Primary special school teachers' knowledge and beliefs about supporting learning in numeracy

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Abstract

This article presents findings from a qualitative study of a group of twelve teachers in primary special schools in Scotland for children with moderate learning difficulties. It sets out an analysis of classroom observations and interviews that explored teachers' knowledge and beliefs about teaching and learning in mathematics with children with moderate learning difficulties. The teachers were interviewed pre- and post-intervention; this was a research-based professional development programme in children's mathematical thinking (Cognitively Guided Instruction) which teachers then developed in their classrooms. The findings showed that prior to the professional development the teachers had a limited knowledge of children's mathematical development with teaching frequently informed by intuitive beliefs and dated and sometimes discredited practices. Most teachers had low expectations of children with learning difficulties. Post-intervention the teachers reviewed this stance and affirmed that a deeper understanding of children's mathematical thinking provided a more secure knowledge base for instruction. They also recognised the extent to which learners were constrained by existing classroom practices. The paper argues for the commonality of this knowledge base and considers the problematic nature of viewing such knowledge as sector specific.

Key words
Moderate learning difficulties; inclusive pedagogy; teachers' knowledge and beliefs; pedagogical content knowledge

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Introduction

This article presents the findings of a study which explored Scottish primary special school teachers' knowledge and beliefs about the teaching and learning of mathematics with children identified as having moderate learning difficulties before and after a period of professional development which focussed on children's mathematical thinking. The study focused specifically on numeracy as an aspect of mathematics rather than other elements of the mathematics curriculum. Discussion of mathematics in this article relates solely to numeracy. Within this context numeracy is recognised as the ability to process, communicate and interpret numerical information (Askew, Brown, Rhodes, et al., 1997, p.11).

There has been a growing body of research evidence, over the last 30 years, in the area of mathematics teaching connecting effective learning with teachers' knowledge of how children are conceptualising (and misconceptualising) mathematical principles (Maclellan, 2012; Jacobs, Lamb & Philipp, 2010; Greer & Meyen 2009; Ball, Thames & Phelps, 2008; Ryan & Williams, 2007; Peterson, Fennema, Carpenter & Loef, 1989; Carpenter, Fennema, Peterson & Carey, 1988).The recognition of this kind of pedagogical knowledge as relevant for all children connects with inclusive arguments which promote the extension of what is commonly available for all as a means of accommodating difference (Ylonen & Norwich, 2012; Florian & Black-Hawkins, 2011; Norwich & Nash, 2011; Hart, Drummond & McIntyre, 2007). This challenges the notion of particular pedagogical knowledge specific to particular groups of learners (Florian & Linklater, 2010; Lewis & Norwich, 2005).

Teachers' pedagogical decisions are driven by the complex interplay between knowledge, beliefs and contextual factors (Lalvani, 2013; Turner, Christensen & Meyer, 2009). Beliefs about the nature of teachers' knowledge, the kind of knowledge which teachers consider desirable and their capacity to support particular learners relate to the teaching of pupils with additional support needs (Florian, 2008) and the area of mathematics teaching (Ma, 1999). However there is little empirical evidence of how these aspects of teacher knowledge and beliefs relate to the instruction of children with moderate learning difficulties (Ylonen & Norwich, 2012; Fletcher-Campbell, 2005).

Knowledge and beliefs

There is an important relationship between teachers' beliefs about the content of their teaching and their actual knowledge of how this content can be structured and taught (Walshaw, 2012; Turner, Christensen & Meyer, 2009; Peterson, Fennema, Carpenter et al., 1989). Instructional practice is not simply determined by teachers' beliefs; other factors contribute to the complexity of this outcome:

curriculum guidelines, policies, pupil behaviour and management systems among other things. Teachers' beliefs, whilst not being exclusive determinants of classroom culture, have a significant bearing on it (Lloyd, 2002; Peterson, Fennema, Carpenter et al., 1989). Teachers' knowledge also has a significant influence on classroom practice (Empson & Junk, 2004; Hiebert, Gallimore & Stigler, 2002) but the boundaries between knowledge and beliefs are not always clear. In the blurring between knowing and believing, belief is perceived as the weaker of the two conditions (Wilson and Cooney, 2003, p131). However, if in the course of confronting new or different pedagogical approaches teachers' beliefs are challenged (Lloyd, 2002), these experiences may lead to a growth in knowledge and a change in prior held beliefs.

