This paper focuses on trajectory design which is relevant for missions that would follow NASA’s Asteroid Redirect Mission (ARM) to further explore and utilize asteroids and eventually human Mars exploration. Assuming that a refueling gas station is present at a given Lunar Distant Retrograde Orbit (DRO), we analyze ways of departing from the Earth to Mars via that DRO. Thus, the analysis and results presented in this paper add a new cis-lunar departure orbit for Earth-Mars missions. Porkchop plots depicting the required $C_3$ at launch, $v_\infty$ at arrival, Time of Flight (TOF), and total $\Delta V$ for various DRO departure and Mars arrival dates are created and compared with results obtained for low $\Delta V$ LEO to Mars trajectories. The results show that low $\Delta V$ DRO to Mars transfers generally have lower $\Delta V$ and TOF than LEO to Mars maneuvers.

**INTRODUCTION**

A Lunar Distant Retrograde Orbit (DRO) is an orbit that exists due to third-body effects. Such trajectory is the solutions of the Circular Restricted Three Body Problem (CR3BP) of a system in which Earths and Moons gravitational attractions are considered. A DRO is possibly a very critical stepping stone, both implicitly and literally, for our ultimate goal: human Mars exploration.

Compared with the other past destinations such as a low-Earth orbit or the Moon, Mars is distant from the Earth and its journey will not be short. We have learned from our history of exploration the importance of logistical considerations for such expeditions. Part of the logistics considerations in space exploration includes the locations of on-orbit propellant depots, in-situ resource utilization (ISRU) plants, and other types of space infrastructure. Multiple recent studies have shown the promising effectiveness of having a propellant depot on the way to or back from the destination.

A DRO has recently been proposed to be one of the most suitable locations to locate those space infrastructure. This is due to its orbital stability and easiness of access in terms of gravity well. For example, a propellant depot can be at a DRO so that a cargo mission can visit that depot to be refueled before heading to its destination. In addition, a space station can also be located at a
DRO, which can be used as a maintenance base or safety haven for any unexpected contingencies in cis-lunar system. Moreover, a DRO can be used for the assembly of large spacecraft. This idea is attractive because it requires only two travels through the Van-Allen Radiation Belts as opposed to other high-Earth orbit (HEO) departure architectures. Furthermore, the recent Asteroid Redirect Mission (ARM) proposed to redirect the asteroid to a DRO and send human there to gain knowledge and experiences for future Mars exploration. A success of this mission would lead to further applications of a DRO.

In order to realize the above scenarios, the efficient transit path from Earth to Mars via a DRO (or the other way around) will be critical. Therefore, this paper aims to consider the Earth - DRO and DRO - Mars transfers. It computes the $\Delta V$ and TOF and generates porkchop plots for a given synodic period.

The analysis of LEO to DRO and DRO to Mars trajectories was supported by the following assumptions:

- $\Delta V$ maneuvers are treated as impulsive maneuvers.
- Earth-Moon dynamics is modelled as the Circular Restricted Three Body Problem (CR3BP) using an Earth-Moon mean distance of 384,400 km.
- Patched conics are used for interplanetary orbital transfers.
- Secular perturbations of other planetary bodies are neglected.

**DRO USED FOR MARS MISSIONS**

In order to show the value of a DRO for Mars exploration, a simple numerical example is shown. The considered scenarios assume a cargo mission, which can be a habitat pre-deployment mission preceding human missions. A propellant depot is assumed to be located at a DRO, and we are interested in the value of having such a propellant depot. More precisely, we want to find how much we can save our initial mass in low-Earth orbit (IMLEO) by refilling the propellant tanks from the depot at a DRO. Note that the cases with a depot cannot be simply compared against those without a depot because the depot development and launch costs are not considered. Instead, the result would give an indication about how much cost we are willing to pay to develop and launch such a propellant depot.

With the representative $\Delta V$ and TOF resulted from later analysis, the following scenarios are considered for the Mars mission.

