

# Simulation of spark dynamic plasma resistance and inductance using PSpice

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## ABSTRACT

This paper presents the results of analyses of the transient resistance and inductance of spark plasma, parameters which have been obtained using the hydrodynamic approach described in [2], and a new model for plasma resistance. A lumped *RLC* model to represent the transient process in the spark-discharge plasma has been built and solved using PSpice simulation software. The dynamic plasma resistance,  $R(t)$ , and inductance,  $L(t)$ , have been used in the lumped-element circuit.

Index terms – Plasma, dynamic, resistance, inductance, hydrodynamic, PSpice.

## 1 INTRODUCTION

This paper presents a PSpice-based solution of the hydrodynamic equations which describe the dynamic electrical properties of spark-discharge plasma. The properties of spark-discharge plasma have been under investigation for over a century [1]. The transient resistance and inductance of plasma in the first quarter-cycle of the current wave-form (rise to peak current) is of particular interest in the pulsed-power community, for applications such as linear transformer drivers (LTDs) where current magnitude and rise-time are critical. Designers of LTDs seek maximum current magnitude and the fastest possible rise-time, both of which are directly affected by the plasma resistance and inductance.

This paper presents the results of analyses of the transient resistance and inductance of spark plasma, parameters which have been obtained using the hydrodynamic approach described in [2], and a new model for plasma resistance. A lumped *RLC* model to represent the transient process in the spark-discharge plasma has been built and solved using PSpice simulation software. The dynamic plasma resistance,  $R(t)$ , and inductance,  $L(t)$ , have been used in the lumped-element circuit. PSpice solves the time-dependant circuit equations and allows a transient solution to be obtained for complex spark-plasma models. Different equations which describe  $R(t)$  and  $L(t)$  for spark-plasmas have been obtained from the literature [1-8] for use in the present analysis, and these equations have been solved and compared with experimental data.

Using PSpice simulations and experimental results, an advanced model of spark-plasma parameters has been

developed. This model will aid further analysis of the transient processes in spark-discharge plasmas.

## 2 PLASMA RESISTANCE MODELS

There are several models for the simulation of plasma resistance, by Toepler [1], Braginskii [3], Rompe and Weizel [4], Barannik *et al.* [5], Popovic *et al.* [6], Demenik *et al.* and Kushner *et al.* [8]. In this paper, models by Toepler, Braginskii, Rompe and Weizel, and Barannik are considered. The equations for channel resistance considered in this paper have been derived from the energy balance equation and in some cases theoretical reasoning supporting their choice of coefficients has been provided [4]. In the models proposed by Toepler, Rompe and Weizel, and Barannik, the spark resistance is inversely proportional to the integral of the current that flows in the channel. The Braginskii model uses the plasma channel radius,  $a(t)$ .

Braginskii's hydrodynamic equation for plasma radius is also used by other authors and has been proved to provide a reasonable fit to experimental data [3, 5, 9, 10]. For the purpose of continuity and comparison with literature results, equation (1) is also used in this paper:

$$a^2(t) = \left( \frac{4}{\pi^2 \rho_0 \zeta \sigma} \right)^{1/3} \int_0^t i^{2/3} dt [\mu m] \quad (1)$$

where  $a(t)$  is the plasma channel radius,  $i$  is the channel current,  $t$  is time,  $\rho_0$  is the initial gas density,  $\sigma$  is the conductivity, and  $\zeta$  is a constant.

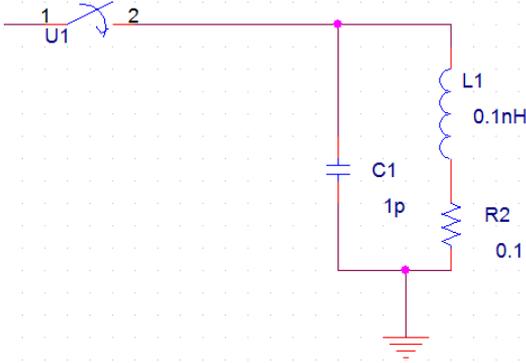
In the literature, plasma-resistance models have been reviewed and compared using a numerical method deriving circuit equations without the use of PSpice [2].

For this purpose, it is necessary to use simple *RLC* circuits to simulate plasma properties, as circuit equations have to be derived. The approach in this paper can use complex circuits attached to plasma *RLC* models, which would be unrealistic to derive by hand, because PSpice solves the circuit equations.

