Impurity Transport Studies on MAST

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Abstract

Impurity transport is a subject of fundamental importance in plasma physics in general and in tokamak physics in particular. The behaviour of the various impurity species and the evolution of their concentration determines, among other things, the fuel dilution and the fusion reaction rate, the plasma radiation pattern and the local energy balance, the plasma effective charge and resistivity and the neutral beam particle and power deposition profile. It is therefore important to develop both a sound experimental base and reliable models to interpret the experimental results and to predict the transport properties of impurities.

Time-dependent helium and methane gas puff experiments have been performed on the Mega Ampere Spherical Tokamak (MAST) during a two point plasma current, $I_p$, scan in L-mode and a confinement scan at constant $I_p$. For the $I_p$ scan, a dimensionless safety factor, $q$, scan was attempted by using a constant toroidal magnetic field and by moderating the beam power to match the plasma temperature. The temperature and magnetic field was also kept constant during the confinement scan to probe the effects of the electron density gradient.

An evaluation of the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) spectral lines, induced by active charge exchange emission and measured using the RGB 2D camera on MAST, indicate that carbon experiences moderately higher rates of diffusion and inward convection than helium in the L-mode high $I_p$ plasma. Lowering $I_p$ in L-mode caused a moderate increase in the helium diffusion and convection coefficients near the plasma edge. Neoclassical simulations were carried out which indicate anomalous rates of helium and carbon diffusion and inward convection in the outer regions of both L-mode plasmas.

Similar rates of helium diffusion are found in the H-mode plasma, however these rates are consistent with neoclassical predictions. The anomalous inward pinch found for helium in the L-mode plasmas is also not apparent in H-mode. An outward flux of helium and carbon is found at mid-radius in H-mode, corresponding the region of positive electron density gradient.

Linear gyrokinetic simulations of one flux surface in L-mode using the gs2 and gkw codes were performed which show that equilibrium flow shear is sufficient to stabilise ion temperature gradient (ITG) modes, consistent with BES observations, and suggest that collisionless trapped electron modes (TEMs) may dominate the anomalous helium particle transport. A quasilinear estimate of the dimensionless
peaking factor associated with TEMs is in good agreement with experiment. Collisionless TEMs are more stable in H-mode because the electron density gradient is flatter. The steepness of this gradient is therefore pivotal in determining the inward neoclassical particle pinch and the particle flux associated with TEM turbulence.


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Chapter 1

Introduction

The international energy outlook report for the year 2013 predicts that the total world energy consumption in the year 2040 is expected to increase by 56%, largely due to the increase in energy demand from developing nations. With over three quarters of the world’s energy currently being supplied from a rapidly declining reserve of fossil fuels that produce harmful greenhouse gases, it is imperative new sources of energy with carbon free emission be developed and perfected to sustain this increasing world energy demand. Renewable electricity has advanced considerably, however wind, solar and wave power plants may suffer from intermittent generation due to their reliance on fluctuating environmental conditions, while the high cost and structural integrity of man-made water dams are a concern for hydro power plants. Nuclear fission currently contributes a significant amount to the world’s energy needs, but faces serious political safety and radioactive waste concerns.

Thermonuclear fusion involves using extremely high temperatures to fuse together different hydrogen isotopes with consequential release of energy. This method of power generation can potentially offer a cleaner, inherently safe and almost limitless supply of nuclear energy. The highest Maxwell averaged fusion cross-section is between deuterium (D) and tritium (T) for temperatures < 1 MeV and is therefore the most commonly planned fuel species in thermonuclear fusion experiments [1]. The D-T reaction generates a 3.5 MeV alpha particle and a 14.1 MeV fast neutron as shown in equation 1.1.

\[
{^2}_1\text{D} + {^3}_1\text{T} \rightarrow {^4}_2\text{He} + {^1}_0\text{n} + 17.6 \text{ MeV}
\] (1.1)

The energy in the form of kinetic energy of the products must be captured. There is an almost limitless supply of deuterium available from the world’s current reserves. However tritium is unstable but can be ‘bred’ by nuclear reaction of the neutrons with lithium. There have so far been very few experiments with D-T fuelling. Most experiments normally operate with D plasmas.

The alpha particles, being charged, can be contained in the reaction region. A fusion power plant is said to ‘ignite’ when the contained alpha particle energy is
sufficient to sustain the nuclear reactions. The sustainability criterion, or Lawson Criterion [1], for the D-T reaction can be expressed in terms of the fusion triple product as

\[ nT\tau_E \geq 10^{21} \text{[m}^{-3}\text{keV s]} \] (1.2)

where \( n \) is the fuel density, \( T \) is the temperature and \( \tau_E \) is the global energy confinement time. There are two main approaches to fulfilling Lawson’s criterion. One method, called inertial confinement fusion (ICF), uses high powered lasers to compress and heat a relatively small, high density frozen fuel pellet for a short period of time until fusion occurs [2]. The other method, known as magnetic confinement fusion (MCF), uses a combination of magnetic fields to confine the fusion fuel in the form of a hot plasma for a sufficient period of time for fusion to occur [1].

MCF systems generally confine the plasma in a toroidal chamber and can operate with a number of different magnetic coil configurations. The most common configurations are the tokamak, described in the next section, and the stellarator, which achieves a steady state plasma using a complex helical coil configuration (see for example Geiger et al. [3]). Current tokamaks can achieve temperatures of the order of 10 keV and densities of the order of \( 10^{20} \text{m}^{-3} \) therefore, from equation 1.2, the main fuel ions must be confined for a time of the order of seconds.

1.1 Tokamaks

The tokamak system is currently regarded as the most promising route to a feasible fusion power plant and is the most developed of MCF systems. An illustration of the key components of a tokamak are shown in figure 1.1. The plasma is created within a vacuum chamber and confined away from the vessel walls in a torus shape using a combination of different magnetic fields. The words ‘toroidal’ and ‘poloidal’ mean the long and short way around the torus respectively, and the low field side (LFS) and high field side (HFS) correspond to the outer and inner parts of a poloidal section of the plasma respectively.

Plasma has a high capacity for cooperative action to escape from confining magnetic fields. In the tokamak design, toroidal field (TF) coils surrounding the vessel produce a toroidal magnetic field, \( B_T \), that is inversely proportional to the major radius of the plasma, \( R \). Two charge dependent vertical particle drifts are associated with the spatially varying field, called the grad-B and curvature drifts. The vertical electric field from these drifts produce a radial \( \mathbf{E} \times \mathbf{B} \) drift directed towards the vessel walls. This \( \mathbf{E} \times \mathbf{B} \) drift can potentially cause a rapid decrease in plasma confinement. To counteract the vertical charge separation, a central solenoid is wound around the centre column to induce a toroidal plasma current, \( I_p \), and therefore a poloidal magnetic field, \( B_P \). \( B_T \) and \( B_P \) add vectorially to give a helical magnetic field that connects the top and bottom of the plasma and which neutralises the charge separation in the plasma. External poloidal field (PF) coils
1.1. TOKAMAKS

Figure 1.1: Schematic representation of a Tokamak, courtesy of the JET image database.

are also needed to stabilise the plasma in the vertical and horizontal directions and modify the shape of the plasma.

$I_p$ also heats the plasma due to the plasma’s finite resistivity, $\eta$. This form of heating is known as ohmic heating. As is well known, $\eta$ decreases as the temperature increases and therefore limits the ohmic heating to temperatures of a few keV. Auxiliary heating methods are used to increase the plasma temperature up to the $10-100$ keV range required for thermonuclear burn. One of the most common forms of auxiliary heating methods, and of most relevance to this thesis, is produced using neutral beam injectors (NBI). Highly energetic ($60-100$ keV) neutral hydrogen atoms from the NBIs penetrate into the plasma core where they ionise and transfer energy to the plasma by collision processes. Neutral beams have an additional diagnostic capability. The charge transfer reaction from the neutral beam hydrogen atom to a plasma ion causes spectral line emission from the recombined ion. This latter point is discussed later in section 1.3.2.

The force balance between the magnetic field, plasma current and pressure gradients, can be defined to first order as

$$\mathbf{j} \times \mathbf{B} = \nabla p$$  \hspace{1cm} (1.3)$$

where $\mathbf{j}$ is the current density and $p$ is the pressure. Taking the scaler product...
of equation 1.3 with $j$ and $B$ proves that there is no current or magnetic field respectively in the direction of the pressure gradient (the radial direction). This creates a magnetic equilibrium consisting of an infinite set of toroidally symmetric nested flux surfaces each with constant magnetic field, current and pressure as shown in figure 1.2. Each flux surface can be characterised according to the magnetic winding number,

$$q = \frac{m}{n} = \frac{aB_T}{RB_P}$$

(1.4)

which denotes the ratio of toroidal revolutions ($m$) a closed field line makes to poloidal revolutions ($n$); $a$ and $R$ are the minor and major radii respectively (see figure 1.2). Rational surfaces with low $q$ values (such as 1, 3/2 or 2) can lead to a plasma instability, therefore $q$ is usually referred to as the safety factor. To avoid large scale plasma instabilities, $q$ must be kept above unity everywhere, and above two near the plasma edge [1].

For the plasma to remain confined, the magnetic field pressure must exceed the plasma pressure. The ratio of these two pressures is called beta, $\beta$, and is quoted (usually as a percentage) using the expression [4]

$$\beta = \frac{2\mu_0 <nkT>}{B_T^2}$$

(1.5)

where $p = <nkT>$, $k$ is the Boltzmann constant and $\mu_0$ is the permittivity of free space. Ideally, tokamaks should operate at high $\beta$ because the fusion reaction rate varies directly with $p$. One can also consider $\beta$ to be inversely proportional to cost because the main running expenditure of a fusion reactor arises from the magnetic field generation. However the maximum beta is limited by magnetohydrodynamic (MHD) instabilities such as the ‘kink’ mode, which occurs when the plasma is carrying excess $I_p$ [1]. The kink mode describes the situation where a toroidal flux
1.2. SPHERICAL TOKAMAKS

Surface deviates from its circular trajectory and folds in on itself causing a rapid loss of confinement. A definition of the \( \beta \) limit will be given in section 1.2.

The global energy confinement time of the plasma is defined as [1]

\[
\tau_E = \frac{\int \frac{3}{2} n(T_e + T_i) d^3x}{P}
\]

(1.6)

where \( P \) is the total power input and \( T_e \) and \( T_i \) are the electron and ion temperature. Unfortunately, a frequently observed feature of tokamaks is that \( \tau_E \) is shorter than predicted values based solely on Coulomb collisions in a toroidal geometry (neoclassical theory). The confinement behaviour is put into four categories: Ohmically heated plasmas and additionally heated plasmas in L (for low), H (for high) and VH (for very high) confinement modes [1]. In low density ohmic plasmas, \( \tau_E \) increases linearly with density up to some critical value. Beyond this critical value \( \tau_E \) remains constant with density. L-mode is characterised by a decrease in \( \tau_E \) with increasing auxiliary heating and features low temperatures and temperature gradients near the plasma edge. H-mode plasmas are characterised by an improved confinement time in the plasma edge and a doubling of the overall confinement time compared to L-mode. The density profile in H-mode plasmas features a flat region in the core and a very steep region in the plasma edge called the density pedestal. VH-mode plasmas cover a variety of plasmas which produce an enhanced confinement in the plasma core with respect to the H-mode plasmas.

The two tokamak configurations of interest for this thesis are the conventional tokamak (CT) and the spherical tokamak (ST). The CT and the ST differ in aspect ratio, \( A \equiv R/a \), as shown in figure 1.3. CTs operate with a value of \( A > 2.5 \) and are the most common tokamak design whereas STs are more compact and allow for a value of \( A \sim 1 \). ITER, a CT currently under construction in France, will be approximately twice the size of the largest tokamak to date (the JET machine in the UK) and aims to demonstrate the physics and engineering technology of a tokamak on the scale of a power station. Following ITER, it is hoped that a demonstration electricity-producing DEMO power plant will be constructed in the year 2030 [5]. Before DEMO, other aspects of a fusion reactor remain to be developed, such as vessel wall and blanket components which convert the neutron power to electrical power. These tests should be carried out in an affordable Component Test Facility (CTF), that can deliver reactor-level neutral and heat fluxes, albeit without producing net fusion power. A prime candidate for a CTF in current planning is the ST due their compact size and affordability [5].

## 1.2 Spherical Tokamaks

The first concept of an ST was put forward in 1986 by Peng and Strickler [6] to extend the concept of a CT into the limit of minimal \( A \). One of the main motivations for reducing \( A \) was to increase the \( \beta \) limit of the plasma. Work by Sykes et al. [7]
Figure 1.3: A conventional tokamak of high aspect ratio compared to a spherical tokamak of low aspect ratio, courtesy of the JET image database.

and Troyon et al. [8] showed that the $\beta$ limit can be written as $\beta_T \leq \beta_N I_N$ where $\beta_T$ is the toroidal $\beta$, $\beta_N = 3.5$ is the Troyon coefficient (assuming $\beta_T$ is a percentage) and $I_N$ is the normalised current $I_N = I_p/aB_T$. $I_N$ is also inversely proportional to $A$ and the safety factor at 95% of the plasma minor radius, $q_{95}$, therefore STs are potentially more efficient than CTs because their smaller $A$ value allows for a higher $\beta$ limit.

Another advantage of STs is that the plasma is inherently more stable than CT plasmas. The natural ST plasma shape, that is of high elongation and triangularity, helps the plasma to remain stable against the magnetohydrodynamic (MHD) modes and thus achieve a larger $\beta$ limit. Elongation of the plasma occurs naturally at low $A$ as a consequence of force balance between the internal plasma currents flowing in opposite directions on either side of the centre column which interact more strongly at low $A$ [9]. The interaction between the two opposite currents creates a vertical force which competes with the natural circular flux surfaces induced by the magnetic field from the centre column. With high vertical elongation of the plasma, the poloidal circumference increases meaning that the particles take longer to complete a poloidal revolution which in turn increases $q_{95}$. As the interaction between the currents on each side of the centre column decreases outwards, the elongation also reduces outwards leading to a triangular, D-shaped plasma. The outward-pointed triangularity of the plasma also acts to increase $q_{95}$.

STs experience a large variation of $B_T$ across the plasma radius compared to CTs meaning that the ratio of $B_T$ and $B_P$ on the HFS is greater than the LFS, resulting in a longer confinement of the particles on the HFS compared to the LFS as shown in figure 1.4. The region of plasma on the HFS, called the good curvature region, experiences increased stability since the centrifugal force acts against the pressure gradient to stabilise it. Conversely, the region of plasma on the LFS, called the bad curvature region, experiences an outward force due to both the centrifugal
1.3. THE IMPACT OF IMPURITIES

Any ion present in the plasma not contributing to the fusion burn is called an impurity. The impurities can be grouped into intrinsic and extrinsic categories, and subcategories of light and heavy, or low and high nuclear charge, $z_0$. Light impurities here are those with $z_0 < 10$. The intrinsic category includes impurities that have migrated into the plasma following erosion of the plasma facing components (PFCs) through sputtering and chemical erosion processes; also a DT plasma has force and the pressure gradient. The increased time the particles spend in the good curvature region was found to improve the stability of certain MHD modes [10]. A strong stabilising factor of turbulent modes can also originate from the large $\mathbf{E} \times \mathbf{B}$ flow shear often found in neutral beam heated ST plasmas [11, 12], and from the strong $\beta$ gradient generally achieved in ST plasmas due to the large central $\beta$ [13].

The first ST designed in the UK, called START [14], achieved a record $\beta_T$ value of $\sim 40 \%$ [15]. After the success of START, two larger scale STs were commissioned: the National Spherical Tokamak eXperiment (NSTX) [16, 17] based at Princeton Plasma Physics Laboratory USA, and the Mega Amp Spherical Tokamak (MAST) [18], based at Culham, UK. The MAST experimental programme makes use of a set of unusually high-resolution and wide-viewing diagnostics making MAST ideal for an impurity transport study. The experimental program on MAST has now finished and research has turned to the upgrade, MAST-U [19].

1.3 The Impact of Impurities

Figure 1.4: Schematic representation of a single field line for a spherical tokamak with safety factor $q = 12$. 
an intrinsic source of helium from thermalised alpha particles created in the plasma core. Extrinsic impurities are purposefully injected into the plasma either to study the subsequent impurity evolution or to alter the radiation pattern of the plasma. One can also make the distinction between recycling and non-recycling impurities depending on the capacity of the plasma wall to retain and/or re-release them. Recycling impurities include the naturally gaseous elements — the intrinsic helium or extrinsic neon or argon for example.

A substantial presence of impurities in the plasma results in fuel dilution\(^1\), which can quench the burn process. More specifically, ignition requires \(\rho = \tau_{He}^*/\tau_E \leq \rho_{\text{crit}}\), where \(\tau_{He}^*\) is the effective thermalised helium confinement time. Simulations carried out by Reiter et al. [20] suggest that \(\rho_{\text{crit}} \sim 15\) for a pure D-T plasma. Furthermore, as the concentration of other plasma impurities increase, the value of \(\rho_{\text{crit}}\) decreases and the ignition constraint in equation 1.2 increases [20].

Impurities also act to cool the plasma. Near the plasma edge, there is a significant temperature gradient where a mixture of neutral and partially ionised impurities exist. Typically low \(z_0\) impurities are fully ionised in the hot plasma core, whereas high \(z_0\) impurities are usually partially ionised. Partially ionised impurities radiate energy through line and continuum emission, while the fully ionised impurities only radiate through continuum emission. Both a decrease in the core plasma temperature due to radiation losses and an increase in the average charge of the plasma cause an increase of plasma resistivity\(^2\). On the other hand, a controlled amount of impurities radiating energy in the plasma edge can help reduce the heat load on the PFCs and increase the stability of the plasma edge.

1.3.1 Impurity Sources

The thin layer of plasma between the vessel walls and the last closed flux surface (LCFS) of the plasma, called the scrape-off-layer (SOL), transports the plasma exhaust onto solid material target surfaces in both divertor or limiter configurations. These target materials must have high heat load capacity. The limiter configuration consists of a toroidally or poloidally symmetric ring of solid material which protrudes from the vessel wall and shields against any plasma interaction. In a divertor configuration, the plasma exhaust is directed onto toroidally symmetric tiles positioned either at the top or bottom (or both) of the vessel. Ideally, the only plasma contact is with the limiters or divertor targets, but in practice, this is not always achieved as unconfined neutral particles can strike all PFCs.

The divertor configuration, illustrated in figure 1.5, is widely accepted as the preferred exhaust solution for future tokamaks and will be used in ITER. The inventory of impurities in the plasma, other than atmospheric and alpha particles,\(^1\)

\(^1\)Fuel dilution describes the ratio \(n_{\text{DT}}/n_e\) where \(n_{\text{DT}}\) is the deuterium/tritium density and \(n_e\) is the electron density.

\(^2\)The plasma resistivity is not only an important parameter when evaluating the limit of ohmic heating, but also when considering the transport of particles due to collisions.
1.3. THE IMPACT OF IMPURITIES

depend on the material choice of the PFCs. The most common material choice for the divertor plates in present tokamaks is carbon, in the form of graphite or carbon fibre composites (CFC). CFC tiles can withstand a much larger power loading than most other materials available, however they suffer from chemical erosion; this is a serious problem when tritiated hydrocarbons form leading to tritium fuel retention [21]. In ITER, the divertor tiles are likely to be made of tungsten to avoid the potential tritium retention found with CFCs, while beryllium will be used in the main chamber [22]. The ITER set up is illustrated in figure 1.5. JET currently has an ITER-like wall installed designed to test these wall materials. Its configuration is also illustrated in figure 1.5.

A number of measures are taken to minimise the impurity content in the vessel chamber. Vacuum pumping of the vessel removes the majority of free molecules in the vessel, but not the trapped molecules absorbed in the vessel surfaces. To aid the vacuum pump, the vessel can be baked at $\sim 200^\circ$C for around 200 hours to evaporate the loosely bound molecules from the vessel surfaces. A technique called gettering involves evaporating a thin layer of metal onto the wall that is chemically reactive (known as a getter), such as titanium, to trap particles such as oxygen and carbon. Boronisation is a similar procedure that deposits a thin film of boron...
onto the wall surface to bind the high \( z_0 \) impurities already present on the wall. Boron is also an effective getter for oxygen. The effectiveness of these two processes quickly deteriorates and are therefore repeated every \( 50 - 100 \) pulses. The other technique used to condition the vessel is glow discharge cleaning (GDC). Energetic ions (often He) bombard the vessel surfaces and remove loosely bound fuel and impurity particles which are then pumped from the machine. GDCs are performed between plasma pulses for a short period of time (\(< 10 \) minutes), and can also be performed following the vessel bake for \( \sim 30 \) minutes.

The ion effective charge, \( Z_{\text{eff}} \), gives a local measure of the impurity concentration in the plasma, averaged over all impurities. The definition of \( Z_{\text{eff}} \) is given in equation 1.7, where the sum is over all ions of element \( i \) and charge state \( z \) with \( n_{i,z} \) denoting the density.

\[
Z_{\text{eff}} = \sum_{i,z} \frac{n_{i,z}z_i^2}{n_e}
\]  

(1.7)

For ITER, it is generally agreed that \( Z_{\text{eff}} \) must be \( \leq 2 \). It is clear that a large inventory of low and high \( z_0 \) intrinsic impurities, such as helium, carbon, beryllium and tungsten, and also atmospheric particles such as oxygen and nitrogen, all contribute to the \( Z_{\text{eff}} \) in a tokamak plasma core and must therefore be carefully monitored. This thesis focuses on \( \text{He}^{2+} \) and \( \text{C}^{6+} \) density measurements.

### 1.3.2 Charge Exchange Spectroscopy

Spectroscopic measurements of impurity spectral line emission can be used to deduce the impurity ion density, from the total intensity of the line, and the impurity temperature and velocity, from the Doppler broadening and wavelength shift of the line; it is the former that is of interest here. NBI hydrogen atoms travelling along the beam path provide a local source of active charge exchange (ACX) emission from the fully ionised low \( z_0 \) impurities in the plasma core. The ACX process is summarised in equations 1.8 and 1.9, where \( A^z \) denotes the fully ionised impurity and \( D^0_b(i) \) is hydrogen beam donor with a single electron in state \( i \) which is usually the ground state but can be in excited states, \( n \) is the principal quantum number and \( l \) is the angular momentum state of the recombined receiver ion.

\[
A^z + D^0_b(i) \rightarrow A^{z-1}(nl) + D^+_b
\]  

(1.8)

\[
A^{z-1}(nl') \rightarrow A^{z-1}(n'l'') + h\nu
\]  

(1.9)

Equation 1.9 represents the process which is the basis for charge exchange spectroscopy (CXS). The electron in the excited state of the receiver atom radiatively decays emitting a photon of energy \( h\nu \) which is equal to the energy difference between the levels \( n'l' \) and \( n''l'' \). There is also passive emission in the vicinity of the ACX spectrum induced by thermal charge exchange between thermal neutral hydrogen plasma atoms and \( \text{He}^{2+} \) and \( \text{C}^{6+} \) ions (often referred to as passive charge exchange
1.3. THE IMPACT OF IMPURITIES

PCX) or collisional excitation of the hydrogen-like He\(^+\) and C\(^5+\) ions located in the colder plasma edge [23].

A filtered 2D camera system called RGB [24, 25] was available on MAST to measure two \(n\)-shell transitions, specifically the HeII \((n : 4 – 3)\) and CVI \((n : 8 – 7)\) at \(\lambda = 468.5\) nm and \(\lambda = 529.1\) nm. Most CXS diagnostics, using a 1D array of discreet viewing chords attached to a spectrometer, separate the passive and active components by measuring a toroidally symmetric region of plasma with no active emission. For a diagnostic such as RGB, the 2D pixel capability can be exploited by selecting pixel regions of the image immediately above and below the beam volume to measure the passive signal. The ACX component of the measured emissivity can be used to infer the He\(^{2+}\) and C\(^{6+}\) densities locally along the beam path using the expression

\[
n_{i,z} = \frac{4\pi e_{i,z}^{(n\rightarrow n')} (n\rightarrow n')^{-1}}{q_{eff}^{(n\rightarrow n')}} \int S n_b ds
\]

where \(S\) is the intersection path length of the line-of-sight and the beam, \(n_b\) is the hydrogen beam density, \(e_{i,z}^{(n\rightarrow n')}\) is the ACX line radiance for the transition \(n \rightarrow n'\) (in ph/s/m\(^2\)/sr) and \(q_{eff}^{(n\rightarrow n')}\) is the effective CX emission coefficient. \(n_b\) is modelled using a code designed during this thesis and \(q_{eff}^{(n\rightarrow n')}\) is interpolated from the Atomic Data and Analysis Structure (ADAS) [26]. The temporal and spatial evolution of the radial impurity ion distribution inferred from CXS directly reflects the impurity transport.

1.3.3 Impurity Transport

The ultimate goal of impurity transport theory is to understand, control and predict the impurity flux across a wide variety of tokamak plasma conditions. The standard ansatz describing the radial impurity flux (perpendicular to the magnetic surfaces) uses a summation of diffusive and convective parts given by

\[
\Gamma_{i,z} = -D_i \frac{\partial n_{i,z}}{\partial r} + n_{i,z} v_i
\]

where \(r\) is the minor radius and \(D_i\) and \(v_i\) are the diffusion and convection transport coefficients respectively. Following a transient event like an impurity gas puff (where \(\Gamma_{i,z} \neq 0\)), the transport coefficients may be determined using two techniques. The first technique reproduces the spatial and temporal evolution of \(n_{i,z}\) using a radial transport code that solves the continuity equation,

\[
\frac{\partial n_{i,z}}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_{i,z}) + S_{i,z}
\]

with a given set of boundary conditions. \(S_{i,z}\) is the source and sink of particles controlled by atomic processes responsible for ionisation and recombination. The second technique fits a linear function to equation 1.11 at each point in space with
the gradient and offset of the fit representing \( D_i \) and \( v_i \) respectively.

A characteristic feature of CT plasmas is that the transport coefficients measured near the plasma edge are usually anomalously higher than those predicted for Coulomb collisions in a cylindrical plasma geometry (classical theory) or a toroidal plasma geometry (neoclassical theory). A large amount of work has gone into classical and neoclassical transport theory in the past and has been well documented [27]. The anomalous transport is caused by highly non-linear turbulence processes, with numerous turbulence drivers and suppression mechanisms. Although the theory for turbulence driven transport is not as complete as neoclassical, the understanding of the physical transport mechanisms has accelerated lately due to the advance in gyrokinetic computer modelling [28–31]. The extrapolation of previous transport studies on CTs (for example [32–34]) to STs is not straightforward, because the ST configuration is generally thought to achieve higher stability of the various plasma instabilities [10–13]. Data obtained from STs will not only improve the transport models but also broaden the international database on the aspect ratio by around a factor of two.

The analysis presented in this thesis focuses on evaluating the transport coefficients during a two point \( I_p \) scan and a plasma confinement mode scan, specifically L-mode and H-mode. The results generally indicate that the helium and carbon transport is neoclassical in H-mode and in the L-mode plasma core, whereas anomalous transport is the dominant mechanism in the outer radii of L-mode plasmas. Three codes are used to predict the transport: nCLASS [35] for neoclassical transport and the gyrokinetic codes GS2 [36, 37] and GKW [38] to give estimates for the level of anomalous transport. The analysis performed with these codes has allowed an explanation to be proposed of the differences between various plasma scenarios and to identify the main drives of neoclassical and anomalous transport.

1.4 Thesis Outline

The structure of this thesis is the following: Chapter 2 includes a review of the MAST vessel and diagnostics; specific focus will be given to the RGB diagnostic used to measure the CX emission, and the neutral beam system. The atomic physics models implemented in the ADAS codes, which are used to determine the effective CX emission coefficients, and the neutral beam stopping coefficient will be described in chapter 3. Chapter 4 provides all the experimental measurements, such as the CX emissivity measurements and the background plasma profiles, and describes the model used to determine the neutral beam and the impurity density. Transport coefficients are derived in chapter 5 and compared to results from other machines. These results are then discussed in light of the neoclassical and linear gyrokinetic simulations in chapter 6. Lastly, an overview and review of certain aspects of the work carried out for this thesis will be summarised in chapter 7.
Chapter 2

MAST and Diagnostics

2.1 Introduction

STs provide a new regime for the study of impurity transport processes in magnetised plasmas; one which must be understood to predict the performance of future devices. In CTs, impurity transport has been studied since the very early days of tokamak research. However, in STs this subject has been explored to a lesser extent because the ST is a configuration developed in more recent times with respect to the CT and also because other topics have been given higher priority so far. MAST and NSTX, two of the world’s leading STs, provide an ideal opportunity to study the transport properties of impurities in high temperature, low aspect ratio and highly elongated plasmas. Experiments have been performed on NSTX aimed at the characterisation of the transport properties of neon, lithium and carbon in H-mode [39, 40] and neon in L-mode [41]. Some work has also been done in the past on MAST on the behaviour of tin [42] and carbon [43]. During the last two MAST experimental campaigns, further experiments have been performed to expand the experimental measurements of light impurity transport and to improve the quality of the measurement of the evolution of the impurity concentration. NSTX and MAST are two complementary machines with similar design specifications, and therefore the present transport analysis is compared with the transport studies performed on NSTX to seek consistent features of impurity transport. The present analysis is also compared with transport experiments performed on Alcator C-Mod [30] (hereafter called C-Mod) to analyse some of the differences between ST and CT impurity transport.

The 2D filtered camera system on MAST, called RGB [24, 25], views the full plasma cross section with VGA sensor resolutions and frame rates of up to 210 Hz. RGB has the capability to simultaneously measure two spectral lines in the visible induced by charge transfer between the injected hydrogen beam atoms and the fully ionised helium and carbon ions in the plasma. These measurements will be used in the present work to infer radial profiles of the He$^{2+}$ and C$^{6+}$ plasma concentration which will be used (in chapters 4 and 5) for the transport analysis.
Transport studies on MAST are supported by simulation codes which require, as input, a description of the bulk plasma parameters. MAST is equipped with a comprehensive set of diagnostics, in particular with respect to profile diagnostics for the electron temperature and density, the ion density and toroidal velocity, and the safety factor, measuring typically every 5 ms with about 1 cm spatial resolution.

This chapter focuses on details of the MAST vessel and the supplementary diagnostics required to constrain the transport models. An overview of the neutral beam design is also given to provide the necessary inputs to the hydrogen beam density model described later (in chapters 3 and 4). A summary of the intrinsic (helium and carbon) and injected impurity gas (helium and methane) influx into the plasma is presented to provide realistic boundary conditions for the impurity transport model detailed later (in chapter 5).

2.2 MAST

MAST is a stainless steel vacuum tank which is cylindrically shaped with a diameter of \( d = 4.0 \) m, and height of \( h = 4.4 \) m. A cross section of the MAST vacuum vessel is shown in figure 2.1. A vertical column, covered with an outer layer of armoured tiles, runs down the centre of the vessel. To allow for a low plasma aspect ratio, the diameter of the centre column is limited to \( d = 0.4 \) m. In the middle of the centre column, a solenoid is wound around a centre rod incorporating 24 water cooled copper conductors which generate the TF, as indicated by the P1 and T7 symbols respectively in figure 2.1. On the magnetic axis, the TF can vary between \( B_T = 0.35 - 0.55 \) T. The TF is highly anisotropic across the plasma diameter and ranges from 2 T on the HFS to 0.25 T on the LFS as \( B_T \propto 1/R \). The solenoid is designed to produce a 1 V flux swing every \(~10\) minutes lasting approximately a second; this sets an upper limit on the plasma pulse length, although in practice various MHD events will terminate the plasma before the solenoid flux runs out. To fuel the plasma, deuterium gas is puffed from the HFS, LFS, bottom and top of the vessel into the plasma edge. For a tokamak such as MAST, the gas generally penetrates deep enough into the plasma to fuel the core. The plasma density throughout the pulse is controlled either by using a feedback system, or by presetting the fuelling times prior to the plasma pulse.

The \( I_p \) induced by the central solenoid produces the primary PF. A total of 5 pairs of PF coils are positioned inside the vessel as shown by the P2 – P6 labels in figure 2.1. Each of these PF coils are encapsulated in a 3-mm thick stainless steel shroud, providing an intrinsic source of iron in the plasma. The P4 and P5 coil pairs provide a vertical field to guard against any abrupt radial displacements, while the P6 coil pairs generate a radial field to provide vertical position control. Typically on MAST, the initial plasma is produced without using the central solenoid by a novel technique called ‘merging compression’ which was first pioneered on START [14].
2.2. MAST

Figure 2.1: Cross-section of MAST vacuum vessel with the internal coils and components labelled on the left, and an example plasma equilibrium on the right.

