times \{P_{k-1}^i + [x_{k-1}^i - x_{k-1}^{0j}][x_{k-1}^i - x_{k-1}^{0j}]^T\}, \tag{17}$$

where x_{k-1}^i and P_{k-1}^i are the state estimated mean and covariance for model i at time step $k-1$.

2) Filtering:

The objective of this step is to simultaneously estimate input and state variables given the sequence of measurements. Major difference from the previous IMM method, in this case no prior knowledge of the input signal is available and thus can be any type, such as non-Gaussian signals.

Simultaneous input and state estimation for each model i is performed as

$$x_k^{-,i} = A_{k-1}^i x_{k-1}^{0j} + a_{k-1}^i, \tag{18}$$

$$u_{k-1}^i = M_k^i (z_k - H_k^i x_k^{-,i}), \tag{19}$$

$$x_k^{+,i} = x_k^{-,i} + B_{k-1}^i u_{k-1}^i, \tag{20}$$

$$x_k^i = x_k^{+,i} + K_k^i (z_k - H_k^i x_k^{+,i}). \tag{21}$$

where $x_k^{-,i}$, $x_k^{+,i}$ represent the state prediction without or with input estimation information, respectively, and u_{k-1}^i represents the input estimate for model i.

The optimal gain matrices are obtained as [21]

$$M_k^i = [(D_k^i)^T (\tilde{R}_k^i)^{-1} D_k^i]^{-1} (D_k^i)^T (\tilde{R}_k^i)^{-1}, \tag{22}$$

$$K_k^i = [P_k^{-,i} (H_k^i)^T + S_k^i](\breve{R}_k^i)^{-1}, \tag{23}$$

where $D_k^i = H_k^i B_{k-1}^i$, $S_k^i = -B_{k-1}^i M_k^i R_k^i$, and

$$\tilde{R}_k^i = H_k^i [A_{k-1}^i P_{k-1}^i (A_{k-1}^i)^T + Q_k^i](H_k^i)^T + R_k^i,$$

$$\breve{R}_k^i = H_k^i P_k^{-,i} (H_k^i)^T + R_k^i + H_k^i S_k^i + (H_k^i S_k^i)^T.$$

The state error covariance matrices at the filtering update step P_k^i, and at the prediction step $P_k^{-,i}$ are defined respectively as

$$P_k^i = E[\eta_k^i (\eta_k^i)^T], \tag{24}$$

$$P_k^{-,i} = E[\bar{\eta}_k^i (\bar{\eta}_k^i)^T], \tag{25}$$

where $\eta_k^i = x_k - x_k^i$, $\bar{\eta}_k^i = x_k - x_k^{+,i}$ and are calculated according to

$$P_k^i = P_k^{-,i} - K_k^i[P_k^{-,i}(H_k^i)^T + S_k^i]^T, \tag{26}$$

$$P_k^{-,i} = \tilde{A}_{k-1}^i P_{k-1}^i (\tilde{A}_{k-1}^i)^T + \tilde{Q}_{k-1}^i, \tag{27}$$

where, $\tilde{A}_{k-1}^i = (I_n - B_{k-1}^i M_k^i H_k^i) A_{k-1}^i$, and $\tilde{Q}_{k-1} = B_{k-1}^i M_k^i R_k^i (B_{k-1}^i M_k^i)^T$.

In addition to estimating the mean and covariance we also compute the likelihood of the measurement for each filter by the Gaussian distribution

$$\Lambda_k^i = \mathcal{N}(\zeta_k^i; 0, P_k^i), \tag{28}$$

where ζ_k^i is the measurement residual for each model i.

The probabilities of each model i at time step k are calculated as

$$c = \sum_{i=1}^{n} \Lambda_k^i \bar{c}_i, \tag{29}$$

$$\mu_k^i = \frac{1}{c} \Lambda_k^i \bar{c}_i, \tag{30}$$

where c is a normalizing factor.

3) Combination

In the final step of the IMM filter, we compute the weighted combination of both input and state estimates produced by all the filters, to yield the final estimates and covariance of the Gaussian density. The weights are chosen according to the probabilities of models which are computed in the filtering step of this algorithm.

The combined estimation for input and state mean are computed respectively as

$$\hat{x}_k = \sum_{i=1}^{n} \mu_k^i x_k^i, \tag{31}$$

$$\hat{u}_{k-1} = \sum_{i=1}^{n} \mu_k^i u_{k-1}^i, \tag{32}$$

and for the state covariance as

$$P_k = \sum_{i=1}^{n} \mu_k^i \times \{P_k^i + [x_k^i - \hat{x}_k][x_k^i - \hat{x}_k]^T\}. \tag{33}$$

Remark 1. Compares with the classical IMM algorithm, the developed IMM filtering for simultaneous state and input estimation make the input estimate (19) for each model by making the two-stages Kalman filter (22)-(23). As state estiamte, the final input estimate is the weighted combination of each model.

