# DESIGN OF A RECONFIGURABLE SATELLITE CONSTELLATION

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This paper provides a fully analytical method to describe a satellite constellation reconfiguration manoeuvre. By making use of low-thrust propulsion and exploiting the Earth's natural perturbing forces it is possible to analytically describe the reconfiguration of a constellation, achieving a desired separation of both Right Ascension of Ascending Node (RAAN) and Argument of Latitude between satellites. An inherent trade-off exists between the time taken for a manoeuvre and the required ΔV, however the analytical solution presented here allows for a rapid visualisation of the trade-space and determination of the ideal transfer trajectory for a given mission. The general method presented can be applied across a range of scenarios, including constellation deployment and repurposing. The results show that for a scenario with an initial orbit semi-major axis of 6878.14km, and a desired final semi-major axis of 6778.14km it is possible to achieve a separation of 180° argument of latitude between a manoeuvring and a non-manoeuvring reference satellite in approximately 68 hours with a ΔV of 200m/s. To achieve the maximum possible RAAN separation of 90° with a ΔV of 200m/s requires a much longer time of over 218 days. Using two manoeuvring satellites with the same total manoeuvre ΔV was found to be more efficient only for short manoeuvre times. This is quantified and for the case considered it is found that using a 2-satellite manoeuvre is advantageous when changing the argument of latitude and when changing the RAAN <10° approximately. The ability to identify this turning point clearly is a distinct advantage of the analytical solution presented.

## I. INTRODUCTION

Satellite constellations, whether for Earth observation or for telecommunications, are traditionally used to achieve global coverage of the Earth. This requires that a large number of satellites be distributed into, and within, a number of orbit planes to allow for continuous or regular observation of selected ground targets. Current methods for deploying constellations vary but generally consist of a number of satellites being launched at once into the same orbital plane and then being distributed within that plane by either the launcher upper-stage or the satellite's own on-board propulsion system [1]. This requires the use of a dedicated launch to populate each orbit plane – a costly method which rapidly reduces the value of constellations with more than a few orbital planes and makes launching a constellation of low-cost small satellites essentially impossible [2].

An alternative deployment option, as demonstrated by the FORMOSAT-3/COSMIC mission in 2006 [3, 4], is to launch multiple satellites into the same plane at the required inclination, and then later distribute them into the required planes. To facilitate this plane change manoeuvre each satellite would need to possess an on board propulsion system and sufficient propellant which would increase the system mass. However, by taking advantage of the Earth's natural $J_2$ effect, the perturbing force experienced by a spacecraft due the oblateness of the central body, the propellant cost required to change the Right Ascension of the Ascending Node (RAAN) can be reduced at the expense of a longer manoeuvre time. A similar technique can be used to distribute the satellites within their final orbit plane, allowing for the complete deployment of a large constellation with only a single launch.

Reconfiguring and repurposing existing constellations can also be carried out using a similar method, allowing constellations to respond to real-time market demands and significantly increase their commercial potential [5].

To date, the problem of manoeuvring satellite constellations has primarily been dealt with numerically [6, 7]. A semi-analytical method has been defined to analyse the deployment of a constellation of small satellites [8], however this requires full knowledge of the mission and constellation parameters and must be iterated to find an appropriate solution.

In this paper an analytical solution to the problem is presented which allows for a complete exploration of the solution space, without complete knowledge of the mission parameters. This allows for the mission trade-offs to be rapidly visualised and the most appropriate constellation deployment strategy selected to fulfil the mission requirements.

## II. METHOD

Two different manoeuvres are considered for the deployment of a satellite constellation. One is to change the position of a satellite within the orbit plane, and the other is to change the RAAN of the orbit plane itself. These manoeuvres are considered individually in the following sections.

### II.I. Argument of Latitude Separation

The position of a satellite within an orbit can be described by the true anomaly or by the argument of latitude, which is a sum of the true anomaly and the argument of perigee. The perturbed rate of change of true anomaly $\theta$ is given in the Gauss version of the Lagrange equations by

$$\frac{d\theta}{dt} = \frac{a^2\sqrt{1-e^2}n}{r^2} - \frac{\sqrt{1-e^2}(e\cos(\theta)+2)F_S}{(e\cos(\theta)+1)(aen)} + \frac{\sqrt{1-e^2}F_R\cos(\theta)}{aen} \quad [1]$$

where $a$ is the semi-major axis, $e$ is the eccentricity, $t$ is time, $n$ is the mean motion, $r$ is the radius of the orbit, $F_R$ is the perturbing acceleration in the radial direction, and $F_S$ is the perturbing acceleration acting in the direction of satellite motion [9].