In this case, the current in the P3 coils is ramped to the maximum value, and then rapidly reduced to zero to create the electric field. Two geometrically opposite plasma rings are created around the P3 coil pairs and then merged together near the vessel equator using the P4 and P5 coil pairs. This method of plasma generation can typically produce an $I_p$ of up to 500 kA. The flux swing of the solenoid is then sufficient to ramp $I_p$ up to 1.0 MA, but more typically to around 600 – 900 kA. The volt-seconds remaining after the ramp up sustains a period of constant $I_p$, known as the current flat-top, for $\sim 500$ ms.

The lower and upper divertor coils, labelled P2 in figure 2.1, create a current in the same direction as $I_p$ causing a null in the PF, called the ‘X-point’, which is illustrated on the RHS of figure 2.1. Generally the magnetic flux surface associated with the X-point is called the last closed flux surface (LCFS), shown by the red dashed line in figure 2.1. Outside the LCFS to the vessel wall, the field lines are open and transport the plasma exhaust ions down to the divertor plates. MAST can operate using either one X-point, called a single null divertor (SND) configuration, or with two X-points at the top and bottom of the vessel, called a double-null divertor (DND) configuration. The experiments in this thesis use a DND configuration during each plasma scenario.
A summary of achieved MAST parameters is given in the first column of table 2.1. The second and third columns show the achieved NSTX and C-Mod [44] parameters to aid comparison of the present analysis with previous transport experiments performed on each of these tokamaks. There is little difference in plasma shape and temperature between MAST and NSTX parameters. C-Mod is a conventional aspect ratio tokamak and operates with a higher magnetic field than MAST.

2.2.1 Neutral Beam Injection

MAST is initially heated from the ohmic heating induced by the resistance to the toroidal plasma current caused by electron-ion collisions. At low temperatures, ohmic heating is quite powerful but is less effective at high temperatures because the resistivity varies with the plasma temperature as $T^{-3/2}$. On MAST, the ohmic heating typically generates a power in the range $0.5 - 1.0$ MW and raises the plasma temperature to $\sim 0.5$ keV. Auxiliary heating from neutral beam injectors (NBI) on MAST is provided by two (JET style) positive ion neutral injectors (PINI) [45, 46], named by their geographical position as the south (SS) and south-west (SW) beams (see figure 2.12). In total, the two PINIs can inject up to 3.8 MW of heating power into the plasma for $\sim 5$ s with typical accelerator voltages in the range of $U_b = 65 - 75$ kV. The energetic beam atoms, which are typically deuterium, penetrate into the plasma core where they ionise and thermalise with the plasma through collisions, raising the plasma temperature up to a few keVs.

The deuterium beam atoms also provide a local source of electron donation to the plasma ions along the beam path. Specifically, the work in this thesis models the density of He\(^{2+}\) and C\(^{6+}\) ions along the beam path using the spectral line emission

| Table 2.1: Achieved Specifications of MAST, NSTX, and Alcator C-Mod |
|-----------------|--------|--------|--------|
|                 | MAST   | NSTX   | C-Mod  |
| Plasma Current (MA) | 1.45   | 1.50   | 2.02   |
| Major Radius (m)    | 0.85   | 0.85   | 0.68   |
| Minor Radius (m)    | 0.65   | 0.67   | 0.22   |
| Aspect Ratio        | 1.31   | 1.27   | 3.91   |
| Elongation          | 2.6    | 2.5    | 1.75   |
| Triangularity       | 0.5    | 0.8    | 0.55   |
| Core Toroidal Field (T) | 0.52   | 0.45   | 8.11   |
| Pulse Length (s)    | 0.7    | 0.55   | 5.0    |
| NBI Power (MW)      | 3.8    | 5.0    | –      |
| Core Electron Temperature (keV) | 2.5    | 2.5    | 2.5    |
| Core Electron Density ($10^{19}$ m\(^{-3}\)) | 5      | 10     | 60     |
| Plasma Volume (m\(^3\)) | 10.0   | 12.5   | 1.0    |
2.2. MAST

Figure 2.2: Cross section of a Positive Ion Neutral Injector (PINI) box. Blue circles represent neutral gas, red circles represent ions, and elongated circles represent energetic particles.

induced by charge exchange between the beam atoms and the impurity ions. The impurity density model requires a model of the hydrogen beam density. The beam density model requires inputs of the geometry of the PINI internal components, the divergence of the beam, the linear beam density per metre entering the plasma, the mixture ratio of the molecular and monatomic atoms and finally the energy of the beam. To understand and provide these inputs, the rest of this subsection gives a summary of the PINI design. Chapter 3 discusses the attenuation of the hydrogen beam atoms in plasma due to atomic ionisation processes and chapter 4 details the beam density model in full.

A schematic drawing of the PINI module is illustrated in figure 2.2. The three primary components of the PINI module are the ionisation source, the acceleration extraction aperture and the neutralisation chamber. The magnetic configuration of the ionisation source determines the mixture ratios of the molecular ions and, to an extent, the divergence of the beamlets, the extraction aperture focuses the beam and the efficiency of the neutralising chamber reduces the beam current entering the plasma.

**Ionisation Source**

Brown [47] and Godden [48] provide a thorough review of the ionisation source. The input gas is normally deuterium for MAST experiments, so the reaction chamber includes the atomic and molecular species D, D$_2$ and D$_3$, in their various permitted states, and their associated ions. These neutral species are promptly ionised by the cathode-anode system within the ionisation source chamber. Partial confinement of the ionised gas is achieved using a multicusp magnet configuration surrounding the
chamber, where magnets of opposite polarity are placed together in each row and column of the walls in a configuration called the checkerboard. Collisions involving the primary energetic electrons emitted from the heated filament cathode ionise the molecular atoms whilst the thermal secondary electrons dissociate the molecular ions. The $D_2^+$ and $D_3^+$ ions lose atoms in the neutralisation chamber becoming $D^0$ atoms with kinetic energies in proportions a half and third respectively to the initial accelerating voltage and therefore travel a smaller distance into the plasma before ionising than the $D^0$ atoms with kinetic energies $E_k = eU_b$.

To increase the monatomic ion yield, another configuration called the supercusp is often used which places two equal polarity linear cusp magnets over the central region of the backplate and along the base of the source walls, as shown in figure 2.2. A long-range magnetic field is created by the supercusp configuration which is strong enough to confine the primary electrons to the region close to the filament cathodes, but weak enough to allow the ions and thermal electrons to drift towards the extraction aperture resulting in a higher monatomic ion yield. One disadvantage of the supercusp configuration is that the long range filter field degrades the uniformity of the plasma. As a result, the beam inherits a higher divergence thus decreasing the overall power delivered to the plasma.

Up until the latest M9 experimental campaign on MAST, both the SS and SW PINIs used the supercusp configuration. Spectroscopic measurements from beam into gas experiments on MAST revealed a fractional injection rate of 88:9:3 (% $E_k : E_k/2 : E_k/3$) at beam energies of 60 keV and a divergence angle (defined as the $1/e$ width of the beam) of 0.5°. For the M9 experimental campaign, the SW PINI was converted to the checkerboard configuration with a measured fractional injection rate of 70:23:7 at 60 keV. The analysis in this thesis focuses on the CX emission induced by the supercusp SS PINI beam; although the neutral beam model can model the beam atoms from either PINI.

**Extraction Aperture**

The extraction aperture consists of 262 beamlet holes within a grid area of 45x18 cm$^2$. This grid is constructed in two halves tilted towards each other producing a vertical focus (VF) length of 14 m. Each beamlet hole is also tilted to the beamlet normal producing a horizontal focus (HF) length of 6 m. The location of the beamlet holes is shown in figure 2.3a. The axes are in beam coordinates ($x_b, y_b, z_b$). To convert to machine coordinates, the coordinates are rotated around $z_b$ by an angle of 64.84° and 4.84° for the SW and SS PINIs respectively. The beam propagates along the equator of MAST (i.e $z_b = Z = 0$).

A voltage is then applied across the aperture to accelerate the ion beam. A typical trace of beam current, $I_b$, and voltage, $U_b$, for the SS and SW PINIs is shown in figure 2.3b. Beam voltage can be verified experimentally by the Doppler shift of the beam emission in the plasma and has been found to be accurate within
2.2. MAST

Figure 2.3: (a) The layout of the PINI extraction aperture beamlet holes. The $x$ and $z$ axes represent the plane perpendicular to the beam axis. (b) Time traces of the beam voltage and current for the SS and SW beams during two different MAST pulses.

±3 kV. A scatter of 5 A is also visible in the current during beam operation. The linear beam density per metre entering the plasma, $n_L^F$, of each species fraction, $F$, is determined using,

$$n_L^F = \frac{I^F \epsilon_N}{v_b^F e} \text{ [m}^{-1}]$$

(2.1)

where $e$ the electronic charge and $v_b$ is the speed of each beam fraction defined as

$$v_b^F = \sqrt{\frac{2eU_b}{m}} = 4.38 \cdot 10^5 \sqrt{E_b} \text{ [ms}^{-1}]$$

(2.2)

where $E_b = eU_b/m$ is the energy of the beam per mass in keV/amu. $\epsilon_N$ is the neutralisation efficiency parameter, which is discussed next.

Neutralisation Chamber

The energetic ions then travel through a neutralisation chamber where a significant proportion will undergo CX reactions with a neutraliser gas. Calculating the efficiency of the neutralisation chamber relies on the knowledge of atomic rate coefficients and a knowledge of the internal heating in the chamber. While $\epsilon_N$ has been extensively studied in previous works [49–51], this thesis will use the widely accepted value on MAST of $\epsilon_N \sim 0.54$. An electromagnet is put in place to deflect the residual fast ions into a cooled ion dump that can withstand heavy ion bombardment. To ensure the purity of the neutral beam, thermal neutrals are removed using vacuum pumps.

The significant parameters listed above are summarised in table 2.2.

| Table 2.2: PINI design specifications required for the beam model |
|------------------|----------------|----------------|----------------|----------------|----------------|
|                  | VF (m) | HF (m) | $\epsilon_N$ (%) | $E_k : E_k/2 : E_k/3$ (%) | Beamlets | 1/e (°) |
| SS PINI          | 6      | 14    | 54               | 88:9:3                   | 262      | 0.5    |
2.3 The RGB Diagnostic

Being able to image the plasma in the visible in two dimensions offers many advantages over typical single chord or a 1D array of discreet viewing chords attached to a spectrometer. For example, with a 2D system, reflections are easier to handle by avoiding the problem regions and an accurate spatial calibration of the diagnostic can be determined by comparing known objects in the field of view with a forward model based on the spatial calibration. An imaging diagnostic of this sort on MAST, called RGB [24, 25], views the full plasma cross section of MAST with VGA sensor resolution (640x480 pixel) and frame rates of up to 210 Hz, simultaneously measuring six pre-selected spectral band-passes in the visible through a single iris. The selected spectral regions are two Bremsstrahlung emissions, $D_\alpha$ beam and non-beam emission and the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) spectral line radiance. The latter two measurements are the primary measurements used for the transport analysis.

The optical layout of RGB is shown in figure 2.4. The design is based on a previous design by Patel et al [24,52]. The primary optics of the diagnostic includes a collection lens with a field stop, a beam splitter cube, two sets of stacked interference filters and two field lenses. In this part of the diagnostic, the plasma light is split into two channels by the beam splitter cube; for convenience these channels are called the upper and lower channels. The field stop, or iris, regulates the spectral throughput while the size of the optics determines the field of view (FOV). A FOV of $51^\circ$ is specified, which is sufficient to measure the horizontal extent of the plasma (see figure 2.12). Furthermore, RGB has been installed $\sim 20$ cm above the vessel equator to access radiation emanating from both the SS and SW beam volumes. The large focal length of the collection lens telecentrically images the plasma onto two primary image planes. Performing image operations in the telecentric region, where the chief rays are parallel to the optical axis, preserves the integrity of the image. The stacked filters, located in the telecentric region, consist of three unique interference filters combined with a set of tailored blocking filters. The interference filter stack
achieves the spectral fine-tuning of the system band-pass. The transmission of the upper and lower interference filter stack, measured following illumination from a broadband light source, is shown in figures 2.5a and 2.5b. A summary of the six pre-selected spectral band-passes are given in table 2.3.

The secondary optics section, shown for the lower channel in figure 2.4, includes a CCD camera system. The upper channel secondary optics are not shown in figure 2.4 as it is identical to the lower channel. The field lens in the primary optics converges the filtered light cone to within the aperture of the CCD camera and reduces the overall length of the diagnostic (to $\sim 0.5$ m). Both CCD cameras are equipped with a Bayer filter mosaic colour array as shown in figure 2.6a. The quantum efficiency of

Table 2.3: RGB spectral band-pass details and physics deliverables.

<table>
<thead>
<tr>
<th></th>
<th>Centre Wavelength (nm)</th>
<th>Effective Bandwidth (nm)</th>
<th>Emission</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Upper</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td>656.11</td>
<td>0.91</td>
<td>$D_\alpha$</td>
</tr>
<tr>
<td>Green</td>
<td>529.51</td>
<td>2.56</td>
<td>C VI ($n = 8 \rightarrow 7$)</td>
</tr>
<tr>
<td>Blue</td>
<td>468.86</td>
<td>2.77</td>
<td>He II ($n = 4 \rightarrow 3$)</td>
</tr>
<tr>
<td><strong>Lower</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red</td>
<td>660.00</td>
<td>3.92</td>
<td>Beam Emission</td>
</tr>
<tr>
<td>Green</td>
<td>561.90</td>
<td>2.21</td>
<td>Bremsstrahlung</td>
</tr>
<tr>
<td>Blue</td>
<td>457.23</td>
<td>2.13</td>
<td>Bremsstrahlung</td>
</tr>
</tbody>
</table>
each Bayer colour filter, as well as the monochrome response, is shown in figure 2.6b. The six pre-selected spectral band-passes are selected to broadly coincide with the high-end of the painted pixel. Grouping each of the separate colour matrix pixels together produces three different colour images for the upper and lower channels. The charges accumulated in both CCDs within the RGB diagnostic are digitised to a 12-bit resolution (12-bit DN/s).

A Bentham integrating sphere produces a broadband light source which uniformly fills the entire spectral band-pass filter, thus providing a spectral band-pass calibration. The spectral radiance of the integrating sphere is in units W/m$^2$/nm/sr and is set by controlling the electric current of the halogen bulbs within the lamp. This calibration therefore produces a conversion factor from the measured 12-bit DN/s to W/m$^2$/nm/sr. To convert from Watts to photons per second, the output is divided by $hc/\lambda_0$, where $\lambda_0$ is the centre wavelength of the spectral line or the centre wavelength of the spectral band-pass for broadband emission (see table 2.3). The band-pass calibration was carried out before the undertaking of this thesis work and was therefore obtained from private communication. To infer the impurity density from the measurements, the output of RGB must be in units of ph/s/m$^2$/sr, therefore the nm$^{-1}$ dependence from the band-pass calibration must be removed. This is done by multiplying the measurement by the effective band-width, $\Delta \lambda_f$, defined as

$$\Delta \lambda_f = \frac{1}{f(\lambda_0)} \int f(\lambda) d\lambda$$  \hspace{1cm} (2.3)$$

where $f(\lambda)$ represents the filter functions shown in figure 2.5.

This expression in equation 2.3 accounts for any attenuation of a spectral line due to the filter response. This is an important point to note for the Doppler shifted beam emission measurement. Due to the change in viewing angle across the beam line, the beam emission emits at a range of wavelengths. For viewing angles that are close to perpendicular to the beam, the spectral line emits at wavelengths close to the edge of the spectral band-pass and are therefore attenuated corresponding to
2.4. IMPURITY SOURCES

One of the methods used to model the impurity transport coefficients (see chapter 5) requires, as input, the source rate of neutral impurity atoms entering the LCFS. The impurity influx may arise from intrinsic or injected impurities. When the plasma comes into contact with plasma facing components (PFC), sputtering and chemical evaporation creates a source of impurities [54]. For sputtering, atoms are ejected from the PFCs if the elastic energy transfer from colliding ions exceeds the surface binding energy. The PFCs on MAST are mainly made of carbon fibre composites (CFC). For deuterium collisions, energies $> 100$ eV are sufficient to produce a high sputtering yield of carbon [55]. Carbon PFCs also can undergo chemical reactions with incident deuterium ions leading to the formation of volatile hydrocarbons which may be released from the PFC surface or sputtered at much lower threshold energies. Chemical erosion dominates at temperatures $< 100$ eV.

The divertor on MAST is of the open configuration [56] so the recycled particles from the divertor are free to migrate around the core plasma. MAST can allow for a larger volume of recycled gas to surround the plasma than most CTs due to the large plasma-wall separation. In particular for MAST, during the plasma current
2.4.1 Gas Influx

A sufficient amount of impurity gas must be injected to perturb the intrinsic impurity density. However injecting too large a quantity may alter the plasma properties. A piezo-electric valve located on the inboard side above the lower divertor is used in the present analysis to puff helium and methane (CH$_4$) gas into the vessel. The
Figure 2.9: The gas flow rate (particles/s) of methane (a) and helium (b) at a plenum pressure of 1 and 1.5 bar respectively. The solid and dashed lines represent measurement and prediction respectively. (c) helium and methane gas flow rates at 1.5 bar compared together.
valve is connected to an impurity plenum held at a constant pressure of 1.5 bar, which is sufficient to puff trace amounts of impurity gas into the machine vessel on relatively short timescales. The flow rate of the impurity gas into the vessel after a voltage was applied to the piezo-electric valve was measured using the screened Bayard-Alpert ionization gauge. Direct pressure readings vary as \( P \propto \frac{1}{R_g} \) where \( R_g \) is the relative sensitivity factor, which is dependent on the gas. The standard gas used by industry for gauge calibration is nitrogen, that is \( R_N = 1 \). Relative sensitivity factors for methane and helium are quoted as 1.4 and 0.18 respectively. The full calibration of the pressure has been performed on MAST for deuterium gas, therefore the correction factor is calculated as \( R_{CH_4,He}/R_{D_2} \) where \( R_{D_2} = 0.35 \).

During the calibration, the ionisation gauge returned the total number of particles (in \( \text{D}_2/\text{m}^3 \)) as a function of time. Taking the derivative of these curves in time and multiplying by the vessel volume (\( \sim 55 \text{ m}^3 \)) provides the flow rate of impurity atoms entering the vessel. Calibrations of the valve were carried out for helium at 1 bar, and methane at 1 bar and 1.5 bar. Flow rate profiles for methane and helium based on a 35 ms gas puff are shown in figure 2.9a and 2.9b respectively. It is seen from figure 2.9a that the 1.5 bar calibration of methane can be estimated by multiplying the 1 bar flow rate by a factor of \( \sim 1.4 \). Importantly, the temporal shape of the flow rate curve is the same at 1.5 bar compared to the 1.0 bar case. This multiplication factor is used to estimate the helium 1.5 bar flow rate. However the gas flow rate does not specifically describe the flow rate of neutral atoms into the plasma. The true flow rate is also dependent on the fuelling efficiency, which is discussed next.

### 2.4.2 Fuelling Efficiency

Cold, undirected neutral particles outside of the plasma do not easily penetrate into the plasma as they are ionised in the scrape off layer (SOL) and promptly directed towards the divertor. The fuelling efficiency, \( \epsilon_F \), is generally quantified by
2.4. IMPURITY SOURCES

Figure 2.11: The fuelling rate of different impurities is shown as a function of the injected quantity. For the transport studies, two different points are shown for carbon and helium which represent results from the $n_e$ and $n_{i,z}$ rise. Results from two previous studies from MAST [61] and ASDEX-U [62] have been used to produce the right hand side (disruption studies) of (a) and the full plot in (b).

comparing the measured rise in density with the expected rise in density from the gas flow rate calibration. One can either analyse the $n_e$ rise measured by Thomson scattering (see next section), or the modelled $n_{i,z}$ rise inferred from charge exchange. Both techniques have been carried out in the present analysis. This section will evaluate $\epsilon_F$ using the $n_e$ measurements, whereas chapter 4 compares the rise of the impurity profile with the modelled rise from the gas injection.

It is noted here that the $n_e$ rise is difficult to measure in this case, because the purpose of a tracer impurity injection is too minimise the $n_e$ perturbation. Nevertheless an attempt at estimating $\epsilon_F$ is made as shown in figure 2.10a and 2.10b for helium and carbon respectively. The electron densities have been integrated over the plasma volume and the modelled injected electron concentrations are calculated from 2xHe and 10xCH$_4$ for a 24 ms helium puff and a 34 ms methane puff. For helium, a value of $\epsilon_F \sim 85\%$ produces the best match between the modelled and experimental $n_e$ rise. The rise in $n_e$ for the methane injection is larger than for the helium injection because there are 10 electrons associated with every methane molecule. A value of $\epsilon_F \sim 8\%$ has been determined for methane, however this does not distinguish whether the electrons have entered attached to hydrogen or carbon.

In an attempt to calibrate these results of $\epsilon_F$ against other work, figure 2.11 shows how these values of $\epsilon_F$ compare to two disruption studies that were carried out on MAST [61] and ASDEX-U [62]. Figure 2.11a shows the results from MAST, including both the results from this thesis ($n_e$ and $n_{i,z}$ rise) and from the disruption studies [61], while figure 2.11b shows the results from the disruption studies on ASDEX-U [62]. One of the main conclusions from these graphs is that fuelling efficiency is dependent on the mass of the injected gas. Light impurities tend to produce higher fuelling efficiencies than heavier impurities. It was noted in the disruption study on MAST that this dependence could be due to the decrease
in propagation speed as mass increases, meaning that the particles take longer to reach the plasma edge [61]. There are two possible reasons why the helium fuelling efficiency is higher for the transport experiments compared to the disruption studies. Firstly, it is expected that as the fuelling rate increases, the gas fills the space between the vessel and plasma where there is minimal contact with the plasma instead of interacting with the plasma [61]. Secondly, it was unclear in the disruption studies whether the plasma was safely extinguished before the gas had a sufficient amount of time to reach the plasma edge.

These results are primarily useful to quantify the impurity source rate into the plasma. As a further result, it was also deduced that gas puffs of $\leq 30$ ms were sufficient to perturb the impurity profile whilst maintaining a relatively unperturbed background plasma equilibrium.

### 2.5 Supplementary Diagnostics

The plasma volume in MAST is typically $\sim 10 \text{ m}^3$, while the vessel volume is $\sim 55 \text{ m}^3$. This large ratio of vessel-plasma volume allows for an exceptional level of diagnostic access compared to other tokamaks around the world. When studying the transport of impurities, it is essential to obtain the background plasma parameters
so as to constrain the transport models. These measurements include the electron temperature, $T_e$, the electron density, $n_e$, the ion temperature, $T_i$, the ion toroidal velocity, $v_{\phi}$, the magnetic pitch angle, $\gamma_p$, the D$_{\alpha}$ radiation, the safety factor, $q$, the effective plasma charge, $Z_{eff}$, and the fluctuation frequency of the density to describe the background turbulence. An overview of the diagnostic layout on MAST is illustrated in the schematic of figure 2.12. A brief description of each diagnostic measurement used in the analysis of the impurity transport is now given.

**Electron Temperature and Density**

Measurements of $T_e$ and $n_e$ are carried out routinely on MAST by analysing the scattering of laser light from the electrons. From a classical viewpoint, this can be described by considering an electromagnetic wave causing an electron to accelerate due to the electric field associated with the wave. The electron emits electromagnetic radiation in all directions as it is accelerated, which corresponds to the scattered wave. This classical description is valid provided that the photon energy is less than the electron energy. The diagnostics used to measure the photon scattering in tokamak plasmas tend to be limited to measurements of visible or longer wavelengths, where the laser light has much less energy than the electrons. In this regime, the scattering process is called Thomson Scattering (TS).

The velocity distribution of the electrons cause a Doppler broadening of the scattered light. If the electrons have a non-relativistic Maxwellian velocity distribution (i.e. Gaussian), then the full width half maximum (FWHM) of the scattered light is related to the electron temperature by (see equation 6.4.8 in Ref. [63])

$$\lambda_{FWHM} = 4\lambda_0 \sqrt{\frac{\ln 2kT_e}{m_ec^2}} = 1.45 \cdot 10^{-1}\lambda_0\sqrt{T_e} \ [\text{nm}] \quad (2.4)$$

where $\lambda_0$ is the wavelength of the laser light in nm, $T_e$ is in keV and the area of the scattered spectrum provides the electron density. For $T_e > 200$ eV, the relativistic effects mean that a Gaussian model centred on $\lambda_0$ is no longer valid. In this case the observed blue-shift and width can be modelled using the analytical Selden equation [64]. For $T_e \geq 5$ keV, higher order relativistic effects become significant and the analytical Naito equation must be used [65].

The common components of a TS system include one, or multiple, lasers and a collection lens which splits the scattered spectrum into wavelength channels using a spectrometer. The original Nd:YAG TS system on MAST [66] was upgraded in 2008 to provide 130 core spatial points at a sampling rate of 240 Hz using eight 30 Hz Nd:YAG lasers [67]. Radial profiles of $T_e$ and $n_e$ given in this thesis are from this new Nd:YAG TS system.
CHAPTER 2. MAST AND DIAGNOSTICS

Ion Temperature and Velocity

Doppler broadening of spectral line emission from partially ionised impurities is used to determine \( T_i \). The diagnostic on MAST, named CELESTE-III, is designed specifically to analyse the visible C VI \((n = 8 \rightarrow 7)\) charge exchange emission (more details of charge exchange are given in chapter 3) with a typical sampling rate of 100 Hz [68]. A high throughput spectrometer is coupled to 64 active toroidal channels viewing the beam volume producing a spatial resolution of \( \sim 1 \) cm over the radial range of \( 0.8 – 1.4 \) m. Passive emission due to the high thermal neutral density in the vessel is subtracted from the active signal induced by the neutral beam using 64 toroidal channels viewing a toroidally symmetric region of the plasma with no active charge exchange signal. CELESTE-III can also be used to measure the \( v_\phi \) of the C\(^{5+}\) ions by measuring the Doppler shift of the emission. The properties of the carbon ions are assumed to be equal to those of the plasma ions because the energy transfer between the species occurs on a faster time scale than the energy confinement.

Effective Charge

The radiation produced by an electron that is accelerated in the electric field of a charged particle is called bremsstrahlung. Assuming the free electrons have a Maxwellian velocity distribution, the bremsstrahlung power per unit volume per meter from a single ion species is given by [69]

\[
\frac{\Delta P}{\Delta \lambda} = \frac{64 \pi}{3c^2} \left( \frac{e^2}{4\pi \epsilon_0} \right)^3 \left( \frac{\pi}{6m_e^3} \right)^{1/2} n_e n_{i,z} z_i^2 < g^{III} > \frac{n_i}{n_e} \left( \frac{T_e}{kT_e} \right)^{1/2} \lambda^2 \exp \left( - \frac{hc}{kT_e \lambda} \right)
\]

where \(< g^{III} >\) is the Maxwell averaged free-free Gaunt factor [70], \( n_{i,z} \) is the density of ion \( i \) with charge \( z \), \( m_e \) is the electron mass, \( \lambda \) is the wavelength of the radiation, \( c \) is the speed of light, \( \epsilon_0 \) is the vacuum permittivity, \( k \) is the Boltzmann constant and \( e \) is the electronic charge. To calculate the continuum emission from all plasma impurities, it is necessary to sum equation 2.5 over all ion species giving

\[
\epsilon_{br} = \frac{\Delta P}{\Delta \lambda} = 1.89 \cdot 10^{-35} n_e \sum n_{i,z} z_i^2 < g^{III} > \frac{n_i}{n_e} \left( \frac{T_e}{T_e \lambda} \right)^{1/2} \exp \left( - \frac{1240}{T_e \lambda} \right) [W/m^3/nm]
\]

where \( n_e \) and \( n_{i,z} \) are in \( m^{-3} \), \( T_e \) is in eV and \( \lambda \) in nm. Recall (from equation 1.7) that the effective charge is defined as

\[
Z_{eff} = \sum n_{i,z} z_i^2 \frac{n_i}{n_e}
\]
where the sum is over all ions of element $i$ and charge $z$ in the plasma. From equations 2.6 and 2.7, an expression for $Z_{\text{eff}}$ can be written as

$$Z_{\text{eff}} = \frac{\epsilon_{br} T_{e}^{1/2} \lambda^2}{1.89 \cdot 10^{-35} n_e^2} << g^{III} >> \exp(-1240/T_e \lambda)$$

(2.8)

where $<< g^{III} >>$ is averaged over all ions.

When measuring the bremsstrahlung radiation, it is important to avoid measuring line radiation. A 2D camera on MAST, called ZEBRA [52], measures the line-integrated emissivity over the entire plasma cross-section using a filter centred on 521.3 nm with a spectral band-pass of $\sim 1$ nm, which is thought to be free of any line radiation. Measuring in the visible spectral region is advantageous since there is a wide range of diagnostic instruments available. One disadvantage of this diagnostic is the fact that the bremsstrahlung spectral intensity falls off exponentially at wavelengths longer than those associated with the $T_e$ ($\sim 1$ nm). It is also unclear the extent to which molecular emission contributes to the signal near the plasma edge.

**Density Fluctuations**

Turbulent fluctuations are thought to induce a cross-field impurity flux greater than predictions based solely on collisions. A goal of turbulence simulations is often to determine the correlation length, or eddy size, of the dominant turbulent modes. The fluctuations are typically in the frequency range of the order of $10^6$ Hz and can be observed in the $D_\alpha$ emission of the injected neutral beam atoms. On MAST, a 2D diagnostic, called the BES [71] diagnostic, was positioned to measure turbulent electron density fluctuations near the plasma edge with a radial and poloidal resolution of $\sim 2$ cm [72]. These measurements are used to confirm the lack of ion scale turbulence found from gyrokinetic simulations in chapter 6.

**Safety Factor and Plasma Shape**

The theoretical models used in chapter 6, describing the stability of the plasma, require a profile of the safety factor and a parametrisation of the plasma shape. On MAST, the equilibrium reconstruction code called EFIT++ [73,74] solves the elliptic second-order, non-linear partial differential Grad-Shafranov equation describing the force balance in the plasma. It is given by

$$\Delta^* \psi = -\mu_0 R^2 \frac{dp}{d\psi} - \frac{dF^2}{d\psi}$$

(2.9)

where $p$ is the pressure, $\psi$ is the poloidal magnetic flux, $F(\psi) = RB_T$ is a function related to the total poloidal current, and $\Delta^* \psi = R^2 \nabla \cdot (R^{-2} \nabla \psi)$ denotes the elliptic operator. TS profiles are mapped to $\psi$ and used as a pressure constraint. A
Figure 2.13: A poloidal cross-section of the magnetic flux geometry using the Miller equations (in black) and using the \texttt{efit++} code (in red) for the #29424 discharge on MAST at $t = 0.24$ s. The Miller flux surfaces shown in (a) are chosen to best represent the \texttt{efit++} magnetic reconstruction, while (b) and (c) demonstrate the effect of each Miller parameter.

Constraint on the plasma current profile and the safety factor $q$ is obtained from experimental measurements of the magnetic pitch angle, which is related to $B_P$, as

$$
\gamma_P = \tan^{-1} \left( \frac{B_p}{B_T} \right) = \tan^{-1} \left( \frac{a}{Rq} \right)
$$

(2.10)

The pitch angle is inferred from measurements of the motional Stark effect (MSE) diagnostic on MAST [75]. This works on the principle that the (Doppler shifted) $D_\alpha$ multiplet from neutral beams is split into degenerate Stark states. Each of these transitions are either polarised parallel or perpendicular to the electric field, denoted by $\pi$ and $\sigma$ components respectively. The orientation of the polarisation direction with maximum emission is measured which reveals the pitch angle of the magnetic field. The poloidal current in the field coils is measured using a solenoid that is wound around the field coil with both ends returning to the same point in space (Rogowski coils). The boundary conditions of $\psi$ are constrained from experimental measurements of $D_\alpha$ emission from the plasma edge obtained from an optical linear camera, called LinCam [76], which measures the plasma $D_\alpha$ emission using a narrow spectral band-pass. An example of the flux surfaces obtained during a discharge on MAST using \texttt{efit++} using the constraints given above is shown in figure 2.13.