Pedagogies that are inclusive and supportive of all learners require more than the development of domain-specific knowledge; the current focus on teachers' subject knowledge in mathematics (Donaldson, 2011; Middleton, 2010; DCSF, 2008) provides an incomplete picture of the kind of development required. Strong mathematical knowledge is not necessarily linked to a deep understanding of children's mathematical thinking (Empson & Junk 2004). However knowledge of children's mathematical thinking can be a powerful instructional pointer (Fennema, Franke, Carpenter et al., 1993) leading to higher achievement (Peterson, Fennema, Carpenter et al., 1989) which facilitates an educational response to the learning needs of pupils who struggle in their mathematical learning (Behrend 2003; Empson 2003). Responding to the needs of individuals on the basis of teachers' knowledge of children's thinking is both challenging and complex and is connected to the type of professional development that teachers undertake (Jacobs, Lamb & Philipp, 2010).

If better outcomes for learners can be achieved by developing teachers' own mathematical competence, such a concern may be deemed by some to be less of an issue for teachers working with those children functioning at the early stages of their mathematical learning. In other words, because children are working with more fundamental mathematical principles, then their teachers only require sufficient mathematical knowledge to support these fundamental understandings. However research evidence would refute this proposition suggesting that all teachers require an informed knowledge of children's mathematical thinking (Jacobs, Lamb & Phillip, 2010**;** Empson & Junk 2004; Fennema, Franke, Carpenter et al., 1993). Shulman's (1986) construct of *pedagogical content knowledge* (PCK) affords a more complex representation of the type of knowledge required by teachers. It centres on the synthesis of knowledge of subject matter, teaching approaches, the curriculum and the learners. It is a uniquely integrated form of knowledge possessed by those who teach as opposed to those who are experts in subject matter. The concept of *PCK* has been elaborated further in the field of mathematics teaching taking into account the type of specialised knowledge involved in understanding children's mathematical thinking and how this knowledge might usefully support student learning (Ball, Thames & Phelps, 2008; Alexander, 2004). Shulman describes PCK as 'most likely to distinguish the understanding of the content specialist from the pedagogue" (1987, p. 8). It is this complex and integrated body of knowledge, further conceptualised in the field of mathematics by Ball, Thames and Phelps (2008) which allows teachers to make informed instructional decisions about how best to present particular mathematical ideas.

A key element of this pedagogical decision-making process is actually knowing what to look for, Jacobs, Lamb and Philipp (2010) describe this as 'professional noticing' which is theoretically informed practice. Unless teachers know what to look for in terms of children's mathematical activity and understand the significance of children's particular strategies, they may recognise certain strategies as inefficient but dismiss these as evidence of lack of proficiency or perhaps even intellectual capacity. So to consider this in an example of a single digit addition problem (7+9): an 11 year old child might consistently achieve correct answers by setting out cubes; first by counting out seven cubes, then nine cubes and then joining both sets and counting from 1 to arrive at 16. The child may complete many problems correctly in this way. This is a commonly used strategy, one which persists with children who struggle in their mathematical learning (Ostad 1997). Without observing and recognising the child's strategy a teacher might be satisfied that all the problems were correctly answered, albeit slowly. The knowledge that allows a teacher to recognise the strategy and understand that it may prove to be problematic when applied to larger number (73+ 96) is specialised, as is knowing how to interpret the child's strategy and knowing how to intervene in ways that are supportive and efficacious. Although this knowledge may be considered to be specialised it is universal in its relevance to all learners.

Teaching children with learning difficulties

Children who struggle in their mathematical learning follow the same trajectory as those who do not struggle (Dowker 2004; Geary 2004) and there is evidence of children with learning difficulties demonstrating the same sense-making strategies as their mainstream peers (Moscardini, 2010; Behrend 2003; Empson 2003; Baroody 1996). This begs the question of what, if anything, is or might be different or unique about the knowledge base of teachers in special schools that allows them to support the mathematical learning of their pupils. Pupils in primary special schools for children with moderate learning difficulties follow the same mathematics curriculum as their mainstream counterparts. In Scotland the content of this is outlined in Curriculum for Excellence (Scottish Government 2010) and previously in the 5-14 National Guidelines (SOED 1991); in England in the National Curriculum (DfEE 1999). These documents set out a pathway for the teaching if not the learning of the mathematics curricula and are relevant to both mainstream and special sectors.