Assuming that all manoeuvres are to be carried out using low-thrust propulsion, it is acceptable to assume that the satellite remains in a circular orbit throughout the manoeuvre ($e = 0, r = a$) and that all applied thrust is in-plane ($F_R$ = 0). In this case, equation 1 is undefined and instead the unperturbed rate of change of true anomaly is applied. As the argument of perigee is undefined for a circular orbit, this gives the rate of change of argument of latitude $u$ as

$$\frac{d\theta}{dt} = \frac{du}{dt} = \sqrt{\frac{\mu}{a^3}}. \quad [2]$$

From equation 2 it is clear that the rate of change of argument of latitude is inversely proportional to the semi-major axis, meaning that satellites in a lower orbit will experience a higher rate of change of argument of latitude compared to those in higher orbits, assuming all other orbital parameters are constant. Thus, by raising or lowering the altitude of a satellite it is possible to utilise these effects to create a separation in argument of latitude between two satellites.

It is of note that the effect of drag is not considered in this investigation as it is assumed to be negligible over the manoeuvre times considered.

#### II.I.I. Governing Equations

For a non-manoeuvring satellite the change in argument of latitude is simply given by

$$u_{ref} = (t_{total} - t_0)\sqrt{\frac{\mu}{a_0^3}} + u_0 \quad [3]$$

assuming that the semi-major axis remains constant throughout, where a subscript of 0 denotes the value at the beginning of the manoeuvre [9].

To describe the motion of the manoeuvring satellite the Gauss version of the Lagrange planetary equations are manipulated. With the assumption that the applied thrust and spacecraft mass remain constant ($F_S = \text{constant}$), this gives the change in semi-major axis over a given thrust period as

$$t_1 - t_0 = \frac{\sqrt{\mu}}{F_S}\left(\frac{1}{\sqrt{a_0}} - \frac{1}{\sqrt{a_1}}\right) \quad [4]$$

where a subscript 1 indicates the value at the end of the manoeuvre period [10]. The change in argument of latitude as a function of the change in semi-major axis is given as

$$a_1 = \left(\frac{1}{a_0^2} - \frac{4}{\mu}F_S\,(u_1 - u_0)\right)^{-1/2}. \quad [5]$$

Any coasting phase in which there is no thrusting will be governed by equation 3.

In order to describe these results in terms of ΔV, the change in velocity,

$$\Delta V = |A| \times t_{thrust} = \left|\sqrt{\frac{\mu}{a_1}} - \sqrt{\frac{\mu}{a_0}}\right| \quad [6]$$

is used where A is the applied thrust in m/s [11]. Note that a positive A value corresponds to an increase in semi-major axis, while a negative A value corresponds to a reduction in semi-major axis.

#### II.I.II. 1-Satellite Manoeuvre

In the case of the 1-satellite manoeuvre, one satellite dubbed the reference satellite performs no manoeuvres while the other manoeuvring satellite varies its altitude to achieve the required argument of latitude separation between them. An initial time $t_0 = 0$ and initial argument of latitude $u_0 = 0$ is assumed for simplicity.

While other manoeuvres are possible, this paper focusses on the most general case. This is a 3-Phase manoeuvre which consists of an initial spiral thrusting manoeuver to either increase or decrease the semi-major axis (Phase 1), a coasting phase during which the semi-major axis is constant (Phase 2), and another spiral thrusting manoeuver to reach the final desired semi-major axis (Phase 3). The reference satellite is assumed to begin in the final desired orbit with the same argument of latitude as the reference satellite.

The total separation in argument of latitude between the manoeuvring satellite and the reference satellite over the entire manoeuvre is described by

$$\Delta u = \frac{1}{8}\Big( \frac{8\gamma\mu}{A a_0^{3/2}} - \frac{2\gamma^4\mu}{A} - \frac{12\gamma^2\mu}{A a_0} + \frac{8\gamma^3\mu}{A\sqrt{a_0}} - \frac{2\mu}{A}\left(\frac{(\mu+\beta a_0)^2}{16\mu^2 a_0^2} - \frac{1}{a_3^2}\right) - 8\sqrt{\frac{\mu}{a_3^3}}t_t + \sqrt{\frac{(\mu+\beta a_0)^3}{\mu^2 a_0^3}}\left(t_t - \frac{\sqrt{\mu}}{A}\left[\gamma + \frac{1}{\sqrt{a_3}} - \frac{1}{2\sqrt{\frac{\mu a_0}{\mu+\beta a_0}}}\right]\right)\Big) \quad [7]$$