- Scenario 1. Direct to Mars: The vehicle directly departs from Earth to Mars
- Scenario 2. Oxygen Refill in a DRO: The vehicle departs from Earth to a DRO, refills its oxygen tank from a propellant depot in the DRO, and heads to Mars.
- Scenario 3. Hydrogen and Oxygen Refill in a DRO: The vehicle departs from Earth to a DRO, refills its hydrogen and oxygen tanks from a propellant depot in the DRO, and heads to Mars.

The computation is simply based on the rocket equation. The assumed parameters are shown in Table 1 and the results are shown in Table 2. The $\Delta V$ and TOF values assume the synodic
period 2035-2036. Chemical rockets with a specific impulse \( (I_{sp}) \) of 450 s is assumed as the vehicle propulsion system. The refill operation is assumed to take 10 days. To represent the IMLEO reduction, the gear ratio is shown, which is defined as the ratio of IMLEO to the landed mass on Mars. In addition, the number of launches is also shown assuming the equivalent payload as NASA Design Reference Architecture 5.0 cargo pre-deployment mission (40 mT on Martian surface\(^7\)) and the launches by the 130 mT Space Launch System (SLS).

Table 1. Assumed parameters (LMO: Low-Mars Orbit)

<table>
<thead>
<tr>
<th>From</th>
<th>To</th>
<th>( \Delta V ) [km/s]</th>
<th>TOF [days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEO</td>
<td>LMO</td>
<td>5.76</td>
<td>202</td>
</tr>
<tr>
<td>LEO</td>
<td>DRO</td>
<td>3.82</td>
<td>6</td>
</tr>
<tr>
<td>DRO</td>
<td>LMO</td>
<td>3.29</td>
<td>206</td>
</tr>
</tbody>
</table>

Table 2. \( \Delta V \) and TOF for each case

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Gear Ratio</th>
<th>TOF [days]</th>
<th>SLS launches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>13.09</td>
<td>202</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>10.56</td>
<td>222</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8.43</td>
<td>222</td>
<td>3</td>
</tr>
</tbody>
</table>

The results show that stopping at the DRO gas station would provide the propellant necessary for part or all of the rest of the journey to Mars, which would effectively reduce the propellant and the size of the tank that need to be launched from Earth. Thus, a smaller number of launches or smaller rockets could be used for the same payload mass to be sent to Mars. In the above case, the number of launches can be reduced from five to three, which can save the launch cost and time significantly. (Note that the current schedule for SLS launch frequency is only once a year.)

An example scenario of utilization of propellant depot in a DRO and lunar ISRU for a Mars cargo mission is shown in Figure 1. The first launch is a crewed mission that delivers the ISRU plant to the lunar surface and the propellant depot to a DRO. During the ISRU operation period, the human crew maintains the ISRU plant and performs science missions at the same time on the lunar surface. After the ISRU propellant generation, the generated propellant is delivered to the propellant depot in the DRO, and the human crew returns to Earth. In the meantime, the Mars cargo is launched and assembled in LEO, stops in a DRO to get refilled, and heads to Mars to deliver the cargo (e.g. habitat pre-deployment for later crew missions). Note that in this scenario, the propellant depot is assumed to be refilled by ISRU, but there are other possibilities for the usage of propellant depots. This scenario provides an example how DROs can be used for Mars exploration, which thus motivates this research to compute the astrodynamics around a DRO.

**EARTH - MOON DRO TRANSFER**

No closed form solution for the DRO in the CR3BP exists. A collection of positions and velocities for points along the orbit can however be obtained by implementing a shooting method. Initial guesses for the shooting method are available in literature.\(^8\) In Figure 2 and Figure 3 a Lunar DRO of 61,500 km amplitude is represented in the rotating reference frame of the CR3BP and in the inertial reference frame centered at the Earth. In Figure 2 the x axis points to the Moon and the y axis is perpendicular to it and both lie in the Earth-Moon plane. In Figure 3 x and y lie on an inertial
plane centered at the Earth-Moon barycenter. An amplitude of 61,500 km was chosen for the DRO based on the work done at the Caltech Space Challenge 2015.  

