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On energy release rates in heterogeneous composite laminates

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Abstract

Composite laminates are usually assumed to be homogeneous when determining the energy release rates (ERRs) associated with inter-ply delamination. This short paper discusses the effect of neglecting this assumption by accounting for inter-ply interface layer thickness and the resulting influence that this may have on the ERRs. A global approach is used to analytically determine ERRs for delaminations subject to mixed mode loading in symmetric double cantilever beam (DCB) samples of a material formed of alternating stiff and compliant layers. In contrast to their homogeneously determined counterparts these ERRs and their mixity are dependent on both sample depth and interface thickness and when compared the conditions under which obvious differences become apparent can be explicitly identified. Some brief conclusions on the application of the analysis to the prescription of practical delamination testing protocols for composite laminates are drawn.

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1. Introduction

Practical high performance composite laminates are usually comprised of stiff layers of resin pre impregnated reinforcing plies bonded together with more compliant interface layers of resin. The thickness of the more compliant interface layers is assumed to be very small compared to that of the reinforcing layers implying that the material is effectively homogeneous. By invoking homogeneity and employing Euler Bernoulli beam theory Williams (1988) was able to derive closed form expressions for the total, mode I and mode II ERRs together with their mixity for a number of layered composite delamination test samples when a variety of loadings are applied. However, the validity of partitioning the total ERR into its mode I and mode II components by this global approach was

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questioned by subsequent experimental evidence such as that of Davidson et al (1997) and Ducept et al (1999), particularly for samples with asymmetric geometry. Other analytical partitioning approaches that account for the local distribution of stress ahead of the crack tip have therefore been reported by for example Wang and Harvey (2012) and Williams (2015) himself. Current consensus acknowledges the equivalence of global and local partitioning approaches for test samples with symmetric geometry but for asymmetric test samples there appears to be no consensus and the applicability of each approach remains contentious. Furthermore, material homogeneity is invariably assumed in both approaches to partitioning.

If homogeneity is assumed then the flexural rigidities of the cracked and undamaged sections of a delamination test sample will vary as the cube of their depth, *d*. Thus, provided that the applied moments, *M*, are scaled according to $(M)^2 \propto (d)^3$, samples of different depths will yield the same ERRs. However, Wheel et al. (2015) have recently demonstrated that a heterogeneous laminate comprised of alternating stiff and compliant layers can exhibit size scaling effects reflecting either those forecast for more generalized elastic continuum theories as discussed by Lakes (1995) for example or, depending on the layer ordering, anomalous effects not forecast by such theories. These size effects can be quite prominent even when the thickness of the compliant layers is relatively small compared to that of the stiff layers and, furthermore, the flexural rigidity will no longer vary with depth cubed. The remainder of this paper uses the heterogeneous laminate model along with a global analysis to determine the ERRs in symmetric delamination tests. These are compared to the ERRs determined on the assumption of homogeneity. The circumstances under which this assumption is suspect can thus be clearly identified.

Nomenclature

b	laminate width
d, d_1, d_2	depth of intact and separating homogeneous laminate sections
E_0, E_1, E_2	flexural modulus of intact and separating homogeneous laminate sections
E, E_A, E_B	flexural modulus of heterogeneous laminate and constituent materials, A and B
G, G_I, G_{II}	total, mode I and mode II energy release rates
I_0, I_1, I_2	second moment of area of intact and separating homogeneous laminate sections
I	second moment of area of heterogeneous laminate
M, M_1, M_2	moments applied to intact and separating homogeneous laminate sections
M_I, M_{II}	globally partitioned moments associated with mode I and mode II delamination
i	integer index
n	number of layers of material A in heterogeneous laminate
t_A, t_B	thicknesses of heterogeneous laminate constituent material layers
y	distance from heterogeneous laminate section neutral axis

2. Size Effects and Energy Release Rates in Heterogeneous Composite Laminates

Figure 1 shows a typical laminate sample of breadth *b* and depth *d* with the laminate being comprised of alternating layers of two different materials, A and B, of moduli E_A and E_B respectively. The corresponding thicknesses of the layers are t_A and t_B. A delamination of length *a* resides within the core layer, comprised of material B. The upper part of the delaminated end of the sample is of depth d_1 while the lower part is of depth d_2 with $d_1 = d_2$ and they are loaded by corresponding bending moments M_1 and M_2 that are not necessarily equal in magnitude or sense. The intact end of the sample must be loaded by a moment, *M*, where $M = M_1 + M_2$. Given that material B forms the core layer then the number of layers of material A must be even. Furthermore, layers of materials B of thickness $\frac{1}{2}t_B$ are located adjacent to the laminate surfaces. Thus the cross sections of the intact laminate and the separating halves are all individually symmetric and the volume fraction of each material is independent of the number of material layers in all three sample parts.