Remark 2. Covariance equations (25)-(26) mean that we obtain the state estimate from the innovation in an optimal way, and show that the input estimate for each model is unbias and indeed optimal.

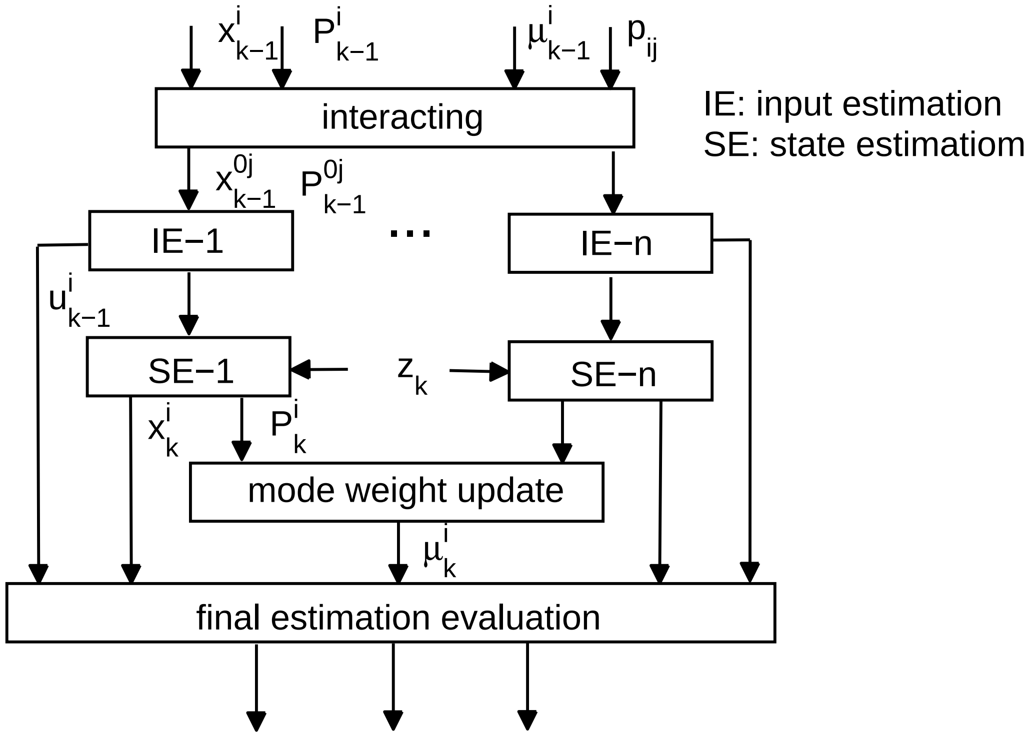

Fig. 3. Algorithm flowchart of IMM filter for simultaneous input and state estimation.

Fig. 3 shows the message passing of the IMM filtering for simultaneous state and input estimation of the stochastic Markovian jump systems.

IV. Empirical study for freeway traffic estimation

In our experiment, the interested freeway link is a section of Interstate 80 East, which is approximately three miles in length at Berkeley, Northern California, CA. This freeway segment is instrumented with loop inductance detectors which are embedded in the pavement along the mainline, the HOV lane, and off ramps, as shown in Fig. 4 (PeMS, [28]).

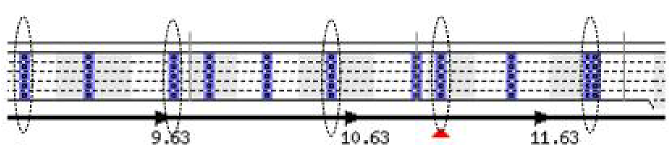

Fig. 4. A freeway link of I80-E divided into four cells (PeMS).

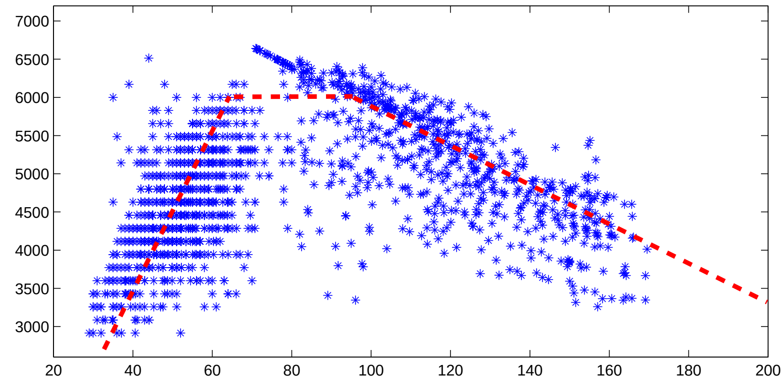

Fig. 5. Calibrated flow-density fundamental diagram of the freeway link.