Notwithstanding issues of definition, the group of children recognised as having moderate learning difficulties (MLD) constitutes the largest group of learners with additional support needs in the UK educational system (DCSF 2009; Norwich & Kelly 2005; Fletcher-Campbell 2005; Crowther, Dyson, Elliot et al., 1998). Statistical evidence shows that a large percentage of this group are educated within the mainstream system (DCSF 2009). In spite of problems of definition (Norwich & Kelly 2005; Crowther, Dyson, Elliot et al.,1998) - and it is beyond the scope of this paper to enter into discussion on the usefulness and relevance of the term *moderate learning difficulties* - it is generally accepted that this is a large and heterogeneous group of learners who are not usually recognised until they enter the school system and whose learning difficulties are non-specific in that they experience difficulty across the curriculum. In Scotland the category of moderate learning difficulties is no longer reported as a discrete category in Scottish Government records. There is an identified 'lack of clarity and consistency' in the collation of data around pupils with additional support needs (HMIE, 2010, p.9). Nevertheless an analysis of Scottish Government Statistical Bulletins, Pupils in Scotland records from 2004 to 2011 shows the learning disability group, however that might be comprised, as the largest group of pupils with additional support needs. The schools that participated in the study were classed as MLD schools, children within the schools reflected this profile of learner and mathematics was but one of the areas of the curriculum which they had difficulty accessing.

Cognitively Guided Instruction

Cognitively Guided Instruction (CGI) (Carpenter, Fennema, Franke et al., 1999) is a research-based framework which aims to help teachers understand and respond to children's mathematical thinking. CGI is built on the thesis that children come to school with intuitive and informal mathematical knowledge which serves as the basis for developing more formal mathematical understanding. Teachers learn to focus on children's understanding; this in turn provides a context for teachers to develop their own pedagogical knowledge. Thus teacher learning becomes a dynamic process situated within classroom interactions and interpretations; by enhancing their knowledge of students' thinking teachers are better placed to design appropriate instructional tasks and to support individual student learning more effectively.

In practice CGI involves the use of mathematical word problems. In the development sessions, during the study, teachers were provided with two related research-based frameworks, one outlining the structure of word-problem types and the second relating to children's solution strategies. The first framework provides teachers with a research-based tool that can be used in practice to design instructional activities. Teachers learn that addition and subtraction can be seen as processes of joining and separating. For example in a problem such as 'There are 3 children on the bus. At the bus stop 5 more children get on. How many children are on the bus now?' the story of the problem dictates a joining action. Whereas in a problem such as 'There are 3 boys and 5 girls on the bus. How many more girls are there than boys?' there is no action in the story that suggests either joining or separating and children may come to a solution by diverse strategies which might include setting out and matching both sets and determining the difference. Understanding how word problems are structured provides teachers with a framework to inform their instruction and guide students' conceptual understanding.

The second framework is research-based knowledge of children's solution strategies. This framework provides teachers with a developmental trajectory of children's mathematical understanding. As pupils engage with particular problems teachers learn to interpret their intuitive solution strategies and use this analysis to inform their teaching. For example, teachers learn to recognise the difference between children using materials to model out problems from children using particular counting strategies or knowledge of number facts to solve problems. In this way, commensurate with a constructivist philosophy, teaching is a dynamic process based on building on the sense that children are making of problems by using knowledge of children's mathematical thinking to inform instruction.

Research question

This article reports findings to the specific question:

What are teachers' in primary special schools expressed beliefs and knowledge (pre- and post-intervention) about teaching and learning in numeracy for pupils with moderate learning difficulties?

Study Design

The study involved 12 primary teachers in three Scottish primary schools for children with moderate learning difficulties. The schools were within the same local education authority. The maximum class size was ten pupils. The sampling was purposeful (Patton 2002) with the involvement of three special schools permitting a replication logic, yielding findings that could be considered more robust. Replication logic involves each case undergoing individual observation and analysis prior to cross-case analysis (Yin 2003). Triangulation was supported through cross-case analyses of: initial interviews; teachers' records and accounts of pupil progress; and post-intervention, teachers' records of pupils' strategies and engagement; classroom observations by the researcher and post-intervention interviews.