Figure 2 shows further details of the cross section of the heterogeneous laminate. Although the number of plies of material A in the intact laminate section is even, the number of layers of this material in the delaminating halves may be either simultaneously odd or even. Both cases are thus depicted in Figure 2. If the number of plies of material A is denoted by n then the flexural rigidity, EI, of the laminate can be determined by summing the products of the layer moduli and their second moments of area about the section neutral axis. For odd n this gives

$$EI = 2\int_0^{t_A/2} E_A b y^2 dy + \sum_{i=1}^{(n-1)/2} 2\int_{(i-1/2)t_A+it_B}^{(i+1/2)t_A+it_B} E_A b y^2 dy + \sum_{i=1}^{(n-1)/2} 2\int_{(i-1/2)t_A+(i-1)t_B}^{(i-1/2)t_A+it_B} E_B b y^2 dy + 2\int_{(n/2)t_A+[(n-1)/2]t_B}^{(n/2)t_A+(n/2)t_2} E_2 b y^2 dy \tag{1}$$

while for even n it yields

$$EI = 2\int_0^{t_B/2} E_B b y^2 dy + \sum_{i=1}^{n/2} 2\int_{(i-1/2)t_A+it_B}^{(i+1/2)t_A+it_B} E_A b y^2 dy + \sum_{i=1}^{n/2-1} 2\int_{it_A+(i-1/2)t_B}^{it_A+(i+1/2)t_B} E_B b y^2 dy + 2\int_{(n/2)t_A+[(n-1)/2]t_B}^{(n/2)t_A+(n/2)t_B} E_B b y^2 dy \tag{2}$$

where y is the distance from the section neutral axis to a given material layer. Evaluating these summations gives the flexural rigidity as

$$EI = \frac{(E_A t_A + E_B t_B) b n^3 (t_A + t_B)^3}{12(t_A + t_B)} + \frac{(E_B - E_A) b n t_A t_B (t_A + t_B/2)}{6} \tag{3}$$

in both cases. Since the depth of the section is $n(t_A+t_B)$, the leading term here represents the rigidity of a homogeneous beam of modulus $(E_At_A+E_Bt_B)/(t_A+t_B)$, this being the mean modulus of the section. The rigidity thus depends on the cube of the section depth in the absence of the second term. This term, however, produces a size effect which depends on the relative magnitudes of E_A and E_B. When material B is stiffer than material A the rigidity will be increased. Such behaviour is consistent with that of Cosserat or micropolar like continuum behaviour as demonstrated recently by Wheel et al. (2015). The variation in rigidity to depth ratio with section depth squared for the case where $E_B = 10E_A$ and $t_B = t_A$ is shown in figure 3. The positive intercept associated with this variation is indicative of such behaviour according to Lakes (1995). The intercept would be coincident with the origin if the material were exhibiting classical, size independent behaviour. A paradoxical size effect that is not anticipated by either classical or the more generalized continuum theories is forecast when material B is more compliant than material A. The case where $E_B = 0.1E_A$ with $t_B = t_A$ is also shown in figure 3 where an apparent increase in flexibility as depth reduces is implied by the negative intercept.