where a subscript 3 indicates the value at the end of the third phase and a subscript t indicates the total value required for the full 3-Phase manoeuvre. Here,

$$\gamma = \left(\frac{1}{\sqrt{a_0}} - \frac{1}{2\sqrt{\frac{\mu a_0}{\mu+\beta a_0}}}\right) \quad [8]$$

and

$$\beta = \left(\sqrt{\frac{\mu}{a_3}} \pm \Delta V_t\right)\left(2\sqrt{\frac{\mu}{a_0}} + \sqrt{\frac{\mu}{a_3}} \pm \Delta V_t\right) \quad [9]$$

where a '+' corresponds to the case where the satellite decreases its semi-major axis in Phase 1 and increases its semi-major axis in Phase 3, and a '-' corresponds to the case where the satellite increases its semi-major axis in Phase 1 and decreases its semi-major axis in Phase 3.

II.I.III. 2-Satellite Manoeuvre

It is also possible to describe the problem by considering two manoeuvrable satellites and no reference satellite. In this case one satellite will initially decrease its semi-major axis while the other increases its semi major axis. The total argument of latitude separation achieved is simply the separation between the two manoeuvring satellites.

While it is possible to vary the semi-major axis of each satellite by a different amount to optimise the manoeuvre, a simple case is considered here in which both satellites begin and end at the same final altitude making use of equal amounts of ΔV. For this case, the total argument of latitude separation can be described by

$$\Delta u = -\frac{\Delta V\left(A^3\Delta V^3 + 2(\delta-\kappa)\mu\right)}{16\mu|A|^4} - \frac{3A\Delta V^2}{2|A|^2 a_0} - \frac{(\kappa-\delta)t_t}{8|A|^3} \quad [10]$$

where

$$\delta = \sqrt{\mu\left(\frac{\left(A\Delta V\sqrt{a_0} - 2\sqrt{\mu}|A|\right)^4}{\mu^2 a_0^2}\right)^{3/2}} \quad [11]$$

and

$$\kappa = \sqrt{\mu\left(\frac{\left(-A\Delta V\sqrt{a_0} - 2\sqrt{\mu}|A|\right)^4}{\mu^2 a_0^2}\right)^{3/2}}. \quad [12]$$

II.II. RAAN Separation

Separating the satellites' orbital planes can be achieved by utilising the $J_2$ effect, causing the RAAN and the argument of perigee of the orbit to drift away from their initial values. The rate of change of RAAN Ω is described by

$$\dot{\Omega} = -\frac{3J_2 n R_e^2 cos(i)}{2a^2\left(1-e^2\right)^2} \quad [13]$$

where $i$ is the orbit inclination and $R_e$ is the radius of Earth [9]. The rate of change of the argument of perigee $\omega$ is given as

$$\dot{\omega} = \frac{3J_2 n R_e^2\left(1 - \frac{5sin^2(i)}{4}\right)}{a^2\left(1-e^2\right)^2}. \quad [14]$$

From equation 13 it is clear that the rate of change of RAAN is inversely proportional to the square of the semi-major axis, meaning that satellites in a lower orbit will experience a higher rate of change of RAAN compared to those in higher orbits, assuming all other orbital parameters are constant. Thus, by raising or lowering the altitude of a satellite it is possible to utilise these effects to vary the RAAN separation between satellites.

In investigating the effectiveness of this technique, it is assumed that all manoeuvres are carried out using low-thrust propulsion, and hence that all satellites considered remain in circular orbits throughout. As such, the rate of change of argument of perigee can be ignored. The effect of drag is considered negligible for the missions analysed and as such is not considered.

II.II.I. Governing Equations

Considering the $J_2$ effect, a non-manoeuvring satellite's change in RAAN over a time $t$ will be given by

$$\Omega_{ref} = -\frac{3}{2}\frac{J_2\sqrt{\mu}R_e^2 cos(i)}{a_0^{7/2}}t + \Omega_0 \quad . \qquad [15]$$

