Pressure-Driven Modelling of Water Distribution Systems

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ABSTRACT

This paper presents a novel method to model water distribution systems (WDS) with insufficient pressure. Methods for the prediction of the performance of a WDS with pressure deficiencies are reviewed. The influence of imposed relationships between nodal heads and outflows is assessed and numerical results are given. A Newton-Raphson technique plus line search is employed for solving the governing equations. It is demonstrated that the approach offers superior results for the hydraulic performance of networks under abnormal operating conditions compared to demand-driven analysis-based models.

KEY WORDS

Demand-driven analysis, head-outflow relationships, pressure-driven analysis, water distribution systems

INTRODUCTION

Despite the interest in developing ever more sophisticated models for water distribution systems (WDS), the applicability of these models to abnormal operating conditions is questionable. For example, during conditions involving excessively high demands for fire fighting or the failure of some network elements, the pressures at some nodes can be too low for the specified demands. When a node has insufficient pressure it may supply only a small proportion of the demand. Unfortunately, the models that are currently available for the analysis of WDS are based on demand-driven analysis (DDA). This means that demands are assumed to be satisfied in full and pipe flow rates and nodal pressures consistent those demands are calculated, with an implicit assumption that there is sufficient pressure in the system. In reality, however, if the demands exceed the capacity of the water distribution system, then DDA results cease to be representative of the hydraulic performance of the system. Therefore, a new approach to model water distribution systems more realistically is urgently required.

This paper describes a new rigorous and efficient modified Newton-Raphson algorithm for analysing water distribution systems with (or without) insufficient pressure. Several examples are used to illustrate the capabilities of the proposed method. The weaknesses of DDA are also demonstrated clearly.

REVIEW OF PREVIOUS APPROACHES

Reddy and Elango (1989) provided a formulation for networks with completely uncontrolled outlets. This formulation, with no upper limit on nodal outflows, is not suitable for municipal water distribution systems. If the network pressures keep rising, the consumer outflows will not necessarily follow that increase as there is a limit to the total amount of water the consumers require at any given time. Head-outflow relationships with an upper limit have, therefore, been formulated on the basis that the nodal demand is fully satisfied when the nodal head is greater than the desired head and zero when the nodal head is less than the minimum head. Note that the minimum head represents the pressure head below which the outflow at that node would be zero or deemed unsatisfactory and the desired head represents the pressure head above which the outflow at that node will be fully satisfied. In the range between the minimum and desired heads several formulations have been assumed. Wagner et al. (1988) and Chandapillai (1991) suggested a parabolic head-outflow formulation (Figure 1(a)), i.e.

$$q_j = \left(\frac{H_j - H_j^{\min}}{H_j^{des} - H_j^{\min}}\right)^{\frac{1}{n_j}} \qquad if \qquad H_j^{\min} < H_j < H_j^{des} \tag{1}$$

where q_j, H_j, $H_j^{\min}$, and H_j^{des} are the ratio of the actual to the required nodal outflow (i.e. nodal demand satisfaction ratio), actual nodal head, minimum nodal head and desired nodal head, respectively. Values of the exponent parameter, n_j, are thought to lie between about 1.5 and 2 (Gupta and Bhave, 1996). This simple relationship appears to offer a good compromise between ease of calibration and realistic predictive capability in this range (Tanyimboh and Tabesh, 1997). Fujiwara and Ganesharajah (1993) proposed a formulation (Figure 1(b)) as

$$q_j = \frac{\int_{H_j^{\min}}^{H_j} (H_j - H_j^{\min})(H_j^{des} - H_j^{\min})dH}{\int_{H_j^{\min}}^{H_j^{des}} (H_j - H_j^{\min})(H_j^{des} - H_j^{\min})dH} \qquad if \qquad H_j^{\min} < H_j < H_j^{des} \tag{2}$$

This highly non-linear function ensures continuity in the derivatives at the minimum and the desired head points, but needs significant computational effort for its evaluation. With reference to equations (1) and (2), the nodal demand satisfaction ratio q_j is set to zero if the nodal head H_j is less than the minimum acceptable head $H_j^{\min}$ or 1.0 if H_j reaches the desired head H_j^{des}. Gupta and Bhave (1996) modified the Germanopolous (1985) formulation to

$$q_j = 1 - 10^{b_j\left(\frac{H_j - H_j^{\min}}{H_j^{des} - H_j^{\min}}\right)} \tag{3}$$

where b_j is a node constant whose value is found by calibration. The relationship of equation (3) is shown in Figure 1(c). Note that the values of n_j, b_j, $H_j^{\min}$ and H_j^{des} used in Figure 1 are 2, 5, 50 m and 60 m, respectively.