Now, according to Williams (1988) the total ERR, G, given by

$$G = \frac{1}{2b}\left\{\frac{M_1^2}{E_1I_1} + \frac{M_2^2}{E_2I_2} - \frac{(M_1+M_2)^2}{E_0I_0}\right\} \tag{4}$$

can be partitioned into mode I and mode II ERRs, G_I and G_{II}, according to:-

$$G_I = \frac{M_I^2}{2b}\left(\frac{1}{E_1I_1} + \frac{1}{E_2I_2}\right) \tag{5}$$

and

$$G_{II} = \frac{M_{II}^2}{2b}\left\{\frac{1}{E_1 I_1} + \frac{(E_2 I_2/E_1 I_1)^2}{E_2 I_2} - \frac{[1+(E_2 I_2/E_1 I_1)]^2}{E_0 I_0}\right\} \quad (6)$$

where E_0I_0, E_1I_1 and E_2I_2 are the flexural rigidities of the intact and lower and upper separating parts of the sample respectively. This partitioning assumes that the moments M_I and M_{II} associated with modes I and II delamination can be expressed in terms of the applied moments M_1 and M_2 via

$$M_I = \frac{(E_1 I_1)M_2 - (E_2 I_2)M_1}{E_1 I_1 + E_2 I_2} \quad (7)$$

and

$$M_{II} = \frac{(M_1 + M_2)(E_1 I_1)}{E_1 I_1 + E_2 I_2} \quad (8)$$

By assuming material homogeneity Williams (1988) was able to simplify these expressions since the flexural rigidities of the three parts of the sample scale as the cube of their depths. It is then possible to demonstrate that both mode I and mode II ERRs along with their mixity will be independent of the separating section depths when the loadings are scaled as indicated previously. This implies that samples comprised of differing numbers of reinforcing plies having a common thickness separated by interfaces of a given depth should all yield the same ERR after scaling. However, this neglects the size effects forecast for a heterogeneous composite laminate by equation 3. This paper thus addresses the question of what influence might material heterogeneity have on the modes I and II ERRs determined by the foregoing global analysis and, moreover, whether it might also affect their mixity resulting from partitioning according to equations 2 and 3.

3. Mixed Mode Loading of the Symmetric Double Cantilever Beam with $E_B < E_A$

When $M_1 = 0$ and $M_2 > 0$ a symmetric DCB sample is subject to mixed mode loading with a mode mixity, G_{II}/G_I, of 0.75 assuming homogeneity. The ratio of interface modulus to that of the reinforcement is now set as $E_B = 0.01E_A$ this being indicative of typical fibre reinforced composite laminates. Since interface layer thickness is less certain the thickness ratio is first prescribed as $t_B = 0.1t_A$ and then $t_B = 0.01t_A$. The moment M_2 is scaled according to $(M_2)^2 \propto (d_2)^3$ so that as the laminate depth is varied both ERRs would remain constant if the material were homogeneous. Figure 4 shows how both the modes I and II ERRs vary with sample depth as quantified by the number of layers of material A, n, when equation 3 is used in determining the flexural rigidities of the heterogeneous material. The ERR variations shown here are normalized with respect to their homogeneous counterparts which are independent of laminate depth when the loading is suitably scaled. When $t_B = 0.1t_A$ both ERRs clearly depend on the laminate depth. When n is large they asymptotically approach the depth independent homogeneous value but as n is reduced both the mode I and mode II ERRs begin to exceed the corresponding homogeneous values. In the case of the thinnest possible laminate the mode I ERR is approximately 20% greater while the discrepancy exceeds 25% for the mode II ERR. Furthermore, figure 4 suggests the mode I and mode II ERRs do not vary proportionately as n is reduced. Thus the mode mixity must also depend on the laminate depth as shown in figure 5. While figure 4 indicates that both ERRs show significant variation as depth is reduced figure 5 implies that the variation in the mode mixity is less pronounced. However, it cannot necessarily be ignored since it deviates from its homogeneous equivalent by nearly 5% in the case of the smallest sample.

Variations in the normalized ERRs with laminate depth when $t_B = 0.01t_A$ but all other parameters remain unchanged are also shown in figure 4. These variations are now noticeably less markedly and for the thinnest sample the disparities are now only around 1% for the mode I ERR and 2% for the mode II ERR. Thus the increased material homogeneity resulting from the reduction in interface thickness diminishes the dependency of the ERRs on depth as expected. Similarly, as seen in figure 5 the variation in the mode mixity with depth also becomes less discernible as interface thickness is reduced.