Given the criteria for admission into moderate learning difficulties schools, the schools and pupils within them could be considered representative of that sector within the particular local authority. Each school was invited to nominate participant teachers for the study. The study conformed to the requirements of the University of Strathclyde's Ethics Committee.

The study was designed over three phases:

Phase 1

The aim was to determine teachers' existing knowledge, beliefs and current practice prior to being introduced to the principles of CGI. This phase involved individual semi-structured interviews and an analysis of current planning and assessment procedures.

Phase 2

Professional development in CGI, which involved eight hours of intensive problem-based learning. The sessions focussed on developing an understanding of two frameworks: word problem types for addition and subtraction and children's solution strategies (Carpenter, Fennema, Franke et al. 1999). Teachers then ran a minimum of ten CGI sessions in their classrooms recording their observations of children's engagement. The brevity of the professional development would be considered a limitation of the study. Time constraints and availability of the participants precluded more extensive professional development.

Phase 3

This was the post-intervention phase. The aim was to determine the effects of the professional development undertaken by the teachers in terms of changes in knowledge and beliefs and impact on practice. This phase involved the use of semi-structured interviews and analysis of classroom-based observations of CGI sessions.

Analysis

Data were analysed adhering to an iterative method, 'Framework', developed at the National Centre for Social Research (UK). Framework is a matrix-based analytic method that permits a rigorous and systematic analysis of data. At each stage of the analysis it is possible to work at increasing levels of abstraction with the original data being accessible at each stage of this process (Ritchie, Spencer and O'Connor 2003). All interviews were transcribed, then read and re-read. Topics were identified and grouped into categories to develop an indexing system. Once all the transcripts were indexed thematic charts were built following the framework outlined by Ritchie, Spencer and O'Connor (2003). The thematic charts allowed data to continue to be analyzed across categories by the participants and also a cross-sectional analysis of each category. To ensure reliability of the indexing, random-sampled transcripts were cross-checked by blind-coding. The final interviews were indexed using the same categories. These data were then ordered within the initial interview thematic charts permitting pre- and post-intervention analysis.

Additional hardcopy and observational data drawn from a range of sources gave an insight into classroom practices and pupil engagement. Hardcopy and observational data comprised: class planning and pupil assessment records; hardcopy and photographic evidence of pupils' work; videoclips of classroom episodes; teachers' fieldnotes; fieldnotes of researcher-observed CGI sessions; researcher journal comments; email correspondence.

Findings

Following the short but intensive period of professional development in CGI the teachers implemented a series of ten CGI sessions in their classrooms. The findings are presented comparatively, pre- and post- professional development in CGI, and under themes that emerged through the data analysis process: subject knowledge; pedagogical knowledge; knowledge of learners and beliefs about learners, learning and teaching. Data are drawn from interviews, classroom observations, fieldnotes and teachers' annotated accounts of the sessions. Pseudonyms have been applied.

Knowledge

Subject Knowledge

Prior to professional development in CGI all twelve teachers felt that their subject knowledge was sufficient for the level of teaching that they were working at with two teachers considering themselves to be 'highly knowledgeable'. The general view expressed was of a knowledge level fit for the level of mathematical instruction that might be expected of teachers working with children with learning difficulties,

'I feel that at this level and also the fact that it is MLD primary... I have a good knowledge.' (Mike, Nevis School)

'... with the maths we do with these children everybody is confident...' (Marjorie, Alder School)

Although growth in subject knowledge was not anticipated following the professional development sessions the implications of knowledge growth in other areas is evidenced below.

Pedagogical knowledge

The initial interviews showed that teachers' knowledge of teaching approaches was mixed but generally limited. Two teachers acknowledged their lack of pedagogical knowledge, one teacher gave a particularly self-deprecating view,

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'I don't know any of it, I know absolutely nothing…you know there are different ways of teaching things, I'm sure there must be lots of different ways.' (Kirsty, Nevis School)

Several teachers talked of the importance of 'practical' approaches but these were not clearly or specifically articulated. Practical approaches generally referred to the use of concrete materials rather than to a specified pedagogical approach. This was also evidenced by the teachers' accounts of their use of resources. For example, pre-intervention Mary gave an account of practice that reflected a behavioural, transmission approach,

' …practical is, well using practical materials, cubes or whatever… and actually showing them moving the cubes exactly what they have to do.' (Mary, Lawers School)

The term 'real-life' was also used by teachers to describe desirable contexts for learning. However this did not appear to translate into practice, as most teachers described working with formal mathematical concepts in order that this understanding might subsequently be transferred into real-life situations. Although several teachers advocated real-life contexts, an analysis of lesson plans and classroom observations showed that no teacher employed real-life scenarios as a context for learning in the classroom.