### 3 PLASMA MODELLING USING PSPICE

#### 3.1 PLASMA RLC MODEL

Plasma is modelled using an *RLC* lumped-element model, representing the plasma resistance, inductance and capacitance. An *RLC* model is the standard approach to modelling plasma. By adding in a basic pulsed-power circuit to this model, a transient analysis of plasma can be conducted. Figure 1 shows the basic *RLC* plasma model, the closure of the switch (*U1*) initiates the plasma breakdown.

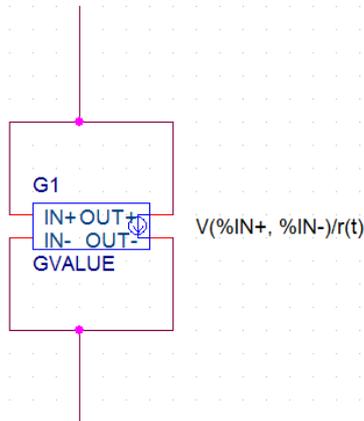


**Figure 1.** RLC lumped element plasma model. *C1*, *L1* and *R2* represent the plasma capacitance, inductance and resistance respectively.

In this paper, the resistor *R2* in Figure 1 will be replaced by a dynamic resistance that is dependent on the current, by solving hydrodynamic equations for plasma resistance. Ultimately this paper will show that the plasma inductance can also be simulated simultaneously with resistance.

#### 3.2 DYNAMIC PLASMA RESISTANCE

In this paper, a time-varying dynamic plasma resistance which is controlled by hydrodynamic models for resistance is used. Voltage-controlled current sources are used to represent a plasma resistance. Using the analogue behaviour modelling (ABM) package in PSpice, the hydrodynamic equations are solved, to control the voltage-controlled current source configured to represent a dynamic resistance. Figure 2 shows how a voltage-controlled current source is configured to represent a dynamic resistance.



**Figure 2.** Voltage-controlled current source representing a dynamic resistance, *R(t)*, controlled by function *r(t)*.

Including the dynamic resistor (Figure 2) as the resistive component of plasma in the *RLC* model, time-varying plasma resistance is considered. Where the resistance model uses current to calculate resistance, *R(t)*, the current-controlled voltage source is used to provide a signal representing time-dependent resistance in this calculation. This method was used to simulate the hydrodynamic models mentioned in Section 2 [1, 3-5].

#### 3.3 DYNAMIC PLASMA INDUCTANCE

This paper also presents a model for simulating the dynamic plasma resistance and inductance using PSpice. Using the ABM package, equation (1) for plasma channel radius can be used to calculate inductance. By assuming that the inductance of a filamentary plasma channel is similar to that of a conductive wire, the inductance can be calculated [11]:

$$L(t) = 2d \left[ \ln \left( \frac{2d}{a(t)} \right) - 0.75 \right] [nH] \quad (2)$$

where: *L(t)* is the time-varying plasma inductance, *d* is the discharge length and *a(t)* is the time-varying plasma channel radius. The inductive voltage drop is obtained as:

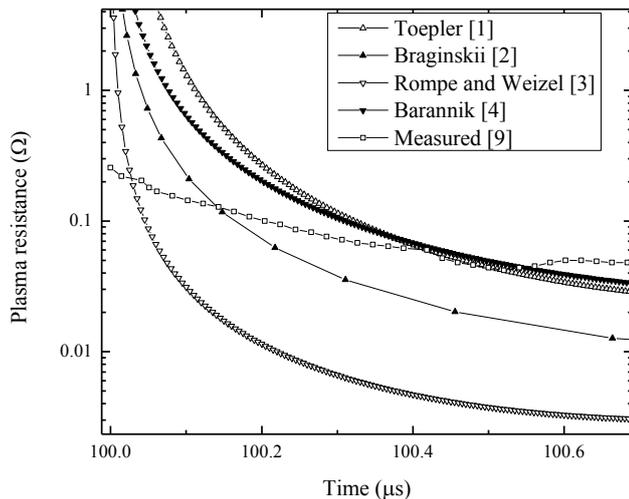
$$V_L = L(t) \frac{di}{dt} \quad (3)$$

where *L(t)* is the inductance and *i* is the circuit current. The ABM package is used to solve equations (1-3) and the inductance is implemented as a voltage-controlled voltage source. A model, which includes both dynamic inductance and resistance, has been solved (Figure 4).

### 4 RESULTS AND DISCUSSION

This paper presents PSpice solutions to hydrodynamic models of plasma resistance and as such the resistance curves are compared against the other models and experimental data taken from literature [3, 10]. For this comparison, the inductance is assumed to be a fixed value of 10 nH. To create a voltage impulse, an 80 nF capacitor was charged in parallel with a 120 kV DC voltage source. The output current pulse has been manipulated, by introducing circuit inductance (1 μH), to be similar to that of Akiyama *et al.* (peak current of ~20 kA and rise time of ~0.5 μs).