From the Gauss version of the Lagrange planetary equations, with the assumption that the orbit remains circular throughout the manoeuvre ($e = 0$, $r = a$) and that all applied thrust is in-plane, it is possible to describe the rate of change of RAAN as a function of the semi-major axis where $F_S$ is the in-plane acceleration (with the positive direction being in the direction of travel) [9]. Assuming that the inclination as well as the applied thrust and spacecraft mass remain constant ($i = \text{constant}$, $F_S = \text{constant}$), this gives

$$\Omega_1 - \Omega_0 = \frac{3}{16}\frac{\mu R_e^2 J_2 cos(i)}{F_S}\left(\frac{1}{a_1^4} - \frac{1}{a_0^4}\right) \qquad [16]$$

where, as before, a subscript 0 indicates the value at the start of the manoeuvre and a subscript 1 indicates the value at the end of the thrusting phase. The time taken for this manoeuvre is given by

$$t_1 - t_0 = -\frac{\sqrt{\mu}}{F_S}\left(\frac{1}{\sqrt{a_1}} - \frac{1}{\sqrt{a_0}}\right). \qquad [17]$$

A coast phase in which no thrusting occurs will be governed by equation 15.

To rewrite the equations in terms of ΔV, equation 6 is used as in the case of the argument of latitude separation.

II.II.II. 1-Satellite Manoeuvre

In the case of the 1-satellite manoeuvre one satellite, dubbed the reference satellite, performs no manoeuvres while the other manoeuvring satellite varies its altitude to achieve the required RAAN separation. An initial time $t_0 = 0$ and initial RAAN $\Omega_0 = 0$ is assumed for simplicity.

While other manoeuvres are possible, this paper focuses on a general 3-Phase manoeuvre which consists of an initial spiral thrusting manoeuver to either increase or decrease the semi-major axis (Phase 1), a coasting phase during which the semi-major axis is constant (Phase 2), and another spiral thrusting manoeuver to reach the required final semi-major axis (Phase 3). The reference satellite is assumed to begin in the final desired orbit with the same RAAN as the reference satellite.

For this 3-Phase manoeuvre, equations 15-17 reduce to give the total final RAAN separation between the manoeuvring satellite and the reference satellite as

$$\begin{aligned}\Delta\Omega = &\frac{3}{256}\sqrt{\mu}\cos(i)J_2R_e^2\Bigg(\frac{\sqrt{\mu}}{16Aa_0^4}\left(\frac{(\mu+\beta a_0)^4}{\mu^4} - 256\right)\\ &- \frac{16\sqrt{\mu}}{A}\left(\frac{1}{a_3^4} - \frac{\left(\beta + \frac{\mu}{a_0}\right)^4}{256\mu^4}\right)\\ &+ \left(\frac{\sqrt{\mu}}{A}\left(\frac{1}{\sqrt{a_0}} - \frac{1}{\sqrt{\frac{\mu a_0}{\mu+\beta a_0}}} + \frac{1}{\sqrt{a_3}}\right) - t_t\right)\left(\frac{\mu a_0}{\mu+\beta a_0}\right)^{-7/2} + \frac{128 t_t}{a_3^{7/2}}\Bigg)\end{aligned} \qquad [18]$$

where, as before,

$$\beta = \left(\sqrt{\frac{\mu}{a_3}} \pm \Delta V_t\right)\left(2\sqrt{\frac{\mu}{a_0}} + \sqrt{\frac{\mu}{a_3}} \pm \Delta V_t\right). \qquad [19]$$

A subscript 3 indicates the value at the end of the third phase and a subscript t indicates the total value required for the full 3-Phase manoeuvre. As in the argument of latitude case, a '+' corresponds to the case where the satellite decreases its semi-major axis in Phase 1 and increases its semi-major axis in Phase 3, and a '-' corresponds to the case where the satellite increases its semi-major axis in Phase 1 and decreases its semi-major axis in Phase 3.

II.II.III. 2-Satellite Manoeuvre

In the case of the RAAN separation manoeuvre it is also possible to describe the problem considering two manoeuvrable satellites. In this case one satellite will initially decrease its semi-major axis while the other will increase its semi major axis. In this case the total RAAN separation achieved is the separation between the two satellites at the end of the manoeuvre.