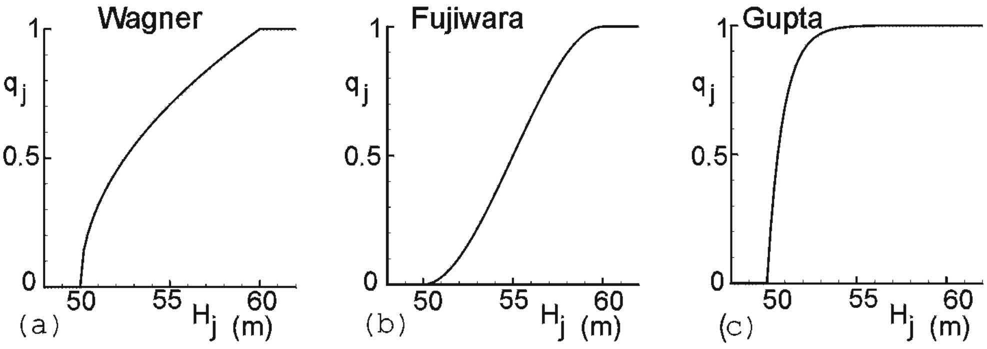

Figure 1 Nodal head-outflow relationships

The system of constitutive equations described later in this paper is solved numerically. There have been a number of attempts at predicting deficient network performance. Bhave (1981) categorised the outflow at a demand node as fully satisfactory if the head was not less than the head required at that node or zero if the head at that node was less than the elevation of the node. All other nodes were modelled as a ground level tanks to determine their outflows. Wagner et al. (1988) and Gupta and Bhave (1996) adjusted nodal outflows between demand-driven analysis solutions of the constitutive equations until the changes in the outflows became insignificant. However, as observed by Tanyimboh and Tabesh (1997), they did not propose any mechanism for adjusting the nodal outflows between successive DDA solutions of the problem. Chandapillai (1991) used the Newton-Raphson method (NRM) for updating nodal outflows between successive demand–driven analyses. The nodal outflows were updated individually and in isolation, which may cause convergence problems.

Fujiwara and Ganesharajah (1993) and recently Ackley et al. (2001) employed non-linear programming techniques, which maximised the sum of available outflows over all demand nodes. However, the practicability of constrained optimisation techniques is yet to be demonstrated on networks of realistic size. Germanopoulos (1985) and Tabesh (1998) used Newton's method to solve the system of equations in which the head-outflow relationship was incorporated. This approach requires a smooth continuously differentiable function of the head-outflow relationship with robust algorithm if it is to work properly. For example, Germanopolous (1985) observed that the function he proposed resulted in nodal outflows which never exceeded 93.2% of the demand. Therefore instability of the algorithm can be present in the region above 93.2%. The Tabesh (1998) algorithm had a step-length parameter whose value was network specific and found by trial and error. This issue and the associated difficulties have been addressed in the algorithm proposed herein by incorporating a line search technique in the Newton-Raphson procedure.

PROBLEM FORMULATION AND SOLUTION

The heads at the nodes and the flows in the elements are the state variables generally used to describe the behaviour of a WDS. The flow in the pipe between two nodes i and j may be related to the heads at those nodes by the Hazen-Williams equation as

$$Q_{ij} = R_{ij}\,\mathrm{sgn}(H_i - H_j)\,|\,H_i - H_j\,|^{0.54}\,; \qquad R_{ij} = 278.54\frac{C_{ij}D_{ij}^{2.63}}{L_{ij}^{0.54}} \qquad \forall ij \in IJ \qquad (4)$$