The gyrokinetic codes use a simplified model of the poloidal cross sectional shape of the plasma, which can be defined following the equations used by Miller et al. [77] (known as Miller parameters). These equations are specified in 2.11 and 2.12 where $\theta$ is the poloidal angle, $r$ is the minor radius, $\kappa$ is the elongation, $\delta$ is the triangularity,
and $S$ is the magnetic axis shift, known as the Shafranov shift \cite{77}.

\begin{align}
R &= R_0 + S + \frac{Sr^2}{a} + r \cos(\theta + \sin^{-1} \delta \sin \theta) \quad (2.11) \\
Z &= Z_0 + \kappa r \sin \theta \quad (2.12)
\end{align}

Figures 2.13a–c demonstrates the effect of changing each Miller parameter. It should be noted that a cylindrical coordinate system can be reproduced with $\kappa = 1$, $\delta = 0.0$ and $S = 0$. Typical plasmas on MAST have Miller geometry values of $\kappa \leq 2.5$ and $\delta \leq 0.5$. An example of a magnetic equilibrium solved explicitly using efit++ is also shown in figure 2.13a to provide comparison with the Miller geometry.

## 2.6 Summary

MAST can produce plasma with a maximum plasma current flat-top of $\sim 1.3$ MA for $\sim 0.5$ s. Plasmas are typically elongated ($\kappa \leq 2.5$) and triangular ($\delta \leq 0.5$) with an aspect ratio of $R/a = 0.85/0.65 = 1.3$. Carbon and helium are intrinsic, the latter introduced from the helium glow discharge cleaning and the former from the plasma facing components. Two neutral beams on MAST, called by their geometrical position (SS and SW), are capable of injecting 3.8 MWs into the plasma. Design specifications of the neutral beams are required for the beam density model described in chapter 4 and they are as follows: the SS PINI uses the supercusp magnetic configuration producing a beam fractional injection rate of 88:9:3 (% $E_k : E_k/2 : E_k/3$), with a typical voltage and current of 70 kV and 60 A. There are 262 beamlets on the PINI grid directed to a horizontal and vertical focal point 14 m and 6 m respectively, each with a divergence of 0.5°. The neutralisation efficiency of the PINI is 0.54.

The primary diagnostic used to measure the impurity emission is called the RGB diagnostic. RGB measures six different narrow spectral band-passes simultaneously through a single iris. These spectral band-passes record the plasma and Doppler shifted beam $D_\alpha$ spectral lines, bremsstrahlung emission in the blue and green regions of the spectrum and the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) spectral lines induced by charge exchange between the hydrogen beam atoms and the He$^{2+}$ and C$^{6+}$ ions. The spatially and absolutely calibrated (ph/s/m$^2$/sr) helium and carbon lines are used later to infer He$^{2+}$ and C$^{6+}$ density profiles, which are used for the transport analysis. Helium and methane gas puffs are used to inject controlled amounts helium and carbon into the plasma. The flow-rate of each gas from the gas nozzle has been measured using the fast ion gauge on MAST. Plasma fuelling estimates from the electron density rise during a gas puff experiment suggest a fuelling efficiency of $\sim 80\%$ for helium, and $\sim 10\%$ for methane. Puffs of $\leq 30$ ms are thought to be sufficient to perturb the impurity profiles whilst not significantly changing the bulk plasma properties.
A number of plasma parameters are required to constrain the theoretical transport models used later (in chapter 6) which include: $T_e$, $n_e$, $T_i$, $v_\phi$, $Z_{\text{eff}}$, $q$ and the Miller parameters. The technique of Doppler broadening is used to measure $T_e$ (from the TS diagnostic) and $T_i$ (from the CELESTE-III diagnostic). $n_e$ is inferred from the amplitude of the scattered spectrum measured by the TS diagnostic. The toroidal speed of the carbon ions are determined by the Doppler shift of the line emission. Bremsstrahlung emission measured by ZEBRA is used to calculate $Z_{\text{eff}}$, however these measurements are less reliable near the plasma edge. The magnetic equilibrium is solved using the EFIT++ code, with constraints provided by the pitch angle (MSE diagnostic), the plasma pressure (TS), and a visible interpretation of the LCFS provided by the D$_\alpha$ emission (from LinCam). An accurate solution of the magnetic equilibrium provides the $q$ profile and is used to fit the Miller parameters.
Chapter 3

Charge Exchange Spectroscopy and Atomic Physics

3.1 Introduction

From the perspective of atomic processes, tokamak plasmas are characterised as ‘electron excited’ because there is no external radiation field and the plasma itself is largely transparent to its own radiation\(^1\). That is thermal atoms and ions in the plasma are primarily excited by collisions with free electrons which are generally close to Maxwellian with an electron temperature, \(T_e\), at each point in the plasma. With \(T_e\) ranging from a few eV in the edge to core values of several keV, line emission occurs from many ionisation stages over a wide spectral range. In this respect the tokamak is like the solar corona plasma, and is often treated in the solar ‘coronal approximation’. The coronal approximation is a zero density approximation balancing collisional excitation/ionisation with radiative decay/recombination. The electron density in the modern tokamak is of order \(10^4\) higher than the solar corona, so the coronal approximation has to be replaced by the more sophisticated collisional-radiative (CR) model which deals with finite density plasma. Furthermore modern tokamak plasmas have regions that are significantly different in behaviour from the coronal case, which are central to this thesis. At the edge of the plasma there is a significant concentration of neutral hydrogen and this allows a charge transfer reaction from the hydrogen donor to an impurity ion receiver which modifies the local state of excitation and spectral emission called passive charge exchange (PCX) spectroscopy. Also in neutral hydrogen beams, the beam hydrogen atoms act as donors to impurity ion receivers in highly ionised states in the core of the plasma, also modifying the local ionisation state and allowing spectral line emission called active charge exchange spectroscopy (ACX).

In the present analysis of CX based emission, the distribution of receiver impurity ions in the plasma matters, and these are determined by the balance between

\(^1\)There is some self absorption of hydrogen Lyman resonance lines in current machines. In ITER this may extend to a limited set of resonance lines of low ionisation stages of light elements
effective recombination and ionisation, influenced also by the transport of the impurity ions across the plasma. CR theory, based on atomic plasma relaxation time constants, determines the handling of the effective coefficients for establishing the ionisation state and emission in the plasma with transport. This thesis uses an elaborated version of CR theory as the underlying basis of the ionisation stage modelling, called generalised collisional-radiative (GCR) theory which takes account of metastable lifetimes. The CR modelling required for impurity ions as PCX and ACX receivers, beam atoms and their emission, and passive thermal plasma emission are different and are addressed in the following sections.

3.1.1 ADAS

The models and effective rate coefficients used for the analysis in this thesis are stored within the Atomic Data Analysis Structure (ADAS) [26]. ADAS incorporates a suite of computer codes, written in FORTRAN and IDL™ programming languages, designed to model the radiating properties of ions and atoms in plasmas. These codes can be accessed through a set of subroutines that may be incorporated into a user’s personal code. Data in ADAS are stored in standard formats, with the various collections given the format names of the form ADFxx, with the xx indicating the data type. Values in the data set are generally specified on a grid of energies, temperatures and densities. On supplying a set of grid points appropriate to a specific user’s calculation, the ADAS extraction routines provide an asymptotic extrapolation of the values stored in the data set. In total, there are more than fifty data types in the ADAS database, however this thesis uses only the specific data types listed in table 3.1: ADF21 and ADF22 data sets provide the beam stopping and emission coefficients described in section 3.4, ADF12 data set provides the effective CX emission coefficients, ADF01 data set is used to provide the n-shell capture cross sections illustrated in figure 3.2, ADF07 and ADF04 data sets provide the electron impact ionisation and excitation rate coefficients used to estimate the ratio of plume to ACX emission and ADF11 provides the various collisional-radiative coefficients required for establishing the ionisation state of impurities.

<table>
<thead>
<tr>
<th>Data type</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>ADF01</td>
<td>Bundle-n and bundle-nl charge exchange cross-sections</td>
</tr>
<tr>
<td>ADF04</td>
<td>Resolved specific ion fundamental data collections</td>
</tr>
<tr>
<td>ADF07</td>
<td>Direct resolved electron impact ionisation data collections</td>
</tr>
<tr>
<td>ADF11</td>
<td>Effective ionisation, recombination and CX rate coefficients</td>
</tr>
<tr>
<td>ADF12</td>
<td>Effective CX emission coefficients</td>
</tr>
<tr>
<td>ADF21</td>
<td>Beam stopping coefficients</td>
</tr>
<tr>
<td>ADF22</td>
<td>Effective beam emission coefficient</td>
</tr>
</tbody>
</table>
3.2 Impurity Ionisation Balance

The primary chord-integrated measurements made in this thesis are of the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) spectrum lines at $\lambda = 468.5$ nm and $\lambda = 529.1$ nm, which are significantly enhanced by ACX. Modelling of the impurity density is based on the ACX component of these measurements, which is localised to the region along the beam path or its immediate vicinity where the primary beam donors or secondary (halo) donors are present. $T_e$ is sufficiently hot that only He$^{2+}$ and C$^{6+}$ ions exist over virtually all of this path. The passive emission contributing to the spectral line emission is localised at the edge of the plasma with contributions from electron impact excitation and from charge transfer from thermal neutral hydrogen in the edge region. Thus it depends on the fractional abundance of He$^+$, C$^{5+}$ and O$^{5+}$ ions\(^2\) for the electron impact component, whereas the PCX component requires the fully ionised impurities and thermal neutral hydrogen populations to overlap in the same region of plasma.

The fractional abundance of each ion charge state is determined by a balance of effective ionisation and recombination rates (the CR rate coefficients) for each ionisation stage in combination with transport. The CR coefficients are obtained by solution of statistical balance equations containing all relevant collisional, radiative and recombination processes that populate and depopulate the atomic levels [78]. CR theory assumes that excited state populations with short relaxation times (ordinary excited states) are in statistical equilibrium with the instantaneous values of the long-lived ground state populations. This assumption can be validated by comparing the approximate expressions for the metastable, $\tau_m$, and ordinary, $\tau_o$, relaxation times shown in equations 3.1 and 3.2 against the ionisation, $\tau_{ion}$, and recombination, $\tau_{rec}$, timescales given in equations 3.3 and 3.4 [79].

\[
\begin{align*}
\tau_m &\sim 10/z^8 \text{ [s]} \quad (3.1) \\
\tau_o &\sim 10^{-8}/z^4 \text{ [s]} \quad (3.2) \\
\tau_{rec} &\sim [10^{11} - 10^{13}](1/(z + 1)^2)(T_e/I_H)^{1/2}(1/n_e) \text{ [s]} \quad (3.3) \\
\tau_{ion} &\sim [10^5 - 10^7](z + 1)^3(I_H/T_e)^{1/2}e\chi/T_e(1/n_e) \text{ [s]} \quad (3.4)
\end{align*}
\]

$T_e$ is in eV, $n_e$ is cm\(^{-3}\), $I_H$ is the ionisation energy of a hydrogen atom (13.6 eV) and $\chi$ is the ionisation potential in eV. Note that atomic convention uses centimetres, however the other chapters in this thesis will use metres. For most tokamak plasmas

\[
\tau_{plasma} \sim \tau_g \sim \tau_m >> \tau_o \quad (3.5)
\]

where $\tau_g$ represents the relaxation time of ground state populations of ions (a composite of $\tau_{rec}$ and $\tau_{ion}$) and $\tau_{plasma}$ represents the time for plasma ion transport across the temperature or density scale lengths. The timescale for redistribution

\(^2\)The O VI ($n = 8 \rightarrow 7$) line occurs at a similar wavelength to the C VI ($n = 8 \rightarrow 7$) line.
of population amongst excited states, $\tau_{\text{red}}$, is complicated due to the very large variation in collisional excitation/de-excitation reaction rates with the $n$-shells of participating states; therefore $\tau_{\text{red}}$ can span the inequalities of equation 3.5. The timescale orderings are considered in greater detail for fast beam atoms and radially moving impurity ions in section 3.5.

The influence of the ordinary states is condensed and projected on to the ground state population to produce stage to stage CR ionisation and recombination rate coefficients. Helium and carbon atoms have metastable states with relaxation timescales somewhere between ordinary states and ground states. GCR theory can provide resolved CR rate coefficients for these metastable states or unresolved stage to stage rate coefficients by condensing the influence of the metastables on to the ground in a similar manner to the ordinary states [79, 80]. ADAS calculations of ADF11 coefficients for light element ions are performed in the more precise GCR treatment at root, with the less precise CR coefficients being condensed from them. The requirement in this section is the unresolved stage to stage CR effective ionisation and recombination coefficients stored in the ADAS ADF11 data set. They are used to determine the ionisation balance of the helium and carbon ions, and to model the source terms for each ionisation stage in the transport equations described later in chapter 5.

In the following paragraphs, a brief summary is made of the relevant parts of GCR/CR theory. Consider the ‘collisional-radiative matrix’, $C_{i,z}^{(k,j)}$, where off-diagonal elements for an ion $i, z$ contain the collisional and radiative rate coefficients linking states $k$ and $j$ and diagonal elements $C_{i,z}^{(k,k)}$ contain the total loss rate coefficients from state $k$. The recombination rate into the level $k$ from the $z + 1$ ion is denoted by $r_{i,z}^{(k)}$. The long-lived time dependent levels are denoted by $\rho, \epsilon$ and the remaining quasi-static ordinary states are denoted by $k$ and $j$. The continuity equation for the population densities in matrix form are given in equation 3.6 where an implicit sum over repeated superscript indices is implied.

$$\frac{dn_{i,z}^{(\rho)}}{dt} = \begin{bmatrix} C_{i,z}^{(\rho,\epsilon)} & C_{i,z}^{(\rho,j)} & n_{i,z}^{(\rho)} & n_{i,z}^{(j)} \\ C_{i,z}^{(k,\epsilon)} & C_{i,z}^{(k,j)} & n_{i,z}^{(k)} & 0 \end{bmatrix} + n_{e} n_{i,z} + 1 \begin{bmatrix} r_{i,z}^{(\rho)} \\ r_{i,z}^{(k)} \end{bmatrix}$$ (3.6)

Expressing the population of quasi-static excited levels as

$$n_{i,z}^{(j)} = -n_{e} n_{i,z} + 1 C_{i,z}^{(j,k)} r_{i,z}^{(k)} - C_{i,z}^{(j,k)} - 1 C_{i,z}^{(k,\rho)} n_{i,z}^{(\rho)}$$ (3.7)

and substituting $n_{i,z}^{(j)}$ into equation 3.6 for $n_{i,z}^{(\rho)}$ allows the influence of the excited state populations to be condensed and projected on to the time dependent states
giving the effective GCR ionisation and recombination rate coefficients as \(^3\).

\[
S_{i,z}^{\text{CD}} = C^{(\rho, \rho)}_{i,z} - C^{(\rho, j)}_{i,z} C^{(j, k)}_{i,z}^{-1} C^{(k, \rho)}_{i,z} \tag{3.8}
\]

\[
\alpha_{i,z}^{\text{CD}} = r^{(\rho)}_{i,z} - C^{(\rho, j)}_{i,z} C^{(j, k)}_{i,z}^{-1} r^{(k)}_{i,z} \tag{3.9}
\]

One component of \(r^{(k)}\) is the CX recombination rate coefficient, \(q^{(k)}_{\text{CX}}\), which is multiplied by \(\zeta = n_H/n_e\), where \(n_H\) is the hydrogen (or donor) density. This is usually separated from \(r^{(k)}\) and quoted as the effective CX recombination rate coefficient, denoted by \(C_{i,z}^{\text{CD}}\) as

\[
C_{i,z}^{\text{CD}} = q^{(\rho)}_{\text{CX}} - C^{(\rho, j)}_{i,z} C^{(j, k)}_{i,z}^{-1} q^{(k)}_{\text{CX}} \tag{3.11}
\]

The time dependence of a population of ions in charge state \(z\) can be written in terms of these GCR coefficients as shown in equation 3.12.

\[
\frac{dn_{i,z}}{dt} = \left( n_e n_{i,z-1} S_{i,z-1}^{\text{CD}} + n_e n_{i,z+1} \alpha_{i,z+1}^{\text{CD}} + n_H n_{i,z+1} C_{i,z+1}^{\text{CD}} \right) - n_{i,z} \left[ n_e S_{i,z}^{\text{CD}} + n_e \alpha_{i,z}^{\text{CD}} + n_H C_{i,z}^{\text{CD}} \right] \tag{3.12}
\]

Note that the terms in (\(\ldots\)) and [\(\ldots\)] denote the source terms referred to later in chapter 5. In equilibrium ionisation balance, the charge state populations are the solutions of the matrix equation

\[
\begin{bmatrix}
-S_{i,0}^{\text{CD}} & \alpha_{i,1}^{\text{CD}} + \zeta C_{i,1}^{\text{CD}} & 0 & 0 \\
S_{i,0}^{\text{CD}} & -(S_{i,1}^{\text{CD}} + \alpha_{i,1}^{\text{CD}} + \zeta C_{i,1}^{\text{CD}}) & \alpha_{i,2}^{\text{CD}} + \zeta C_{i,2}^{\text{CD}} & 0 \\
0 & S_{i,1}^{\text{CD}} & \ldots & \ldots \\
0 & 0 & \ldots & \ldots
\end{bmatrix}
\begin{bmatrix}
n_{i,0} \\
n_{i,1} \\
n_{i,2} \\
\ldots
\end{bmatrix} = 0 \tag{3.13}
\]

subject to the normalisation \(n_T^i = \sum_z n_{i,z}\).

Dashed lines in figure 3.1b and 3.1c represent the fractional abundance of the helium and carbon ionisation stages calculated in ADAS using equation 3.13 with the temperature and density profiles illustrated in figure 3.1a. He\(^{2+}\) and C\(^{6+}\) ions are dominant over the range \(0 \leq r/a \leq 1\) and \(0 \leq r/a \leq 0.8\) respectively. Transport also influences the distribution of charge states within the plasma, and this can be accounted for by expanding the total time derivative in equation 3.12 in terms of the particle flux \(\Gamma_{i,z}\) as shown in equation 3.14.

\[
\frac{dn_{i,z}}{dt} = \frac{\partial n_{i,z}}{\partial t} + \nabla \cdot \Gamma_{i,z} \tag{3.14}
\]

To model the ionisation balance including transport, a 1.5D transport code named

\(^3\)The suffix \(\text{CD}\) indicates that the coefficients include dielectronic recombination rates. Note the difference between plasma and atomic physics notation. Usual atomic physics notation uses a superscript \(z\) and a subscript \(\text{cd}\).
Figure 3.1: (a) Time averaged $n_e$ and $T_e$ profiles from MAST discharge #29424 used to calculate the ionisation balance of helium (b) and carbon (c). The dashed lines are calculated using ADAS assuming zero particle flux and the solid lines are calculated using SAND assuming a particle flux based on the $D_i$ and $v_i$ profiles from an L-mode discharge (see chapter 5).
3.3. CX EFFECTIVE EMISSION COEFFICIENT

The population model required to predict the rate of spectral line emission is a special type of CR model. It includes ACX capture from hydrogen beam atoms into all excited states up to some cut-off principal quantum shell. Tabulations of experimental or theoretical state selective charge exchange cross-section data, stored in the ADAS adf01, span the important range of principal quantum shells \( n_0 < n < n_1 \). The method of solution used is called the ‘l-redistributive cascade model’. It progresses the solution recursively downwards from the cut-off \( n \)-shell using these CX cross-sections in association with spontaneous emission coefficients and both electron and ion impact re-distributive cross-section between \( l \)-substates. The population solution therefore progresses downwards in \( n \) with lower levels populated by CX and cascading electrons from higher states; no collisional excitation from lower to higher \( n \)-shells is allowed although direct collisional-ionisation losses are allowed. The ADAS solution used here works with \( nl \)-subshell populations. An alternative ADAS \( lj \)-redistributive cascade model works with \( nlj \)-sublevels but results are little different and the \( nl \)-method results are the most widely used [82].

At this point, it is worth discussing the state selective CX cross-section data
to explain the main difference between thermal and non-thermal CX. Figure 3.2 illustrates these cross-sections as a function of \( n \)-shell capture for a thermal and non-thermal collision energy and an \( n = 1 \) and \( n = 2 \) donor state population. Thermal hydrogen donors in the \( n = 1 \) shell tend to transfer into a critical \( n \)-shell of the impurity ion, \( n_{\text{crit}} \sim Z^{3/4} \). Shells around \( n_{\text{crit}} \) are called the dominant receiver shells. Donation from the \( n = 2 \) shell populates \( n \)-shells approximately \( \sim 2n_{\text{crit}} \).

Levels of the receiver ion above \( n_{\text{crit}} \) are called sub-dominant levels. Visible spectral line emission from the receiver is between sub-dominant levels, typically with upper level around \( 2n_{\text{crit}} \). At low donor atom speeds, typical of thermal plasma, the selectivity is very strongly into \( n_{\text{crit}} \) from the ground state donor. Thus thermal CX into the sub-dominant states requires an excited donor (in states \( n > 1 \)). As the beam energy of the donor increases towards that of typical beams, the \( n \)-shell selectivity of the capture becomes less pronounced and capture cross-sections to sub-dominant levels approach a fall-off with \( \sigma \propto n^{-3} \). Thus at full beam energies it is generally the ground state donor which is most effective in populating the receiver levels emitting visible transitions. Because there are fractional energy components in the beam, both ground and excited donor levels are included.

The chord-integrated ACX line radiance (in \( \text{ph/s/m}^2/\text{sr} \)) can be calculated using the expression

\[
\epsilon_{i,z}^{(n \rightarrow n')} = \sum_{l,l'} \epsilon_{i,z-1}^{(nl \rightarrow n'l')} = \frac{1}{4\pi} \int_S \sum_{l,l'} A^{(nl \rightarrow n'l')} n_{i,z-1}^{(nl)} ds
= \frac{1}{4\pi} \int_S \left( \sum_{l,l'} \frac{A^{(nl \rightarrow n'l')} n_{i,z-1}^{(nl)}}{n_b n_{i,z}} \right) n_b n_{i,z} ds
= \frac{1}{4\pi} \int_S q_{\text{eff}}^{(n \rightarrow n')} n_{i,z} n_b ds
\approx \frac{1}{4\pi} q_{\text{eff}}^{(n \rightarrow n')} n_{i,z} \int_S n_b ds
\]

(3.15)

\( S \) is the intersection path length of the line-of-sight and the beam and \( A^{(nl \rightarrow n'l')} \) is the spontaneous decay rate. The expression (\( ... \)) is evaluated in the \( l \)-redistributive CR model. It depends on local plasma conditions, but is independent of \( n_b n_{i,z} \). It is the local CX effective emission coefficient, \( q_{\text{eff}}^{(n \rightarrow n')} \). If the approximation is made that \( T_e, n_e, B_T \) etc. are constant along the sight-line \( S \), \( q_{\text{eff}}^{(n \rightarrow n')} \) can be taken outside the integral. The term \( \int_S n_{i,z} n_b ds \) is called the ‘emission measure’. The further assumption may be made that \( n_{i,z} \) is constant along \( S \). The total \( q_{\text{eff}}^{(n \rightarrow n')} \) is dependent on the fraction of donor atoms in the \( n = 1, 2, ..., \infty \) shells. However, the fraction of donor atoms in \( n = 3 \) is of the order of \( 10^{-4} \) across most of
3.3. CX EFFECTIVE EMISSION COEFFICIENT

Figure 3.3: The CX effective emission coefficient for the He II \((n = 4 \rightarrow 3)\) (a) and C VI \((n = 8 \rightarrow 7)\) (b) spectral lines are shown as a function of collision (or beam) energy. The contributions from the \(n > 2\) donor atoms are ignored in this analysis.

The temperature and density ranges of interest, therefore the contribution from the \(n > 2\) donor atoms is ignored in this analysis. The total \(q_{e_{\text{eff}}}^{(n \rightarrow n')}\) values provided by the ADAS \textsc{adf12} dataset for the He II \((4 \rightarrow 3)\) and C VI \((8 \rightarrow 7)\) spectral lines are illustrated in figures 3.3a and 3.3b respectively; these graphs have accounted for the fraction of donor atoms in the \(n = 1\) and \(n = 2\) shells using data from the \textsc{adf22} dataset.

This model is only used to provide the effective emission rate for the ACX process. Passive emission from electron excitation and thermal CX is separated from the ACX signal using an experimental subtraction technique (discussed later in chapter 4). There are, however, two secondary sources of active emission induced by NBI that are not accounted for in this model which are difficult to separate experimentally from the primary ACX. Firstly, the beam is surrounded by a diffusive ‘halo’ of thermal neutrals created following the charge transfer between the hydrogen beam donor atoms and the thermal hydrogen ions in the plasma. This thermal halo acts as a secondary donor source for the impurities. The second source of emission stems from the hydrogen-like impurity ions that are created following the ACX reactions. The impurity electron decays to the ground state almost instantaneously and the resultant ion then continues to follow the field lines; these ions are referred to as ‘plume’ ions [83]. As the plume ions travel around the plasma, there is a probability of secondary emission due to electron excitation which can contribute to different ACX sight-lines. A model for these two processes is presented in the following subsections.

3.3.1 Beam Halo

The ratio of the averaged density of halo atoms to beam atoms (assuming \(Z_{\text{eff}} = 1\) and neglecting CX ionisation losses for simplicity) is given by \(n_h/n_b = q_{\text{CX}}^{\text{TOT}}/q_e^{n \rightarrow \infty}\), where \(q_{\text{CX}}^{\text{TOT}}\) and \(q_e^{n \rightarrow \infty}\) are the total CX rate from a donor beam atom into the receiver halo atom and electron impact ionisation rate respectively. In the core
plasma of MAST ($Z_{eff} \sim 1.2$, $T_e = T_i \sim 1$ keV and $n_e \sim 4 \times 10^{19}$ m$^{-3}$), $q_e^{(n\rightarrow\infty)} \sim 2.2 \times 10^{-8}$ cm$^3$s$^{-1}$ and $q_{CX}^{\text{TOT}} \sim 4.6 \times 10^{-8}$ cm$^3$s$^{-1}$ which gives a ratio of $\sim 2.1$. This ratio will decrease with increasing beam energy and electron temperature.

An estimate of the fraction of $n = 2$ halo atoms can be made by considering a balance of the main populating and de-populating processes of the $n = 2$ halo atom, as shown in equation 3.16 where $n_i$ is the plasma proton density, $q_{CX}^{(k \rightarrow n)}$ is the CX rate coefficient from donor state $k$ into receiver shell $n$, $q_e^{(n \rightarrow n')}$ is the electron impact excitation rate coefficient, and $n_b^{(k)}$ and $n_h^{(n)}$ are the hydrogen beam and halo densities in state $k$ and $n$ respectively.

$$n_i n_b^{(2)} q_{CX}^{(2 \rightarrow 2)} + n_e n_h^{(1)} q_e^{(1 \rightarrow 2)} = n_h^{(2)} A^{(2 \rightarrow 1)} + n_e n_h^{(1)} q_e^{(1 \rightarrow 2)}$$

$$n_h^{(2)} = \frac{n_i n_b^{(2)} q_{CX}^{(2 \rightarrow 2)} + n_e n_h^{(1)} q_e^{(1 \rightarrow 2)}}{A^{(2 \rightarrow 1)} + n_e q_e^{(2 \rightarrow \infty)} n_b^{(2)}}$$

$$n_h^{(2)} \approx \frac{n_e q_{CX}^{(2 \rightarrow 2)}}{A^{(2 \rightarrow 1)} n_b^{(2)}}$$

The first approximation in equation 3.16 is made because the CX rate from the $n = 2$ beam atom donor into the $n = 2$ halo atom receiver is of the order of $10^{-6}$ cm$^3$s$^{-1}$, while the electron collisional excitation rate for $n = 1 \rightarrow 2$ is of the order $10^{-12}$ cm$^3$s$^{-1}$. The second assumption in equation 3.16 is made because, according to equations 3.2 and 3.4, the spontaneous decay rate of the $n = 2$ shell is of the order of $10^8$ s$^{-1}$, while the electron ionisation rate from $n = 2$ is of the order of $10^7$ s$^{-1}$. It is therefore reasonable to assume that the density of halo atoms in level $n = 2$ is less than or equal to the density of beam atoms in the level $n = 2$, that is $n_h^{(2)} \leq n_b^{(2)}$.

With this approximation, the graphs in figure 3.3 can be used to provide an estimate of the halo atom contribution to the active emission. At thermal energy (1 keV), $q_{CX}^{(n \rightarrow n')}$ is approximately two orders of magnitude less than the value at typical beam energies for the He II ($n = 4 \rightarrow 3$) line and approximately an order of magnitude less for the C VI ($n = 8 \rightarrow 7$) line. One could expect the halo to increase the helium and carbon emission by $\sim 2\%$ and $\sim 20\%$ respectively. This has been tested explicitly (see figure 4.12 in chapter 4) by including a fourth thermal beam fraction representative of the halo within the impurity density calculations.

### 3.3.2 Plume Ions

Next, consider a plume ion created along the beam path at a radius $R_1$. The ratio of ACX to plume emission at $R_1$ can be calculated using the expression

$$\frac{\epsilon_{ACX}^{(n \rightarrow n')}}{\epsilon_{PL}^{(n \rightarrow n')}} = \frac{\int_S n_i z q_{eff}^{(n \rightarrow n')} n_b ds}{\int_S n_m q_e^{(1-n)} b^{(n \rightarrow n')} n_e ds}$$

(3.17)
where \( b^{(n \rightarrow n')} = A^{(n \rightarrow n')} / \sum_{n'} A^{(n \rightarrow n')} \) is the dimensionless branching ratio and \( n_{pl} \) is the plume density which can be modelled using the expression derived by Bell et al. [84]. The average plume density, \( \langle n_{pl} \rangle \), can be estimated by assuming the plume cloud to have a linear extent of the beam width, \( w \sim 15 \text{ cm} \), plus the ionisation length, \( \lambda_{i,z} = v_\phi / n_e q_{\infty}^{1 \rightarrow \infty} \), while the ACX signal is assumed to only originate from the region \( w \). This gives the expression

\[
\langle n_{pl} \rangle = n_{i,z} \frac{n_b q_{tot}^{acx}}{n_e q_{\infty}^{1 \rightarrow \infty} w + \lambda_{i,z}}
\]  

(3.18)

where \( q_{tot}^{acx} \) is the total CX rate coefficient from all beam donor states into all plume receiver states.

If the integrals in equation 3.17 are approximated by multiplying the mean values of the corresponding densities and rate coefficients by \( w \), then the expression for \( \langle n_{pl} \rangle \) in equation 3.18 can be substituted into equation 3.17 to give

\[
\frac{\epsilon_{ACX}^{(n \rightarrow n')}}{\epsilon_{PL}^{(n \rightarrow n')}} = \frac{q_{eff}^{n \rightarrow n'}}{q_{tot}^{acx} q_{\infty}^{1 \rightarrow \infty} b^{(n \rightarrow n')}} \frac{w + \lambda_{i,z}}{w}
\]  

(3.19)

The direct rate coefficients for electron ionisation and excitation are provided by the ADAS ADF07 and ADF04 data sets respectively and illustrated in figure 3.4a. Branching ratios for the \( n = 4 \rightarrow 3 \) and \( n = 8 \rightarrow 7 \) transitions are 0.298 and 0.127 respectively. Figure 3.4b illustrates the dependence of \( \lambda_{i,z} \) on \( v_\phi \) for three different values of \( n_e \) and \( T_e \). The total rate coefficients for CX into the He\(^+\) and C\(^{5+}\) plume ions are both of the order of \( 10^{-7} \text{ cm}^3/\text{s} \). Consider a point along the beam path with \( T_e = T_i = 1 \text{ keV} \), \( n_e = 3.5 \times 10^{19} \text{ m}^{-3} \) and a parallel velocity equal to the thermal velocity. The C\(^{5+}\) plume ions have relatively long ionisation lengths and low electron excitation rate coefficients meaning that the plume brightness is negligible compared to the ACX brightness. On the other hand, for He\(^+\) the ACX signal is only around five times brighter than the plume signal.
There is then the issue of the plume ions travelling along the field lines and emitting in sight-lines measuring ACX emission from different regions along the beam path. The ACX emission is dimmer in the core due to the exponential decay of the neutral beam donor atoms, while the plume brightness decays at a rate proportional to $\exp(-L/\lambda_{i,z})$, where $L$ is the distance along the flux surface. In addition, $C^{5+}$ plume ions complete at least one revolution around the plasma and therefore emit in front of and behind the beam volume. However $He^{+}$ plume ions do not complete one revolution and therefore only radiate behind the beam volume (assuming anti-clockwise toroidal $I_p$ flow). If the pitch angle of the magnetic flux surfaces is such that the plume ions don’t physically intersect the active sight-lines, then plume emission can be disregarded. A review of the magnetic geometry and RGB sight-lines with respect to the plume emission is given in section 4.3.