The equation coefficients have been selected from Engel *et al.* [2] for comparison with their simulations and to compare with the experimental data from Akiyama *et al.* [9]. Engel *et al.* normalised the resistance curves to be equal to the measured curve at time *t*=0.5 μs. Braginskii's model was not solved by Engel *et al.*, but similar experimental parameters were used in this paper. The integrator in the model by Rompe and Weizel required an initial condition which was set to  $1 \times 10^{-6}$ . Figure 3 shows the resistance curves for each model alongside an experimental curve.

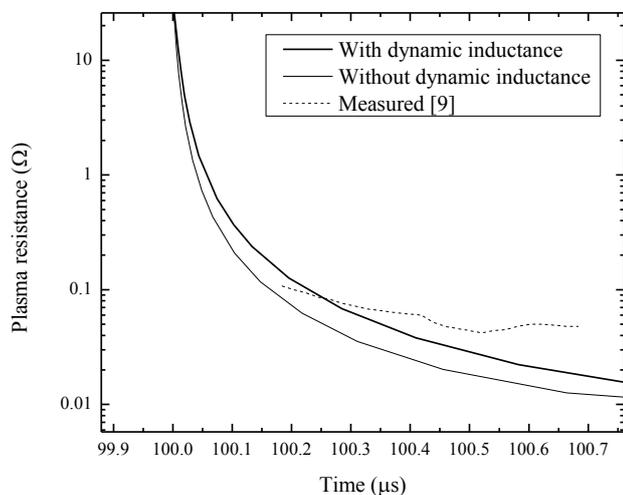


**Figure 3.** Plasma resistance taken from PSpice simulated hydrodynamic equations for resistance.

Shown in Figure 3, the plasma resistance curves for Toepler, Braginskii, Rompe and Weizel, and Barannik are compared against experimental data. Figure 3 shows that PSpice can be used to solve hydrodynamic equations for plasma resistance in a self-consistent model. The results are similar to the measured resistance at  $t=0.5\mu\text{s}$  for the models by Toepler and Barannik *et al.* However, the model by Rompe and Weizel was an order of magnitude different. The results for Toepler and Barannik models are similar to the results shown by Engel *et al.*, confirming the PSpice approach with traditional approaches to solving plasma resistance.

Using PSpice has the advantage that it could allow for much more complex circuits attached to the plasma resistance models. Many of the applications which are most interested in plasma-resistance simulations use complex pulsed-power circuits and it would be useful for them to include plasma resistance into their circuit simulations.

Using the method described in Section 3.3, the dynamic inductance of plasma is included in Braginskii's model alongside the dynamic plasma resistance. Figure 4 shows the plasma-resistance curve alongside the voltage and current wave-forms for Braginskii's equation for plasma resistance [3].



**Figure 4.** Comparison of plasma resistance from Braginskii's model [3] with and without dynamic inductance.

Using the dynamic plasma resistance and inductance, Figure 4 shows the plasma resistance with and without time-varying dynamic inductance alongside the measured plasma curve. This simulation shows that the dynamic resistance and inductance properties of plasma can be simulated simultaneously. The plasma-resistance curve from the model with dynamic inductance is closer to the measured plasma resistance.

## 5 CONCLUSION

In this paper, the plasma-resistance models in literature have been simulated using PSpice utilising the analogue behavioural modelling (ABM) package. It has been shown how dynamic resistance and inductance of a plasma channel can be simulated using voltage-controlled current sources and voltage-controlled voltage sources. Various models from the literature have been implemented using the PSpice approach. The results have been compared to determine which model provides the closest fit to an experimental curve taken from the literature.

The results of the PSpice simulations have been compared with different approaches to modelling the same hydrodynamic models and it has been shown that the PSpice model provides similar results and can be assumed to be a viable method for simulating plasma resistance.

An advanced model for simulating plasma properties was developed with dynamic resistance and inductance simulated simultaneously. Comparing Braginskii's model for plasma resistance with a fixed inductance and with a dynamic inductance showed that using time-varying inductance can increase the accuracy of plasma-resistance simulations.

Future work in this area will include further investigation of different plasma-resistance models in literature, dynamic inductance included in all models, comparison between the advanced models with dynamic inductance and fixed inductance, and comparison with previously published results. A new model for plasma resistance could be developed using new experimental data and the results from the PSpice simulations.

## ACKNOWLEDGEMENTS

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