Questions which probed pedagogy resulted with seven of the teachers giving an account of their knowledge of resources. One teacher expressed a desire to learn more about teaching methods. Overall the teachers displayed a limited knowledge of pedagogical approaches. The use of IT was not discussed in any detail by any teacher. Specific interventions such as Maths Recovery or Numeracy Recovery were not mentioned. Although some teachers indicated the importance of counting, there were few examples of discussion of children's counting. No teacher described a systematic approach to the teaching of counting.

In order to determine how teachers were teaching for understanding and whether they were supporting children to make connections between their procedural and conceptual understanding, the following question was asked in the initial interview.

What do you consciously build into your practice that enables children to make connections between procedures and their conceptual understanding?

Nearly every teacher found this question challenging, with some teachers unable to answer it at all even with prompting. Responses included:

'I don't know, I have never thought about that.' (Shona, Lawers School)

'I don't know if I do anything consciously, I just have a gut feeling what will work for a child.' (Helen, Nevis School)

' *How do I make it meaningful? ….I really can't think.'* (Mary, Lawers School)

> '*…ehm, well I don't know if I do much of that at all … I think it is just presumed that it is really too difficult for our children…I don't sort of think about that very much.'* (Paul, Alder School)

The uncertainty conveyed by the teachers in the initial interviews contrasted notably with their responses following professional development in CGI and the application of CGI in their practice. The teachers' records, accounts and classroom observations revealed that most teachers had grasped fundamental principles relating to CGI. They were also able to adapt this learning into existing classroom routines without the need for any organisational restructuring. Most teachers worked with whole classes, and a few with small groups of 3 or 4. Every teacher stated that it was a positive and beneficial experience with 10 teachers emphasising their own learning. Although the teachers quickly became familiar with problem types and found the pedagogical framework CGI provided to be useful, they understood that the professional development period was brief and recognised that deeper learning would require more time,

Mike: ' …when I think now about what I know about CGI, I think what I did in the past masqueraded as problem solving because it was merely contextualised problems lifted from the workbook we were doing'

Paul :'…it was good to know exactly what kinds of problems they are solving and to have a better idea of the strategies they are using to solve them'

Mary: ' This was condensed and short… I feel that I still don't know enough'

Kirsty's growth in knowledge was notable. From feeling that she knew '*absolutely nothing'* she now stated,

' I am actually more aware of what I can put into my teaching where I didn't before, I just knew I had to cover certain areas… now I can be more specific about each child'

Although working in CGI encouraged teachers to focus more on the learning than teaching, few teachers used their observations of children's strategies to design problems specifically to extend mathematical understanding. Some teachers saw CGI as about problem-solving as opposed to a means of engaging in mathematical sense-making activity. Two teachers, Harry and Marjorie, conceptualised CGI idiosyncratically, in these cases word-problems were administered as tests of factual knowledge followed by a demonstration of what Marjorie described as *'the proper way to solve it'*. Both these teachers held onto a transmission view of teaching.

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Knowledge of learners

This related to teachers' knowledge of individual children within their class, in general as well as specific to mathematics; it also related to their general knowledge of children's mathematical development.

Prior to professional development in CGI, every teacher had both specific and generic knowledge of their pupils, in the broadest sense they knew their pupils well. They were aware of particular areas of difficulty for particular children; they were also able to articulate an understanding of areas of difficulty that were reflective of children with moderate learning difficulty. They had a good knowledge of what had been covered in the curriculum by children in their class; this was evidenced by assessment records. However they were less clear on children's mathematical understanding. They were unable to specify how individual children might solve particular mathematical problems. All the teachers displayed only a limited knowledge of children's mathematical development. They struggled to explain how children might solve a problem such as 6+3. Several teachers described how they would teach this but when probed had difficulty in explaining what children might do.

Mary: 'I would just be looking for the way they're actually doing it. I can't even think…ehm… I don't know'.