Again, while a more general case is possible, here a simple 3 Phase manoeuvre is considered in which the satellites both begin and end at the same altitude and use equal amounts of ΔV. Assuming that the satellites have the same thrust magnitude A but applied in opposite directions, the total RAAN separation can be described by

$$\begin{aligned}\Delta\Omega = &\frac{3}{256}\sqrt{\mu}\cos(i)J_2R_e^2\Bigg(\frac{16\sqrt{\mu}}{Aa_0^4}\Bigg(\frac{2\left(\sqrt{\mu} - \frac{A\Delta V\sqrt{a_0}}{2|A|}\right)^8}{\mu^4}\\ &+ \frac{2\left(\sqrt{\mu} + \frac{A\Delta V\sqrt{a_0}}{2|A|}\right)^8}{\mu^4} - 4\Bigg)\\ &+ \frac{\frac{\Delta V}{|A|} - t_t}{|A|^7}\Bigg(\left(\frac{\mu a_0}{\left(A\Delta V\sqrt{a_0} - 2\sqrt{\mu}|A|\right)^2}\right)^{-7/2}\\ &- \left(\frac{\mu a_0}{\left(A\Delta V\sqrt{a_0} + 2\sqrt{\mu}|A|\right)^2}\right)^{-7/2}\Bigg)\Bigg)\end{aligned} \qquad .[20]$$

## IV. RESULTS

Table 1 gives the value of the orbital constants used in the evaluation of the following results, assuming an Earth orbiting satellite. Table 2 gives the assumed acceleration of the propulsion system as well as the value used for the initial and final semi-major axes.

| Parameter | Symbol | Value | Units |
|---|---|---|---|
| Gravitational Parameter | μ | 3.986E14 | $m^3/s^2$ |
| Radius of Earth | $R_e$ | 6.371E3 | km |
| $J_2$ Parameter | $J_2$ | 1.0827E-3 | - |

Table 1: Orbital Constants

| Parameter | Symbol | Value | Units |
|---|---|---|---|
| Propulsion acceleration | A | ± 0.001 | $m/s^2$ |
| Initial semi-major axis | $a_0$ | 6878.14 | km |
| Final semi-major axis | $a_3$ | $a_0$ ± 100 | km |

Table 2: Mission Parameters

### IV.I. Argument of Latitude Separation

Using the equations defined in Section II, the achievable argument of latitude separation for a given ΔV and transfer time can be determined analytically.

#### IV.I.I. 1-Satellite Manoeuvre

For a given initial and a final semi-major axis it is possible to calculate the achievable argument of latitude separation as a function of the ΔV and the manoeuvre time for the 3-Phase manoeuvre described in Section II.I. Fig. 1 and Fig. 2 show this up to a maximum separation of ±180° for the case in which the semi-major axis is lowered in Phase 1 and the case in which the semi-major axis is increased in Phase 1 respectively.

In order to reflect only manoeuvres which are physically possible, the graphs here are plotted for cases in which the total manoeuvre time is greater than the time taken for the two thrust phases. In addition, the results plotted here are only for cases in which the ΔV applied is sufficient to reach the required final semi-major axis and also give a positive or negative change in argument of latitude as required. This is because even if the satellite reaches the required final orbit, it may have lagged behind the reference satellite over all, when in fact it was required to lead, and vice versa.

These results show that with an initial semi-major axis of 6878.14km, and a final semi-major axis of 6778.14km it is possible to achieve a separation of 180° argument of latitude in just over 68 hours with a ΔV of 200m/s in the case where the semi-major axis is decreased in Phase 1. For the case in which the manoeuvring satellite increases its semi-major axis in Phase 1, a separation of -180° can be achieved with a ΔV of 200m/s in just over 69 hours with an initial semi-major axis of 6878.14km, and a final semi-major axis of 6978.14km.

As demonstrated by the below graphs, there is a significant trade-off between the ΔV cost and the time taken for the manoeuvre, with a lower ΔV necessitating a longer manoeuvre time to achieve the same argument of latitude separation. For example, with a ΔV of 100m/s the time required to achieve a separation of 180° increases to just over 130 hours in the decreasing altitude case and just under 130 hours in the increasing altitude case.

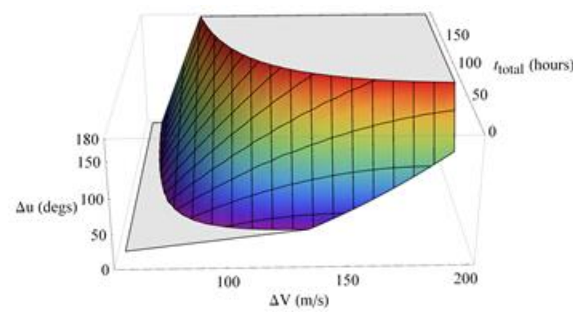


Fig. 1: Δu for 3-Phase Manoeuvre separating argument of latitude, decreasing altitude