where Q_{ij} is the pipe flow rate, $\mathrm{sgn}(X)$ is the sign of X, H_i and H_j are nodal heads, C_{ij} is the Hazen-Williams friction coefficient, D_{ij} and L_{ij} are the diameter and length for the pipe in meters, respectively, and IJ is the set of all links in the network. The nodal flow continuity equations are expressed in terms of the nodal heads as

$$F_i(H_i, H_j) = \sum_{j\in N_i} R_{ij}\,\mathrm{sgn}(H_i - H_j)\,|\,H_i - H_j\,|^{0.54} - Q_i(H_i) = 0 \qquad i = 1\ldots, N \qquad (5)$$

where Q_i, a function of H_i e.g. equations (1)-(3), is the outflow at node i, N_i represents all the nodes connected to node i and N is the number of nodes in the network. Note that, in the case of the traditional demand-driven analysis (DDA) Q_i is constant i.e. the demand. A numerical procedure is used to solve the resulting system of non-linear equations. An algorithm that combines the rapid local convergence of Newton's method with global convergence will guarantee some progress towards the solution at each iteration. A technique is used in the present work, based on a line search procedure, in each iteration of the Newton-Raphson method (NRM) to establish the optimum value of the step size as shown in Box 1. The proposed algorithm is referred to herein as the Newton-Raphson Line Search Algorithm (NRLSA).

Box 1 Newton-Raphson plus line search algorithm

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Step 0. Initialisation. Set $k = 0$. Set initial heads $\boldsymbol{H}^{(k)}$.
Step 1. Test for convergence. Compute $\boldsymbol{F}(\boldsymbol{H}^{(k)})$. If $\|\boldsymbol{F}(\boldsymbol{H}^{(k)})\|$ is less or equal
        a predefined tolerance, Exit.
Step 2. Evaluation. Compute the Jacobian $\mathrm{J}(\boldsymbol{H}^{(k)})$.
Step 3. Solve $\mathrm{J}(\boldsymbol{H}^{(k)})\,\delta\boldsymbol{H}^{(k+1)} = -\boldsymbol{F}(\boldsymbol{H}^{(k)})$ and obtain $\delta\boldsymbol{H}^{(k+1)}$, i.e. the nodal heads
        corrections.
Step 4. Line-search algorithm. Compute the value of $\lambda$ to minimise
        $\|\boldsymbol{F}(\boldsymbol{H}^{(k)} + \lambda\delta\boldsymbol{H}^{(k+1)})\|$ with respect to $\lambda$.
Step 5. New iterate. Set $\boldsymbol{H}^{(k)} = \boldsymbol{H}^{(k)} + \lambda^{(k)}\delta\boldsymbol{H}^{(k+1)}$ and go to Step 1.
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DEMONSTRATION EXAMPLES

Example 1

A well-known four-loop network (Figure 2(a)) with twelve designs was chosen to demonstrate the results obtained using the new NRLSA algorithm and to highlight some of the weaknesses of demand-driven network analysis. The pipe diameters are given in Table 1 (Tabesh, 1998). For all the pipes a Hazen-Williams coefficient of 130 is used. The minimum and required heads are 0 and 30 m, respectively, for all nodes. The demand or outflow required at each node is shown in Figure 2(a) and the source head is 100 m. Several sets of analyses were performed the results of which are shown in Table 2 and Figure 2(b). For the HDA, equation (1) was used with an exponent parameter value of $n_j = 2$.

Table 1 Pipe diameters

Pipes	Diameters (mm) for designs 1-12											
	1	2	3	4	5	6	7	8	9	10	11	12
9-1, 9-3	250	250	250	250	250	250	250	250	250	250	255	255
1-2, 3-6	175	175	180	180	180	185	185	185	190	190	190	190
1-4, 3-4	145	145	145	145	145	145	145	145	145	150	150	155
2-5, 6-7	115	115	115	120	125	125	130	135	140	140	140	140
4-5, 4-7	100	105	105	105	105	105	105	105	105	110	115	120
5-8, 7-8	100	100	100	100	100	100	100	100	100	100	100	100

Table 2 illustrates the nodal head results for Design 1 obtained by Tabesh (1998), EPANET 2 (Rossman, 2000) and the NRLSA. Columns 2 to 4 show the results of demand driven analyses using the demands shown in Figure 2(a). The agreement between the three sets of results is satisfactory, therefore demonstrating the accuracy of the present algorithm. The conventional DDA results clearly show that heads at nodes 5, 7 and 8 are negative implying that the demands cannot be fully satisfied.