'I really don't know what they would be doing… they just say 6 and forget about the process'. (Rita, Alder School)

Teachers did not use knowledge of children's mathematical thinking to inform planning. With the exception of one teacher whose planning was based on her own recordings and assessments of children's understanding, planning was informed by the next step set out in curriculum planners. Some teachers described planning as being informed by 'instinct', or 'gut feeling'. The need to develop knowledge in the domain of children's mathematical thinking was identified by several teachers.

No teacher demonstrated an understanding of the developmental progression in children's solution strategies involved in single digit addition and subtraction problems. Although some recognised the importance of counting they struggled to explain how this was connected to children's solution strategies. For five teachers children's emerging strategies were seen as a deficit, for example, children's use of manipulatives was used to illustrate what they were unable to do. Two teachers' knowledge of children's mathematics was particularly confused: counting was synonymous with addition and conservation of number was confused with object permanence.

Following professional development in CGI the teachers' accounts in interview and their annotated observations showed that they were attending to children's solution strategies although they found interpreting them challenging. Teachers were beginning to identify the strategies that individual children were using with varying degrees of accuracy. This aspect of teacher learning is recognised through CGI research as requiring significant time.

Mary: 'There are things that the children are doing that (still) baffle me'

Several teachers were surprised at pupils' ability to use their own strategies and explain their solutions. Post-intervention all teachers had an increased understanding of children's mathematical thinking as well as having a language that allowed them to articulate this understanding,

Mary: ' This is probably the first programme that has given me a progression as to how to give them the tools to work out problems.'

Paul: ' It gives [me] an idea of what level children are operating at…I can see the ones who are still direct modelling and that shows me that their understanding of number isn't quite as high as I perhaps thought it was.'

Importantly the teachers were focussing more deliberately on the children's strategies, they were more aware of what to look for. Some were redefining their role and beginning to question transmission models of teaching.

Rita: 'I am more conscious of watching them now rather than thinking about myself and how am I going to put it across to them… we were doing the same old thing that we had done for years …we are too busy giving them facts and I don't think we know enough about how they think'

Mary: 'It made me observe more closely whether they count or direct model [represent both sets in a problem usually with materials] … it does focus your attention to detail which surprised me'

This enhanced knowledge of children's understanding was also recognised as a useful framework for assessing and conveying information about learners,

'...it would give colleagues a fair idea of what ability the child had' (Anne, Nevis School)

Beliefs

Beliefs about learners and learning

Pre-intervention, in the initial interviews, the majority of teachers had limited or low expectations of children with learning difficulties. Learning with understanding emerged as a significant issue in these interviews with most teachers stating that learning with understanding was particularly difficult for children with learning difficulties. Only one teacher expressed a positive view of the children's potential believing that they had the ability to learn for themselves and from each other. A polarised view of learning potential emerged with some indication that the few teachers of a constructivist orientation connected the quality of learning potential to pedagogy, while those aligned to a behaviourist tradition expressed views that saw learning as a function of children's cognitive ability.

There was a frequently expressed view of children with moderate learning difficulties as unique or in some way qualitatively different from children without learning difficulties. This was evidenced by phrases such as *'our children'* preceding anecdotal descriptions or unsubstantiated claims such as *'our kids…are unable to manipulate ideas'* or *'I find that with MLD children, they learn best when they are moving'*. This uniqueness was highlighted in terms of the relationship between procedural and conceptual understanding,

Paul: 'I think that in the MLD sector children know how to go through the routine of a sum without perhaps having a real grasp of the number concepts…'.

Post-intervention every teacher felt that the pupils had benefitted. This was evidenced by their engagement and ability to explain their reasoning. A significant finding was that in the final interviews eleven of the twelve teachers stated that they had underestimated children's potential, particularly in their ability to explain; one teacher maintained a strong 'within-child deficit' stance. There was surprise that children with moderate learning difficulties were using the same strategies to solve problems as children without learning difficulties.

Paul: 'What it has shown me is that children are working with their understanding of number… it highlights the range and ability in a different way…I can see the ones who really do need support to make those number connections'

Beliefs about teaching

Pre-intervention most teachers believed practical approaches were of value for pupils with moderate learning difficulties. In line with the teachers' knowledge of pedagogy presented above, these 'practical' approaches although considered important, were generally undefined. 'Practical' in terms of teachers' expressed pedagogies rested more on the use of materials rather than on any contextual settings or real-life scenarios. 'Variety' was a word frequently used to describe what was believed important in teaching children with moderate learning difficulties. However when unpackaged this usually referred to a diversity of resources rather than to a range of pedagogical approaches.