The orbit selection process for the transfer from Earth to Lunar DRO often comes to the trades between TOF and ΔV. Human missions typically prefer a trajectory with a short TOF even if the ΔV is not the minimum. This is due to crew health problems such as radiation dose limit and psychological problems. Chemical propulsion systems are often used for such missions. This trend is true for part of the robotic missions that requires a fast delivery as well. Most robotic missions, on the other hand, typically prefer a trajectory with a small ΔV even though their TOF is longer. For these missions, a low-thrust electric propulsion system, possibly with a low-energy transfer, is preferred to save the propellant.
For fast human missions, where a high thrust propulsion system (e.g. chemical) is preferred, the following transfer types have been considered:

- Direct transfer with lunar far side injection. This transfer requires two maneuvers, one to depart from LEO and one to insert into DRO, with the injection occurring tangentially along the $x$ axis of the Earth-Moon rotating reference frame. This transfer is shown in Figure 4.

- Prograde or retrograde powered lunar gravity assist. These transfers requires three maneuvers: departure from LEO, periselenium maneuver and injection maneuver, which can occur either with a prograde or retrograde motion. Retrograde transfers are typically less expensive in terms of injection $\Delta V$. 

![Figure 3. 61,500 km amplitude DRO in the inertial Earth-centered reference frame](image)

![Figure 4. Direct Earth-DRO transfer](image)
A direct transfer was chosen because of the reduced TOF and because the absence of Moon flyby maneuvers makes it safer for a human mission. The direct transfer from Earth to the DRO can be realized in 5.86 days; it requires a burnout velocity at the Earth of 10.91 km/s and a $\Delta V$ for the injection into the DRO of 619.82 m/s.

**MOON DRO - MARS TRANSFER**

In order to know the characteristic energy ($C_3$) the spacecraft needs to achieve to leave the Earth-Moon system and arrive at Mars at a certain arrival date, Earth-Mars porkchop plots were generated. The abscissa and ordinate on a porkchop plot represent the Earth-Moon departure and Mars arrival dates, respectively. The key parameters that are plotted on porkchop plots and used to investigate possible transfer orbits are the Earth departure characteristic energy, $C_3$, the Mars arrival $v_\infty$, and TOF. These parameters are plotted as contour lines of constant values and allow mission designers to decide the best launch and arrival windows for interplanetary missions. Figure 5 shows an example of a porkchop plot for Earth-Mars transfers during the 2035-2036 synodic period. In order to determine the position and velocity vectors of Earth and Mars used to generate porkchop plots, ‘Keplerian Elements for Approximate Position of the Major Planet’ by Standish was used.11

Figure 5. Porkchop plot illustrating Earth-Mars transfer for the 2035-2036 timeframe.
Figure 5 shows departure maneuvers for any dates falling within the time period that was considered (February 1, 2035 to November 15, 2035) while in reality one must take into account the geometry of the Earth and the Moon. Thus, $\vec{C}_3$ vectors were transformed from the J2000 Ecliptic (Ecl) frame to the Earth-Moon rotating reference frame. Such transformation was done by using the following Direction Cosine Matrices:\textsuperscript{12}

- $R_{Ecl-EME}$: transforms from J2000 Ecliptic to Earth Mean Equatorial (EME) frame using the obliquity of the Earth, $\varepsilon = 23.43928$ deg.

- $R_{EME-EMI}$: transforms from EME to Earth-Moon Inertial (EMI) perifocal frame using the inclination, right ascension of the ascending node, and argument of periapsis of the Moon’s orbit derived from its ephemeris.

- $R_{EMI-EMR}$: transforms from EMI to Earth-Moon Rotating (EMR) reference frame using the true anomaly of the Moon’s orbit derived from its ephemeris.

Consequently, $\vec{C}_3$ vectors can be converted from Ecl to EMR using the relationship:

$$\vec{C}^{EMR}_3 = R_{EMI-EMR} R_{EME-EMI} R_{Ecl-EME} \vec{C}^{Ecl}_3$$

(1)

Using Equation (1) and the resulting $\vec{C}^{EMR}_3$, right ascension $\alpha$ and declination $\delta$ of departure $\vec{C}_3$ vectors can be computed. Right ascension and declination are defined in the EMR frame as shown in Figure 6.