The results of HDA in Table 2 (columns 5-8) illustrate that all nodal demands are fully satisfied except for node 8. These results demonstrate the capability of the proposed model in terms of its ability to provide accurate information on the locations and actual magnitudes of deficiencies in flows and pressures. The conventional DDA model, therefore, is not able accurately to represent the behaviour of the network. Tabesh (1998) arrived at similar conclusions. However, it can be seen that for the head-driven analysis results, the values of head and outflow at node 8 reported by Tabesh differ significantly from those of NRLSA. To resolve these differences the actual nodal outflows from the two solutions were entered into EPANET2 as nodal demands. The results of the feasibility check in columns 9-10 show that there is much better agreement for the NRLSA (5.29 cf. 5.27 m) than Tabesh (7.75 cf. 4.82 m).

Table 2 Results for design 1 for source head of 100 m

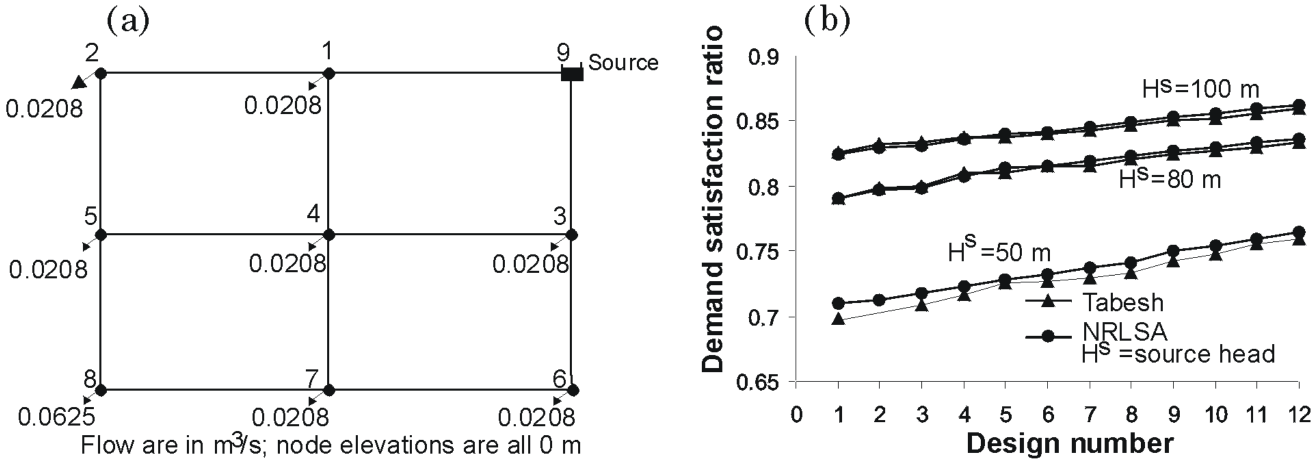
Nodes	Conventional DDA Heads (m)			HDA				Feasibility Check Heads (m)	
				Heads (m)		Outflows (m^3/s)			
	NRLSA	Tabesh	EPANET	NRLSA	Tabesh	NRLSA	Tabesh	NRLSA	Tabesh
1, 3	83.17	83.17	83.19	88.20	88.02	0.0208	0.0208	88.21	88.30
2, 6	57.11	57.11	57.14	71.36	70.94	0.0208	0.0208	71.38	71.63
4	56.78	56.78	56.82	71.99	71.58	0.0208	0.0208	72.01	72.27
5, 7	-20.34	-20.34	-20.25	36.68	35.33	0.0208	0.0208	36.71	37.63
8	-177.63	-177.63	-177.46	5.27	4.28	0.0262	0.0255	5.29	7.75

Figure 2(b) shows the hydraulic behaviour of the 12 designs of Table 1 for values of source head of 100, 80 and 50 m. Overall there is reasonable agreement between the NRLSA and Tabesh results. Confirmation of both accuracy and hydraulic feasibility of the NRLSA results has therefore been achieved.