Nine of the twelve teachers believed that effective teaching of children with moderate learning difficulties required a transmission approach through explicit strategy instruction. One teacher, advocated discovery approaches. Although several teachers believed it was important to encourage children to make connections in their mathematical thinking and to be able to transfer and apply this knowledge in real-life situations this was not reflected in their practice.

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Following the intervention teachers were beginning to question their previously held views about transmission approaches and were beginning to talk in terms of mediated learning, although one teacher held onto a transmission view.

' It has made me realise that we tend to meddle too much in children's learning… we want to do it for them but now you can stand back and let them get on with it and only really step in if they have a major problem… you still mediate, you still encourage them' (Lianne, Lawers school)

Helen: '… it has opened my mind to the way children think. There is too much pressure put on children ...this way you are actually encouraging children to play with numbers and you are creating an environment for them.'

Procedural competency was considered important by many of the teachers. For one teacher it was sufficient in itself. He viewed the ability to compute without conceptual understanding as a satisfactory outcome. Five teachers viewed it as a pre-requisite to be able to attend to problems. An analysis across categories revealed that the teachers who placed the most importance on procedural skills were those whose pedagogy and didactics followed a transmission approach.

Following the development of CGI in their classrooms most teachers were beginning to reveal the importance that they were now placing on children's mathematical thinking; the notion of children making 'connections' permeated the final interviews. This was a marked shift away from previously expressed views of procedural competency as sufficient. The impact that this was having on their teaching was apparent in terms of the importance that the teachers were now giving to accessing this thinking, some teachers were recognising that previously this had not been the case.

Kirsty: 'I am starting to know a lot more, a lot more than I had before ...as I said before, I didn't really understand a lot of what was going on… I actually lost the kids and I couldn't see what they were doing.'

Rita: 'It has been good for me, it has given me more understanding of watching how their minds work, if that makes sense.'

Discussion

The findings are consistent with previous American studies (Carpenter et al, 1999; Carpenter et al. 1988) in finding that prior to professional development teachers were operating at an intuitive level with fragmented knowledge of children's mathematical thinking. Pre-intervention there was no evidence of the type of specialised pedagogical knowledge unique to mathematics teaching called for by Ball, Thames and Phelps (2008), which allows teachers to look for and understand student errors and misconceptions and which in turn informs teaching. It also reflects concerns with mathematics education in the UK arising due to a 'lack of pedagogy' (Ryan & Williams, 2007, p.5), a systemic failure to connect theory and practice and a misplaced focus on improving teachers rather than teaching (Hiebert & Morris, 2012; Hiebert, Gallimore & Stigler, 2002). The challenge of learning with understanding for children with learning difficulties should be seen not as a within-child deficit, but as a problem of pedagogy and the requisite pedagogical knowledge (Ball, Thames & Phelps, 2008; Florian, 2008; Anthony & Walshaw, 2007).

Although the current investigation was a small-scale study and the results cannot be generalised, the findings showed that, prior to professional development, the participating teachers did not demonstrate a depth of knowledge about children's mathematics that would position them to support children's mathematical learning effectively. Post-intervention however, the positive message from the study was that the participants were highly receptive to the professional development and they recognised the value of this to their practice. It should be noted that the study related to a particular area of mathematics teaching with children with learning difficulties. No claims are being made about the generalisability of the findings to other areas of mathematics or to other domains of learning. Consideration might also be given to the extent to which it may be useful for teachers to develop a deeper understanding of psychological aspects of learning, particularly for pupils with more significant difficulties in learning.

The findings show that pre-intervention, in terms of 'professional noticing' (Jacobs, Lamb & Philipp, 2010) which is that capacity to recognise, understand and respond to children's conceptualisations, the participating teachers did not know what to look for. They knew the curriculum programme and they could identify next steps on this basis but they lacked knowledge of children's developmental trajectory in mathematics. Most participants believed that their subject knowledge was sufficient but they were generally unaware of the body of research-based knowledge of children's mathematics that could be applied in their classrooms. This is not to apportion blame on the teachers. There are issues of professional development at every level, from initial teacher education to post-qualifying that require to be considered (Ryan & Williams, 2007). In the absence of this research-based knowledge about learning, teachers were reliant on the intuitive beliefs described by Turner, Christensen and Meyer as stemming from 'common sense' and experiences in education (2009, p.361)