![Figure 6](image.png)

**Figure 6. Geometry of Right Ascension and Declination of departure C3 vectors. Source: NOAA.\textsuperscript{13}**

$\vec{C}_3$ changes for each Earth departure date and Mars arrival date combination. Thus, from Equation (1) it is possible to calculate right ascension and declination for each $\vec{C}_3$ as a function of departure and arrival dates. Figure 7 and Figure 8 show such time-dependent relationship for right ascension and declination respectively.
Figure 7. Right ascension (in degrees) of departure $\vec{C}_3$ vectors for the 2035-2036 Earth-Mars synodic period.
While right ascension varies almost uniformly from 0 to 360 degrees roughly once per month, declination is less predictable. On the other hand, right ascension is a limiting factor for Earth-Moon departures to Mars making such opportunities occur once or twice a month. Declination can significantly influence the required departure $\Delta V$ but it does not represent a limiting factor in terms of timing.

Departure $\Delta V$ was calculated by adding both DRO-Earth and Earth flyby propulsive maneuvers. An Earth flyby altitude of 300 km was chosen to avoid atmospheric drag and to maintain the spacecraft at a safe distance from the Earth’s surface. Starting from a combination of departure and arrival dates, $\vec{C}_3$ magnitudes and orientation (\(\alpha\) and \(\delta\)) were determined as mentioned previously. Using the given Earth flyby altitude of 300 km and $\vec{C}_3$ magnitude, one can calculate the perigee velocity a spacecraft needs ($v_{p,n}$) in order to achieve such $\vec{C}_3$. Right ascension $\alpha$ provides information on whether or not a departure trajectory exists based on the Earth-Moon geometry. Taking into account the TOF from DRO to Earth and the TOF from perigee to the Earth Sphere Of Influence (SOI), it was found that $\alpha$ values falling in the range of approximately 205 to 240 degrees yield feasible
trajectories two days every month.

The right ascension $\alpha$ of the vector $\vec{C}_3$ can be related to the right ascension $\theta$ of the spacecraft perigee in its trajectory from the DRO to the Earth (Figure 9). In Figure 9, $\gamma/2$ represents half of the turn angle due to the flyby.

\[ \theta = \alpha - 90^\circ - \arcsin \left[ \left( \frac{r_p}{\mu} v_{p,n}^2 - 1 \right)^{-1} \right] + \frac{TOF_{hyp}}{T_{EM}} \cdot (360^\circ) \]  

where $r_p$ is the perigee radius of the Earth flyby, $v_{p,n}$ is the perigee velocity the spacecraft needs at $r_p$ in order to achieve the given $C_3$, $TOF_{hyp}$ is the TOF of the departure hyperbolic orbit from $r_p$ to the Earth’s SOI and $T_{EM}$ is the Earth-Moon orbital period. Note that the term
arcsin \left( \frac{v_p^2}{\mu v_{p,n}^2} - 1 \right)^{-1} \] represents half of the turn angle \( \gamma \) obtained from the Earth flyby maneuver and the term \( \frac{\text{TOF}_{hyp}}{T_{EM}}(360^\circ) \) takes into account the rotation of the Earth-Moon rotating reference frame while the spacecraft is escaping the Earth-Moon system.

Figure 10. \( \Delta V \) and TOF from a DRO as function of the departure point on the DRO.

Figure 11. Velocity \( v_p \) and right ascension \( \theta \) at Earth perigee, \( v_p \).
Once \( v_{p,n} \) and \( v_p \) are found, the Earth flyby \( \Delta V \), \( \Delta V_{EF} \), can be computed using the law of cosines:

\[
\Delta V_{EF} = \sqrt{v_p^2 + v_{p,n}^2 - 2v_pv_{p,n} \cos \delta}
\]  

(3)

Therefore, the departure \( \Delta V \), \( \Delta V_{dep} \), can be computed as follows:

\[
\Delta V_{dep} = \Delta V_{DRO} + \Delta V_{EF}
\]

(4)

Figure 12 shows an example of feasible departure \( C_3 \) vectors in the EMR frame.