Table 3 shows results for a source head of 50 m where the NRLSA model gives exact flows and pressures for all nodes. By contrast, the DDA results are not particularly meaningful.

Table 3 Results for design 1 for source head of 50 m

Nodes	DDA Heads (m)	NRLSA			Feasibility Check Heads (m)
		Heads (m)	Outflows (m^3/s)	q_j	
1, 3	33.17	41.09	0.020800	1.00	41.09
2, 6	7.10	29.40	0.020589	0.99	29.40
4	6.78	30.26	0.020800	1.00	30.26
5, 7	-70.35	14.00	0.014210	0.68	14.00
8	-227.65	1.88	0.015657	0.25	1.88



Flow are in m3/s; node elevations are all 0 m

Figure 2 (a) Sample network (b) Network demand satisfaction ratio for each design

Example 2

The network is shown in Figure 3(a), while pipe diameters are shown in Table 4. For all the pipes, lengths of 1000 m, Hazen-Williams coefficients of 140 and required nodal heads of 60 m were used. The minimum nodal heads were 45 m for nodes 3 and 4 and 50m for nodes 1, 2, 5 and 6. Thirty-five hypothetical source heads between 80.0 m and 46.0 m were used to demonstrate the influence of system pressures on nodal outflows. This example also provides further evidence of both the robustness and accuracy of the proposed algorithm. The results were compared to the results reported by Ackley et al. (2001) based on the maximisation of nodal outflows (MO). The head-outflow relationship of equation (1) was used.

Table 4 Pipe diameters

Pipes	Source-1	1-2	1-3	3-4	3-5	5-6	2-4	4-6
Diameters (m)	0.5	0.4	0.4	0.25	0.25	0.25	0.4	0.25

The results are shown in Figures 3(b) and 4(a). Note that the results are shown for nodes 1,3 and 6 only for clarity of the graph. The model gives identical results to the outflow maximisation approach as shown in Figures 3(b) and 4(a). However unlike MO, the NRLSA algorithm predicts the network performance down to a demand satisfaction ratio of zero corresponding to a source head of 45.1 m. The maximisation approach did not produce results for source heads less than 51 m because of computational difficulties. Also, confirmation of the hydraulic feasibility of the results was achieved by using EPANET 2 as explained in Example 1.

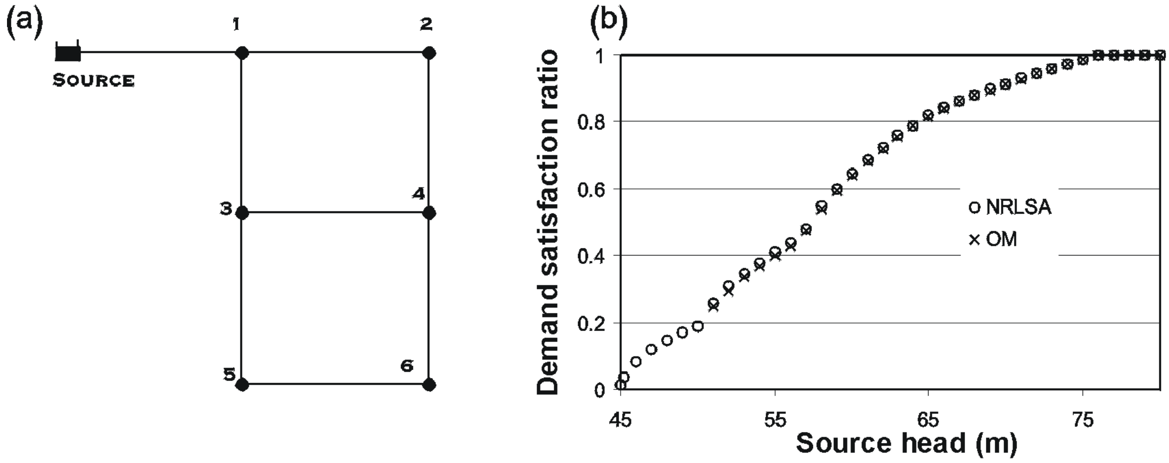


Figure 3 (a) Sample network (b) Network demand satisfatcion ratio for source head range

Finally, equations (2) and (3) were used in the NRLSA model to illustrate their effects on the network. Figure 4(b) illustrates the sensitivity of the model to changes in the head-outflow relationship and its general flexibility and robustness. Equation (3) suggests a much better hydraulic performance than equation (1) and (2) for source heads around 57 m or less.