4. Discussion and Conclusions

Previous analyses of delamination within fibre reinforced composite laminates have invariably assumed that the interface layers bonding two adjacent reinforcing plies are of negligible thickness. The analysis presented in this short paper incorporates finite interface thickness thus making it possible to determine how, as a consequence, the ERRs associated with interfacial delamination are influenced. In conclusion, it appears that the usual assumption of material homogeneity is valid when the ratio of interface to ply thickness is $O(10^{-2})$ since there is little influence on the ERRs. However, when this ratio is increased to $O(10^{-1})$ then material heterogeneity starts to have a noticeable influence on both of the ERRs and also their mixity. This influence becomes more marked as sample depth is decreased through reducing the number of ply and interface layers.

These conclusions may have important repercussions for standardized delamination testing methods for composite laminates. Such methods are well established in for example ISO (2001, 2014) standards for pure modes I and II delamination and an analogous ASTM (2001) standard for mixed mode testing. However, when used to test laminates in which interface thickness cannot be neglected then caution may need to be exercised when applying an analysis that assumes homogeneity to thin samples comprised of just a few reinforcing plies otherwise erroneous ERRs may be obtained.

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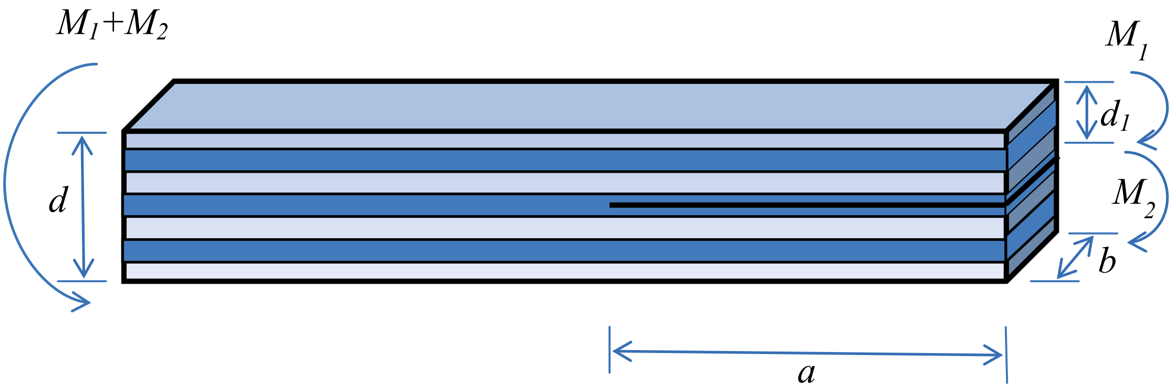