### 3.4 Beam Stopping and Emission

The final ingredient required to model the impurity density is the line-of-sight integrated hydrogen beam donor density. In chapter 2, an expression was derived for the number of beam atoms entering the plasma based on the beam current and voltage. An attenuation model is required to determine how far these hydrogen atoms penetrate into the plasma before ionising (or charge exchanging). This distance can be determined by calculating the CR ionisation coefficient (including loss by charge transfer) of the hydrogen beam atom at each point along the beam path, which in literature is commonly referred to as the beam stopping coefficient. The CR model described in section 3.2 models the hydrogen atoms in a beam by incorporating the beam atom translational velocity in the integrals of cross-sections over Maxwellian distributions. Only populations of complete $n$-shells are evaluated (the bundle-$n$ approximation) since redistribution among $l$-substates of the beam atoms is complete, establishing a statistical population of $l$-substates. More precisely, it is noted that the hydrogen beam atoms experience a large motional Stark electric field which also mixes $l$-substates of different parity. The substates of $n$-shells of the moving beam atom are properly represented as Stark manifold states in $nkm$ quantisation. The Stark manifold populations of an $n$-shell are very close to statistical. Observations of the $D_\alpha$ beam emission show that the radiative transitions are spectrally resolved into motional Stark multiplets. Although there is the capability in ADAS to expand the bundle-$n$ solution over low $n$-shell Stark states, this is not required in this study since only beam stopping and integral $n \rightarrow n'$ emission are of concern.

Since ionised beam atoms are lost from the beam, there are no recombination processes for the beam. With a steady state neutral beam source ($\partial/\partial t = 0$), the expression in equation 3.12 can be simplified for the beam density along the beam
3.4. BEAM STOPPING AND EMISSION

Figure 3.5: The shine through factor for the SS beam with energy 35 keV/amu travelling through the (time-averaged) L-mode plasma #29424.

path, with coordinate \( l \), as

\[
\nabla \cdot \mathbf{\Gamma}_b = v_b \frac{dn_b}{dl} = -n_e S_{b}^{cd} n_b
\]

(3.20)

with a general solution for each beam density fraction, \( F \), written as

\[
n^F_b = n^F_L \exp \left( -\int_L^n n_e v_b S_{b}^{cd} \, dl \right) \Phi^F
\]

\[
= n^F_L \Lambda^F \Phi
\]

(3.21)

where \( L \) is the path length of the beam atoms, \( n^F_L \) is defined in equation 2.1, \( v_b^F \) is the beam speed defined in equation 2.2 and \( \Lambda^F = \exp \left( -\int_L^n n_e v_b S_{b}^{cd} \, dl \right) \) is called the shine through factor. \( \Phi \) includes the divergence and focussing of the beam and has units of \( \text{m}^{-2} \). A derivation of \( \Phi \) is given in section 4.4.

Impurity collisions with the beam atoms play the central role in determining the effective ionisation rate coefficient. The CR model used within ADAS includes arbitrary mixtures of impurity ion colliders. However the associated multi-dimensional look-up tables are unsuitable for fast analysis. Rather, the code is executed for each light impurity species (from H\(^+\) to Ne\(^{10+}\)) treating the plasma as composed of a pure species (and its associated electrons). A mixed species beam stopping coefficient is constructed as a linear superposition from these pure solutions as described and assessed by Anderson et al. [85]

\[
S_{b}^{cd}(E_b, n_e, T_i) = \sum_i z_i f_i S_{b,i}^{cd}(E_b, n_{equiv,i}^{equiv}, T_i)
\]

(3.22)

where \( f_i \) is the fraction of impurity ion \( i \). Since it is the ions driving the collisionality primarily and not the electrons, the density of impurities is derived from \( n_e \) using
the equivalent electron density, \( n_{ei}^{\text{equiv}} \), defined as
\[
    n_{ei}^{\text{equiv}} = \frac{n_e \sum_k z_k^2 f_k}{z_i \sum_k z_k f_k}
\]
(3.23)

Concentrations of helium and carbon in MAST are generally < 5% and < 0.5 % respectively (see chapter 4), therefore the inclusion of each impurity in the ionisation calculation is only a minor correction. Figure 3.5b illustrates the \( \Lambda \) factor for a hydrogen beam atom entering the plasma with a beam speed of 35 keV/amu, using the plasma profiles from figure 3.1a. For a pure hydrogen plasma, around half the population of beam atoms ionise after they reach the plasma core. A pure helium or carbon plasma causes an enhanced attenuation of the beam, as shown by the green and blue lines respectively in figure 3.5b, however a difference of < 1% is found when using typical helium and carbon concentrations found in MAST, as shown by the red line in figure 3.5b.

A benchmark of this beam density model is carried out by comparing the modelled and measured beam \( D_\alpha \) (\( n = 3 - 2 \)) spectral line from the SW beam in MAST (see chapter 4). The intensity of the spectral line driven by excitations may be written as
\[
    \epsilon_b^{(n-n')} = \sum_{y=1}^{3} \int_S A^{(n-n')} n_b^{(n)} x \frac{n_e n_b^F}{n_e n_e^F} ds
    = \sum_{y=1}^{3} \int_S q_{\text{bis}}^{(n-n')} n_e n_b^F ds
\]
(3.24)
where \( q_{\text{bis}}^{(n-n')} \) is the effective beam emission coefficient stored in the ADF22 data set.

### 3.5 Atomic, Transport and Beam Timescales

The primary assumption used to obtain the CR rate coefficients is that ordinary states relax rapidly compared to the rate at which the temperature and density are changing spatially and temporally in the plasma. Consider first the time for plasma parameters to evolve in a typical MAST discharge, \( \tau_{\text{plasma}} \), compared to the spontaneous relaxation time of the ordinary states, \( \tau_o \). \( \tau_{\text{plasma}} \) is also used later to define the time for particles to travel across the plasma pressure scale length. According to equation 3.2, the ordinary excited states relax in timescales of \( \leq 10 \) ns. For MAST, the typical current flat-top and diffusion times are of the order of 100 ms and the global energy confinement time of the order of 10 ms. Fluctuations in the electron density caused by plasma turbulence generally have a wave like character with frequencies of the order of 1 \( \mu \)s [12]. Thus it may be assumed that \( \tau_{\text{plasma}} \gg \tau_o \) for plasma ions travelling along closed field lines, where the pressure is spatially constant (see equation 1.3). Transport in MAST causes the impurity ions to move across closed field lines and therefore the pressure is not spatially constant.
3.5. ATOMIC, TRANSPORT AND BEAM TIMESCALES

Similarly, NBIs direct hydrogen beam atoms towards the plasma core meaning that the pressure changes rapidly in space. Another concern is that an excited beam atom may travel a certain distance across the diagnostic field of view before radiatively decaying. It must be determined whether or not this distance is greater than the spatial resolution of the diagnostic.

The spontaneous decay time, $1/\lambda_{n \rightarrow n'}$, provides an upper limit to the lifetime of each excited atomic state. Collisional excitation/de-excitation processes may decrease the lifetime of the state depending on the plasma collisionality and the $n$-shell of the participating state. The reciprocal of the diagonal components of the CR matrix created within ADAS provides the exact lifetime of each state and is therefore used in the following paragraphs, along with the spontaneous decay times, to provide a comparison of these various timescales and distances discussed in the paragraph above.

Let the time for particles to travel across the pressure gradient scale length be expressed generically as

$$\tau_{\text{plasma}} = \frac{L_p}{v_t}$$

(3.25)

where $L_p = -p(\partial p/\partial r)^{-1}$ is the plasma pressure scale length and $v_t$ is the total particle speed defined separately for impurity ions and beam atoms as

$$v_t = \begin{cases} 
\frac{D_i}{L_{n_i,z}} + v_i & \text{Impurity ions} \\
 v_b & \text{Beam atoms}
\end{cases}$$

(3.26)

$D_i$ and $v_i$ are the radial impurity diffusion and convection coefficients, $L_{n_i,z}$ is impurity density scale length and $v_b$ is the beam speed defined in equation 2.2.

Consider a He$^{2+}$ ion being transported radially across the plasma cross-section with a constant impurity diffusivity of 1 m$^2$/s and convection of -20 m/s in the plasma edge (the minus indicating inward transport) and similarly a hydrogen beam atom injected towards the plasma core with an energy of 35 keV/amu. An illustration of $\tau_{\text{plasma}}$ is given in figure 3.6a for these two cases during an L-mode plasma. It is noted that $L_p$ in H-mode plasmas is typically longer than in L-mode plasmas (except in the steep density pedestal of the H-mode plasma edge). Radial transport of impurity ions across $L_p$ occurs in timescales many orders of magnitude longer than $\tau_o$. On the other hand, fast hydrogen beam atoms travel across the plasma pressure gradient scale length in timescales similar to $\tau_o$.

The exact lifetime of the $n = 2, 3, 4$ shells of the hydrogen beam atom, provided by the diagonal elements of the CR matrix generated in the ADAS model described in section 3.4, are illustrated as a function of $n_e$ in figure 3.6b. At typical MAST densities ($10^{19}$ m$^{-3}$), the $n = 3$ shell is the longest lived excited state with a lifetime approximately an order of magnitude less than $\tau_{\text{plasma}}$ in the plasma core. The finite distance travelled by the $n = 3$ hydrogen beam atom during its lifetime is illustrated in figure 3.6c. For MAST beam energies and densities, the distance is of the order
Figure 3.6: (a) The time for He$^{2+}$ ions and D$^0$ beam atoms to travel across the plasma pressure scale length in the L-mode discharge #29424 as a function of normalised radius. The solid lines in (b) represent the lifetimes of the D$^0_n$ excited states calculated using ADAS and the dashed lines represent the spontaneous decay rates. (c) The distance travelled by the $n = 3$ beam atom before relaxing to the ground state as a function of beam energy.
of 1 cm. This is sufficient for the purposes of this thesis where the diagnostic spatial resolution is also of the order of 1 cm. It is also noted that, for the expected energies used by diagnostic beams on ITER, the predicted spatial displacement of the beam emission is closer to $\sim 1$ mm assuming a higher density of the order of $10^{20}$ m$^{-3}$. The spatial displacement of the beam emission may increase up to $\sim 10$ cm at densities of the order of $10^{19}$ m$^{-3}$ and beam energies of 1 MeV which are predicted for the ITER heating beams.

### 3.6 Summary

This chapter has discussed three ADAS models that are used to determine the fractional abundance of helium and carbon charge states in the plasma, the CX effective emission rate coefficient and the attenuation and emission rate of the hydrogen beam atoms. The models suggest that the He$^{2+}$ and C$^{6+}$ ions exist over virtually all of the beam path, however a significant fraction of lower ionisation stages exist near the plasma edge for carbon (and to a lesser extent helium) which may reduce the ACX signal in the region $0.8 \leq r/a \leq 1.0$. The amount of secondary emission induced by the beam halo atoms and plume ions has been estimated by considering the balance of the primary atomic processes at play. Emission induced by the beam halo atoms is thought to increase the intensity of the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) spectral lines by $\sim 2\%$ and $\sim 20\%$ respectively. Emission from the electron excited plume ions at their point of creation is almost two orders of magnitude less than the ACX emission, however as they travel along the closed field lines and drift in front of different sight-lines their emission may become comparable to the ACX signal. This latter point is addressed further in the next chapter.

Lastly, focus was given to the assumption made in the CR models regarding the ordering of timescales for spontaneous decay time of the excited atomic states, $\tau_o$, and the time for the excited atoms to travel across the plasma pressure gradient scale length, $\tau_p$, that is $\tau_p >> \tau_o$. Radial transport of impurity ions across the plasma pressure gradient scale length occurs in timescales many orders of magnitude longer than $\tau_o$. On the other hand, fast hydrogen beam atoms travel across the plasma pressure gradient scale length in timescales similar to $\tau_o$. An illustration of the exact lifetimes of the excited beam atom states, provided by the CR matrix generated in ADAS, showed that the electron density in MAST is high enough such that collisional processes decrease the lifetime of the excited states to values around half that expected from the spontaneous decay time. The analysis also showed that $n = 3$ shell is the longest lived state and, with a beam energy of 35 keV/amu, may travel $\sim 1$ cm across the diagnostic field of view before emitting.
Chapter 4

Plasma Scenarios and Measurements

4.1 Introduction

The global energy confinement time, $\tau_E$, is a central parameter in the design of tokamak fusion reactors. The relationship between the thermal helium confinement time, $\tau_{He}$, (which is directly related to the transport) and $\tau_E$ can also indicate whether plasma scenarios will be viable for future DT plasma [20]. Previous findings from CTs suggest that $\tau_E/\tau_{He} \sim 1$ [57, 86, 87], however this relationship in an ST has not been studied. It is commonly found in CTs that $\tau_E$ is around a factor of two greater in the core of H-mode plasmas compared to L-mode plasmas and scales almost linearly with $I_p$ and weakly with inverse aspect ratio, $\epsilon$, in both L-mode and H-mode plasmas [88]. However previous results from MAST indicate that $\tau_E$ exhibits a weaker dependence on $I_p$ in H-mode [89] and stronger dependence on $\epsilon$ in L-mode [90]. In the present analysis, time-dependent impurity transport experiments, which involve puffing trace amounts of helium and methane (CH$_4$) gas into the plasma edge, have been carried out to study the behaviour of helium and carbon during a two point $I_p$ scan in L-mode and a confinement mode scan, specifically L- and H-mode, at constant $I_p$. $T_e, T_i$ and $B_T$ have been matched in both scans in an attempt to produce essentially one dimensional scans of $q_{95}$ (from the $I_p$ scan) and the $n_e$ gradient (from the confinement mode scan). Although results from MAST previously indicated that $\tau_E$ scales weakly with $q_{95}$ in H-mode [91], these scalings have not been tested for impurity transport in L-mode.

The RGB diagnostic produces 2D pixel frames of emission measured through two narrow spectral band-pass filters. Pixels capturing ACX and passive emission in the vicinity of the beam volume are mapped to the plasma major radius by calculating the intersection point between the pixel line of sight and the beam axis. Radial measurements of the background plasma parameters (like $n_e$, $T_i$ and $Z_{eff}$) are interpolated onto this radial map to extract the CX effective emission coefficients. The emission captured in pixels immediately above and below the beam and outside
of the halo region is averaged to estimate the magnitude of the passive emission within the beam volume. The beam density is modelled separately in 3D machine coordinates and combined with the diagnostic sight-lines to provide the impurity density model with line-integrated beam densities. The following sections will review the plasma scenarios and scans used in the analysis and provide further detail of the steps involved in modelling the impurity density.

4.2 Plasma Parameter Scans

Both plasma parameter scans in this thesis begin from the same reference plasma scenario: an L-mode plasma with \( I_p = 900 \) kA, \( B_T = 0.55 \) T, additional heating power from the SS NBI of \( P_{NBI} = 2.1 \) MW, on-axis \( n_e = 3.5 \times 10^{-19} \) m\(^{-3}\), \( R_0 = 0.83 \) m, \( a = 0.6 \) m, \( \kappa = 1.93 \) and \( \delta = 0.4 \). Continuous NBI heating of the plasma is applied from \( t = 0.05 \) s onwards, meaning that CXS is also available from this time onwards. The plasma has an unconnected double null divertor configuration with deuterium as the working gas. This first scenario is based on the MAST H-mode plasma pulse #22664 described by Valović et al. [91]. To keep the plasma in L-mode, the vertical position of the magnetic axis was shifted above the equatorial plane of the machine by \( \sim 5 \) cm; this technique is particular to MAST and is not general practice on other CTs. Ideally two repeat plasmas should be performed directly after each other, one without and one with a gas puff (and in that order), to obtain the unperturbed plasma profiles. In practice, this was not always achievable due to operation restrictions. Repeat plasmas were run when possible. In a number of the scenarios, the reference for the helium gas puff plasma was taken from the plasma with the methane gas puff, and vice versa for the methane gas puff plasma. This was possible because the gas puffs did not perturb the background plasma. A summary of the plasma parameters listed above are given in the first column of table 4.1. The second and third columns show parameters of two other plasma scenarios discussed in subsections 4.2.1 and 4.2.2.

Time traces of the plasma parameters for the first scenario are illustrated in

<table>
<thead>
<tr>
<th>Parameter</th>
<th>L-mode, High ( I_p )</th>
<th>Low ( I_p )</th>
<th>H-mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_p ) (kA)</td>
<td>900</td>
<td>600</td>
<td>900</td>
</tr>
<tr>
<td>Confinement Mode</td>
<td>L-mode</td>
<td>L-mode</td>
<td>H-mode</td>
</tr>
<tr>
<td>On-axis ( n_e ) (10(^{19}) m(^{-3}))</td>
<td>3.5</td>
<td>4.2</td>
<td>3.8</td>
</tr>
<tr>
<td>On-axis ( B_T ) (T)</td>
<td>0.55</td>
<td>0.55</td>
<td>0.55</td>
</tr>
<tr>
<td>( P_{NBI} ) (MW)</td>
<td>2.1</td>
<td>3.2</td>
<td>2.1</td>
</tr>
<tr>
<td>( q_{95} )</td>
<td>5.5</td>
<td>6.3</td>
<td>6.3</td>
</tr>
<tr>
<td>Helium pulse # (ref.)</td>
<td>29261 (29424)</td>
<td>29417 (29420)</td>
<td>29275 (29276)</td>
</tr>
<tr>
<td>Methane pulse # (ref.)</td>
<td>29424 (29261)</td>
<td>29421 (29417)</td>
<td>29425 (29426)</td>
</tr>
<tr>
<td>Helium gas timing (s)</td>
<td>0.195 – 0.219</td>
<td>0.175 – 0.189</td>
<td>0.265 – 0.279</td>
</tr>
<tr>
<td>Methane gas timing (s)</td>
<td>0.185 – 0.219</td>
<td>0.160 – 0.189</td>
<td>0.240 – 0.279</td>
</tr>
</tbody>
</table>
Figure 4.1: Time traces for the plasmas with helium (left) and methane (right) gas puffs. The averaging time periods and gas puff timings are shown at the bottom of both columns.
figure 4.1 by the green lines. Both helium and methane gas puffed plasmas have similar time histories except for the edge $D_\alpha$ emission, where a temporary increase is found during the methane puff but not during the helium puff plasma. This is probably due to the increase in the edge neutral hydrogen density following the dissociation of the injected hydrocarbon. The plasma evolves rapidly during the $I_p$ ramp-up, as shown by the $T_e$ traces, therefore the timing of the gas puff was triggered at the start of the $I_p$ flat-top at $t \sim 0.2$ s. A sharp rise in the He II ($n = 4 \rightarrow 3$) emissivity is found at mid-radius indicating that the helium profile was successfully perturbed by the gas puff. A smaller rise in C VI ($n = 8 \rightarrow 7$) emissivity is found at mid-radius after the methane puff which is possibly due to an overall outflux of intrinsic carbon from the plasma as shown by the decrease in the dotted line representing the intrinsic signal.

The absence of edge localised modes (ELMs) in L-mode allows study of neoclassical and turbulent transport without the perturbations induced by the intermittent MHD phenomena [92]. However MHD activity is always present from a certain time onwards during MAST pulses, as is illustrated in figures 4.2a and 4.2b using the Fourier spectrogram from the inboard Mirnov coils. Frequency sweeping modes, known as fishbone instabilities, evolve at $t \sim 0.26$ s into the internal kink instability known as the ‘long-lived mode’ (LLM) which occurs at frequencies around $F \sim 20$ kHz [93]. Fishbones eject fast ions from the plasma reducing the heating efficiency of the neutral beam injection [94]. It is not thought that the fishbone instability significantly affects the impurity evolution. LLMs typically occur in MAST plasmas when the core safety factor is $\leq 1.3$ and can cause a decrease in toroidal rotation and electron and ion confinement [93]. Since there is no model currently available to analyse the transport of impurities associated with MHD, the time window available for the time-dependent transport experiment is in practice reduced to $50 - 100$ ms so as to avoid the LLM. The analysis times are shown explicitly by the shaded regions at the bottom of figure 4.1.

Profiles of $T_e$ and $n_e$ from TS, the $q$-profile solved by EFIT++, $Z_{eff}$ from the deuterium, helium and carbon ions (see equation 4.16) and $T_i$ and $v_\phi$ measured by CX are time averaged over their respective analysis windows and illustrated by the green lines in figure 4.3. The figure shows only the profiles for the helium puff plasmas. Results from the carbon puff plasmas are similar within the error bars. The radial grid is $\rho = \sqrt{\phi_N}$, where $\phi_N$ is the toroidal flux normalised to the value at the LCFS. Both neoclassical and gyrokinetic codes used in chapter 6 operate in $\rho$ space and therefore it is helpful in this chapter to map the background plasma profiles, required as inputs for the codes, and the impurity density profiles, required to determine the transport coefficients, in $\rho$ space. Raw emissivity and neutral beam density profiles are still presented in $R$ space, but note that the conversion between $\rho$, $R$ and $r/a$ for each plasma is demonstrated later in figures 5.5, 5.8 and 5.11. Although the analysis time window only captures the very beginning of the LLM,
4.2. PLASMA PARAMETER SCANS

Figure 4.2: The Fourier spectrogram from the inboard Mirnov coils is used to highlight the MHD activity found through the L-mode high $I_p$ plasma (top), the low $I_p$ plasma (middle) and the H-mode plasma (bottom) with helium (left) and methane (right) injections. The sharp spikes in frequency found at the beginning of each scenario are caused by the fishbone instability. These fishbones evolve into a LLM which occurs at $\sim 20$ kHz. Lower frequency modes are caused by slowly rotating locked modes, like the neoclassical tearing mode (NTM).
flattening of \( n_e \) is evident within \( \rho < 0.5 \). The reason for the flattening is currently unclear, however it is probably caused by MHD activity.

### 4.2.1 Current Scan

A two point \( I_p \) scan in L-mode at fixed \( B_T \) was performed by lowering the \( I_p \) of the 900 kA L-mode plasma to 600 kA; the 600 kA plasma is referred to as the low \( I_p \) scenario. To match \( T_e \) and \( T_i \) from the high \( I_p \) plasma, both the SS and SW NBIs were used, increasing \( P_{\text{NBI}} \) from 2.1 MW to 3.2 MW. The gas puff timing was triggered earlier at low \( I_p \) to coincide with the earlier \( I_p \) flat-top. The main gas fuelling rate of the low \( I_p \) plasma was increased because \( n_e \) is lower at the beginning of the \( I_p \) flat-top compared to the high \( I_p \) plasma. A sharp rise in the mid-radius He II (\( n = 4 \rightarrow 3 \)) emissivity is found following the helium gas puff, however the C VI (\( n = 8 \rightarrow 7 \)) emissivity remains approximately unchanged following the methane gas puff.

Although the aim of this thesis is not to analyse the effect of impurities on MHD, or vice versa, it is noted that the duration of the low \( I_p \) plasma, without a disruption, is increased for the methane injection compared to the helium injection. \( Z_{\text{eff}} \) remains relatively unchanged during both gas puffs. After \( t \sim 250 \text{ ms} \) in low \( I_p \) plasma with the helium gas puff, there is evidence of a slowly rotating magnetic island (\( F \sim 5 \text{ kHz} \)) caused by a neoclassical tearing mode (NTM), which results in a locked mode as shown in figure 4.2c. For the same plasma, the NTM is replaced by the LLM during the methane injection, as shown in figure 4.2d. The reason the NTM grows into a locked mode during the helium gas puff, but not during the methane gas puff, may be caused by a difference in the local plasma pressure gradient within the magnetic island; a smaller plasma pressure gradient has been shown to increase the size of the NTM [95, 96].

As stated previously, there is no model at present to predict the transport caused by the LLM or NTM, therefore the impurity evolution is only analysed from \( t = 0.2 - 0.25 \text{ s} \) during the low \( I_p \) plasma. Background plasma parameters averaged over this time period are shown in figure 4.3. The value of \( q_{95} \) increases moderately during the low \( I_p \) plasma. This \( q_{95} \) dependence can be attributed to the scaling law given in equation 4.1 [91].

\[
q_{95} = \frac{2\pi a^2 \kappa B_T}{\mu_0 I_p} \tag{4.1}
\]

\( T_e \) and \( T_i \) profiles are very similar in the both high and low \( I_p \) L-mode plasmas because of the additional \( P_{\text{NBI}} \) applied during the low \( I_p \) plasma. Although the volume integrated \( n_e \) in both plasmas is similar, a higher value of on-axis \( n_e \) is found for the low \( I_p \) plasma compared to the high \( I_p \) plasma (4.2 : \( 3.5 \times 10^{19} \text{ m}^{-3} \)).
Figure 4.3: The background plasma profiles time averaged over the transport analysis window for the L-mode high $I_p$ plasma (green #29261), the low $I_p$ plasma (blue #29417) and the H-mode plasma (red #29275). The spatial coordinate is $\rho = \sqrt{\phi_N}$. 
4.2.2 Confinement Mode Scan

The final scenario, referred to as the H-mode scenario, varied the confinement of the plasma by inducing a controlled L-H mode transition at \( t \sim 0.25 \) s during the L-mode high \( I_p \) plasma. The vertical position of the magnetic axis in L-mode was lowered by \( \sim 3 \) cm to induce the transition into an ELMy H-mode. The ELMs can be seen from the red trace in figure 4.1 as sharp, frequent spikes in the edge \( D_\alpha \) emission. In this case, instead of triggering the impurity gas puff at the start of the \( I_p \) flat-top, it is triggered at the beginning of the H-mode period at \( t \sim 0.25 \) s. A rise in the He II \((n = 4 \rightarrow 3)\) emissivity is measured following the helium gas puff, while the C VI \((n = 8 \rightarrow 7)\) emissivity remains again unchanged following the methane gas puff. This shows that a transport analysis of helium is possible in each scenario, while it is only possible in the 900 kA L-mode plasma for carbon.

A difference to the duration of the H-mode phase is found, depending on the choice of the injected impurity gas. The ELMy H-mode period continues for longer during the methane gas puff compared to the helium gas puff, although the total duration of the plasma pulse remains the same during both gas puffs. MHD activity is unavoidable during the H-mode period as shown in figures 4.2e and 4.2f. Similar to the low \( I_p \) plasma, an NTM is excited and eventually locks from \( t = 0.28 - 0.32 \) s during the helium gas puff, while the LLM and NTM coexist together after the methane gas puff until a large sawtooth crash occurs at \( t = 0.33 \) s. The sawtooth is probably caused by the \( q \) profile falling below unity.

Focus is only given to the region of the plasma where MHD effects are expected to be minimal, therefore the evolution is only tracked for \( t = 0.27 - 0.32 \) s. The increased particle confinement associated with H-mode causes a steep rise in the plasma volume integrated \( n_e \), as shown in figure 4.1. \( T_e \) and \( T_i \) remain fairly constant since no additional heating was applied during the L-H mode transition. Compared to the L-mode plasma, the time averaged \( n_e \) profile is flatter near the outer region of the H-mode plasma in the range \( 0.5 < \rho < 0.9 \). At the very edge of the plasma from \( \rho \geq 0.9 \), the density profile is very steep and typically called the pedestal region. The density pedestal is characteristic of most H-mode plasmas. A decrease in \( v_{\phi} \) is found in the region \( \rho < 0.4 \), which is probably due to the presence of the LLM [93].

4.3 Emissivity Measurements

The 2D pixel frames from the RGB diagnostic span the entire plasma cross-section with a pixel resolution of 640x480 (VGA) and a sampling rate of 200 Hz. The temporal and spatial resolution of RGB must be high enough to ensure that the relevant transport phenomena are captured fully. On the other hand, an advantage of a lower spatial and temporal resolution is that the statistical noise is averaged. Temporal resolution is largely dependent on the impurity particle confinement time, which is assumed here to be \( \sim \tau_E \). For MAST, \( \tau_E \) in H-mode plasmas is approximately 50
4.3. EMISSIVITY MEASUREMENTS

ms, while L-mode plasmas are found to be approximately 0.7 times shorter than the H-mode plasmas [90]. Reducing the temporal resolution from 200 Hz to 100 Hz, by averaging every two frames, is acceptable for the predicted confinement times.

To translate the pixel resolution in a radial resolution, the 2D pixel frame must be mapped to the relative R and Z of the SS beam axis. The intersection points in the xy-plane between the pixel (L_1) and SS beam (L_2) line-of-sight are found by solving the two simultaneous parametric line equations 4.2, where \( P_1 = (x, y) \) for the pixel line-of-sight with unit vectors \( \hat{P}_1 \), and \( P_2 = (x, y) \) for the beam line-of-sight with equivalent unit vectors \( \hat{P}_2 \).

\[
L_1 = P_1 + t\hat{P}_1 \\
L_2 = P_2 + s\hat{P}_2
\]  

(4.2)

Solving for \( t \) and \( s \) when \( L_1(t) = L_2(s) \) allows the intersection points to be calculated. The radial resolution of RGB along the SS beam axis is \( \sim 3 \) mm. The resolution is not a constant along the beam, since each pixel line-of-sight views the beam at a different angle, however it is reasonable to assume that each pixel line-of-sight has a perpendicular view of the SS beam. This technique can also be carried out for the SW beam, when making comparisons of the modelled and experimental SW beam emission in section 4.4.2. Impurity density profiles from the SW beam are not treated in this analysis because the SW was not operational during the L-mode low \( I_p \) plasma, whereas the SS beam was operational during all the plasma scenarios.

The required radial resolution for transport is more complex since it depends on the transport regime. In a classical straight cylindrical plasma with a uniform magnetic field, the diffusion step size, \( \Delta r \), is of the order of the Larmor radius,

\[
\rho_a = \frac{m_a v_{th}}{q_a B_T}
\]  

(4.3)

for species \( a \) where \( v_{th} \) is the thermal velocity, \( m_a \) the species mass and \( q_a \) the species charge. In neoclassical transport theory, it is shown in chapter 6 that the toroidal magnetic geometry can increase \( \Delta r \) above classical predictions. Since \( \rho_C \approx \rho_{He} \approx 1 \) cm, each 2D RGB pixel frame is rebinned over 5 pixels giving a pixel resolution of 128x96 and radial resolution of \( \sim 1.5 \) cm. A typical frame of helium and carbon ACX emission, with respect to the R and Z of the SS beam axis, is illustrated in figures 4.4a and 4.4b.

The PCX component is observed across the entire plasma cross-section, while the ACX component follows the Gaussian shape of the neutral beam in the vertical direction. To separate the ACX and PCX emission associated with the SS beam, the emissivity \( \sim 30 \) cm above and below the SS beam axis is averaged for each \( R \) column to identify the PCX component. This distance is chosen because it is out-with the ACX region and it avoids the brighter emission around the P5 coils. After the PCX has been subtracted, a least squares vertical Gaussian fit of the remaining
Figure 4.4: Emissivity contours, measured at $t = 0.22$ s during the L-mode 600 kA plasma (left: #29417 and right: #29421) using the two RGB band-passes centred on 468.5 nm and 529.51 nm ($\Delta \lambda \sim 3$ nm), are illustrated in (a) and (b) respectively; $R$ and $Z$ are with respect to the SS beam axis. The vertical Gaussian fits for the SS and SW ACX contributions at $R = 1.3$ m are indicated by the red and blue lines respectively in (c) and (d). Respective radial ACX profiles (for $Z = 0$) are shown in (e) and (f).
4.3. EMISSIVITY MEASUREMENTS

Figure 4.5: A plot of the pitch angle in radians, measured by the MSE diagnostic during the low $I_p$ plasma at $t = 0.23$ s.

ACX emissivities along each $R$ column is performed. Two Gaussian distributions are used in the fitting procedure when the SW beam is operation because it contributes to the ACX emission from the SS beam. An example of the Gaussian fit for one $R$ column with both beams operating is illustrated in figures 4.4c and 4.4d for the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) emission respectively. The ACX emission associated with pixels corresponding to $Z = 0$ is then stored as a function of $R$, as shown in figures 4.4e and 4.4f.