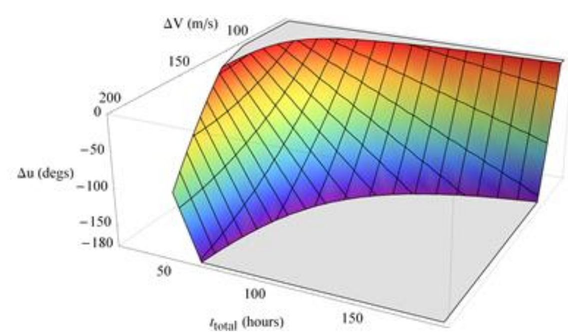


Fig. 2: Δu for 3-Phase Manoeuvre separating argument of latitude, increasing altitude

IV.I.II. 2-Satellite Manoeuvres

In the case where a 2-satellite manoeuvre is used to separate the argument of latitude, it is assumed that both satellites begin and end their manoeuvres at the same altitude, with one satellite initially lowering its semi-major axis whilst the other raises its semi-major axis. In this case the total separation is the sum of the separation achieved by each satellite in relation to the starting reference point.

Fig. 3 shows the achievable argument of latitude separation for such a manoeuvre as a function of the total ΔV (i.e. each satellite requires ΔV/2), and the manoeuvre time up to a maximum separation of 180°. Fig. 4 shows the same results (blue) plotted against the results for a 1-satellite manoeuvre using the same ΔV and the same manoeuvre time (green) for a case in which the semi-major axis is initially lowered. Fig. 5 gives the same comparison but for the 1-satellite case in which the semi-major axis is initially raised. It is clear from Fig. 5 that the 2-satellite manoeuvre will always be more efficient than the 1-satellite altitude raising manoeuvre. However, Fig. 4 shows that if an argument of latitude separation manoeuvre of greater than approximately 5000° was required, then a 1-satellite manoeuvre with a decreasing altitude would be most efficient. This is because in the 1-satellite manoeuvre there is a greater relative difference in the rate of argument of latitude drift between the manoeuvring and the non-manoeuvring satellite, than there is between the two manoeuvring satellites in the 2-satellite case. Over a certain time period this higher variation in rate of drift overrides the advantages offered through the use of two satellites. However, as a separation of >360° corresponds to multiple revolutions, the concept of a >5000° separation is purely theoretical.

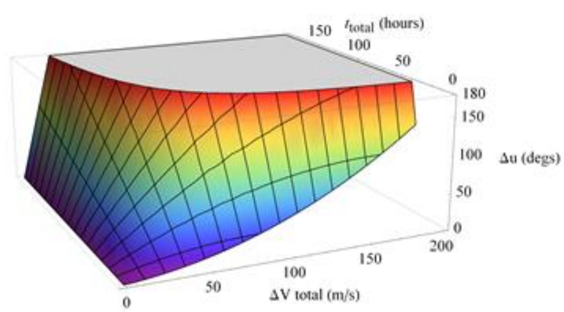


Fig. 3: Δu for 2-satellite manoeuvre, separating argument of latitude

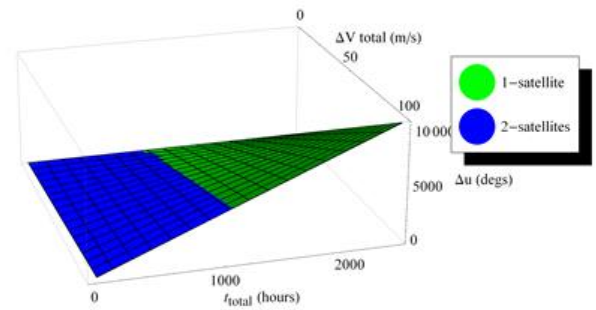


Fig. 4: Comparison of 2-satellite (blue) and 1-satellite decreasing altitude (green) manoeuvres, separating argument of latitude

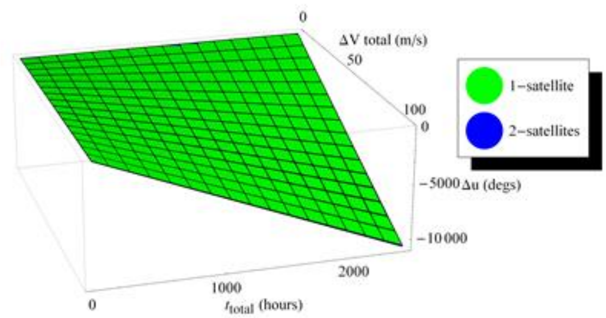


Fig. 5: Comparison of 2-satellite (blue) and 1-satellite increasing altitude (green) manoeuvres, separating argument of latitude

IV.II. RAAN Separation

Using the equations defined in Section II, the achievable RAAN separation for a given ΔV and transfer time can be determined analytically.