Figure 1 Composite laminate with delamination of length a

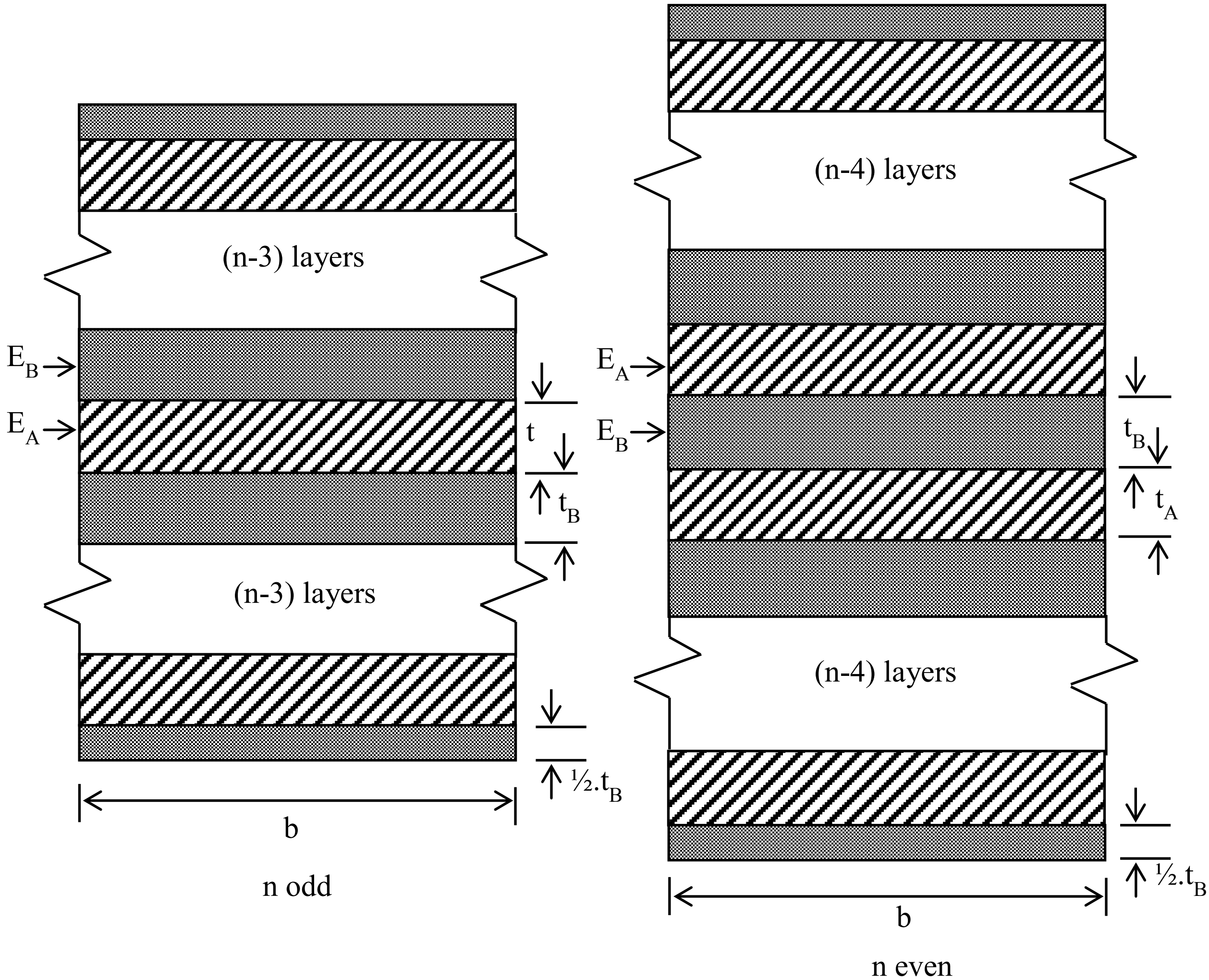

Figure 2 Details of laminate cross section

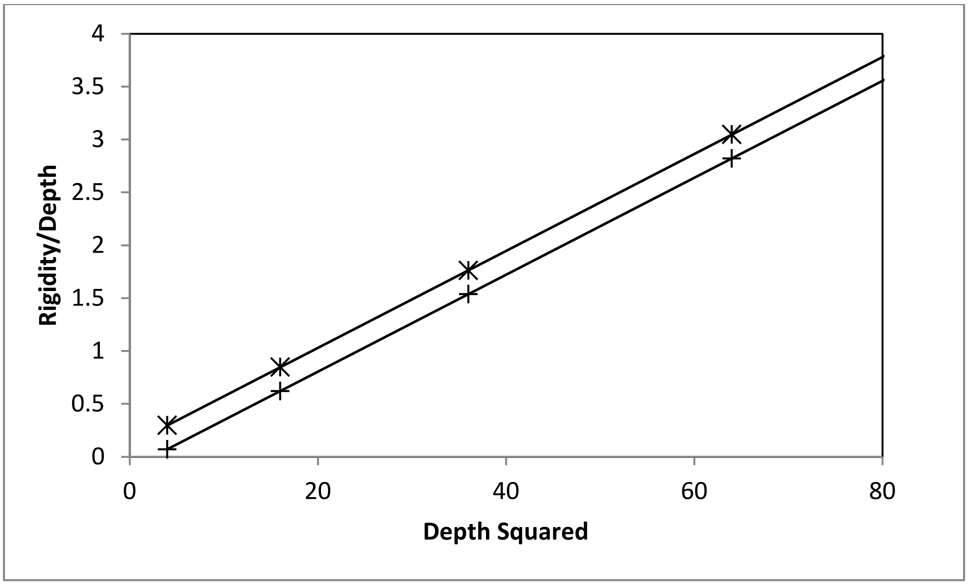

Figure 3 Variation in rigidity/depth with depth squared when $t_B = t_A$, $E_B = 10E_A$ (stars) and $t_B = t_A$, $E_B = 0.1E_A$ (crosses)