Each pixel line-of-sight views the SS neutral beam approximately along the vessel equator. If a plume ion is vertically displaced by more than $\sim 10$ cm along the magnetic field line before it has travelled across a different line-of-sight in the $xy$-plane, it can be assumed that it does not contribute to the ACX emission. For a plume ion travelling approximately 1 m along the field line, the vertical displacement of the plume ion can be approximated by the tangent of the pitch angle, $\gamma_P$, meaning that $\gamma_P < 0.15$ radians is required for a plume ion to intersect another sight-line. An example of $\gamma_P$, measured by the MSE diagnostic during the MAST pulse #29417 at $t = 0.23$ s, is illustrated in figure 4.5 and shows that $\gamma_P$ falls below 0.15 radians within $R < 1.0$ m. The effect of the plume emission in this region is observed experimentally in figure 4.4e. The impurity density should be approximately constant in the region around the magnetic axis ($R \sim 0.93$ m), so one should expect to observe a decrease in ACX emissivity due to the exponential decay of the beam density. Instead, the He II ($n = 4 \rightarrow 3$) and C VI ($n = 8 \rightarrow 7$) emission profiles flatten within $R < 1.0$ m. To predict the radial plume emissivity profile, either a model which tracks the magnetic field lines around the plasma or a dedicated experiment (such as the experiment carried out by Bell [84]) would be needed; both of which are beyond the scope of this thesis. Radial $n_{He2+}$ and $n_{C6+}$ profiles are therefore only analysed in the range $R \geq 1.0$ m ($\rho > 0.2$).
CHAPTER 4. PLASMA SCENARIOS AND MEASUREMENTS

Figure 4.6: A plan drawing of the SS beam trajectory in beam coordinates \((x_b, y_b)\) and in machine coordinates \((x_m, y_b)\). The horizontal focal point of the beamlets is shown by \(y_h\). An example of the grid of points used to interrogate the beam density in the NEBULA code is shown by the dashed rectangle.

4.4 Beam Density Model

To derive the impurity densities from the emissivity profiles, the line-of-sight neutral beam density must be determined. A general description of the beam density, \(n_b^F\), of each beam fraction \(F\) is given in equation 3.21. Attention here is given to deriving an expression for the divergence and focusing factor \(\Phi\). A significant part of the work of this thesis was devoted to simulating a 3D deposition grid of \(n_b\) within the plasma using the IDL\textsuperscript{TM} programming language. The basic equations of photometry and beam shape relevant to neutral beams are explained by Tournianski [97]. The code designed to implement this was titled: NEutral Beam Universal Line-integrated Analysis (NEBULA). A benchmark of NEBULA will be made against experimental measurements from the RGB band-pass filter centred on \(\lambda_0 = 660\) nm, that is on the Doppler shifted \(D_\alpha\) emission from the SW beam.

NEBULA begins by creating a beam geometry coordinate system, \((x_b, y_b, z_b)\), as shown in figure 4.6. The PINI grid consists of 262 beamlet slits, the trajectories of which are parametrised in terms of the horizontal, \(y_h\), and vertical, \(y_v\), focal lengths of the beam as follows:

\[
(x_b, z_b) = \left( \left[1 - \frac{y_h}{y_b}\right] x_0, \left[1 - \frac{y_b}{y_v}\right] z_0 \right), \quad (4.4)
\]

At any given point in the plasma, \(P(\theta)\), the factor \(\Phi\) for each beamlet, \(i\), can be
expressed as
\[ \Phi(P) = \sum_i \frac{I_i}{r_i^2} \cos \theta_i \text{ [m}^{-2}] \] (4.5)

where \( r_i \) is the distance \( OP \) and \( \theta \) is the angle which \( OP \) makes to the normal of the beamlet. \( I_i \) denotes a dimensionless quantity that accounts for the Gaussian divergence of the beamlet and has the form
\[ I_i(P) = \Delta_0 \Delta_1 e^{-\left(\frac{\theta_i}{\alpha_i}\right)^2} \] (4.6)

\( \Delta_0 \) and \( \Delta_1 \) are normalisation constants and \( \alpha_i \) is the divergence angle of each beamlet.

The two normalisation constants, which divide \( \sum_i I_i \) between one hemisphere of solid angle, \( \Omega \), and the total number of beamlets, \( b_{tot} \), are defined as:
\[ \Delta_0 = \frac{1}{2\pi \int_0^\pi e^{-\left(\frac{\theta_i}{\alpha_i}\right)^2} \sin \theta d\theta} \]
(4.7)

\[ \Delta_1 = \frac{1}{b_{tot}} \] (4.8)

By substituting the expressions for \( \Delta_0 \) and \( \Delta_1 \) into Eq. 4.6, \( \Phi \) may be written as
\[ \Phi(P) = \sum_i \frac{e^{-\left(\frac{\theta_i}{\alpha_i}\right)^2} \cos \theta_i}{2\pi b_{tot}^2 \int_0^\pi e^{-\left(\frac{\theta_i}{\alpha_i}\right)^2} \sin \theta d\theta} \] (4.9)

Using the expression for \( n^F_b \) defined in equation 3.21, and substituting the values of \( n^F_L \) and \( \Phi \) from equations 2.1 and 4.9 respectively, the full expression for \( n^F_b \) is
\[ n^F_b(P,t) = \frac{I^F_b(t) e_N}{e^V_b(t) e} \sum_i \frac{e^{-\left(\frac{\theta_i}{\alpha_i}\right)^2} \cos \theta_i}{2\pi b_{tot}^2 \int_0^\pi e^{-\left(\frac{\theta_i}{\alpha_i}\right)^2} \sin \theta d\theta} \Lambda^F_i(P,t) \] (4.10)

An illustration of the beam density output from NEBULA in beam geometry is shown in figure 4.7.

It is necessary to convert from beam geometry into machine geometry \((x_m, y_m, z_m)\) for two reasons. Firstly, values of \( S^C_b \) are interpolated from the ADAS ADF21 data set with local inputs of \( n_e \) and \( T_e \) from TS measurements converted from machine geometry, and secondly, beam densities are integrated over the line-of-sight using machine geometry. The machine coordinates, which have the same origin as the beam coordinates, can be expressed using the 3D rotation equations,
\[ x_m = x_b \cos \sigma_R + y_b \sin \sigma_R - z_b \sin \sigma_R \sin \sigma_T \] (4.11)
\[ y_m = -x_b \sin \sigma_R + y_b \cos \sigma_R - z_b \cos \sigma_R \sin \sigma_T \] (4.12)
\[ z_m = y_b \sin \sigma_T + z_b \cos \sigma_T \] (4.13)
Figure 4.7: Results from NEBULA for a beam power of 1.8 MW are shown by a density contour plot, in beam geometry, through the centre of the neutral beam in the vertical and horizontal plane in (a) and (b) respectively. The red lines indicate the beamlet focal points and the circles represent the PINI grid, where the beamlets originate.

where $\sigma_R$ is the rotation angle around $z_b$ specified as 64.84° and 4.84° for the SW and SS PINIs respectively and $\sigma_T$ represents the rotation angle around $x_b$.

With the RGB line-of-sight unit vectors and camera location, the line-of-sight neutral beam density, defined as $\int n_b^F ds$ where $ds$ is the path integral of the line-of-sight through the beam volume, can be calculated and used to obtain the impurity density profile. An example of each line-of-sight beam density fraction, for the L-mode high $I_p$ plasma at $t = 0.22$ s, is shown in figure 4.8a. One point to note from this graph, is that the second and third beam fractions are attenuated at a faster rate than the main beam fraction. This is illustrated by figure 4.8b, which shows the ratio of each line-of-sight beam density fraction compared to the total line-of-sight beam density. The main beam density fraction is most significant (> 85%) throughout the entire plasma cross section. This is an important point when considering the contribution to the total ACX signal from the neutral beam atoms and the thermal beam halos.

4.4.1 Narrow Beam Approximation

A secondary requirement of NEBULA was the capability of executing between consecutive MAST pulses, requiring that the processing time be less than $\sim 5$ minutes. Reducing the number of modelled beamlets on the PINI grid from 262 to 15 equally spaced beamlets is a simple approximation which can be applied to reduce the processing time of NEBULA without significantly altering the beam density profile. With this approximation, it was found using the IDL function SYSTIME that the code took $\sim 0.06$ s per grid point ($y_b, x_b, z_b$). If the neutral beam density is modelled during the entire duration of the pulse ($\sim 400$ ms), with a temporal resolution of 10 ms and a grid size of 15x11x11 giving a spatial resolution along the beamline of
Figure 4.8: This figure illustrates the chord integrated beam density simulations made using NEBULA with respect to the RGB diagnostic. These densities are shown explicitly in (a) for each beam fraction during the L-mode high $I_p$ plasma. The ratio of each integrated beam fraction compared to the total integrated beam density is plotted in (b) and the relative difference between the total integrated beam density modelled using the narrow beam approximation (NBA) and the full model is shown as a percentage in (c).
~ 10 cm, then the code takes ~ 1 hour to run.

Rather than going for a parallel optimisation of the code, a second simplification has been implemented. The assumption, known as the narrow-beam approximation (NBA) [98–100], is that $\Lambda_f(P,t)$ can be characterised by following the attenuation of one artificial central beamlet line, with $n_b$ defined in beam geometry in equation 4.14.

$$n_b(P,t)^y = n_f^y(P) \sum_i \Phi_i(P) \Lambda_f^y(y_b, t) \quad (4.14)$$

Two approximations are present in the NBA model: the PINI grid is considered a point source with respect to a point in the plasma and each $x_bz_b$ plane of the beam represents a single magnetic flux surface. Figure 4.8c shows the ratio of the line-of-sight neutral beam profile calculated by the NBA and by the full model. The difference between the two models is $\leq 1\%$. Implementing the NBA model in NEBULA reduced the calculation time per grid point to 1.7 ms which, for the same example described in the previous paragraph, gives a total processing time of $\sim 2$ minutes.

4.4.2 Beam Emission

To separate the contribution to the $D_\alpha$ spectral line at $\lambda = 656.1$ nm from hydrogen beam atoms and thermal neutral hydrogen in the plasma edge, RGB observes the beam at an angle so that the wavelength of the beam emission is Doppler shifted. However, the viewing angle changes across the beam line meaning that the spectral line is emitted through a range of wavelengths. Consequently the measurement, which is in units of ph/s/m$^2$/sr/nm, cannot be multiplied simply by one effective filter width to remove the nm$^{-1}$ dependence. The RGB (lower red channel) bandpass filter is shown in figure 4.9a. Although the shifted line wavelength of the main beam energy fraction lies consistently within the centre of the filter, the emission from the second and third beam fractions lie at the edge of the filter for sight-lines observing the beam near the edge of the plasma. The filter attenuation is accounted for in the model by the following method: the effective beam emission coefficient provided by the ADAS adf22 data set is used to determine the number of photons in units of ph/s/m$^3$/sr emitted from each hydrogen beam atom (see equation 3.24). The beam emission is integrated through each pixel line-of-sight giving units of ph/s/m$^2$/sr. The angle between the pixel line-of-sight and the centre of the beam is used to calculate the effective width of the filter using equation 2.3 to give values in units of ph/s/m$^2$/sr/nm. Lastly, the emission from each beam fraction is summed together and then compared directly with the measured beam emission.

A comparison of the modelled and experiment beam emission from the SW NBI is illustrated in figure 4.9b. A good match in the radial shape of the beam emission is found for $R > 1.1$ m. This is of most importance to transport studies which rely primarily on the impurity density shape rather than the absolute concentration. As
4.5 Impurity Density Model

To calculate the impurity density from CXS, equation 3.15 is rewritten in terms of $n_{i,z}$ as

$$n_{i,z} = \frac{4\pi\epsilon_{i,z-1}^{(n\rightarrow n')}}{\sum_{F=1}^{3} q_{eF} F(n\rightarrow n') \int n_{i} F ds} \quad (4.15)$$

First, the effect of the derived impurity concentration on the attenuation of the neutral beam is discussed. For the first iteration of the impurity density deduction, the plasma is assumed to be 100% deuterium. In the second iteration, the derived $n_{He^{2+}}$ and $n_{C^{6+}}$ profiles are included in the calculation of the beam attenuation in the plasma. These iterative steps are continued until convergence is reached. $Z_{eff}$ is close to unity in most plasmas on MAST, therefore the convergence of the calculation occurs after $\sim 3$ iterations. Figure 4.10 shows the final plasma volume integrated impurity concentrations, $\int n_{He^{2+}}/n_{e} dV$ and $\int n_{C^{6+}}/n_{e} dV$, after the third iteration of the impurity density calculation for each plasma.

The intrinsic He$^{2+}$ and C$^{6+}$ concentrations increase during the $I_{p}$ ramp-up.
Figure 4.10: The temporal evolution of the plasma volume integrated $n_{He^2+}/n_e$ (left) and $n_{C6^+}/n_e$ (right) is shown for the L-mode high $I_p$ plasma (top), the low $I_p$ plasma (middle), and the H-mode plasma (bottom). In each plot, the impurity concentration is plotted for the plasma with and without a gas puff. The red line indicates the simulated injected impurity concentration based on the gas influx calibration with a pre-defined fuelling efficiency.
4.5. IMPURITY DENSITY MODEL

Figure 4.11: A comparison of the $Z_{\text{eff}}$, measured from the Bremsstrahlung radiation, and the $Z_{(D,He,C)}$, calculated using the modelled $n_{He^{2+}}$ and $n_{C^{6+}}$ profiles, for the L-mode high $I_p$ plasma.

Phase mainly due to the strong interaction between the plasma and the centre column. During the $I_p$ flat-top, the plasma is moved away from the centre column and the intrinsic He$^{2+}$ generally remains approximately constant while the C$^{6+}$ concentrations either remain constant or decrease. For the L-mode high $I_p$ plasma, the He$^{2+}$ concentration remains constant whereas the C$^{6+}$ decreases. For the low $I_p$ plasma, the He$^{2+}$ concentration continues to increase during the $I_p$ flat-top, while the C$^{6+}$ concentration remains constant. The transition to H-mode does not significantly affect the He$^{2+}$ concentration which remains constant, but it does cause a change to the C$^{6+}$ concentration, where a decrease is found in L-mode and a constant concentration is found in H-mode. These results all indicate a higher recycling rate of helium into the plasma compared to carbon.

A comparison of the concentration with and without the impurity gas puff in figure 4.10 can be used to demonstrate the fuelling efficiency of the gas puff. Intrinsic impurity concentrations on MAST are $n_{He^{2+}}/n_e \sim 0.04$ and $n_{C^{6+}}/n_e \sim 0.004$. A $\sim 85 - 100$ % fuelling efficiency is found for the helium gas puff, which agrees quite well with the fuelling efficiency derived from the TS measurements in section 2.4.2. For the methane gas puff, a fuelling efficiency of the C$^{6+}$ ions is $\sim 1.0 - 2.5$ %, moderately lower than the 10 % fuelling efficiency determined from the TS measurements. In each case, this rise in impurity concentration induced by the gas puff is small enough for the injected impurity to be considered as trace and therefore does not influence the equilibrium background plasma parameters.

To test whether the dominant impurities have been taken into account during the attenuation of the beam, it is important to compare the overall $Z_{\text{eff}}$ inferred from Bremsstrahlung measurements with the $Z_{\text{eff(D,He,C)}}$ contribution from the
deuterium, helium and carbon ions defined as

\[
Z_{\text{eff}}(D,He,C) = 1 + \sum_{i,z} n_{i,z} \frac{z_i(z_i - 1)}{n_e}
\] (4.16)

A comparison of \(Z_{\text{eff}}\) and \(Z_{\text{eff}}(D,He,C)\) during the L-mode high \(I_p\) plasma with no gas puff is shown in figure 4.11. Both measurements suggest an effective charge close to unity in the plasma core, however \(Z_{\text{eff}}\) increases rapidly near the plasma edge. It is noted that the diagnostic used to measure the Bremsstrahlung also captures molecular emission near the plasma edge which could account for this rise [52]. A similar behaviour is also found the 600 kA L-mode plasma and the H-mode plasma.

The thermal halo atoms situated around the neutral beam can induce secondary ACX emission leading to an overestimation of the impurity concentration. The discussion in section 3.3.1 stated that the halo population is \(\sim 2.1 n_b\) and the fraction of \(n = 2\) halo atoms is approximately equal to the fraction of \(n = 2\) beam atoms. Therefore an estimation of the halo contribution is made by creating an artificial 4th beam fraction with thermal energy. Figure 4.12 shows the percentage difference between the \(n_{He^{2+}}\) and \(n_{C^{6+}}\) concentrations with and without the addition of this 4th beam fraction. The results suggest that the \(n_{He^{2+}}\) and \(n_{C^{6+}}\) profiles may be overestimated by \(\sim 4\%\) and \(\sim 20\%\) respectively, however the halo light does not lead to significant changes of the evaluated impurity density profile shape.

### 4.5.1 Peaking Factor

A study of the peaking factor, \(1/L_{n_{i,z}} = -(1/n_{i,z}) \partial n_{i,z}/\partial r\), provides an indication of where the impurities accumulate, whereas the evolution of the injected impurities allows an understanding of the underlying transport mechanisms. A positive peaking factor indicates an inward impurity density gradient and therefore an accumulation

![Figure 4.12: A simulation of \(n_{He^{2+}}\) and \(n_{C^{6+}}\) has been carried with and without an artificial 4th beam fraction representing the halo population around the beam. This plot illustrates the percentage difference between the two simulations for each density.](image-url)
4.5. IMPURITY DENSITY MODEL

Figure 4.13: Time averaged profiles of $n_{He^{2+}}$ (a) and $n_{C^{6+}}$ (b) are shown for the reference plasmas with no gas puff. The peaking factor, $1/L_{n_{i,z}}$, for both impurities is shown respectively in (c) and (d). These profiles are based on the fits illustrated by the solid lines in (a) and (b).

The peaking factor indicates a positive impurity density gradient and therefore a hollow impurity profile. In the limit of zero flux, which is the assumption made for the intrinsic impurities in the plasma without a gas puff, the peaking factor can be shown (see equations 5.1 - 5.3) to be equal to the ratio of $-v_i/D_i$. If the impurity gas puff does not change the transport properties of the plasma, and it is assumed that it does not, then the zero flux peaking factor should be equal in magnitude to $-v_i/D_i$ derived in the next chapter. It is shown later in figure 6.6 that good agreement is found in general between the transport coefficients and the zero flux peaking factors.

Figures 4.13a and 4.13b illustrate the time averaged intrinsic $n_{He^{2+}}$ and $n_{C^{6+}}$ profiles as a function of $\rho$ during each plasma. The respective peaking factors are also plotted in figures 4.13c and 4.13d. There is little difference between the peaking factors for the high and low $I_p$ L-mode plasmas, however it is clear that a larger concentration of impurities penetrate into the low $I_p$ plasma compared to the high $I_p$ plasma. In H-mode, the impurity profiles are hollow between $0.2 \leq \rho \leq 0.6$ suggesting a significant difference in transport between L-mode and H-mode plasmas. The trends are fairly similar for helium and carbon, however it can be
Figure 4.14: Contour plots of the modelled $n_{\text{He}^2+}$ (left) and $n_{\text{C}^6+}$ (right) profiles as a function of $\rho$ and $t$ injected from the gas puff during each plasma. The intrinsic $n_{\text{He}^2+}$ and $n_{\text{C}^6+}$ profiles has been subtracted to illustrate the evolution of the injected impurities from the plasma edge into the core.
seen that the negative peaking factor in H-mode is larger for carbon compared to helium. In addition, the difference in magnitude between the carbon profiles in each plasma is greater compared to the helium profiles.

4.5.2 Injected Impurity Evolution

The evolution of $n_{\text{He}^2+}$ and $n_{\text{C}^6+}$ associated with the helium and methane gas puffs, with the subtracted intrinsic impurity profiles from the reference plasmas, is illustrated in figure 4.14. The evolution of the $n_{\text{He}^2+}$ profile following the gas puff is clearly observed in the range $0.2 \leq \rho \leq 0.8$ in all three plasmas. The same can be said for the $n_{\text{He}^2+}$ profile in the L-mode 900 kA plasma, but not the L-mode 600 kA plasma nor the H-mode 900 kA plasma. The resulting evolution of the $n_{\text{C}^6+}$ profiles during the latter two plasmas are inconclusive as the methane gas puff did not produce a significant perturbation.

For the L-mode high $I_p$ plasma, the carbon seems to penetrate into the mid-radius of the plasma in a shorter time scale than helium, however both impurities reach the inner core on similar time scales. This would suggest a region of higher transport for carbon in the region $0.5 \leq \rho < 0.8$ compared to helium, but similar rates of transport between the two impurities in the region $0.2 \leq \rho < 0.5$. For the $I_p$ scan in L-mode, helium seems to penetrate into the core in a shorter timescale at low $I_p$ compared to high $I_p$ suggesting higher rates of transport at lower $I_p$. In H-mode, it is difficult to draw any conclusions within $\rho \leq 0.4$, but it is evident that a significant concentration of helium builds up around $\rho \sim 0.6$ in H-mode. An MHD event occurs 60 ms after the gas puff and redistributes helium around the plasma.

4.6 Summary

This chapter has described the plasmas designed to analyse the helium and carbon transport during a two-point scan of $I_p$ in L-mode (600 kA, 900 kA) and a confinement scan (L-H) at 900 kA. The analysis aimed to avoid MHD activity, however towards the end of the L-mode plasmas and throughout the inner core of the H-mode plasma this was not possible. The helium gas puff provided a significant perturbation to the helium density in all three scenarios, however a sufficient perturbation of the carbon density was only achieved in the high $I_p$ L-mode plasma. Gaussian fits are used to distinguish between the ACX and PCX components along each column of the pixel image. This provides a reliable background subtraction of the passive emission. Radial profiles are derived by taking the pixel value of the vertical Gaussian fit closest to $Z = 0$ and storing this for each radial column. The Gaussian fitting does not account for plume or halo emission. Estimates made in this thesis suggest that the plume emission is only significant within the range $\rho < 0.2$, and therefore profiles are not used within this range. Halo emission can cause a 20% over estimation of the carbon profile, and is less significant for helium.
A benchmark of the NEBULA code, designed specifically for this thesis, was carried out by comparing the modelled and experimental beam emission. The degree of attenuation along the beam trajectory is well matched by NEBULA, however the magnitude of the simulated beam emission from NEBULA is greater than experiment. Core values of $Z_{eff}(D,He,C)$, calculated from the modelled impurity concentrations, are in reasonable agreement with the $Z_{eff}$ profile in the core. The zero flux peaking factors of the intrinsic $n_{He2^+}$ and $n_{C6^+}$ profiles were fairly similar during both L-mode plasmas, but moderately different in H-mode. In the range $0.2 \leq \rho \leq 0.4$, the peaking factors in L-mode are close to zero, but increase with radius in the range $0.4 < \rho \leq 0.8$. In H-mode, both impurities have a hollow profile in the range $0.2 \leq \rho \leq 0.6$, with a moderately inward peaking factor in the range $0.6 < \rho \leq 0.8$. The magnitude of the inward peaking factor in H-mode is generally smaller than the L-mode plasma. The magnitude of the peaking factor in the hollow region is larger for carbon than helium.
Chapter 5

Impurity Transport Coefficients

5.1 Introduction

Transport studies usually characterise the evolution of the impurity flux in terms of radial diffusion, $D_i$, and convection, $v_i$, profiles. They provide an estimate of the impurity confinement time in the core plasma which is compared against $\tau_E$ to predict whether the fusion produced thermal helium particles quench the fusion burn. Neoclassical and gyrokinetic impurity fluxes can be written in terms of $D_i$ and $v_i$ and compared to the transport coefficients determined from experiment. This allows specific driving mechanisms for transport to be identified and provides a basis for understanding and controlling transport in future tokamaks.

Two methods are used to determine $D_i$ and $v_i$ following the injection of a trace amount of impurity gas into the plasma. ‘Trace’ implies that the perturbation does not alter the transport or any other background plasma parameters. Shortly after the gas puff, at each point in space, there is an influx phase where the impurity flux is negative and followed by an outflux phase with a positive impurity flux. Plasma pulses on MAST are not long enough to observe this outflux phase. During the influx phase at each point in space, the rate at which the impurity flux decays compared to the impurity density gradient can be interpreted in terms of a $D_i$ and $v_i$ coefficient. This method is advantageous as it does not require any knowledge of the gas fuelling rate into the plasma, however the impurity flux can only be determined in plasma regions where the atomic source coefficients, that is the effective ionisation and recombination rate coefficients, are negligible. Therefore this interpretative method cannot be used to determine the transport coefficients near the plasma edge.

A predictive model, that solves the particle continuity equation with a given set of boundary conditions, can be used to reproduce the impurity density evolution inferred from CXS by using the $D_i$ and $v_i$ coefficients as free parameters in a least squares fit. This model uses the unresolved CR ionisation and recombination coefficients described in chapter 3 to model the atomic source terms and therefore can be used to determine the $D_i$ and $v_i$ coefficients in the plasma edge. However, in reality, the ACX signal from the impurities is weak in the plasma edge, which
typically gives rise to a high degree of uncertainty in the edge transport coefficients. One disadvantage of this method is that, as with any least squares fitting algorithm, there may exist multiple local minima in $\chi^2$ space, so care must be taken to ensure that the solution is physically plausible. The following chapter describes both models in greater detail and presents (and compares) the helium and carbon transport coefficients for each plasma scenario.

### 5.2 Transport Model

The continuity equation for the impurity ions was defined in equation 3.12, with particular attention on the atomic source rate coefficients. Focus is now given to the impurity flux which contains the physics of impurity transport. Equation 3.12 is re-written more compactly as

$$\frac{\partial n_{i,z}}{\partial t} = -\nabla \cdot \mathbf{\Gamma}_{i,z} + S_{i,z} \tag{5.1}$$

with $S_{i,z}$ representing the sources from ionisation, recombination and CX. The common ansatz describing the impurity flux, $\mathbf{\Gamma}_{i,z}$, is written as

$$\mathbf{\Gamma}_{i,z} = -D_i \nabla n_{i,z} + v_i n_{i,z} \tag{5.2}$$

A negative flux implies that the impurity ions are flowing towards the core of the plasma and the assumption is made that $D_i$ and $v_i$ are the same for all charge states of the impurity. $D_i$ is always positive and therefore acts in the opposite direction to the impurity density gradient, whereas $v_i$ can be positive or negative depending on the physical driving mechanism for the transport.

Using equation 5.2, the zero-flux ($\mathbf{\Gamma}_{i,z} = 0$) peaking factor, defined as $1/L_{n_{i,z}}$, can be written as

$$\frac{1}{L_{n_{i,z}}} = \left| \frac{\nabla n_{i,z}}{n_{i,z}} \right| = \frac{|v_i|}{D_i} \tag{5.3}$$

where $|\nabla n_{i,z}| \sim \partial n_{i,z}/\partial r$ and $|v_i| \sim v_i(r)$. The magnitude of the peaking factor informs on the amount of accumulation and the sign reveals the direction of the particle convection since $D_i$ is always positive. For zero-flux, only the ratio of $v_i/D_i$ can be determined from experiment. This is known as the ‘ambiguity problem’. A transient event, such as a sawtooth, ELM or short gas puff, disturbs the impurity equilibrium ($\mathbf{\Gamma}_{i,z} \neq 0$) and allows $D_i$ and $v_i$ to be evaluated separately, rather than as a ratio. The two models that determine $D_i$ and $v_i$ use equations 5.1 and 5.2 and are discussed next in section 5.2.1 and 5.2.2.
5.2. TRANSPORT MODEL

5.2.1 UTC-SANCO Approach

The first method of determining the $D_i$ and $v_i$ coefficients uses a 1.5D\(^1\) radial transport code, called sanco [81], to solve equation 5.1 for the impurity density with a given set of boundary conditions and a description of the impurity flux (in terms of equation 5.2). One of the benefits of sanco is that it uses the CR ionisation, recombination and CX rate coefficients provided by the ADAS adf11 data set to determine the atomic source terms for each ionisation stage. To interpolate these coefficients from the data set, $T_e$, $T_i$, $n_e$ and $n_i$ as a function of $r/a$ and $t$ must be supplied as inputs to sanco. A simplification of the magnetic equilibrium is used in sanco. Firstly, the plasma profiles and transport coefficients are assumed to be flux surface averages and secondly, a cylindrical geometry is used. The former simplification assumes that there is no asymmetry between the HFS and LFS of the plasma. This is justified on MAST, since no significant asymmetries are found in the TS or SXR measurements. The latter simplification could be cause for error on MAST due to the triangular shape of the plasma, however the error is not thought to exceed the typical error bars of the transport coefficients. In cylindrical geometry, the transport equations are re-written as

$$\frac{\partial n_{i,z}(r,t)}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r}(r \Gamma_{i,z}(r,t)) + S_{i,z}(r,t)$$  \hspace{1cm} (5.4)

$$\Gamma_{i,z}(r,t) = -D_i(r,t) \frac{\partial n_{i,z}(r,t)}{\partial r} + v_i(r,t)n_{i,z}(r,t)$$  \hspace{1cm} (5.5)

where the radial coordinate $r = \sqrt{V/2\pi^2R_0}$ is based on the enclosed plasma volume of a flux surface, $V$. $D_i$ and $v_i$ are given as a function of $r$ and $t$, however usually these coefficients are assumed to be constant in time over the analysis window; in fact, the second method of determining the transport coefficients relies on this assumption.

An arbitrary (but realistic) set of parametrised radial $D_i$ and $v_i$ profiles are specified by the user as an initial match to the experimental and modelled impurity density evolution. The Universal Transport Code (utc) [58,103] uses the Levenberg-Marquardt least-squares fitting algorithm to determine best fit values of the free parameters, $p_i$, which include the $D_i$ and $v_i$ profiles and also $\epsilon_F$. The algorithm proceeds by varying the free parameters by a small amount, $\delta p_i$, (typically 10 \%) to obtain the partial derivatives,

$$\frac{\partial f_n}{\partial p_i} = \frac{f_n' - f_n}{\delta p_i}$$  \hspace{1cm} (5.6)

where $f_n$ corresponds to the predicted quantity. Corrections to $\delta p_i$ are then obtained

\(^1\)SANCO works in 1.5D because it includes a simplified SOL where ions may travel poloidally along the field lines to and from the divertor. This simplified SOL regulates the influx rate of particles into the LCFS.
by inverting $M_{i,j} \Delta p_i = b_i$. The matrix $M_{i,j}$ and vector $b_i$ are defined as

$$M_{i,j} = \sum_{n=1}^{N} \frac{\partial f_n}{\partial p_i} \frac{\partial f_n}{\partial p_j} w_n$$

(5.7)

and

$$b_i = -\sum_{n=1}^{N} \frac{\partial f_n}{\partial p_i} (f_n - y_n) w_n$$

(5.8)

where $w_n$ signifies the weighting of each data point and $y_n$ corresponds to the measured quantity. The normalised covariance matrix, $C^N$, is defined as

$$C^N_{i,j} = \begin{cases} \frac{C_{i,j}}{\sqrt{C_{i,i}C_{j,j}}} & i \neq j \\ \sqrt{C_{i,i}} & i = j \end{cases}$$

(5.9)

where $C_{i,j} = M_{i,j}^{-1}$. Diagonal and non-diagonal elements reflect the errors and covariance in the free parameters respectively. However, these errors only represent the range of $p_i$ which provide the same solution of $\chi^2$. If the solution is not the global minimum, then the physical solution may not be within the limits of the error bars.

SANCO models the flux of neutral impurities entering the plasma by creating a simplified SOL and plasma edge description, which is illustrated in figure 5.1. There are five main inputs to the model: the neutral particle influx, $\Gamma_{\text{gas}}$, the parallel loss time, $\tau_{||}$, the effective recycling coefficient, $R_{\text{rcy}}$, the fuelling efficiency, $\epsilon_F$, and the thermal kinetic energy of the neutral atoms $E_{\text{th}}$. $\Gamma_{\text{gas}}$ and $\epsilon_F$ are described in section 2.4 of chapter 3. An accurate description of the remaining parameters requires complex models which are not considered in this work. Instead, various estimates are made based on physical assumptions.
Atoms and molecules leaving the gas valve have thermal velocities corresponding to room temperature. As they interact with the SOL plasma, the molecules dissociate at a threshold $T_e$ forming atoms with Franck-Condon, or breakup energies, of a few eV. The atoms are kinetically energised by elastic collisions and CX reactions with plasma ions, however their ionisation times are too short for them to thermalise fully with the SOL plasma (generally around $10 - 20$ eV on MAST), therefore the assumption is made that $E_{th} \sim 2$ eV [58, 104]. The value of $E_{th}$ determines the distance that the neutral particles travel into the plasma before ionising. For $E_{th} = 2$ eV, the neutrals travel $\leq 10$ cm into the core plasma. Underestimating $E_{th}$ will cause an overestimation of $D_i$ and $v_i$ over the range in $r$ up to the maximum penetration depth of the neutrals.