In the empirical studies, this freeway link is spatially partitioned into four cells with lengths ranging from 0.65, 0.82, 0.59 and 0.71 miles, respectively. The blue points along the freeway link denote where the loop detectors are installed. Each loop detector gives flux, speed, and occupancy measurements every 30 seconds. The actual flux information is used to validate traffic parameters of the fundamental diagram and compare with estimated values with filtering. The vehicle densities are calculated using the occupancy measurements from the detectors inside of each cell divided by the g-factor (about 25 feet in PeMS [26]) in which the g-factor is the effective vehicle length for the detector.

The fundamental diagrams of CTM model are roughly calibrated using linear regression method [12], based on the historic traffic data of all detectors collecting in this segment of one week in Oct. 2013. The first validated traffic parameter is the free flow speed v. Then subsequently calibrate the maximum allowable flow p_{max}, the critical density ρ_J, and the congestion wave speed ω. The calibrated trapezoid chart of the fundamental diagram is shown in Fig. 5. Some simulation and model calibrated parameters are displayed in Table I. The noise covariances of system model and observation model are assumed to be 5 veh/mile per time interval.

In this simulation, we estimate some unknown vehicle densities of cells, and upstream and downstream boundary fluxes of the freeway link in two cases, the case of using our developed IMM filtering with stochastic hybrid traffic model, and the case of using EKF filtering based on the CTM model. The utilized measurements are vehicle occupancy of 4 hours (7:00 am-11:00 am) of cell 1 and 4, including the morning rush-hour congestion on Oct. 20, 2013. Since our research focus on the algorithm performance of filtering, we simplify the observation model by changing the direct occupancy measurement into the

TABLE I

CALIBRATED MODEL AND SIMULATION PARAMETERS

Symbol	Name	Value	Unit
ρ_J	Critical density	320	$veh/mile$
q_{max}	Maximum allowable flow	5860	veh/h
v	Free flow speed	72	$mile/h$
ω	Congestion wave speed	24	$mile/h$
Δt	Time interval	30/3600	h
Q_k^i	Modeling error	5	$veh/mile/30s$
R_k^i	Measurement error	5	$veh/mile/30s$
N	Cell number	4	–
T	Total time step	240	–

vehicle densities in advance. Besides using the above mentioned g-factor, other transfer method includes using calibrated speed-density diagram with the speed measurement [10]. Therefore, we may assume the system observation matrix is

$$C_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{34}$$

In the simulation of IMM filtering, the prior probability distribution μ_0 and the mode transition matrix Π are firstly solved with system identification methods [11]. The prior probability distribution are set to be $\mu_0^1 = 0.9$ and $\mu_0^2 = 0.1$, (model 1: free-flow mode; model 2: the full congestion mode), which are fitted using the initial vehicle densities of four cells. The mode transition matrix is fitted using the data of two adjacent sample sets of vehicle densities and set to be

$$\Pi = \begin{bmatrix} 0.92 & 0.08 \\ 0.08 & 0.92 \end{bmatrix}, \tag{35}$$

which means both modes are most likely to stay and have equal probability to jump to another. According to the IMM filtering algorithm, μ_0 have no effect on the state estimate. So do the input estimate.

In the simulation of EKF filtering with CTM model, since the freeway link is divided into four cells, the CTM model will include sixteen sub-modes to describe the dynamics of traffic flow [6]. Then, the predictive model is indeed a piecewise affine nonlinear system with state-induced switching rules. In this case, as a contrast, we use EKF filtering to estimate vehicle densities of cell 2 and 3, and driving-in and driving-out fluxes, simultaneously. Filtering algorithm refers to our previous work [20].

TABLE II

PERFORMANCE EVALUATION OF SIMULTANEOUS VEHICLE DENSITY AND BOUNDARY FLUX ESTIMATION

	MAPE		RMSE	
	CTM-EKF	SHM-IMM	CTM-EKF	SHM-IMM
Cell 2	0.0721	0.0997	4.7236	6.2465
Cell 3	0.0946	0.1187	5.0076	7.5531
Driving-In Flux	0.1534	0.2072	5.6124	7.6763
Driving-Out Flux	0.1297	0.1410	5.3648	6.4529

The simulations are performed in MATLAB 7.0 environment using an Intel 2.40 GHz processor with 512 MB of RAM and under Microsoft Windows XP operating systems. The simulations have been repeated 50 times, and averagely the developed IMM filtering algorithm takes about 0.79 seconds while the simulation time of EKF filtering with CTM model requires about 2.86 seconds. The estimated traffic densities and boundary fluxes are depicted in Fig. 6 and Fig. 7, respectively, against the historical data over the selected time period. The corresponding MAPEs and RMSEs of the traffic densities of cell 2 and cell 3 and the driving-in and driving-out boundary fluxes are indicated in Table 2.