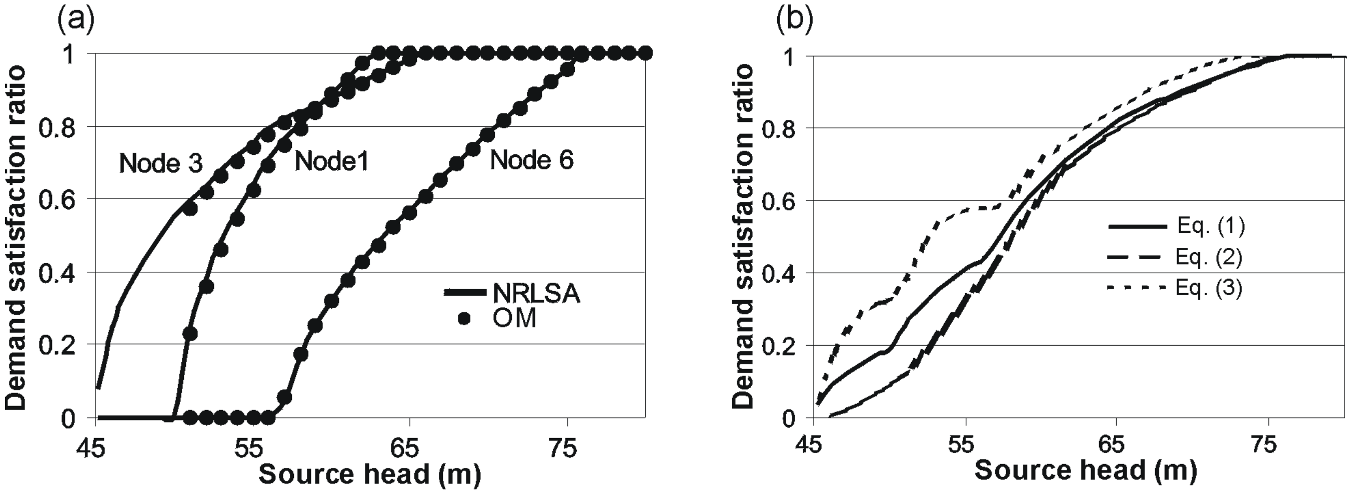


Figure 4 (a) Nodal demand satisfaction ratio for source head range (b) Network demand satisfaction ratio for three head-outflow relationships

Example 3

The new NRLSA algorithm was also used on the serial network illustrated in Figure 5(a) (Gupta and Bhave, 1996). The diameters of pipes 1 to 4 are 400, 350, 300 and 300 mm, respectively. The outlet elevations of nodes 1 through 4 of 90, 88, 90 and 85 m, were taken as the respective minimum nodal heads $H_j^{\min}$. The demand nodes 1 through 4 have required flows of 2, 2, 3 and 4 m^3/min. The lengths and the Hazen-Williams coefficients for all pipes are 1000 m and 130, respectively. The values of H_j^{des} were obtained from (Tanyimboh and Tabesh, 1997)

$$H_j^{des} = H_j^{\min} + R_j (Q_j^{req})^{n_j} ; \qquad R_j = 0.1, \qquad n_j = 2 \qquad (6)$$

To confirm both the accuracy and robustness of the NRLSA algorithm and to demonstrate the effects of head-outflow relationships, the source head for this network was varied from 85 to 115 m. Figure 5(b) shows that for this network equations (1-3) give similar results in general. The Gupta and Bhave (1996) results are slightly higher in general, perhaps because no head-outflow relationship was imposed beyond the stipulation of the minimum and desired nodal heads.