A neutral impurity atom ionised in the SOL via CX reaction forms part of the neutral influx into the core plasma otherwise, as ions, they will be transported along the field lines towards the divertor plates. It is important to estimate the impurity ion transit time to the divertor plates to predict the number of recycled impurities re-entering the plasma. This transit time, $\tau_{||}$, can be estimated using the expression

$$\tau_{||} = \frac{R q_{95}}{\sqrt{2 E_{th}/m}}$$

(5.10)

where $R q_{95}$ is the connection length. For example, in the L-mode high $I_p$ plasma, the connection length is calculated using $R \sim 1.4$ m and $q_{95} \sim 15$ to give 21 m, so that a He$^+$ ion with $E_{th} = 2$ eV will take $\sim 2$ ms to travel to the divertor.

The fraction of particles recycled into the plasma is denoted as $R_{rcy}$. This parameter is principally dependent on the divertor configuration and difficult to predict. In most divertor configurations, helium is primarily recycled back into the plasma with $R_{rcy} \rightarrow 1$. With a cryopump applied with an argon frost layer, the recycling rate has been shown to reduce to $R_{rcy} = 0.85 - 0.95$ [105, 106]. The divertor in MAST does not have a cryopump and therefore the recycling is expected to be close to unity for helium. Conversely, carbon is extremely reactive and tends to stick to the divertor material and the assumption is made that $R_{rcy} \rightarrow 0$.

The edge model coefficients are only a basic estimate of the complex processes occurring in the plasma edge and SOL. These simplifications are a weakness of the present use of sanco, although sanco can be coupled to edge models that seek to model these coefficients with more accuracy, but these models are not used in this thesis. Instead, the transport coefficients are compared with a second model (described in the next subsection) which does not rely on a model of the plasma edge parameters.

### 5.2.2 Flux Gradient Approach

The second method of determining the transport coefficients uses the interpretative approach described in two previous studies [57, 87]. Reducing the problem to
cylindrical coordinates and integrating equation 5.4 over \( r \) with the expression for \( \Gamma_{i,z} \) given in equation 5.5 yields

\[
\frac{1}{r n_{i,z}(r,t)} \frac{\partial}{\partial t} \int_0^r n_{i,z}(r,t) r' dr' = D_i(r,t) \frac{1}{n_{i,z}(r,t)} \frac{\partial n_{i,z}(r,t)}{\partial r} - v_i(r,t)
\]

\[+ \frac{1}{n_{i,z}} \int_0^r S_{i,z}(r,t) dr' \quad (5.11)\]

Consider a radial region of plasma where \( S_{i,z} \approx 0 \) and where the plasma transport is constant in time (i.e. \( D_i(r,t) \equiv D_i(r) \)). At each point in space, equation 5.11 can be re-written in the linear form \( y = mx + c \), where the term on the LHS of equation 5.11 is \( y \), the spatial gradient is \( x \), the gradient \( (m) \) is \( D_i \) and the \( y \)-intercept \( (c) \) is \( -v_i \). Both \( y \) and \( x \) are determined directly from impurity density measurements inferred from CXS and fitted as a function of time at each point in space, with the fit providing the \( D_i \) and \( v_i \) coefficients. This method will be referred to as the Flux Gradient (FG) method.

Although the FG and UTC-SANCO methods both deduce the transport coefficients from fits to the experimental impurity density evolution, the FG method is a more direct approach as it does not need to solve the transport equation or rely on any simplified edge model. However one disadvantage of the FG method is that it breaks down in regions where \( S_{i,z} \) becomes non-negligible. An attempt is made in this analysis to provide a robust evaluation of the transport coefficients based on results from the FG and UTC-SANCO methods. One should find agreement between the two codes, within the error bars of the transport coefficients, in the limit of \( S_{i,z} = 0 \). Disagreement may occur if the solution of the least squares fit during the UTC-SANCO method is not the global minimum. The FG method is extremely sensitive to noise in the impurity density profiles, since the transport coefficients are determined from the evolution of two gradients. Therefore it is necessary to smooth the density profiles in time and space to average out the noise. It has been found that a representation of \( n_{i,z} \), which is smooth in time and space, can be made by fitting the following temporal functions to each spatial location [57]:

\[
n_{i,z}(r,t) = \begin{cases} 
A_1 + A_2 \tanh[A_3(t - A_0)] & r/a \leq 0.5; \ t > 0 \\
A_1 + A_2 \exp[-A_3(t - A_0)^2] & r/a > 0.5; \ t \leq A_0 \\
A_1 + A_2 \exp[-A_4(t - A_0)^2] & r/a > 0.5; \ t > A_0 
\end{cases} \quad (5.12)
\]

Once values of \( A_1, A_2, A_3 \) and \( A_4 \) have been determined for every spatial point, a 5\(^{th}\) order polynomial fit is made to the spatial profile for every moment in time. Carrying out the fitting in this order produced satisfactory smooth gradients in both time and space. The density evolution inside the region \( \rho = 0.2 \) is treated by interpolating the fit, whilst ensuring a density greater than zero.
5.2. TRANSPORT MODEL

Figure 5.2: The FG code is benchmarked against the UTC-SANCO method. The red line shows the $D_{He}$ and $v_{He}$ profile used by SANCO to simulate $n_{He^{2+}}$. The $D_{He}$ and $v_{He}$ coefficients determined by applying the FG method to $n_{He^{2+}}$ are illustrated by the black lines.

5.2.3 Flux Gradient Benchmark

The UTC-SANCO method has been used and documented in a number of studies (for example by Giroud et al. [58]) and can therefore be considered benchmarked. A code has been developed for this thesis to determine the transport coefficients using the FG method and therefore requires benchmarking. Consider an $n_{i,z}$ profile modelled using SANCO with an arbitrary $D_{He}$ and $v_{He}$ profile based on the expected neoclassical predictions described in chapter 6 (illustrated by the red line in figures 5.2a and 5.2b respectively). Applying the FG method to the simulated $n_{i,z}$ should return the same pre-defined $D_{He}$ and $v_{He}$ profile. For this benchmark, the edge influx conditions and source terms for $n_{i,z}$ are calculated using the L-mode high $I_p$ plasma conditions (described in table 5.1). As described in chapter 4, $\rho = \sqrt{\phi_N}$, where $\phi_N$ is the normalised toroidal flux. The conversion between $\rho$, $R$ and $r/a$ is given in figure 5.5.

The FG method produces a good match in $D_{He}$ and $v_{He}$ in the range of $\rho < 0.7$, illustrated by the comparison of the red (SANCO) and black (FG) lines in figures 5.2a and 5.2b respectively. The increase in $D_{He}$ and $v_{He}$, calculated by the FG method in the range $\rho \geq 0.7$, is due to the non-negligible source terms. In coronal equilibrium, one would not expect significant sources as far into the plasma as $\rho = 0.7$. In this case however, the energy of the injected neutrals is sufficient to allow penetration further into the plasma. Since the simulated density has been created with conditions designed to represent the experimental plasma conditions, the FG $D_{He}$ and $v_{He}$ will only be quoted for the region of $\rho < 0.7$. The source terms for C$^{6+}$ are significant further into the plasma, therefore FG $D_C$ and $v_C$ coefficients are only quoted for $\rho < 0.6$. 
Figure 5.3: A comparison of the fitted and experimental $n_{H_2^+}$ (left) and $n_{C_6^+}$ (right) profiles for the L-mode high $I_p$ plasma as a function $\rho$ (top), $t$ (middle) and $\rho$ and $t$ (bottom).
5.3. IMPURITY CHARGE SCAN

5.3 Impurity Charge Scan

A transport analysis of both $n_{He^{2+}}$ and $n_{C^{6+}}$ is possible in the L-mode high $I_p$ plasma. An example of the fitted and experimental $n_{He^{2+}}$ and $n_{C^{6+}}$ profiles following the helium and methane gas puffs is shown in figure 5.3. In the plasma core, the $n_{C^{6+}}$ rises faster than $n_{He^{2+}}$, suggesting a larger inward flux for carbon. The temporal evolution of $-\Gamma_{He^{2+}}$ and $-\Gamma_{C^{6+}}$ are shown as a function of $\partial n_{He^{2+}} / \partial r$ and $\partial n_{C^{6+}} / \partial r$ in figures 5.4a and 5.4b respectively. The arrows on the linear fits indicate the direction in time following the gas puff. The bottom two graphs of figure 5.4 show the fit between the experimental and modelled impurity density from Sanco. Table 5.1 lists the edge model parameters used in Sanco for each impurity. A moderately higher fuelling efficiency of $\epsilon_1 = 1.0$ and $\epsilon_2 = 0.04$ was needed for helium and carbon respectively compared to the values illustrated in figure 4.10 of $\epsilon \sim 0.85$ and $\epsilon \sim 0.025$. $\tau_{||}$ is determined using $E_{th} = 2$ eV. As expected, a value of $\eta_F$ close to unity was also required to produce the best fit.

The $D_i$ and $v_i$ coefficients deduced from both methods are shown in figures 5.5a
CHAPTER 5. IMPURITY TRANSPORT COEFFICIENTS

Figure 5.5: On the left, the $D_i$ (top) and $v_i$ (bottom) coefficients for helium and carbon, determined using the UTC-SANCO (diamonds) and FG (squares) method in the L-mode 900 kA plasma, are illustrated as a function of $R$, $r/a$ and $\rho$. On the right, the experimental Ne transport coefficients (grey shaded region) taken from a study on NSTX [41] during an L-mode 1 MA plasma, are illustrated as a function of $r/a$. Over the range where the sources are negligible, the two methods agree within error bars for both $D_i$ and $v_i$ giving confidence in the methods evaluation of the transport coefficients. In the inner region of the plasma ($\rho < 0.4$), helium and carbon both experience low transport with $D_i \sim 0.5 \text{ m}^2\text{s}^{-1}$ and $v_i \sim -1 \text{ ms}^{-1}$. Outside this radius ($\rho \geq 0.4$), the transport of both impurities increase with radius. $D_C$ and $v_C$ increase up to $\sim 6 \text{ m}^2\text{s}^{-1}$ and $\sim -30 \text{ ms}^{-1}$ respectively, while $D_{He}$ and $v_{He}$ increase to values of $\sim 2 \text{ m}^2\text{s}^{-1}$ and $\sim -15 \text{ ms}^{-1}$ respectively.

Similar plasma conditions on NSTX (L-mode, $I_p = 1 \text{ MA}$ and $B_T = 0.45 \text{ T}$) have been studied with neon [41]. $D_{Ne}$ and $v_{Ne}$ coefficients from this study are shown in figures 5.5b and 5.5d respectively. Note that this graph is given as a function of $r/a$, not $\rho$. A scale matching $R$ and $r/a$ to $\rho$ is shown at the bottom of figure 5.5c to aid comparison between the radial grids. The region of low transport found for both impurities in the plasma core ($\rho < 0.4$) in the present work seems to be consistent with the NSTX neon transport results. At mid-radius, the rapid increase
in $D_i$ found in the present analysis is also found for $D_{Ne}$. Somewhat higher rates of $v_C$ are found compared to $v_{Ne}$, although the magnitude of $v_{Ne}$ near the plasma edge is not given in their results.

From these comparisons, it can be concluded that the magnitude of $D_i$ and $v_i$ increases with $z$ in the outer region of the plasma ($\rho > 0.4$). For the inner core of the plasma, no obvious trend with $z$ is found.

5.4 Current Scan

For the low $I_p$ plasma, it was shown in section 4.5.1 that the methane gas puff did not significantly perturb the background $n_{C6^+}$ concentration, therefore transport analysis is only carried out for $n_{He^{2+}}$. Comparisons of the fitted and experimental $n_{He^{2+}}$ are shown in figure 5.6. A rise in $n_{He^{2+}}$ can be seen over the entire plasma cross-section, however the MHD activity 80 ms after the gas puff ($t > 0.27$ s) rapidly ejects the He$^{2+}$ ions towards the plasma edge and therefore the time window is limited to 80 ms.

The fits produced with the FG and UTC-SANCO methods are shown in figure 5.7. Similar to the previous scenario, the FG linear fits show a clear change in transport, in space but not time, and the evolution of $n_{He^{2+}}$ has been reproduced by SANCO within error bars. Identical edge model parameters to those for the L-mode high $I_p$ plasma are used in SANCO, as shown in table 5.1. The $D_{He}$ and $v_{He}$ coefficients are shown in figures 5.8a and 5.8c respectively. Both the FG and UTC-SANCO methods show agreement within error bars for $\rho < 0.6$. The $D_{He}$ and $v_{He}$ for the high $I_p$ plasma are over-plotted in both graphs to aid the comparison. Decreasing $I_p$ causes an increase in $D_{He}$ within $\rho < 0.5$. There is also a suggestion that $D_{He}$ increases nearer the plasma edge at low $I_p$, but the large error bars qualify the certainty of this conclusion. $v_{He}$ follows the same trend as found in the high $I_p$ plasma, where the large inward pinch near the plasma edge reduces almost to zero in the core. Decreasing $I_p$ causes a small increase in the magnitude of $v_{He}$.

These results suggest that reducing $I_p$ in L-mode acts to increase the magnitude of both $D_{He}$ and $v_{He}$. This is a result that has also been inferred from argon transport measurements made during a four point $I_p$ scan in L-mode plasmas on C-mod [30]. The $D_{Ar}$ and $v_{Ar}$ coefficients for the latter study are shown in figures 5.8b and 5.8d respectively. For the C-Mod study, $B_T$ was kept constant at $\sim 8$ T

<table>
<thead>
<tr>
<th>$\tau$ (ms)</th>
<th>Helium</th>
<th>Carbon</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{rcy}$</td>
<td>[3, 3, 3]</td>
<td>[2, / , /]</td>
</tr>
<tr>
<td>$\epsilon_F$</td>
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<td>[0.10, / , /]</td>
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<tr>
<td>$E_{th}$ (eV)</td>
<td>[1.0, 1.0, 1.0]</td>
<td>[0.04, / , /]</td>
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<td></td>
<td>[2, 2, 2]</td>
<td>[2, / , /]</td>
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</table>
CHAPTER 5. IMPURITY TRANSPORT COEFFICIENTS

Figure 5.6: A comparison of the fitted and experimental $n_{He^2+}$ profiles for the low $I_p$ plasma as a function $\rho$ (top), $t$ (middle) and $\rho$ and $t$ (bottom).
5.5. **CONFINEMENT MODE SCAN**

The largest difference between the L-mode and ELMy H-mode plasmas occurs in \( n_e \) in the region \( 0.5 \leq \rho \leq 0.9 \). For the H-mode plasma, the \( n_e \) profile is hollow in this range compared to the steep gradient in L-mode. Only helium transport is evaluated because the methane gas puff failed to perturb the intrinsic \( n_{C_6^+} \). Comparisons of the fitted and experimental \( n_{He^{2+}} \) profile are illustrated in figure 5.9. The fit is limited to the region \( 0.4 \leq \rho \leq 0.8 \) because of the onset of significant MHD activity 60 ms after the gas puff.

Figure 5.7 shows the FG and UTC-SANCO fits respectively. The FG linear fits show a clear change in transport in space but not time and the evolution of \( n_{He^{2+}} \) has been reproduced by SANCO within error bars. The experimental \( n_{He^{2+}} \) profile has been produced by UTC-SANCO using the same edge parameters as the L-mode case. The \( D_{He} \) and \( v_{He} \) coefficients are shown in figures 5.11a and 5.11c respectively, with the results from the L-mode plasma also plotted for comparison. Within \( 0.4 \leq \rho \leq 0.7 \) there is agreement within error bars between the FG and UTC-SANCO transport coefficients. The magnitude of \( D_{He} \) is marginally smaller near the edge of the H-mode plasma. There is a more obvious difference found in \( v_{He} \) between the two scenarios, with the inward pinch in L-mode becoming less significant in H-mode. In fact, \( v_{He} \) changes direction from inwards to outwards to vary the \( q \) profile, however the temperature profiles were not matched in each plasma. An increase in \( D_{Ar} \) is observed from mid-radius towards the plasma edge \( (\tau/\alpha > 0.3) \) as \( I_p \) is decreased. In this same radial region, the inward \( v_{Ar} \) increases in magnitude as \( I_p \) is decreased. Although these trends show striking similarities to the helium transport on MAST, it cannot be concluded unambiguously that the same underlying transport mechanisms are responsible for the trends in each study.
Figure 5.8: On the left, the $D_{\text{He}}$ (top) and $v_{\text{He}}$ (bottom) coefficients, determined using the UTC-SANCO (diamonds) and FG (squares) method in the 900 kA and 600 kA L-mode plasmas, are plotted as a function of $R$, $r/a$ and $\rho$. On the right, the experimental Ar transport coefficients taken from a study on C-Mod [30] during an L-mode $I_p$ scan are shown as a function of $r/a$.

within $\rho < 0.6$ in the H-mode plasma. This could explain why an out-flux of He$^{2+}$ (and also C$^{6+}$) is observed in the intrinsic $n_{\text{He}^{2+}}$ and $n_{\text{C}^{6+}}$ profiles in figure 4.13.

A different study on NSTX determined neon transport coefficients in an ELMy H-mode plasma during an $I_p$ scan at 1 MA and 1.2 MA [39]. In this case, $B_T$ was varied between 0.45 T and 0.55 T meaning that $q_{95}$ was kept constant. $D_{\text{Ne}}$ and $v_{\text{Ne}}$ coefficients from this study are shown in figures 5.11b and 5.11d respectively. It is more accurate to compare the results from the 1.2 MA H-mode plasma since the $n_i$ gradient becomes positive at mid-radius in the NSTX study; a phenomenon also observed in the MAST H-mode plasma. A region of positive $v_i$ coincides with this region of positive $n_i$ gradient in both studies. The comparison between the 1 MA H-mode NSTX discharge in figures 5.11b and 5.11d and the 1 MA NSTX L-mode discharge shown in figures 5.5b and 5.5d also shows a small difference in $D_{\text{Ne}}$. On the other hand, there is also only a very small difference between $v_{\text{Ne}}$ in both cases at 1 MA. But, a change in the direction of $v_{\text{Ne}}$ is found in the NSTX study when
Figure 5.9: A comparison of the fitted and experimental $n_{\text{He}^2+}$ profiles for the H-mode plasma as a function $\rho$ (top), $t$ (middle) and $\rho$ and $t$ (bottom).
5.6 Extrapolation to ITER

The ratio of the $\tau_E$ and the effective helium confinement time, $\tau^*_{He}$, must exceed a critical value, generally of the order of 0.1 [20]. This ratio is often defined as $\eta$ and is expressed in equation 5.13, where $\tau^0_{He}$ and $\tau^\text{edge}_{He}$ are the confinement times of helium within the core and edge plasma respectively.

$$\eta \equiv \frac{\tau_E}{\tau^*_{He}} = \tau_E \left( \tau^0_{He} + \tau^\text{edge}_{He} \frac{R_{\text{recy}}}{1 - R_{\text{recy}}} \right)^{-1} > \eta_{crit} \quad (5.13)$$

The first term in brackets on the RHS of equation 5.13 shows that the transport of helium from the plasma core to the plasma edge must occur on a time scale faster than its production rate. The second term in brackets on the RHS of equation 5.13 shows that, even if the transport from the plasma core to the plasma edge is rapid, an inefficient pumping rate or low edge transport could still lead to the build up of helium within the plasma core.

It is not realistic to calculate $\tau^*_{He}$ in this thesis because the high helium recycling rate in MAST would make the second term in brackets on the RHS of equation 5.13 unphysically large compared to a machine like ITER, which is envisaged to operate
5.6. EXTRAPOLATION TO ITER

Figure 5.11: On the left, the $D_{\text{He}}$ (top) and $v_{\text{He}}$ (bottom) coefficients, determined using the UTC-SANCO (diamonds) and FG (squares) method in the L-mode and H-mode plasmas, are plotted as a function of $R$, $r/a$ and $\rho$. On the right, experimental Ne transport coefficients from an H-mode $I_p$ scan on NSTX [39] are shown as a function of $r/a$. This figure does not show the equivalent L-mode results for the NSTX study.

with enhanced helium pumping. Furthermore, a lack of reliable transport coefficients for the plasma edge mean that the predicted $\tau_{\text{He}}^{\text{edge}}$ will have a relatively high degree of uncertainty. Results from this thesis can however be used to examine $\tau_{\text{He}}^0$ by using the relation [107]

$$\frac{\tau_{\text{He}}^0}{\tau_E} = \frac{\chi_T^{\text{eff}}}{D_{\text{He}}} + P_V$$

(5.14)

where $\chi_T^{\text{eff}}$ is the total effective thermal diffusivity and $P_V$ essentially plays the role of the zero flux profile peaking factor. Equation 5.14 shows that the amount of helium in the plasma core is directly related to the core helium transport coefficients. A reasonable estimate of $P_V$ on MAST would suggest a value in the range of 0 and 10 based on the ratio $v_i/D_i$. It is expected that the transport in a CT like ITER will be predominantly driven by $D_i$, leading to relatively smaller values of $C_V$. Low-Z impurities generally experience a pinch of the order of the Ware pinch ($|v_i| < 4$ m/s) [58, 108].
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Figure 5.12: The magnitude of $\chi_{T,i}^{\text{eff}}$ is shown on the left for each plasma in the range where the $T_e$ and $T_i$ gradient is well defined. The ratio of $D_{He}/\chi_{T,i}^{\text{eff}}$ is shown on the right in the same radial range.

The definition of $\chi_{T,i}^{\text{eff}}$ can be written as [87]

\[
Q_e = -\chi_{e}^{\text{eff}} n_e \frac{\partial}{\partial r} T_e \\
Q_i = -\chi_{i}^{\text{eff}} n_i \frac{\partial}{\partial r} T_i \\
Q_e + Q_i = -\chi_{T}^{\text{eff}} \left( n_e \frac{\partial}{\partial r} T_e + n_i \frac{\partial}{\partial r} T_i \right)
\]

where $Q_e$ and $Q_i$ are the electron and ion heat fluxes respectively. Generally, $\chi_{T}^{\text{eff}}$ is only analysed in the mid-radius region where the temperature gradients are well defined. For MAST, this range is $\rho \approx 0.35 - 0.6$. Radial $\chi_{e,i,T}^{\text{eff}}$ profiles are calculated for each plasma using the predictive transport code JETTO [109, 110]. An input of the time-averaged experimental inputs of $n_e$, $T_e$, $T_i$ and $Z_{\text{eff}}$ mapped to a magnetic equilibrium are required by JETTO. Time averaged profiles of $n_e$, $T_e$ and $T_i$ are used for each scenario as shown previously in figure 4.3. A constant value of 1.2 is used over the plasma radius for $Z_{\text{eff}}$, which is a reasonable estimate based on the $n_{He2+,C6+}$ profiles calculated in chapter 4.

$\chi_{T,i}^{\text{eff}}$ profiles and the $D_{He}/\chi_{T,i}^{\text{eff}}$ ratios are shown for each scenario in figures 5.12a and 5.12b respectively. For the range shown, $\chi_{T}^{\text{eff}}$ follows the same trend as $D_{He}$ in each scenario with a magnitude $\sim 4$ times greater than $D_{He}$. The agreement in magnitude between $\chi_{i}^{\text{eff}}$ and $D_{He}$ is far closer than $\chi_{T}^{\text{eff}}$ with $D_{He}/\chi_{i}^{\text{eff}} \sim 1$. On the other hand, $\chi_{i}^{\text{eff}}$ does not strictly follow the same trend as $D_{He}$ in H-mode, as $\chi_{i}^{\text{eff}}$ increases in the H-mode plasma. This figure also shows that $\chi_{e}^{\text{eff}} > \chi_{i}^{\text{eff}}$, a result also found previously on MAST and NSTX [111, 112]. $\chi_{i}^{\text{eff}}$ is typically of the same order of magnitude as neoclassical predictions, whereas $\chi_{e}^{\text{eff}}$ is predominantly driven by turbulence [113].

A number of CT studies have tried to quantify $D_{He}/\chi^{\text{eff}}$. Two helium gas puff studies on TFTR showed that values of $D_{He}$ in the plasma core agree in magnitude...
5.7. SUMMARY

with $\chi_i^{eff}$, but also found that $\chi_i^{eff} > \chi_e^{eff}$ [86,87]. Similarly on DIII-D, $D_{He}/\chi_i^{eff}$ is found close to unity [57]. Although these studies state that $D_{He}/\chi_i^{eff}$ ~ 1, this is due to the fact that $\chi_i^{eff} > \chi_e^{eff}$. The ratio of $D_{He}/\chi_i^{eff}$ ~ 0.25 found in this thesis has also been observed for the main gas species on JET [114]. A moderately larger ratio of $D_{He}/\chi_i^{eff}$ ~ 0.4 was found on MAST [90]. This would indicate that the confinement time of helium is approximately equal to the confinement time of the main gas species. This is encouraging because it suggests that helium will be ejected from the core plasma in a time scale approximately equal to that of the main ion gas. If helium was confined for a longer period of time than the main gas, then this would lead to a build up in the overall concentration of helium.

For a larger D-T spherical tokamak, or even a next step device such as MAST-U where helium is still intrinsic from the HeGDC, the role of $P_V$ must be taken into account when analysing the $\eta$ ratio. Results from the present study indicate that the large inward pinch found in the L-mode plasmas could cause a large accumulation of helium in the plasma core. $P_V$ is less significant in H-mode because the pinch is reduced and actually enhances the outward flux of impurities at mid-radius.

5.7. Summary

Two models, one using a predictive approach (UTC-SANCO) and one using an interpretative flux gradient approach (FG), have been used to determine the $D_i$ and $v_i$ coefficients for the plasma scenarios described in chapter 4. A code was developed to implement the FG model and benchmarked by reproducing the arbitrary transport coefficients used by sanco to model a helium density profile. Each plasma scenario has a region of low transport within $\rho \leq 0.4$, which increases with radius towards the plasma edge. A correlation was found between the transport coefficients and each of the scanning parameters, consistent with other impurity transport studies carried out on NSTX and C-mod. These trends are summarised as follows: in the range of $\rho \geq 0.4$, the carbon transport was greater than the helium transport. A peak of $\sim 3 \text{ m}^2\text{s}^{-1}$ was found for $D_{He}$, while a peak of $\sim 6 \text{ m}^2\text{s}^{-1}$ was found for $D_C$. The convection was directed inwards for both impurities with a peak of $-20 \text{ ms}^{-1}$ for $v_{He}$ and $-35 \text{ ms}^{-1}$ for $v_C$. No significant difference between $D_{He,C}$ and $v_{He,C}$ was found within $\rho < 0.4$. Reducing $I_p$ caused an increase in $D_{He}$ and $|v_{He}|$ in the range of $0.3 < \rho < 0.7$. Due to the large error bars of $D_{He}$ and $v_{He}$ near the plasma edge, it is unclear whether there is any correlation with $I_p$ for $\rho \geq 0.8$. The results also suggest that there is no correlation of $D_{He}$ and $v_{He}$ with $I_p$ within $\rho \leq 0.3$. For the confinement scan, the main difference in $D_{He}$ occurs between $0.6 < \rho < 0.8$, where $D_{He}$ is found to marginally decrease in the ELMy H-mode. A clearer correlation with the confinement mode is found for $v_{He}$. The absolute magnitude of the inward pinch found in L-mode is significantly reduced in H-mode for $\rho > 0.6$ and changes direction (from inwards to outwards) within the $\rho \leq 0.6$. 


The ratio of $D_{He}/\chi_{T,e,i}^{eff}$ was discussed. It was found that $\chi_T$ closely followed the same trend as $D_{He}$ throughout the parameter scans, but produced a ratio of $D_{He}/\chi_{T}^{eff} \sim 0.25$. On the other hand, $\chi_{i}^{eff}$ was more closely matched in magnitude to $D_{He}$, with $D_{He}/\chi_{i}^{eff} = 0.5 - 1.5$. $\chi_{i}^{eff}$ followed the same trend as $D_{He}$ for the $I_p$ scan in the L-mode plasma, but the opposite trend was found during the confinement mode scan at constant $I_p$. The next chapter will compare these experimental findings with neoclassical and gyrokinetic simulations.
Chapter 6

Theoretical Transport Modelling

6.1 Introduction

The ultimate goal of any transport theory is to derive an expression for the particle flux. Radial transport in magnetically confined toroidal plasma can either originate from Coulomb collisions, or from the effects of short-range fluctuations in the plasma parameters. The study of the particle flux generated by Coulomb collisions in a cylindrical geometry forms the basis of classical transport. In this model, it is not practical to treat each particle separately; rather averages over larger distributions of particles are handled using macroscopic fluid equations [115]. It can readily be shown that the moments of the Vlasov equation, with a Fokker-Plank collision operator (Vlasov-FP), yield expressions for the conservation of particles (zeroth moment), momentum (first moment), and energy (second moment) [116]. The classical particle flux due to collisions is determined by analysing the momentum balance equation.

Tokamaks operate with a toroidal plasma geometry, which causes a geometrical enhancement of the transport coefficients. To distinguish between the physics of the classical cylindrical plasma geometry and the toroidal plasma geometry found in tokamaks, the term ‘neoclassical transport’ is used.

In tokamaks, the actual impurity flux usually exceeds the neoclassical predictions. The additional transport is referred to in literature as ‘anomalous transport’. Neoclassical predictions assume that the plasma parameters are stationary but plasmas are rarely quiescent as fluctuations in the plasma parameters arise spontaneously. Plasma turbulence is the result of non-linear interaction between small scale parameter fluctuations of the order of the ion Larmor radius, $\rho_i$, which are driven unstable mainly by the plasma currents and the spatial gradients in temperature and density. These types of instability are called microinstabilities and the study of plasma behaviour on spatial scales comparable to the gyroradius is the subject of gyrokinetic theory. Microinstabilities are generally thought to cause the anomalous transport found in CTs [117], although an important feature of the ST is
that the equilibrium flow shear, driven by the strong toroidal rotation, is predicted to stabilise the turbulence in both L-mode and H-mode plasmas [12].

Linear gyrokinetic simulations yield the properties of the most unstable microinstabilities. Strictly, more demanding non-linear gyrokinetic simulations are required to predict the properties of the saturated turbulence. A quasilinear approach is used in the present analysis, exploiting the properties of the dominant linear modes to model the saturated turbulence. This chapter summarises the neoclassical and gyrokinetic models and compares the diffusion and convection coefficients inferred from the transport models with the results deduced from CXS.

6.2 Neoclassical Transport

The classical transport model analyses the radial particle flux induced by collisions in the simplest confined plasma; a straight cylindrical plasma with a uniform magnetic field. In this simplest form, the plasma can be described by Braginskii’s fluid-like equations [115]. Consider a fluid species, \( a \), with mass, \( m_a \), number density, \( n_a \), and charge, \( z_a \), undergoing collisions with background fluid species denoted by \( b \). The momentum balance equation, derived from the first moment of the Vlasov equation, can be written as (Helander and Sigmar equation 2.16 [116])

\[
R_a = n_a m_a \frac{d\mathbf{v}_a}{dt} + \nabla p_a + \nabla \cdot \pi_a - q_a n_a (E + \mathbf{v}_a \times B)
\]  

(6.1)

where \( \mathbf{v}_a \) denotes the mean fluid velocity, \( p_a \) is the pressure, \( \pi_a \) is the viscosity tensor and \( E \) and \( B \) are the average electric and magnetic fields respectively. \( \frac{d}{dt} = (\frac{\partial}{\partial t} + \mathbf{v}_a \cdot \nabla) \) is the convective derivative which describes the rate of change with respect to a moving coordinate system with the fluid element. \( R_a \) is the total force exerted on species \( a \) due to Coulomb collisions from the different species in the plasma, defined as \( R_a = \sum_b R_{a,b} \).