IV.II.I. 1-Satellite Manoeuvre

For a given initial and a final semi-major axis it is possible to calculate the achievable RAAN separation as a function of the ΔV and the manoeuvre time. Fig. 6 and Fig. 7 show this up to a maximum RAAN separation of ±90° for the case in which the semi-major axis is lowered in Phase 1 and the case in which the semi-major axis is increased in Phase 1 respectively.

As in the argument of latitude case, the graphs here are plotted for cases in which the total time is greater than the time taken for the two thrust phases. and in which the ΔV applied is sufficient to reach the required final semi-major axis and also give a positive or negative change in RAAN as required.

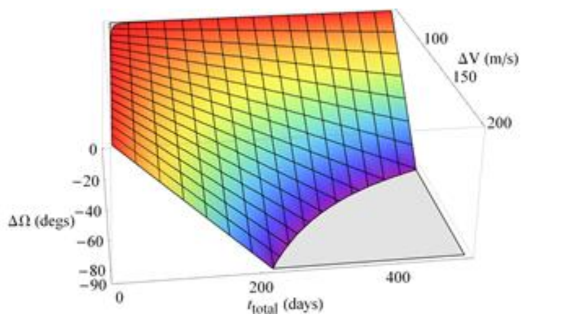


Fig. 6: ΔΩ for 3-Phase Manoeuvre separating RAAN, decreasing altitude

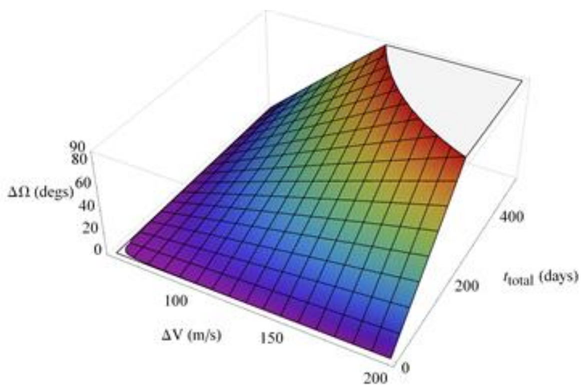


Fig. 7: ΔΩ for 3-Phase Manoeuvre separating RAAN, increasing altitude

These results show that with an initial semi-major axis of 6878.14km, and a final semi-major axis of 6778.14km it is possible to achieve a separation of -90° RAAN in 218 days with a ΔV of 200m/s in the case where the semi-major axis is decreased in Phase 1. For the case in which the manoeuvring satellite increases its semi-major axis in Phase 1, a separation of 90° can be achieved with a ΔV of 200m/s in 249 days with an initial semi-major axis of 6878.14km, and a final semi-major axis of 6978.14km.

As before, there is a trade-off between the ΔV cost and the time taken for the manoeuvre as demonstrated in the graphs, with a lower ΔV necessitating a longer manoeuvre time to achieve the same argument of latitude separation. For example, with a ΔV of 150m/s the time required to achieve a separation of ±90° increases to almost 336 days in the decreasing altitude case and over 375 days in the increasing altitude case.

IV.II.II. 2-Satellite Manoeuvres

In the case where a 2-satellite manoeuvre is used to achieve the RAAN separation, it is assumed that both satellites begin and end their manoeuvres at the same altitude, with one satellite initially lowering its semi-major axis whilst the other raises its semi-major axis. Here the total RAAN separation is the sum of the separation achieved by each satellite in relation to the starting reference point.

Fig. 8 shows the achievable RAAN separation up to a maximum of -90° for the 2-satellite manoeuvre as a function of the total ΔV (i.e. each satellite requires ΔV/2) and the manoeuvre time. Fig. 9 shows the same results (blue) plotted against the results for a 1-satellite manoeuvre using the same ΔV and the same manoeuvre time (green) for a case in which the semi-major axis is initially lowered. Fig. 10 gives the same comparison but for the 1-satellite case in which the semi-major axis is initially raised. It is clear from Fig. 10 that the 2-satellite manoeuvre will always be more efficient than the 1-satellite altitude raising manoeuvre. However, Fig. 9 shows that when a small RAAN separation manoeuvre is required (in this case less than approximately 10°) a 2-satellite manoeuvre is most efficient, while for RAAN separations larger than this a 1-satellite manoeuvre in which the altitude is decreased is more effective. This is because in the 1-satellite manoeuvre there is a greater relative difference in the rate of RAAN drift between the manoeuvring and the non-manoeuvring satellite, than there is between the two manoeuvring satellites in the 2-satellite case. As in the argument of latitude case, this results in a turning point for the RAAN separation beyond which the 1-satellite manoeuvre is in fact faster than the 2-satellite manoeuvre for the same total ΔV.