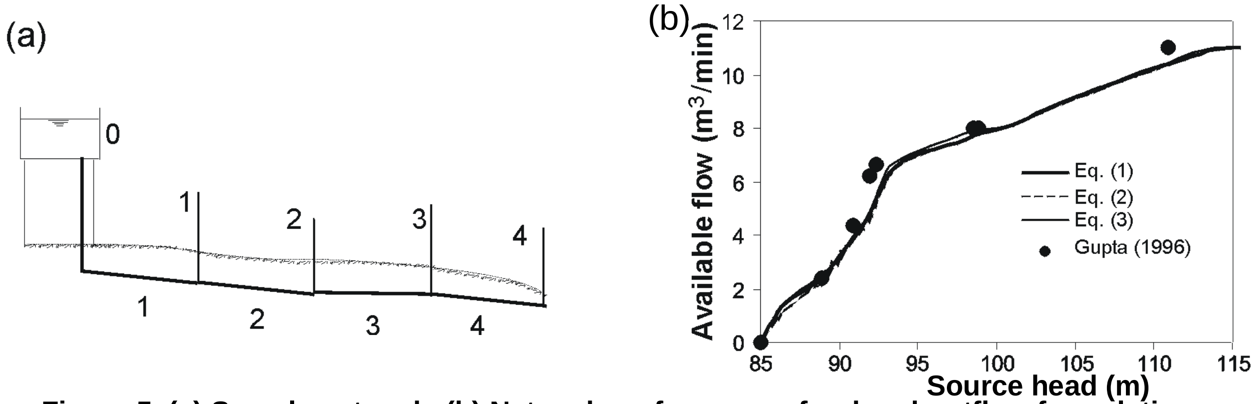


Figure 5 (a) Sample network (b) Network performance for head-outflow formulations

CONCLUSIONS

A new algorithm for head-driven analysis for water distribution systems has been demonstrated. A line search is used in the Newton-Raphson method to improve convergence. An important feature of the proposed method is its flexibility to incorporate a range of the head-outflow relationships into the constitutive equations. The performance of the algorithm has been tested by comparing the results with published works. Confirmation of both accuracy and hydraulic feasibility of the results has been achieved. It has been noticed in all the examples tried, so far that the algorithm is stable and robust. All the results presented were obtained on a PC (Pentium 3, 800Mhz) using a double precision Fortran 90 program. Results obtained so far suggest that the HDA algorithm has similar execution run times (CPU) to its DDA counterpart.

The results also show how the use of different assumed head-outflow relationships can lead to differences in the predicted network performance under subnormal conditions. Therefore, a head-outflow relationship based on field data is urgently required.

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REFERENCES

Ackley J R L, Tanyimboh T T, Tahar B and Templeman A B (2001), "*Head-driven analysis of water distribution systems", Water Software Systems: Theory and applications, B Ulanicki, B Coulbeck and J Rance (Eds),* Volume 1, Research Studies Press Ltd, UK, 183-192.

Bhave P R (1981), "*Node flow analysis of water distribution systems*", *J. Transportation Engineering*, ASCE, 107 (July), 457-467.

Chandapillai J (1991), *"Realistic simulation of water distribution system", J. Transportation Engineering*, ASCE, 101 (4), 258-263.

Germanopolous G (1985), *"A technical note on the inclusion of pressure dependent demand and leakage terms in water supply network models", Civil Engineering Systems*, 2 (September), 171-179.

Fujiwara O and Ganesharajah T (1993), "*Reliability assessment of water supply systems with storage and distribution networks", Water Resources Research*, 29 (8), 2917-2924.

Gupta R and Bhave P R (1996), "*Comparison of methods for predicting deficient network performance", J. Water Resources Planning and Management,* 122 (3), 214-217.

Reddy L S and Elango K (1989), "*Analysis of water distribution networks with head-dependent outlets", Civil Engineering Systems,* 6 (3), 102-110.

Rossman L A (2000), *"EPANET 2 Users Manual", United States Environmental Protection Agency, Cincinnati, Ohio, US*.

Tabesh M (1998), *"Implications of the pressure dependency of outflows on data management, mathematical modelling and reliability assessment of water distribution systems*", PhD Thesis, University of Liverpool, UK.

Tanyimboh T T, Tabesh M (1997), *"Discussion of comparison of methods for predicting deficient network performance", J. Water Resources Planning and Management*, ASCE, 123 (6), 369-370.

Wagner J M, Shamir U and Marks D H (1988), *"Water distribution reliability: simulation methods", J. Water Resources Planning and Management*, ASCE, 114 (3), 276-294.

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