Taking the vectorial product of equation 6.1 with \( B \) gives an expression for the perpendicular fluid velocity as

\[
\mathbf{v}_{a,\perp} = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{1}{n_a m_a \Omega_{ac}} \mathbf{b} \times \left( \nabla p_a + \nabla \cdot \pi_a - R_a + n_a m_a \frac{d\mathbf{v}_a}{dt} \right)
\]  

(6.2)

where \( \Omega_{ac} = z_a B/m_a \) is the gyrofrequency and \( \mathbf{b} = \mathbf{B}/B \) is the unit vector in the direction of the magnetic field. When the distribution of particles is close to a Maxwellian, the terms in equation 6.2 can be ordered in terms of their relative contribution. The leading order terms are

\[
\mathbf{v}_{a,\perp}^0 = \frac{\mathbf{E} \times \mathbf{B}}{B^2} + \frac{\mathbf{b} \times \nabla p_a}{n_a m_a \Omega_{ac}}
\]  

(6.3)

where the first term on the RHS is known as the \( \mathbf{E} \times \mathbf{B} \) drift and the second term is the diamagnetic drift which causes a drift of particles in the poloidal direction.
6.2. NEOCLASSICAL TRANSPORT

The reduction of equation 6.3 can be justified by the following considerations. If the fluid motion is constant in time ($\partial v_a/\partial t = 0$) and space ($v_a \cdot \nabla v_a = 0$), then the convective derivative term is negligible. For a rapidly rotating plasma in toroidal geometry, the centrifugal force can create a significant fluid drift and therefore the convective term becomes significant. This point will be addressed in section 6.2.3. The viscosity term can be neglected if the gradient scale length of the macroscopic plasma parameters is greater than the collision mean free path. In a cylindrical plasma, this approximation is acceptable since the collisional mean free path is given by the Larmor radius, $\rho_a$, as shown in the next subsection ($\rho_a$ is defined previously in equation 4.3), but in a toroidal plasma, this approximation does not always hold as the collisional mean free path is enhanced by the toroidal geometry. Lastly, when the gyrofrequency is larger than the collision frequency (see equation 6.7 below), then the perpendicular force can be neglected. The gyrofrequency is smaller for low $B_T$ operation, as is typical in STs, since $\Omega_{ac} \propto B$ and therefore the collision force can become significant. It is this collisional force which is of interest for the present analysis.

6.2.1 The Role of Collisions

The fluid species $a$ experiences a perpendicular collision force due to the combination of the friction force caused by the velocity difference with fluid species $b$, denoted by $R_{a,b\perp}^{Fr}$, and the thermal force induced by the temperature gradient, denoted by $R_{a,b\perp}^{Th}$. These terms are summarised as (Helander and Sigmar equations 4.37 – 4.38 [116])

\[
R_{a,b\perp}^{Fr} = m_a n_a \nu_{a,b} (v_{b\perp} - v_{a\perp}) = \frac{n_a \nu_{a,b} kT_a}{\Omega_{ac}} b \times \left( \frac{z_a T_b \nabla p_b}{z_b T_a p_b} - \frac{\nabla p_a}{p_a} \right) \quad (6.4)
\]

\[
R_{a,b\perp}^{Th} = \frac{3}{2} \frac{n_a \nu_{a,b} k}{\Omega_{ac}} b \times k \nabla T \quad (6.5)
\]

so that

\[
R_{a\perp} = - \sum_b \frac{n_a \nu_{a,b} kT_a}{\Omega_{ac}} b \times \left( \frac{\nabla p_a}{p_a} - \frac{z_a T_b \nabla p_b}{z_b T_a p_b} - \frac{3}{2} \frac{\nabla T_a}{T_a} \right) \quad (6.6)
\]

The collision frequency, $\nu_{a,b}$, is defined as (Helander and Sigmar equation 1.4 [116])

\[
\nu_{a,b} = \frac{4\sqrt{2\pi}}{3(4\pi e_0)^2} \frac{z_a^2 z_b^2 \ln \Lambda_{a,b}}{n_a (kT)^{3/2}} \quad (6.7)
\]

where $\ln \Lambda_{a,b}$ is the Coulomb logarithm and $k$ is the Boltzmann coefficient. The expression for $v_{a\perp}^0$ in equation 6.3, is substituted into the initial definition of $R_{a,b\perp}^{Fr}$ in equation 6.4 to give the final expression in equation 6.4. Note the $E \times B$ term in equation 6.3 is the same for all species and therefore cancels out in equation 6.4 and the perpendicular component $R_{a\perp}$ is directed in the poloidal direction. Coulomb collisions obey the conservation of momentum, therefore $\sum_a R_a = 0$. 
The plasma ions experience friction with both electrons and impurities; collisions between like species cause a net zero friction force. The relative magnitude of the frictional forces from electrons and ions can be estimated by the expression (Helander and Sigmar equation 5.8 [116])

$$\frac{R_{i,e}^{Fr}}{R_{i,i,z}^{Fr}} \approx \frac{1}{\alpha} \sqrt{\frac{m_e}{m_i}} << 1 \quad (6.8)$$

where $\alpha = n_{i,z}z_i^2/n_{i}z_i^2$ is the impurity strength parameter. In the previous chapters, the subscript $i$ has denoted all ions in the plasma, but for this chapter $i$ is limited only to the primary ion species (i.e. D$^+$) and the subscripts $i, z$ represents all the remaining $z$ charged impurity ions in the plasma. The friction between the plasma ions and electrons is weak compared to the impurities. For a pure plasma, the plasma ion flux is constrained to the electron particle flux to conserve quasi-neutrality.

The perpendicular plasma ion particle flux is calculated from $\Gamma_{i,\perp} = n_i v_{i,\perp}$ (including the smaller terms of $v_{i,\perp}$ in equation 6.2). In cylindrical geometry, focus is only given to the part of $\Gamma_{i,\perp}$ induced by collisions. Since $R_i = -R_{i,z}$, the radial impurity particle flux is related to the plasma ion flux as $\Gamma_{i,z} = -\frac{1}{z_i} \Gamma_i$.

Making the assumption that $T_i = T_{i,z}$ and expressing the pressure gradient as $\nabla p_a = k(n_a \nabla T + T \nabla n_a)$ allows $\Gamma_{i,z,\perp}$ to be written in terms of the density and temperature gradients as

$$\Gamma_{i,z,\perp} = \frac{1}{z_i} b \times \frac{R_{i,\perp}}{m_i \Omega_{ic}} \left( -\nabla n_{i,z} + z_i \left( \frac{\nabla n_i}{n_i} - \left( \frac{1}{2} + \frac{1}{z_i} \right) \frac{\nabla T}{T} \right) \right) \quad (6.9)$$

Comparing equation 6.9 with equation 5.5 and using equation 6.7, the classical radial diffusion, $D_{i,z}$, and convection, $v_{i,z}$, coefficients can be defined as

$$D_{i,z}^{Cl} = \left( \frac{4\sqrt{2\pi m_i}}{3(4\pi\varepsilon_0)^2} \right) \ln \Lambda_{i,i,z} n_i \frac{n_i}{\sqrt{T B_T^2}} = 2.24 \cdot 10^{-4} \ln \Lambda_{i,i,z} n_i \frac{n_i}{\sqrt{T B_T^2}} \quad (6.10)$$

$$v_{i,z}^{Cl} = D_{i,z}^{Cl} z_i \left( \frac{\partial \ln n_i}{\partial r} - \left( \frac{1}{2} + \frac{1}{z_i} \right) \frac{\partial \ln T}{\partial r} \right) \quad (6.11)$$

$T$ is in keV, $n_i$ is in $10^{19}$ m$^{-3}$, $B$ is in T and $D_{i,z}^{Cl}$ and $v_{i,z}^{Cl}$ are in m$^2$/s and m/s respectively.

Attention is drawn in equations 6.10 and 6.11 to the parameter dependencies. The diffusion can be described as a random walk process with $D \sim \Delta r^2/\Delta t$, using the mean free path $\Delta r \sim \rho_i$, and the collision time step, $\Delta t \sim \nu_{i,1,z}$. The $1/B^2$ dependence in equation 6.10 implies a significant difference in radial diffusion across the plasma in an ST, due to the anisotropic $B_T$. Also $D_{i,z}^{Cl}$ is independent of impurity charge. For $v_{i,z}^{Cl}$, a steep main ion density profile causes a particle pinch (inward convection), while a steep temperature gradient screens the particles from the core.
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6.2.2 Toroidal Geometry

The classical particle flux is purely the result of perpendicular friction. In a toroidal system, the classical $\Gamma_{i,z\perp}$ is enhanced by parallel friction contributions resulting from various mechanisms, depending on the collisionality regime. The full derivation of the neoclassical terms is given in the overview by Hirshman and Sigmar [118]. Since the parameter dependencies of the neoclassical transport coefficients are relevant to this thesis, a brief description of the underlying transport processes is given, along with the key equations from Hirshman and Sigmar. These equations have also been summarised by Fussman et al. [119].

The collisionality, $\nu_*$, defined as the ratio of the collision frequency and the guiding-centre transit frequency, $\Omega_t$, (see equation 6.12 where $v_{th}$ is the thermal velocity) identifies the different physical regimes involved in neoclassical transport.

$$\nu_* \equiv \frac{\nu}{\Omega_t} = \frac{qR}{v_{th}} \sim \frac{d_{||}}{\Delta r} \begin{cases} \gg 1 & \text{Pfirsch-Schlüter Regime} \\ \ll 1 & \text{Banana-Plateau Regime} \end{cases}$$

When $\nu_* \gg 1$, the mean free path $\Delta r = v_{th}/\nu$ is shorter than the parallel distance around a flux surface $d_{||} \sim qR$ and the Braginskii fluid equations may still be used to analyse the transport. This collisionality regime is called the 'Pfirsch-Schlüter' (PS) regime. In the opposite limit, $\nu_* \ll 1$, referred to as the 'Banana-Plateau' (BP) regime, the fluid equations are no longer applicable and a kinetic description of the plasma must be used. For the plasma scenarios used in this thesis, it can be seen from figure 6.1 that $D^+$ and $\text{He}^{2+}$ ions are within the BP regime over the entire plasma cross section, whereas $C^{6+}$ ions lie between the PS and BP regime.

**Pfirsch-Schlüter Regime**

Consider first the $C^{6+}$ ions in the PS regime near the edge of the plasma. In this regime, the parallel motion along a field line is purely diffusive, since the guiding-centre orbits are constantly interrupted by collisions. From the random walk picture, the parallel diffusivity can be estimated as $D_{||} \sim \Delta r^2 \nu \sim v_{th}^2/\nu$, which allows the (diffusive) transit frequency to be expressed as

$$\Omega_{ps}^t \sim D_{||}/(qR)^2 \sim \frac{1}{\nu} \left( \frac{v_{th}}{Rq} \right)^2$$

As the fluid diffuses in the parallel direction, the vertical particle drifts, $v_d$, caused by the grad-B and curvature drift lead, to a random walk in the radial direction with a mean free path

$$\Delta r \sim \frac{v_d}{\Omega_{ps}^t} \sim \frac{p_a v_{th}}{R} \frac{1}{\Omega_{ps}^t}$$

The expression for $v_d$ is obtained from Helander and Sigmar [116] equation 6.20. The random walk estimate for the perpendicular transport can be written using the
Figure 6.1: The variation of the effective collisionality and bounce frequency ratio with $\rho$ for the $D^+$, $He^{2+}$ and $C^{6+}$ ions is shown for L-mode high $I_p$ plasma (a), the low $I_p$ plasma (b) and the H-mode plasma (c). Dashed lines represent the cut-offs for each neoclassical regime: Banana, Plateau and Pfirsch-Schlüter.
expressions in equations 6.13 and 6.14 as

$$D \sim \Delta r^2 \Omega_p^T \sim q^2 \rho_n^2 \nu$$

(6.15)

which is larger than the classical diffusion coefficient by a factor $q^2$.

This enhanced transport in the PS regime, over the classical value, is because the perpendicular diamagnetic current, $j_{\perp} = b \times \nabla P/B$, is not divergence free (i.e. $\nabla \cdot j_{\perp} \neq 0$). The magnetic field strength varies radially across the plasma approximately as

$$B_T = \frac{B_0}{1 + (r/R_0) \cos \theta}$$

(6.16)

where $B_0$ is the field at $R_0$ and $\theta$ is the poloidal angle. The addition of a parallel current produces a divergence free total current and conserves quasi-neutrality (i.e. $\nabla \cdot (j_{\perp} + j_{||}) = 0$). It is shown in Helander and Sigmar (see equation 8.18 [116]) that this parallel current is the combination of a small PS current and a larger Ohmic current, the latter induced by the central solenoid, neutral beams or the so-called bootstrap-current. A small electric field is required to drive the parallel PS current against friction, which causes a radial $E_p^T \times B$ drift and therefore contributes to the radial particle flux. The full PS diffusion coefficients listed in equations 20 and 21 of Fussman et al. [119] are

$$D_{T,i,z}^{ps} = 2Kq^2D_{T,i,z}^{cl}$$

(6.17)

$$v_{T,i,z}^{ps} = D_{T,i,z}^{ps}z_i \left( \frac{\partial \ln n_i}{\partial r} + \frac{H}{K} \frac{\partial \ln T}{\partial r} \right)$$

(6.18)

where $H$ and $K$ are functions of the impurity strength parameter and the collisionality. The values of these two coefficients for a low collisionality case with low impurity concentrations are $H \sim -0.5$ and $K \sim 1$ as shown by Hirshman and Sigmar on page 1138 [118]. The PS transport is therefore similar to the classical case, except for the $\sim q^2$ enhancement of $D_{T,i,z}^{cl}$.

### Banana-Plateau Regime

The transport of the He$^{2+}$ and C$^{6+}$ ions in the BP regime is now considered. Recall that, in this regime, a kinetic description of the plasma must be used. When the particles travel along a field line from the point of lowest magnetic field, $B_T^{min} \propto (R_0 + r)^{-1}$, into the region of higher field, the mirror force acts to repel a certain fraction of particles back into the region of low magnetic field, as shown by the bold line in figure 6.2a. The fraction of reflected and unreflected particles are known as the ‘trapped’ and ‘passing’ particles respectively.

The fraction of trapped particles can be determined by noting that, when moving from one region of magnetic field to another, the kinetic energy, $W = \frac{m}{2}(v_\perp + v_{||})$, and magnetic moment, $\mu = W_\perp / B$, of the particles are conserved (in the absence of E fields) [1]. For trapping to occur, the magnetic field required to produce $v_{||} = 0$
Figure 6.2: A poloidal cross section showing the magnetic flux surfaces and the trajectory of a trapped particle (shown in bold) without (a) and with (b) radial drifts included. The minimum and maximum magnetic field along the particle trajectory is denoted by $B_T^{\text{min}}$ and $B_T^{\text{max}}$ respectively, while the magnetic field at the bounce point, where $v_{||} = 0$, is denoted by $B_T^b$. The displacement distance of the particle due to the radial drifts is denoted in (b) by $\Delta r$.

must be less than $B_T^{\text{max}}$ and therefore, from the constancy of $\mu$, the requirement for trapping can be written as (see Wesson equation 3.11.3 [1])

$$\frac{v_{||0}}{v_{\perp0}} < \left( \frac{B_T^{\text{max}}}{B_T^{\text{min}}} - 1 \right)^{1/2} = \left( \frac{2\epsilon}{1-\epsilon} \right)^{1/2}$$

(6.19)

The 0 subscript denotes the speeds at the point where $B_T = B_T^{\text{min}}$ and $\epsilon$ denotes the inverse aspect ratio, $r/R_0$. If the particles have achieved a Maxwellian distribution, then using the expression in 6.19, the fraction of trapped particles can be written as (Wesson equation 3.11.5 [1])

$$f = \left( \frac{v_{||0}}{v_{\perp0}} \right)_{\text{crit}} = \left( \frac{(v_{||0}/v_{\perp0})^2}{1 + (v_{||0}/v_{\perp0})^2} \right)^{1/2}$$

(6.20)

and therefore $f \propto \sqrt{\epsilon}$.

From the expression for $v_{||}$ in equation 6.19, the transit frequency for the trapped particles, often referred to as the bounce frequency, can be written as

$$\Omega_t^b \approx \frac{v_{th}\sqrt{2\epsilon}}{qR}.$$ 

(6.21)

Furthermore, the effective collision frequency in the Banana regime is given by $\nu_{ eff} = \nu/\epsilon$. The trapped particle guiding-centre orbits therefore only exist for
\( \nu_s \ll e^{3/2} \). In the collisionality space between the Banana and PS regimes, the trapped particle guiding-centre orbits are regularly interrupted, whereas the passing particle guiding-centre orbits are collisionless; this regime is known as the Plateau regime. The \( \text{He}^{2+} \) ions are well within the Banana regime, whereas the \( \text{C}^{6+} \) ions span all three regimes.

As the trapped particles traverse around their orbits, a radial drift is induced due to the vertical curvature and centrifugal effects, similar to the PS case. When incorporating these drifts into the trapped particle orbit, a so-called ‘banana orbit’ is traced out in the poloidal plane, as shown in figure 6.2b. The radial distance travelled by the trapped particles, \( \Delta r \), can be approximated by dividing \( v_d \) by \( \Omega_b^t \), as

\[
\Delta r \approx \frac{\rho_a v_{th}}{R} \frac{R q}{v_{th} \sqrt{\epsilon}} = \rho_a \frac{q}{\sqrt{\epsilon}}
\]  

(6.22)

The collision mean free path of the particles is now larger than the classical value \( (\rho_a) \) by a factor of \( q/\sqrt{\epsilon} \).

Using the random walk estimation, with \( \Delta r \) and \( \nu/\epsilon \), the Banana diffusivity coefficient is approximately \( q^2/\epsilon^{3/2} \) times greater than \( D_{i,z}^{cl} \). The complete calculation of the particle flux in the Banana regime takes into account the parallel friction force induced by the viscosity anisotropy. Equation 27 and 28 from Fussman [119] give \( D_{i,z}^b \) and \( v_{b,i,z} \) in the Banana regime (explicitly for the \( \text{He}^{2+} \) ions) as

\[
D_{i,z}^b \approx 1.52 \times 10^{-23} \ln \Lambda \frac{n_i}{\sqrt{T B^2 \epsilon^{3/2}}} \quad [\text{m}^2 \text{s}^{-1}] \]  

(6.23)

\[
v_{b,i,z} = 2 D_{i,z}^b \left( \frac{\partial \ln n_i}{\partial r} - 0.12 \frac{\partial \ln T}{\partial r} \right) \quad [\text{ms}^{-1}] \]  

(6.24)

where \( T \) is in keV, \( B \) in T and \( n_i \) in \( 10^{19} \) m\(^{-3} \). The convection is independent of impurity charge with a moderately reduced temperature screening.

The transport of \( \text{C}^{6+} \) ions in the Plateau regime is now briefly discussed. The effective collision frequency is defined as \( \nu_e \equiv (v/v_\parallel)^2 \nu \), since it is the small angle scattering which plays the important role in the Plateau regime. This means that the particle flux is essentially due to particles with \( v_\parallel \approx 0 \). The fraction of particles in this regime is given by the ratio \( f_p \sim v_\parallel/v_{th} \) [116]. Furthermore, the particle flux is greatest when a resonance occurs between the effective collision frequency and \( \Omega_t \) (Helander and Sigmar equation 10.5 [116]). A particle with this resonant velocity therefore drifts a radial distance, \( \Delta r \sim v_\parallel/\Omega_t \sim q\rho_a \), which allows the random walk estimate to be written as \( D \sim f_p \Delta r^2 \Omega_t \sim qv_{th}\rho_a^2/R \). Equations 25 and 26 from
Fussman [119] give $D_{\text{p},z}^i$ and $v_{\text{p},z}^i$ in the Plateau regime as

$$D_{\text{p},z}^i \approx \frac{4.04}{R} \frac{q}{z_i^3} B^2 \frac{A_{i,z} T^{3/2}}{[\text{m}^2\text{s}^{-1}]} \tag{6.25}$$

$$v_{\text{p},z}^i = D_{\text{p},z}^i z_i \left( \frac{\partial \ln n_i}{\partial r} + \frac{3(z_i - 1) \partial \ln T}{2z_i} \right) \quad [\text{ms}^{-1}] \tag{6.26}$$

where $T$ is in keV, $A_{i,z}$ is the unit mass and $B$ in T. Note that $D_{\text{p},z}^i$ is now charge dependent and the temperature gradient now causes an inward pinch of impurities into the plasma.

The total radial particle flux is calculated by summing the radial fluxes in each regime $\Gamma_a^r = \Gamma_{\text{cl}}^r + \Gamma_{\text{ps}}^r + \Gamma_{\text{bp}}^r$. An illustration of the total diffusivity is shown by the schematic in figure 6.3. The definitions described above are only estimates because they describe a simplified transport regime with a high aspect ratio and a circular poloidal cross-section. MAST operates in a regime of high triangularity and elongation, meaning that the diffusion and convection coefficients must be multiplied by a geometrical factor related to the flux surface average; this factor has been shown to reduce the transport coefficients by up to a factor of two in D-shaped plasmas [120].

### 6.2.3 Neoclassical Simulations

The neoclassical transport properties of a multi-species axisymmetric plasma of arbitrary aspect ratio, geometry and collisionality is implemented in a FORTRAN module called nCLASS [35]. This code is integrated into the JETTO code suite described in section 5.6. However, in order to run nCLASS, JETTO requires the $q$ profile as well as the $T_e$, $T_i$, $n_e$ and $Z_{\text{eff}}$ profiles. Values of $n_i$ are calculated in an interpretative manner within JETTO using $n_e$ and $Z_{\text{eff}}$. Helium and carbon are assumed as the only two impurities in the simulations.
The neoclassical diffusion, $D_{i,z}^{NC}$, and convection, $v_{i,z}^{NC}$, coefficients, calculated by nCLASS for each plasma, are shown in figure 6.4 and demonstrate the neoclassical parameter trends. In general, $D_{i,z}^{NC}$ is largest in the inner core and decreases rapidly until $\rho \sim 0.3$, where it then increases with radius towards the plasma edge. Increasing $z$, from $z = 2$ (helium) to $z = 6$ (carbon), acts to decrease the magnitude of $D_{i,z}^{NC}$ and $v_{i,z}^{NC}$ most significantly in the region of $0.4 < \rho < 0.8$, while a change of direction is found in $v_{i,z}^{NC}$, from inwards to outwards, within $\rho < 0.4$. Decreasing $I_p$ in L-mode acts to increase moderately $D_{He}^{NC}$ and $v_{He}^{NC}$ by a factor of $\sim 1.3$. Moving from L-mode to H-mode causes an increase in $D_{He}^{NC}$ mainly in the region $0.5 < \rho < 0.8$ and a decrease in the magnitude of $v_{He}^{NC}$ in the region $\rho > 0.6$ as well as a change in direction in the region $0.4 < \rho \leq 0.6$.

The large rates of $D_{i,z}^{NC}$ found within $\rho \leq 0.3$ are mainly due to the dependence of $D_{He}^{He}$ on $e^{-3/2}$, as shown in equation 6.23. An increase in the magnitude of $D_{C}^{NC}$, compared to $D_{He}^{NC}$, could be due to Plateau $C^{6+}$ ions following equation 6.25, which is dependent on $z^{-2}$. The difference between $v_{He}^{NC}$ and $v_{C}^{NC}$ is most probably due to the dependence $v_{i,z}^{NC} \propto z$ in all collisionality regimes. For $\rho < 0.5$, an increase in $n_e$ at low $I_p$ is observed experimentally which, from equation 6.23, could explain the increase in $D_{He}^{NC}$ compared to the high $I_p$ case. Towards the edge of the plasma, although $n_e$ is marginally lower at low $I_p$, the larger values of $q$ at low $I_p$ will act to increase $D_{He}^{NC}$. The $n_e$ profile is predominantly causing the difference in transport in the confinement scan since $n_e$ increases near the plasma edge in H-mode compared to L-mode plasmas, which will increase $D_{i,z}^{NC}$ and decrease $v_{i,z}^{NC}$ due to the lower density gradient.

For the $z$ scan, the trend of $D_{i,z}^{NC}$ and $v_{i,z}^{NC}$ is mainly in disagreement with the experimental findings. In figure 5.5, it is shown that $D_{i,z}$ and $v_{i,z}$ increase with $z$ in the range $\rho > 0.4$. This increase therefore cannot be a neoclassical effect. The small increase in both $D_{He}^{NC}$ and $v_{He}^{NC}$ found at low $I_p$, compared to the high $I_p$ plasma, is in reasonable agreement with the experimental trends shown in figure 5.8. The
Figure 6.5: Comparisons of the radial neoclassical (red dashed line) and experimental (solid black lines) diffusion and convection coefficients, calculated in each plasma scenario, are shown on the left and right respectively.
6.2. NEOCLASSICAL TRANSPORT

Figure 6.6: Peaking factor calculated using the $D_{He}$ and $v_{He}$ profiles from the UTC-SANCO analysis (solid line), the zero flux peaking factor from the plasma without a gas puff (dash-dot line) and the neoclassical $D_{He2+}^{nc}$ and $v_{He2+}^{nc}$ coefficients from NCLASS.

confinement scan does highlight a significant difference with the experimental trend in $D_{He}$, but agrees reasonably well for $v_{He}$, as shown in figure 5.11. Experimentally, moving from L-mode to H-mode causes a moderate decrease in $D_{He}$, whereas the neoclassical results would suggest the opposite.

So far the neoclassical and experimental parameter trends of the impurity transport have been compared. It is also necessary to evaluate whether the absolute magnitudes correspond to determine whether the impurity transport is responding predominantly to the neoclassical physics or to the background plasma turbulence. A comparison of the magnitudes of the transport coefficients for each scenario are illustrated in figure 6.5. For the L-mode high $I_p$ plasma, the experimental $D_{He,C}$ and $v_{He,C}$ coefficients are both similar in magnitude to the neoclassical predictions within $\rho \leq 0.4$. Towards the edge of the plasma, the experimental $D_{He,C}$ and inward pinch are larger in magnitude than neoclassical predictions, more so for carbon than helium. At low $I_p$, the experimental values of $D_{He}$ are higher than neoclassical predictions over the range $\rho > 0.3$, whereas $v_{He}$ is larger in magnitude in the range $\rho > 0.4$. Therefore, it can be concluded that in both L-mode discharges (high and low $I_p$), the impurity transport is neoclassical up to $\rho = 0.4$, and the rates found between $0.4 < \rho < 0.8$ are anomalous. In the H-mode plasma, the picture is different and the neoclassical predictions, within the range $0.4 < \rho < 0.8$, agree within the error bars of the experimental results.

The helium zero flux peaking factor and the UTC-SANCO and neoclassical $-v_{He}/D_{He}$ ratio is shown for the L-mode 900 kA plasma in figure 6.6. There is good agreement between the $-v_{He}/D_{He}$ determined from UTC-SANCO and the zero flux peaking factor, which suggests the gas puff is not significantly changing the transport properties of the plasma. The UTC-SANCO and neoclassical $-v_{He}/D_{He}$ are very similar over the range of $0.2 \leq \rho \leq 0.8$. The behaviour is also similar for the L-mode
and H-mode plasmas. This shows that neoclassical theory can be used to predict where the impurities accumulate in the plasma after the impurity flux becomes zero. However the separate magnitudes of $D_{He}$ and $v_{He}$ suggest that neoclassical theory does not predict the time scales in which the impurities dynamically evolve near the plasma edge in L-mode. In other words, the peaking factor analysis would lead to the (incorrect) conclusion that the plasma is neoclassical over the entire plasma radius in L-mode, whereas the gas puff experiments show that anomalous transport is dominant near the L-mode plasma edge. This demonstrates one of the main motivations for carrying out gas puff experiments.

One limitation of nCLASS is that it does not include rotational effects. In a strongly rotating plasma, such as those generated in MAST, this limitation must be addressed. More specifically, one can no longer assume that $d/dt \approx 0$ in equation 6.2 since the Coriolis and centrifugal forces cause a significant perpendicular drift. The contribution of the rotation to the transport is summarised in equations 6.27–6.29 (Delgado-Aparicio equations 3–6 [121]):

\[
D_{1,z}^{\text{Rot}} = D_{1,z}^{\text{np,ps}} \left( 1 + \Gamma_{1,z} M_{D} R_{0}^{2} R_{2}^{2} \right)^{2} \quad (6.27)
\]

\[
\Gamma_{1,z}^{A} = \frac{A_{1,z}}{2} \left( 1 - \frac{2 z_{i}}{A_{1,z}} \left[ \frac{T_{e}}{T_{e} + T_{i}} \right] \right) \quad (6.28)
\]

\[
M_{D}^{2} = \frac{v_{\phi}^{2}}{v_{\text{th}}^{2}} \approx 1.05 \cdot 10^{-5} \frac{v_{\phi}^{2}}{T_{i}} \quad (6.29)
\]

\[T_{i}\text{ is in keV and } v_{\phi}\text{ is in km/s. The ratio of } D_{1,z}^{\text{rot}} / D_{1,z}^{\text{nc}} \text{ has been plotted as a function of } \rho \text{ in figure 6.7 for the L-mode high } I_{p} \text{ plasma. It can be seen that the correction from the rotation will act to increase the neoclassical rates for helium by } \sim 20\% \text{ in the plasma core and by } \sim 1\% \text{ near the plasma edge. This correction is amplified}
\]

Figure 6.7: The magnitude of the approximate estimate of the radial neoclassical diffusion coefficient enhanced by rotation and value calculated by nCLASS is shown for carbon and helium during the L-mode high $I_{p}$ plasma.
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for higher charge elements like carbon, where an increase of \( \sim 60\% \) is found in the plasma core and \( \sim 5\% \) in the plasma edge. This enhancement to \( D_{\text{nc}}^{\text{ic}} \) decreases during the low \( I_p \) and H-mode plasmas and therefore has not been shown. The conclusions given previously from the nCLASS simulations stated that the transport is mainly anomalous in the region of \( \rho > 0.4 \) and the minor correction from the rotation does not change this conclusion.

6.3 Anomalous Transport

If the fluctuations in electrostatic potential, \( \phi \), and electromagnetic potential, \( A_{||} \), are out of phase with the density fluctuations, then a radial flux is induced due to the net \( \nabla \phi \times B \) drift and the parallel motion along the perturbed field line. It is this fluctuation-driven radial flux that is thought to be responsible for the anomalous transport. The electrostatic potential fluctuations, \( \phi \), have a wave like character and are generally represented in Fourier form as

\[
\phi = \sum_k f_k \exp \left( i [k \cdot r] - i \Omega_k t \right)
\]

(6.30)

where each Fourier component has a unique wavenumber, \( k \), an associated complex frequency, \( \Omega_k = \omega_k + i \gamma_k \), and an amplitude \( f_k \). Gyrokinetic codes are typically used to show the growth-rate, \( \gamma_k = \text{Im} (\Omega_k) \), and real frequency, \( \omega_k = \text{Re} (\Omega_k) \), of the turbulent modes as a function of the wavenumbers parallel and perpendicular to \( B \), denoted as \( k_{||} \) and \( k_{\perp} \) respectively.

There are a number of different microinstabilities that can occur in tokamak plasmas [122]. The fundamental source of most instabilities is the temperature and density gradients. In CTs, the dominant electrostatic instabilities are the Ion Temperature Gradient (ITG) mode, the Trapped Electron Mode (TEM) and the Electron Temperature Gradient mode (ETG) [123]. The ITG instability, often called the \( \eta \) instability [124], rotates in the ion diamagnetic direction at long wavelength scales in the range of \( k_{\perp} \rho_i < 1 \), whereas the TEM and ETG rotate in the electron diamagnetic direction at medium wavelength scales (\( 1 < k_{\perp} \rho_L < \rho_i / \rho_e \)) and short wavelength scales (\( k_{\perp} \rho_i > 3 \)) respectively. When a particles gyroradius is much larger than the mode’s correlation length, the effect of the mode is averaged out, hence ETG modes are not expected to affect significantly the ion motion and ITG modes are not expected to affect significantly the electron motion.

ITGs are destabilised when \( \eta_i = d(\ln T_i) / d(\ln n) > 1 \) [125]; therefore the ion temperature gradient drives the ITG unstable whereas the density gradient stabilises the ITGs. At low collisionality, the (collisionless) TEMs are driven primarily by the electron temperature gradient and/or the electron density gradient [126]. As the collisionality increases, the trapped particles become detrapped and hence the (dissipative) TEMs become stabilised [127, 128]. TEMs (and ETGs) are thought to
be responsible for the observed anomalous electron heat transport on MAST [12]. Electromagnetic effects must also be considered for high $\beta$ tokamaks like MAST, as they can produce longer wavelength modes, such as tearing modes (TM) and Kinetic Ballooning Modes (KBM) [12,129].

### 6.3.1 Microstability Analysis

To model $\gamma_k$ and $\omega_k$ of the dominant microinstabilities, it is necessary to investigate the stability of a particular plasma equilibrium to small field perturbations of the form shown in equation 6.30. Gyrokinetic codes are used to study the evolution of $\phi$, following an initial perturbation, by linearising the Vlasov equation in the following limits:

$$\frac{\omega_k}{\Omega_c} \approx \frac{C_a}{\Omega_c} \approx \frac{k||}{k_\perp} \approx \frac{\rho_i}{L_p} << 1 \quad (6.31)$$

These orderings simply state that the gyromotion, $\Omega_c$, is rapid compared to the fluctuating mode frequency, $\omega_k$, and the model collision frequency, $C_a$. The instability is predominantly elongated in the parallel direction due to the rapid parallel motion of the particles and the pressure scale length is large compared to the Larmor radius.