![Figure 12. Example of feasible departure \( C_3 \) vectors.](image)

Table 3 summarizes the key parameters for such departure opportunities using an Earth flyby altitude of 300 km.

**Table 3. Key parameters for an example of departure opportunities.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum Value</th>
<th>Maximum Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Departure Dates</td>
<td>July 19, 2035</td>
<td>July 20, 2035</td>
</tr>
<tr>
<td>Arrival Dates</td>
<td>January 14, 2036</td>
<td>February 8, 2036</td>
</tr>
<tr>
<td>( C_3 ) at Launch [km(^2) / s(^2)]</td>
<td>14.683</td>
<td>15.8523</td>
</tr>
<tr>
<td>( \alpha ) [deg]</td>
<td>204.97</td>
<td>226.96</td>
</tr>
<tr>
<td>( \delta ) [deg]</td>
<td>9.041</td>
<td>10.278</td>
</tr>
<tr>
<td>Departure ( \Delta V ) [km/s]</td>
<td>2.5783</td>
<td>2.8705</td>
</tr>
</tbody>
</table>
Figure 13 shows a sample trajectory a spacecraft would undertake to leave the DRO and reach the Earth’s SOI.

Figure 14 and Figure 15 respectively show $\Delta V_{dep}$ and $\Delta V_{tot}$ for feasible departure trajectories during the 2035-2036 Earth-Mars synodic period. White space means that no feasible departure trajectory exists. $\Delta V_{tot}$ was calculated considering a 200x200 km altitude final Mars orbit for all cases. $\Delta V_{tot}$ does not take into account the $\Delta V$ necessary to reach the DRO from LEO; this is because a refueling depot exists in such DRO.

Table 4 summarizes the minimum $\Delta V_{tot}$ case for this synodic period and compares the key parameters with a LEO-LMO trajectory.14

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value for DRO-LMO</th>
<th>Value for LEO-LMO</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta V_{tot}$ [km/s]</td>
<td>3.2904</td>
<td>5.7582</td>
</tr>
<tr>
<td>$C_3$ at Launch [km$^2$/s$^2$]</td>
<td>10.351</td>
<td>10.397</td>
</tr>
<tr>
<td>$v_\infty$ at Arrival [km/s]</td>
<td>2.6516</td>
<td>2.6283</td>
</tr>
<tr>
<td>Departure Date</td>
<td>June 23, 2035</td>
<td>June 28, 2035</td>
</tr>
<tr>
<td>Arrival Date</td>
<td>January 15, 2036</td>
<td>January 16, 2036</td>
</tr>
<tr>
<td>Time of Flight [days]</td>
<td>206</td>
<td>202</td>
</tr>
</tbody>
</table>

Table 5 summarizes all minimum $\Delta V_{tot}$ cases for each Earth-Mars synodic period from 2020 to 2040 and compares $\Delta V_{tot}$ and TOF with LEO-LMO trajectories.14 Absolute lowest $\Delta V_{tot}$ for each
Figure 14. $\Delta V_{dep}$ for the 2035-2036 Earth-Mars synodic period.

Trajectory type is in bold. From Table 5, it is clear that DRO-LMO maneuvers can save from 1.1 to 2.5 km/s of the $\Delta V_{tot}$. On the other hand, departure opportunities are limited to a maximum of two days roughly every month and the use of high thrust propulsion is necessary to successfully achieve the required $\Delta V$ during the Earth flyby maneuver.

Table 5. $\Delta V_{tot}$ and TOF comparison between DRO-LMO and LEO-LMO trajectories.14