It may be seen that IMM filtering provides a relatively satisfied estimation for both vehicle densities and boundary fluxes. Although, compared with EKF filtering, predicted with CTM model, the developed IMM filtering slightly reduces the accuracy of estimation within about 20 percents. An obvious improvement (with simply model magnitude) is that the algorithm speed is faster than using CTM model.

V. CONCLUSION

In this paper, we have investigated the simultaneous estimate of traffic densities and boundary fluxes of freeway traffic with the stochastic hybrid traffic model by taking advantages of SCTM and SMM models. In order to estimate the boundary flux with on-road traffic measurement, we have developed IMM filtering for simultaneous input and state estimation of discrete-time Markovian switching systems. The performance of the developed filtering algorithm is investigated in the empirical studies. The developed approach would has practical implications for the control of freeway traffic, since both on-ramp metering and variable speed control need real-time traffic information.

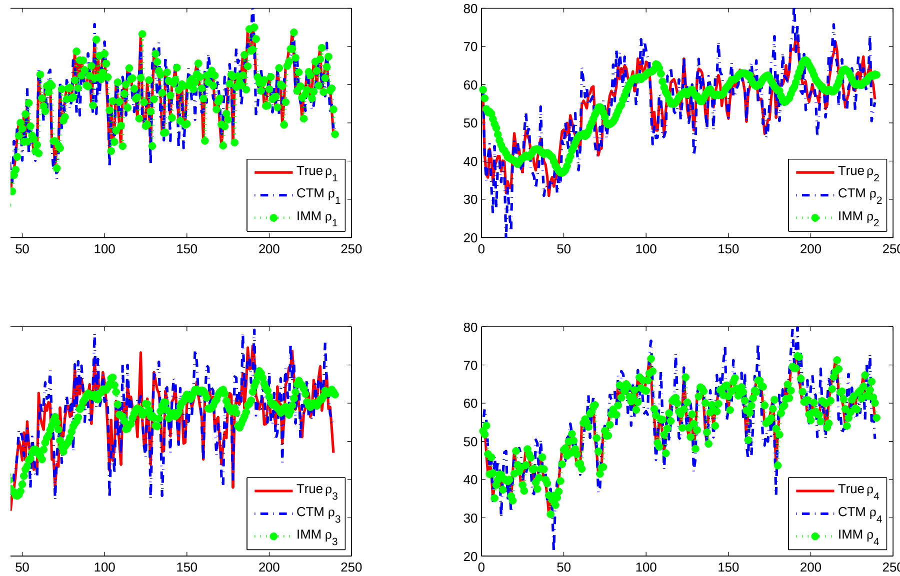

Fig. 6. The estimated traffic densities of freeway traffic.

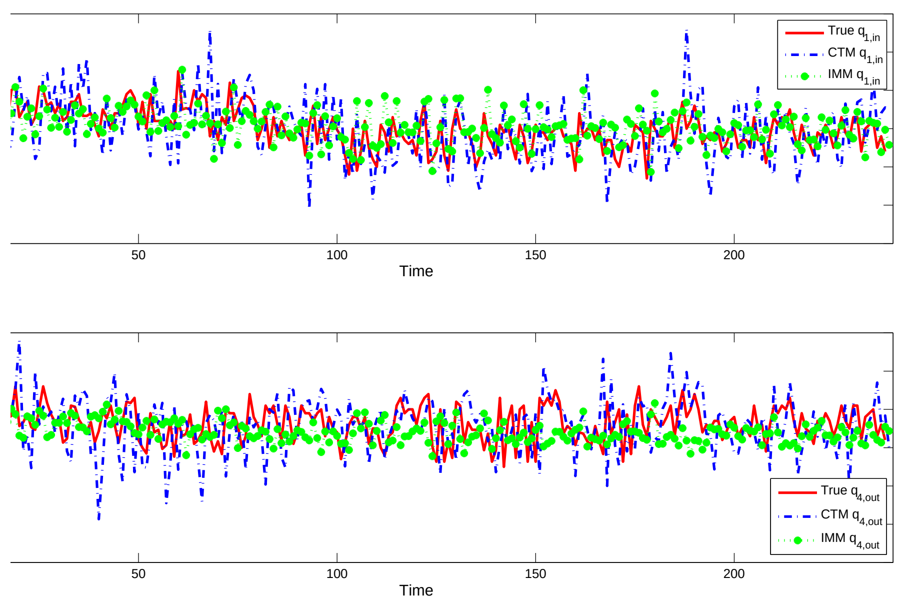

Fig. 7. The estimated driving-in and driving-out fluxes of freeway traffic.

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