Within the limits of equation 6.31, the kinetic equation can be averaged over the small and rapidly gyrating Larmor orbits to give the linearised electromagnetic gyrokinetic equation [12]:

$$\left[ \frac{\partial}{\partial t} + (v_d + v_{||} b) \cdot \nabla + C_a \right] g_a = \left[ z_a \frac{\partial f_{0a}}{\partial \theta} \frac{\partial}{\partial \theta} + \frac{1}{2} k_\perp \times b \cdot \nabla f_{0a} \right]$$

$$\times \left[ (\phi - v_{||} A_{||}) J_0(d) + \frac{v_{||}}{k_\perp} B_{||} J_1(d) \right] \quad (6.32)$$

where $g_a$ is the perturbed distribution function for species $a$, $f_{0a}$ is the Maxwellian particle distribution function, $C_a$ is the model collision operator and $E$ is the particle kinetic energy per unit mass. The perpendicular drift velocity term, $v_d$, contains the drifts due to the inhomogeneous magnetic field, such as the grad-B and curvature drift, the $E \times B$ drift, and the inertial drifts due to the Coriolis and centrifugal force. The parallel motion along the unperturbed field is contained in $v_{||} b$. Lastly, the terms $J_0$ and $J_1$ are first and second Bessel functions representing the gyroaverage of the perturbations with argument $d = k_\perp \rho_i$.

Two local flux-tube electromagnetic gyrokinetic codes, gs2 [36,37] and GKW [38], are used to solve the linear gyrokinetic equation to yield the properties of the unstable microinstabilities as a function of $k_y \rho_i$. Here, $k_y$ is the in-magnetic-surface perpendicular wavenumber. The flux-tube codes follow the field lines around the plasma from the outboard mid-plane to the inboard mid-plane and assume that the densities and temperatures and their gradient scale lengths are constant during the simulation. A proper account of the transport should include the non-linear saturation of the modes, however non-linear gyrokinetic simulations demand large
computing resources and therefore only the linear gyrokinetic equation is analysed in this thesis. A quasilinear approach is adopted, where the properties of the dominant linear modes are used to model the saturated turbulence. Knowledge of $\gamma$ allows an approximation for the thermal diffusivity, called the mixing length thermal diffusivity:

$$\chi_{mix} \approx \gamma/k_y^2$$

(6.33)

The dominant modes are inferred in the wavelength region where $\chi_{mix}$ is maximised.

The flux surface $\rho = 0.7$ is analysed for each plasma, since this surface provides reliable experimental transport coefficients thought to be dominated by turbulence in the L-mode plasmas. The magnetic equilibrium in GS2 and GKW is set up using the Miller parameters described in chapter 2. The magnetic flux reconstruction in each plasma has been calculated using the EFIT++ code described in chapter 2, constrained by the MSE diagnostic pitch angle and the $D_\alpha$ emission. Profile information for both GS2 and GKW input files were extracted from TRANSP [130] analysis to ensure consistency of the data. TRANSP provides information for six different plasma species (e, D, H, C and fast H, D) but, to reduce the complexity of the simulation, only electrons and (bulk) plasma ions were considered. Therefore the pressure profile and density of the plasma ions were modified to match the electron pressure profile density with $Z_{eff}$ forced to unity in the input file.

Table 6.1 lists the relevant plasma equilibrium parameters of each plasma at $\rho = 0.7$, normalised by the sound speed, $c_s = \sqrt{T_i/m_i}$, and the minor radius, $a$. These parameters include the safety factor, $q$, the magnetic shear, $\hat{s} = (r/q) dq/dr$, electron collisionality, $\nu^* = \nu_e a/c_s$, plasma beta, $\beta$, plasma triangularity, $\delta$, plasma elongation, $\kappa$, the normalised gradients $a/L_{T_i}$, $a/L_{T_e}$, $a/L_{n_e}$ and $a/L_{u_i}$, and the equilibrium flow shear, $\gamma_E \propto u'/q$, where $u'$ is the spatial gradient of the toroidal ion rotation velocity normalised by $-a/c_s$. The largest difference between the input values occur for the L- and H-mode plasmas where the H-mode plasma experiences a significant reduction in $a/L_{n_e}$ as well as an increase in $\beta$. Only minor differences are found between the two L-mode plasmas at different $I_p$.

On a spherical tokamak like MAST, the stabilising perpendicular component of the sheared toroidal flow, induced by strong neutral beam heating, dominates over the destabilising parallel component, and is thought to stabilise the long wavelength ITG instabilities [12]. This can be written quantitatively in terms of the sheared flow stabilisation criterion $\gamma_E > \gamma_{max}$ [131], where $\gamma_{max}$ is the maximum linear growth-rate. Growth-rates of the dominant instabilities have been computed in the absence of sheared flow with both codes. Effective linear growth-rates that account for the stabilising influence of flow shear have been calculated with GS2 using the method described in [113]. In this method, firstly a pre-set amplification factor, $f_{NL}$, determines the amount by which the initial value of $\phi$ must grow by to be considered unstable. This amplification factor is chosen to represent the nonlinear saturation level. An estimate of this value, based on previous literature
Figure 6.8: $\phi$ amplitude versus $t$ in linear electromagnetic calculations for the $\rho = 0.7$ surface in the L-mode high $I_p$ plasma for a stable and unstable mode, following the inclusion of flow shear, at (a) $k_y \rho_i = 1.11$ and (b) $k_y \rho_i = 1.53$. Horizontal grey lines indicate the amplitude range, denoted by $f_{NL}$, used to obtain the effective growth-rate $\gamma$. The red dashed line indicates the linear fit of the unstable mode in (b).

[113], suggests $f_{NL} = 100$, however a smaller value of $f_{NL} = 20$ seems to be needed in the simulations described above to preserve unstable linear modes at wavelengths greater than $k_y \rho_i > 1$. Once the mode has grown by this factor, any subsequent linear evolution of $\phi$ is disregarded. The effective growth-rate of the mode is then determined by fitting $\phi$ over the range of $t$ that achieved the required mode amplification. An example of the temporal evolution of $\phi$ including the fitted function is illustrated at two different wavenumbers in figure 6.8 for the gs2 simulation of the L-mode high $I_p$ plasma.

The parallel component of the sheared toroidal flow, as mentioned above, can act to destabilise certain modes due to the Coriolis drift and centrifugal force [132, 133]. The derivation of the Coriolis drift [134] and centrifugal force [135, 136] is included in GKW by writing the gyrokinetic equations in the co-moving frame [137]. GS2 also has the ability to include these terms, however the derivations have not been

<table>
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<th>Table 6.1: GS2 input parameters for $\rho = 0.7$</th>
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<tr>
<td>L-mode high $I_p$</td>
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<tr>
<td>$c_s$ (m/s)</td>
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<tr>
<td>$a$ (m)</td>
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<tr>
<td>$\kappa$</td>
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<td>$\sin^{-1} \delta$</td>
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<td>$\beta$</td>
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<td>$u'$</td>
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<td>$\gamma_E$</td>
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Figure 6.9: Linear growth-rate spectrum for the flux surface $\rho = 0.7$ in the L-mode (a) and H-mode (b) plasmas at high $I_p$. Calculations performed with and without flow shear using the GKW (circles) and GS2 (squares, triangles) codes respectively. The dashed lines represent the dominant linear mode in each case. Symbols along the x-axis represent the stable modes found. The mixing length estimates for the simulations with flow shear included is shown respectively for each case in (c) and (d).

The growth-rate spectrum from the microstability analysis of the L-mode and H-mode high $I_p$ plasmas using GS2 and GKW are shown in figures 6.9a and 6.9b respectively for the range $0.1 < k_y \rho_i < 10$. Due to the similarity of input parameters for the low $I_p$ and high $I_p$ plasmas, the growth-rate spectrum for the low $I_p$ plasma is not shown. For the L-mode high $I_p$ plasma, both codes show unstable modes across the $k_y$ spectrum with $\gamma_E = 0$. A similar growth-rate spectrum is observed in the H-mode plasma with $\gamma_E = 0$, however modes in the region $1 < k_y \rho_i < 2$ are stable. A similar stable region was reported for an earlier MAST H-mode discharge in [113]. The main difference between the local equilibria in L- and H-mode is that $a/L_n_e$ is lower in H-mode. The sensitivity of TEM stability to the density gradient was also recently demonstrated in the microstability analysis of MAST discharges with pellet injection [138].

With $\gamma_E$ set to the experimental value in the simulation, the plots in figures 6.9a and 6.9b suggest that unstable long wavelength ITG modes become stabilised in L- and H-mode at $\rho = 0.7$. The mixing length estimation, $\sim \gamma/k_y^2$, is shown in figures
6.9c and 6.9d with a maximum corresponding to $k_y \rho_i \sim 2$ and $\sim 4$ for L- and H-mode respectively. The BES electron density fluctuation signal, $\delta n_e/n_e$, averaged over 4 poloidal channels at each radius and time averaged over 2 ms ($\pm 1$ ms either side of 0.24s), is plotted in figure 6.10a. The values of $k_y \rho_i$ observed by the BES diagnostic correspond to ITG turbulence. For $\rho = 0.7$, the electron density fluctuation signal is approximately 0.4 %, very close to the detection threshold. This suggests that ITG modes are indeed stable for this flux surface, but does not prove or disprove the existence of TEMs. The BES measurements are consistent with ITG modes being stable on this equilibrium surface of the 900 kA L-mode discharge. Towards the edge of the plasma the BES signal increases indicating a rising amplitude of ITG turbulence. This could be explained by the fact that the equilibrium flow shear decreases with radius as shown in figure 6.10b.

Another point is that the carbon transport is larger than neoclassical predictions by a greater amount than the helium transport. This is possibly because the Larmor radius of a C$^{6+}$ ion is smaller than a He$^{2+}$ ion by an amount $\sqrt{m_{He}/m_C} \sim 0.6$. Carbon ions are therefore affected by shorter wavelength fluctuations than the helium ions. This is significant because the long wavelength modes are stabilised by the toroidal flow shear, leaving only the shorter wavelength modes.

### 6.3.2 Quasilinear Peaking Factor

Quasilinear theory is now exploited to analyse the helium particle flux from GKW in terms of the transport coefficients associated with the dominant modes. From gyrokinetic theory, the helium particle flux can be written, assuming that $T_{He^{2+}} = T_i$ as (Angioni equation 4 [29])

$$\frac{a \Gamma_{He^{2+}}^{GKW}}{n_{He^{2+}}} = D_{He}^{GKW} \left( \frac{a}{L_{He^{2+}}} + C_T \frac{a}{L_{T_i}} + C_u u' + C_p \right) \quad (6.34)$$

where a negative impurity flux again represents an inward motion of impurities. The multiplier outside the brackets on the RHS of equation 6.34 represents the diffusivity, which is mainly dominated by the $E \times B$ advection, while the combination of the latter three dimensionless terms represent the convection coefficient. The trace impurity transport thermodiffusive ($C_T$) [139,140], rotodiffusive ($C_u$) [132,133] and convective ($C_p$) [141] dimensionless coefficients are functions of the impurity mass, charge, temperature, and the growth-rate and real frequency of the background turbulence. The latter dependence means that the direction of each term is dependent mainly on the modes direction of rotation, which for TEMs and ITG are different in sign (TEM is negative and ITG is positive). Summarising figure 2 of Ref. [142] for the impurities, the $C_T$ term, which scales as $1/z$, and the $C_u$ term are directed inwards for TEMs and outwards for ITG modes, while the $C_p$ term has a term proportional to the magnetic shear which is always inwards and a term proportional to $1/q^2$ which is outwards for TEMs and inwards for ITG modes.
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Gyrokinetic simulations using gs2 and gkw, assuming \( n_e = n_i \) and with helium set as a trace species (i.e. not affecting the background turbulence), can be used to determine quasilinear estimates of the particle flux described in equation 6.34. In this case, since the toroidal rotation may play a significant role in MAST plasmas, the particle flux will be investigated using gkw only in order to include the \( C_u \) term in equation 6.34. Since gkw simulations have not included the stabilising effect of the toroidal flow shear, the particle flux contribution from wavenumbers that are associated with stable modes found using gs2 after the inclusion of the sheared toroidal flow are shown but not considered in the discussion.

The dimensionless coefficients \( C_T \), \( C_u \) and \( C_p \) are determined as a function of \( k_y \rho_i \) using gkw by calculating the gyrokinetic particle flux of four trace amounts of helium with a pre-determined orthogonal set of equilibrium gradients [31]. This is demonstrated more clearly in table 6.2. Figure 6.11a illustrates the direction and relative magnitude of each term as a function of \( k_y \rho_i \) for the unstable and stable modes in the L-mode high \( I_p \) plasma at \( \rho = 0.7 \). Note that, in this graph, the negative value of the transport coefficients are shown and therefore a positive value
corresponds to an inward direction. This has been done because of the definition of the peaking factor, as will be discussed next. It can be seen from the graph in figure 6.11a that in the region of \( k_y \rho_i \sim 2 \), \( C_T \) is largest and directed inwards while the \( C_u \) and \( C_p \) terms are comparably less and also directed inwards.

These dimensionless impurity transport coefficients \( (C_T, C_u \text{ and } C_p) \), combined with the experimental values of \( a/L_{T_i} \) and \( a/L_{u_i} \) (measured by CX), allow an estimate of the zero flux steady state peaking factor to be made as

\[
\frac{a}{L_{n_{He}^{2+}}} = -(C_T \frac{a}{L_{T_i}} + C_u u' + C_p) \tag{6.35}
\]

The quasilinear impurity peaking factor is finally obtained by assuming that impurity transport is dominated by modes at \( k_y = k_{y, max} \) where the mixing length diffusivity is maximised. Figure 6.11b illustrates the peaking factor for the L-mode high \( I_p \) plasma calculated with \( \gamma_E = 0 \). The filled triangles represent the points that can be assumed to be zero if flow shear is included. Encouraging agreement with experiment is found in both direction and amplitude of the peaking factor around \( k_y \rho_i = 2 \). The contribution from each of the different linear components of the flux are shown by the dashed lines. The major contribution to the peaking factor comes

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<td>He Trace 1</td>
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<td>He Trace 2</td>
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<td>He Trace 3</td>
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<td>He Trace 4</td>
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from the ion temperature gradient, with the \( C_T \) term essentially determining only the direction. The \( C_u \) and \( C_P \) terms are also directed inwards but contribute weakly to the overall peaking factor.

Quasilinear estimates of the diffusion and convection coefficients, for the 900 kA L-mode discharge at \( \rho = 0.7 \), can be made using

\[
\Gamma_{QL}^{He2+} = \Gamma_{GKW}^{He2+} \frac{\chi_{\text{eff}}}{\chi_{\text{GKW}}} \tag{6.36}
\]

where \( \chi_{\text{eff}} = Q_{\text{GKW}}^{He} T_e / (a/L_T) \). The value of \( \chi_{\text{eff}} \) is interpolated from figure 5.12a at \( \rho = 0.7 \) and the value of \( \chi_{\text{GKW}} \) is chosen around \( k_y^{\text{max}} \). This gives values of \( D_{\text{GKW}}^{He} \) of the order of \( 1 - 10 \) m\(^2\)/s, similar to the experimental diffusivity. Furthermore, a previous study by Casson et al. [31] showed that this quasilinear method of obtaining of the peaking factor gives good agreement with full non-linear gyrokinetic simulations.

There are a number of points that must be considered when comparing these results to ITER helium transport predictions. One major difference is that the sheared toroidal flow will not be sufficient on ITER to stabilise the ITG modes. This also indicates a more subtle difference namely that \( C_u \sim 0 \). Consider the modes associated with \( k_y \rho_i < 1 \), as shown in figure 6.11a. The \( C_P \) term has the larger impact on the overall peaking factor, while the \( C_T \) term is less significant. This result has also been demonstrated in a previous study which provides a polynomial fit for the \( C_P \) and \( C_T \) terms as a function of \( \rho \) expected for ITER (see Angioni equations 8 and 9 [142]).

Although the toroidal shear is not expected to stabilise the ITG modes, it has been shown in an experiment on ASDEX-U that the TEMs may be excited in ITER plasmas by applying strong electron cyclotron heating (ECH) [143]. The findings from ASDEX-U were that, when ECH heating was applied to the central region of the plasma, the convection direction of Si impurities reversed direction from inwards to outwards. This may seem contradictory to the results from this thesis, which suggest that the TEMs are driving an inward pinch, but the results can be explained by considering the dependencies of the two terms that make up \( C_P \); that is the curvature and parallel compression terms. The curvature term is proportional to \( \hat{s} \) and is directed inwards for both TEMs and ITG modes, whereas the compression term is proportional to \( 1/q^2 \) and is outwards for TEMs and inwards for ITG modes. In the central region of the plasma \( 1/q^2 \gg \hat{s} \) therefore, when the ECH is used to excite the TEMs, the outward compression term dominates over the inward curvature term. Analysis in this thesis has focused on the region close to the plasma edge, where \( \hat{s} > 1/q^2 \), and therefore the inwards \( C_P \) term only enhances the inwards \( C_T \) term. An experimental conclusion regarding the inner core from an ST point of view is not possible, since the impurity transport is dominated by neoclassical physics in this region of the plasma.
6.4 Summary

Neoclassical radial profiles of $D^{\text{NC}}$ and $v^{\text{NC}}$, calculated using the NCLASS code, are found in every scenario to be comparable in magnitude to the experimental transport coefficients up to a certain radius. In the L-mode plasmas, this region incorporates the plasma core up to $\rho = 0.5$ at high $I_p$ and $\rho = 0.3$ at low $I_p$. From the plasma mid-radius to the edge, the experimental diffusivity and absolute magnitude of the inward pinch in the L-mode plasmas is greater than neoclassical predictions. For the H-mode plasma, the anomalous inward pinch near the edge of the plasma decreases to a magnitude comparable with neoclassical predictions suggesting that the turbulence has been suppressed.

A microstability analysis has been performed using the GS2 and GKW codes on $\rho = 0.7$ using the L-mode and H-mode high $I_p$ plasma parameters to determine the dominant modes in L-mode. A quasilinear approach has been adopted where the properties of the dominant modes are used to model the saturated turbulence. Without including flow shear, the growth-rate spectrum from both codes indicate unstable modes in the ITG and TEM wavelength region. There are a number of results that suggest that the toroidal flow shear stabilises the ITG modes. An effective growth-rate calculation that includes the stabilising effects of the toroidal flow shear has been performed using GS2. These results indicate that unstable ITG modes in the region $k_y \rho_i < 1$ are suppressed. The BES diagnostic records $n_e$ fluctuations at the wavelength scale equal to $k_y \rho_i \sim 0.5$ representative of ITG modes. At $\rho = 0.7$, the signal is close to the detection threshold indicating that ITG modes are minimal. The main difference between the growth-rate spectrum of the L- and H- mode plasmas is the stabilisation of modes in the region $1 < k_y \rho_i < 3$ due to the shallow electron density gradient. This would suggest that the modes responsible for the anomalous transport in L-mode are found in this wavelength region associated with TEMs. Gyrokinetic quasilinear calculations of the peaking factor in L-mode agree best with the experimental peaking factor, in the region $k_y \rho_i \sim 2$. This also highlighted that the anomalous L-mode pinch is mainly driven by the thermodiffusion term, with smaller contributions from the rotodiffusion and convective term.

In conclusion, the electron density gradient plays a crucial role in the transport of light impurities in spherical tokamaks; a steep electron density gradient causes an inward pinch of helium directly from neoclassical transport and from turbulence associated with collisionless TEMs. Generally the helium pinch caused by turbulence dominates over the neoclassical pinch in L-mode. To avoid a significant accumulation of helium in the plasma core, it is recommended to keep the electron density gradient low; a condition that is met in the ELMy H-mode plasmas on MAST.
Chapter 7

Conclusions and Future Work

7.1 Thesis Objectives and Review

The work presented in this thesis has focused on characterising the helium and carbon transport in three MAST plasma scenarios: two L-mode plasmas with a current of 900 kA and 600 kA and an H-mode plasma with a current of 900 kA. The plasma temperature and magnetic field have been matched in each scenario in an attempt to create essentially one dimensional scans of the safety factor (from the current scan) and the spatial electron density gradient (from the confinement mode scan). Carbon and helium are both intrinsic impurities, the latter introduced from helium glow discharge cleaning and the former from the carbon wall components. Although plasma pulses in MAST are short (< 0.5 s) compared to most conventional tokamaks (> 10 s), the energy confinement time is of the order of 10 ms allowing sufficient time to measure the evolution of the impurity flux in the plasma following a short (< 30 ms) gas puff of helium or methane. A transport analysis of helium was performed in all three plasmas, while an analysis of the carbon transport was only performed for the L-mode 900 kA plasma. For the 600 kA and H-mode plasmas, the perturbation to the intrinsic carbon density profile from the methane gas puff was not sufficient to allow a transport analysis. An impurity charge scan was therefore carried out during the 900 kA L-mode plasma.

An elaborated collisional-radiative model, implemented within the ADAS framework, was used in the analysis to determine the fractional abundances of the impurities across the temperature gradient. Helium and carbon are both fully ionised over the majority of the plasma radius, except for the cold plasma edge region where lower ionisation stages may exist. Excited He$^+$ and C$^5+$ ions are formed along the neutral beam injection path following active charge exchange between the hydrogen beam atoms and the He$^{2+}$ and C$^6+$ ions. The spectral line radiance from these excited ions was measured by the RGB diagnostic, which was built, calibrated and installed on the MAST vessel prior to the start of this work.
RGB Diagnostic

RGB has the ability to measure simultaneously six pre-selected spectral bandpasses in the visible. Incident emission from all wavelengths passes through a single iris. Image splitting and spectral filtering using several interference filters occurs in the telecentric region of the optics. The filtered emission along each channel is collected using two identical camera chips with 640x480 pixel resolution and frame rates of up to 210 Hz. The pixels are coloured to allow the emission to be separated into red, green and blue channels. The selected emissions are beam D\textsubscript{α} emission, non-beam D\textsubscript{α} emission, two Bremsstrahlung emissions and the He II \((n = 4 \rightarrow 3)\) and C VI \((n = 8 \rightarrow 7)\) spectral lines at \(\lambda = 468.5\) nm and \(\lambda = 529.1\) nm respectively induced by active charge exchange with the neutral beam atoms. The transport analysis focuses on the latter two measurements of helium and carbon.

Impurity Density Model

Three pieces of information were required to convert the 2D pixel frames of emission measured by RGB into radial profiles of the He\textsuperscript{2+} and C\textsuperscript{6+} density: an estimate of the passive emission contributing the total emission measured within the beam volume, the charge exchange effective emission coefficients and the line-of-sight integrated neutral beam density. The emission from a narrow horizontal range of pixels above and below the beam volume was averaged (top and bottom) along each pixel column and used to estimate the passive charge exchange emission. Each pixel was mapped to \(R\) and \(Z\), with respect to the beam line, to determine the experimental background plasma parameters in the centre of the beam volume for each line-of-sight. These plasma parameters were used to interpolate values from the ADAS data set containing the charge exchange effective emission coefficients. The line-integrated neutral beam density was determined using a model, designed specifically for the work in this thesis, which combined the attenuation from the beam divergence and the atomic ionisation processes occurring between the neutral beam atoms and the plasma particles.

The neutral beam model was benchmarked by producing a forward model of the neutral beam emission measured by RGB. Good agreement between the model and experiment was found for the shape of the beam emission profile, however the experimental measurements were dimmer than the model by \(\sim 40\%\). The cause of this discrepancy is currently unknown and is the subject of further work. However this was not an immediate issue for the impurity transport analysis, which relied primarily on the impurity density shape rather than the absolute concentration. The concentrations are more important when deducing the effective charge of the plasma, which was used as input to the theoretical transport models. Using the most recent calibrations of RGB, the helium and carbon concentrations in MAST were \(\sim 5\%\) and \(\sim 0.5\%\) respectively. This agreed broadly with the effective charge inferred from the Bremsstrahlung measurements from the ZEBRA diagnostic.
Emission induced by excited plume ions and thermal charge exchange between the impurity ions and the beam halo atoms may cause an over-estimation in magnitude of the impurity density profiles; these contributions are not taken into account using only the passive charge exchange subtraction technique. A basic model of these two processes suggested that the beam halo contribution could increase the helium and carbon signal by $\sim 5\%$ and $\sim 20\%$ respectively, while plume emission in the diagnostic sight-lines viewing the plasma core could be similar in magnitude to the active charge exchange signal. Impurity density profiles were only considered outside the range in which plume emission was significant, which is $\rho > 0.2$.

**Transport Models**

A predictive and an interpretative method was used to determine the diffusion and convection coefficients for both impurities. The predictive technique used the 1.5D SANCO transport code to solve the particle continuity equation with a given set of boundary conditions and a parametrised radial diffusion and convection profile. SANCO was combined with ADAS to allow deduction of the atomic source terms (ionisation, recombination and charge exchange) in the continuity equation. The modelled and experimental impurity density evolutions were matched by setting the diffusion and convection coefficients as free parameters in the least squares fitting algorithm implemented in the UTC code. The interpretative approach extracted the $D_i$ and $v_i$ coefficients from a linear fit to the temporal evolution of the measured impurity flux as a function of the impurity density gradient. This model is insensitive to the impurity influx into the plasma but breaks down in regions of non-negligible atomic source rates. SANCO and UTC have been used in previous studies, but a new a code was designed for this thesis to implement the interpretative method, called the Flux Gradient method, which required benchmarking. This was done by determining the diffusion and convection coefficients from a helium density profile modelled by SANCO. The diffusion and convection coefficients deduced using the Flux Gradient method matched the values used by SANCO in the region $\rho < 0.7$ for helium and $\rho < 0.6$ for carbon; towards the plasma edge the source terms become significant.

For each plasma scenario, an analysis of the neoclassical transport coefficients was carried out using the NCLASS code, while two local flux-tube electromagnetic gyrokinetic codes, GS2 and GKW, were used to simulate the linear growth rate spectrum of the turbulent modes at $\rho = 0.7$ for the L-mode and H-mode plasmas at 900 kA. A summary of the experimental and theoretical findings is given in the next section.
7.2 Results Summary

A general feature of all three plasma scenarios was a region in the plasma core where the convection was mostly insignificant while the diffusivity was $\sim 0.5 - 1 \text{ m}^2/\text{s}$. From $0.4 < \rho < 0.8$, both impurities experienced an inward convection of $5 - 30 \text{ m/s}$ and a diffusivity of $1 - 5 \text{ m}^2/\text{s}$. The results suggest that this outer region of plasma is most susceptible to the impurity charge and confinement mode scan: the diffusion and convection coefficients increased with impurity charge and decreased in H-mode compared to L-mode. There was also evidence from the intrinsic impurity profiles that an outward convection occurs in the inner core of the H-mode plasmas. The current scan found little difference in transport near the plasma edge, however lowering the plasma current produced a higher diffusion and inward convection at mid-radius. These trends are in broad agreement with the findings of impurity transport studies carried out on NSTX and C-Mod.

In previous helium transport studies on conventional tokamaks, the ratio of the helium diffusivity and the effective total heat diffusivity, denoted by $\eta$, was often examined to indicate whether a certain plasma scenario would be viable for plasma producing helium from fusion burn. Profiles of the effective heat diffusivity were inferred from the total power balance of the plasma calculated by the predictive jetto code. The results suggested that $\eta \sim 0.1 - 0.3$, and is due to the anomalous heat diffusivity driven by the electrons. The helium and effective ion heat diffusivity were similar in magnitude. For the plasma current and confinement scans, a similar trend in helium diffusivity was found compared to the effective total heat diffusivity, however the opposite trend in helium diffusivity compared to the effective ion heat diffusivity was found during the confinement scan.

Neoclassical Analysis

Over the plasma radius, the He$^{2+}$ ions are mainly in the Banana collisionality regime, while the C$^{6+}$ ions span the Banana, Plateau and Pfirsch-Schlüter collisionality regimes. The neoclassical simulations did not reproduce the impurity charge and confinement mode trends, but did show the moderate increase in diffusion and convection found at low current. The experimental diffusivity and convection in each scenario were similar in magnitude to the neoclassical predictions up to a certain radius. In the L-mode plasmas, the transport was anomalous in the range $0.5 < \rho \leq 0.8$ and $0.3 < \rho \leq 0.8$ for the 900 kA and 600 kA plasmas respectively. For the H-mode plasma, the magnitude of the inward pinch near the plasma edge was similar to the neoclassical predictions suggesting that the turbulence had been suppressed. The outward flux of impurities observed at mid-radius is a neoclassical effect caused by the region of positive main ion density gradient.
Gyrokinetic Analysis

Linear gyrokinetic simulations of the L-mode plasma at $\rho = 0.7$ showed unstable modes across the wavenumber spectrum when flow shear was not taken into account. A similar growth rate spectrum was observed in the H-mode plasma, however modes in the trapped electron mode (TEM) region in H-mode were stable, presumably because the electron density gradient is lower in H-mode. Including flow shear in the simulations stabilised the ion temperature gradient (ITG) modes in both the L-mode and H-mode plasmas. The (ion scale) density fluctuations measured by the BES diagnostic at $\rho = 0.7$ were close to the detection threshold, which agreed with the theoretical conclusion that the ITG modes were stabilised. It was noted that the BES signal increased nearer the plasma edge indicating unstable ITG modes, which was probably due to the lower flow shearing rate near the edge. Considering that short wavelength electron temperature gradient modes are unlikely to affect the impurity flux, the simulations therefore suggested that TEMs were responsible for the anomalous transport. A quasilinear estimate of the (dimensionless) helium peaking factor calculated by GKW agreed in both magnitude and in direction in the TEM region. The main contribution to the magnitude and direction of the peaking factor came from the ion temperature gradient and the thermodiffusion coefficient respectively, with smaller contributions from the rotodiffusion, parallel compression and curvature terms.

In summary, the electron density gradient plays a crucial role in the transport of light impurities in spherical tokamaks; a steep electron density gradient causes an inward pinch of helium and carbon from the turbulence associated with collisionless TEMs in L-mode.

7.3 Further Research

Gas Puff Timing

The timing of the gas puff was tailored during the experiments to maximise the contrast between the injected and intrinsic impurity density without significantly disturbing the background plasma. Durations of 24 ms and 14 ms were used for helium and 34 ms and 24 ms for methane however, in hindsight, a 30 ms helium gas puff is recommended for all plasmas. An alternative method of injection is probably needed to introduce carbon more effectively into the 600 kA and H-mode plasmas. Reference plasmas (with no gas puff) should be carried out immediately before the gas puff plasmas to obtain a better measurement of the intrinsic impurity profiles.

Expand the MAST Impurity Transport Database

With more machine time, the impurity transport would also have been studied during a collisionality scan and with nitrogen impurity. An initial design of the plasmas
required for the collisionality scan has been made but needs further tailoring to allow for a transport analysis. Nitrogen is adjacent in charge to carbon and therefore fully ionised over the majority of the plasma, however it is an extrinsic impurity and so the gas puff would not have to compete with an intrinsic concentration. Nitrogen was injected into the 900 kA L-mode and H-mode plasmas and measured using 10 chords (5 active and 5 passive) attached to a spectrometer. Calibrations of the measured nitrogen active charge emission were not obtained before the end of this work and therefore the results have not been considered in this thesis.

Poloidal Asymmetries

The effect of the toroidal rotation on the poloidal symmetry of impurities has not been mentioned in this thesis. A flux surface average of the impurity density in the presence of a poloidal asymmetry may lead to a different solution of impurity transport coefficients and therefore requires attention in future work. Charge exchange spectroscopy cannot be used to study the inboard-outboard distribution of impurities because the active charge exchange emission is weaker than the passive emission on the inboard side of the plasma due to the exponential decay of the hydrogen beam atoms. Inversion codes were designed during this work to study the emission measured from the tangential soft x-ray (SXR) camera. There was little evidence from this study to suggest any significant poloidal asymmetries. However the results were inconclusive because of inaccuracies in the relative chord-chord calibrations. This is probably because the SXR diagnostic was not shuttered during boronisation causing an uneven deposition of particles on the camera sensors. The SXR camera port window on MAST-U will be shuttered to protect the camera sensors, which may allow for a better description of the poloidal distribution of impurities in the future.
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