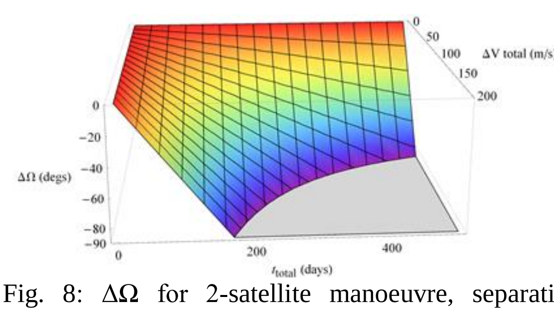


Fig. 8: ΔΩ for 2-satellite manoeuvre, separating RAAN

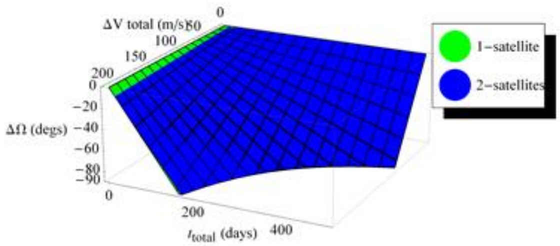


Fig. 9: Comparison of 2-satellite (blue) and 1-satellite decreasing altitude (green) manoeuvres, separating RAAN

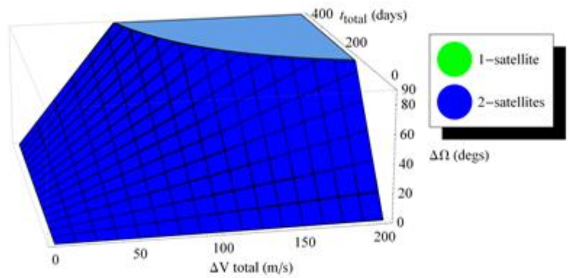


Fig. 10: Comparison of 2-satellite (blue) and 1-satellite increasing altitude (green) manoeuvres, separating RAAN

### IV.III. Combined Argument of Latitude and RAAN Separation

As both the RAAN and the Argument of Latitude Separation manoeuvres are performed by varying the satellite’s semi-major axis, it is impossible to vary one value without affecting the other. To investigate this, the resultant RAAN change is calculated for the case in which the argument of latitude is changed by 180°. This is the maximum separation that would realistically be required and as such is a worst case.

The results are shown in Table 3 and demonstrate that the RAAN angle drifts by less than 0.5° even during the largest possible argument of latitude manoeuvre.

| ΔV | Total time | Argument of latitude separation | RAAN Separation |
|---|---|---|---|
| 200 m/s | 68 hrs | 180° | -0.459° |

Table 3: RAAN separation during maximum argument of latitude separation manoeuvre

## V. CONCLUSIONS

It is possible to analytically describe a 3-Phase manoeuvre to separate either the RAAN or the argument of latitude of two satellites. There is a distinct trade-off to be had in both cases between the amount of propellant used and the total manoeuvre time.

Separating two satellites by 180° argument of latitude will take less than 1 week for a ΔV value of 50-200m/s when using a 1-satellite manoeuvre. This time can be significantly reduced using a 2-satellite manoeuver. While the results show that for very large argument of latitude separations (in this case >5000°) a 1-satellite manoeuvre is faster, this would never be required in reality and so for all argument of latitude separation manoeuvres a 2-satellite manoeuvre is recommended.

The RAAN separation manoeuvre takes a much longer time than the argument of latitude separation, with a manoeuvre designed to separate two satellites by 90° taking 6-18 months for a ΔV value of 100-200m/s when using a 1-satellite manoeuvre. For smaller manoeuvres (in this case <10°) a 2-satellite manoeuvre can offer a reduction in manoeuvre time for a given ΔV, however for values above this the 1-satellite manoeuvre is more efficient.

Whilst it is impossible to perform either an argument of latitude or a RAAN separation manoeuvre in isolation from the other, the change in RAAN during an argument of latitude manoeuvre will be relatively small since the time required to change the RAAN is much larger than the time required to change the argument of latitude. As such, effective deployment of a constellation could be achieved by first obtaining the required RAAN separation and then adjusting the argument of latitude to the required value.

## VI. ACKNOWLEDGEMENTS

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