<table>
<thead>
<tr>
<th>Synodic Period</th>
<th>$\Delta V_{tot}$ [km/s]</th>
<th>TOF [days]</th>
<th>$\Delta V_{tot}$ [km/s]</th>
<th>TOF [days]</th>
</tr>
</thead>
<tbody>
<tr>
<td>DRO-LMO</td>
<td>DRO-LMO</td>
<td>LEO-LMO</td>
<td>LEO-LMO</td>
<td></td>
</tr>
<tr>
<td>2020-2021</td>
<td>3.4430</td>
<td>210</td>
<td>5.8921</td>
<td>207</td>
</tr>
<tr>
<td>2022-2023</td>
<td>3.8884</td>
<td>241</td>
<td>5.9536</td>
<td>347</td>
</tr>
<tr>
<td>2024-2025</td>
<td>3.9823</td>
<td>265</td>
<td>5.7166</td>
<td>333</td>
</tr>
<tr>
<td>2026-2027</td>
<td>4.0035</td>
<td>293</td>
<td>5.6799</td>
<td>311</td>
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<tr>
<td>2028-2029</td>
<td>3.5863</td>
<td>299</td>
<td>5.8722</td>
<td>300</td>
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<tr>
<td>2030-2031</td>
<td>5.0580</td>
<td>186</td>
<td>6.2268</td>
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<td>2032-2033</td>
<td>4.0130</td>
<td>229</td>
<td>6.0583</td>
<td>200</td>
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<tr>
<td>2035-2036</td>
<td>3.2904</td>
<td>206</td>
<td>5.7582</td>
<td>202</td>
</tr>
<tr>
<td>2037-2038</td>
<td>3.8789</td>
<td>240</td>
<td>6.1000</td>
<td>348</td>
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<tr>
<td>2039-2040</td>
<td>4.1087</td>
<td>253</td>
<td>5.7862</td>
<td>340</td>
</tr>
</tbody>
</table>
CONCLUSION

As discussed in previous literature, a variety of Earth orbit departures such as High Earth Orbits (HEO) with Low Perigee (LP) or High Perigee (HP) and cis-lunar departure orbits have been analysed. Such orbits allow spacecraft to obtain the necessary $C_3$ to reach destinations such as Mars and Near Earth Asteroids (NEA) of interest. The analysis and results presented in this paper add a new cis-lunar departure orbit, namely from a DRO, to escape the Earth-Moon system and arrive at Mars. DRO-LMO trajectories have the advantage of needing less $\Delta V_{tot}$ than LEO-MLO trajectories. Lower $\Delta V_{tot}$ means fewer or smaller launch vehicles are needed and thus the launch cost is reduced. Additionally, the assembly of large spacecraft in a cis-lunar environment such as a DRO means that the crew would need to travel through the Van-Allen Radiation Belts two times as opposed to other HEO departure architectures.

A few disadvantages of DRO-LMO trajectories exist. In fact, since the necessary $\Delta V$ to depart from the Earth-Moon system starting from a DRO is highly dependent on the position of the Moon at departure and the desired interplanetary transfer trajectory to arrive at Mars, DRO-LMO trajectories are limited to roughly once or twice every month. Additionally, since the Moons orbital elements change in time, DRO-LMO low $\Delta V_{tot}$ opportunities do not repeat as regularly as LEO-LMO low $\Delta V_{tot}$ opportunities. Conversely, because Moon DROs possess a much higher orbital energy than
LEOs, $\Delta V_{dep}$ for DRO-LMO is generally much lower than that for LEO-LMO. Additionally, even though DRO-LMO trajectories require the spacecraft to reach a DRO in the first place and hence add TOF to the total mission duration, DRO-LMO interplanetary TOFs are almost always lower than LEO-LMO interplanetary TOFs and never exceed 300 days. In this paper, only a LMO of 200x200 km altitude was considered because the focus was to decrease $\Delta V_{dep}$. Thus, considering other Martian orbits or other orbital insertion maneuvers at Mars (e.g. aerobraking or aerocapture) can lower $\Delta V_{tot}$ even further. Another consideration is that the analysis developed to create the results presented in this paper can easily be used to develop departure DRO maneuvers to achieve other desired $C_3$ to target other celestial bodies.

ACKNOWLEDGMENT

Davide, Marilena and Koki would like to thank the California Institute of Technology and the Caltech Space Challenge 2015 organizers, mentors and staff members. In March 2015, they gave us the opportunity to work on and present the design of the human portion of the Asteroid Redirect Mission. This paper shows how a captured asteroid in a DRO can be used to facilitate human missions to Mars.

REFERENCES