The lack of detailed knowledge of what children do in mathematics was evidenced by the particular difficulty teachers had in explaining what children might do when adding 6+3, along with their struggle to account for their pedagogy in terms of supporting conceptual understanding. Such knowledge may be deemed to be 'deep' knowledge and an indicator of quality of practice in schools, at least as far as guidance disseminated to Scottish schools is concerned (HMIE 2011), but without a clear account in specific domains of what characterises 'deep' knowledge the term is little more than rhetoric that fails to inform and support teachers and consequently pupils. Yet there is a body of research that indicates what this deep knowledge entails in terms of children's mathematical thinking (Baroody & Dowker 2003; Carpenter et al. 1999; Fuson 1988; Steffe et al. 1983) and how such knowledge might inform practice (Carpenter Franke & Levi 2003; Fosnot & Dolk 2001; Anghileri 2000). The challenge is not in delineating this knowledge base, it is in developing its use in practice through professional development activity that is both effective and sustainable.

It is interesting to note that all of the teachers believed their subject knowledge was sufficient for teaching children with learning difficulties but following professional development they recognised that the required knowledge was more complex. This specialised knowledge is domain specific (Ball Thames & Phelps, 2008) rather than sector specific. Further research would be required to determine the extent of the existence, or absence, of this knowledge base with the special sector. It brings into question the belief that specialized knowledge resides within a sector rather than recognising this as about individual teachers having developed a deep and integrated body of knowledge in particular domains, in this case numeracy. A deep understanding of how children conceptualise and misconceptualise mathematical principles allows teachers to make instructional decisions that are supportive and responsive to the particular understandings of the individual. This supports the concept of inclusive pedagogy in recognising the capacity that children have to make sense of their learning, in other words the 'transformability' described by Hart, Drummond and McIntyre (2007). In this respect the growth in knowledge demonstrated by the teachers reflected a change in beliefs (Lloyd, 2002) about the learners, and in particular about their capability. Prior to developing CGI in their classrooms the teachers displayed commonly held views of within-child deficits with no indication of the need to question pedagogy (Lalvani, 2013).

The special education system has been described as a mechanism for the identification and removal from mainstream schools of children whose needs might be best met elsewhere (Dyson 2001; Barton 1997; Tomlinson 1982) with an expectation of support that is in some way qualitatively different from that available in a mainstream setting (Florian, 2008). This separatist function sits uneasily with the counter-proposition that there is no distinct pedagogy required that is unique to children with moderate learning difficulties (Fletcher-Campbell, 2005; Lewis & Norwich, 2005). The tension generated by this anomaly is a real one, neatly expressed by Thomas and Loxley, (2007) who state that 'children who are difficult to teach have become by default 'special' children and teachers have begun to believe that they are not skilled enough to deal with 'special' children' (p.27). An extension of this logic is that if some teachers in mainstream schools feel ill-equipped to support the learning of some children in core curricular areas then it would be reasonable to assume that these children would benefit from the expertise of teachers in the special sector and that such expertise resides there. This is in line with the argument that the justification of a category rests on the educational benefits gained from any additional or different provision that results from that categorisation (Norwich & Kelly 2005, p.36). On this basis an expectation of educational benefit through effective support in core curricular areas such as literacy and numeracy would not seem unreasonable. The unique expertise believed to reside in the special sector is frequently used as a justification for the continued existence of segregated provision. Arguably the issue is that the absence of such knowledge constrains inclusive practice regardless of the setting.

Conclusion

The development of a knowledge base that permits an inclusive pedagogy is of relevance to all teachers and should not be seen as sector specific, teachers need to be and feel equipped to support all learners. It involves having depth of knowledge and understanding in specific domains and refraining from seeing expertise as residing elsewhere. It recognises the importance of learning and development from the point of view of the teacher as well as the child; learning and teaching is seen as a dynamic process situated in teacher-pupil interactions. Such interactions afford teachers an insight into children's conceptualisations that can then inform teaching purposefully. The relevance of a knowledge base which supports this kind of practice and the effect this may have on teachers' beliefs was clearly expressed in the final interview by one teacher,

'It has been quite an eye-opener for me I must admit….I can teach them and they can learn it but they might not understand it. I never gave that a second thought before. Quite honestly if they could do it I was happy, I'm not now."

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