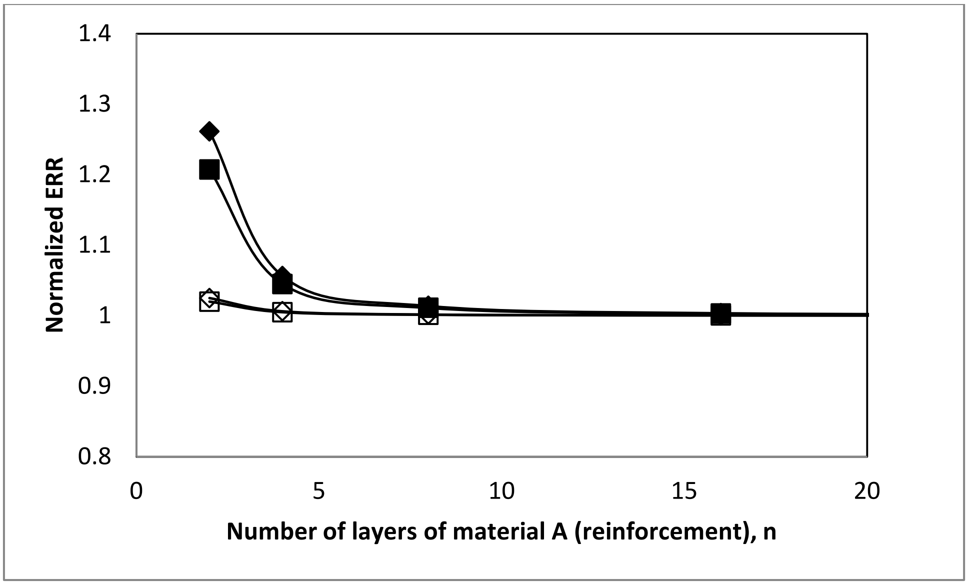

Figure 4 Unevenly loaded symmetric sample: variation in normalized mode I (squares) and mode II (diamonds) ERRs with number of layers of material A (reinforcement) for $t_B = 0.1t_A$ (filled markers) and $t_B = 0.01t_A$ (open markers)

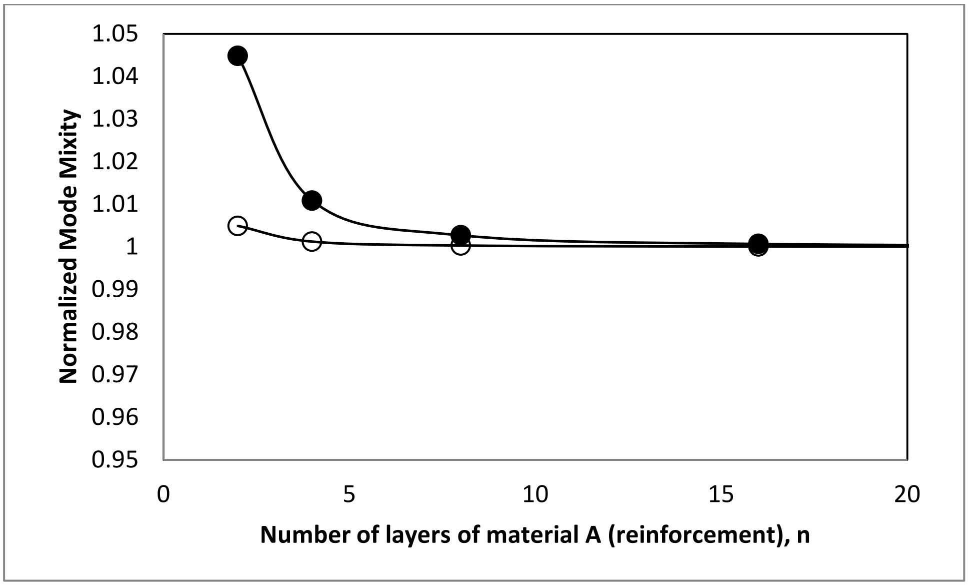

Figure 5 Unevenly loaded symmetric sample: variation in mode mixity with number of layers of material A (reinforcement) for $t_B = 0.1t_A$ (filled circles) and $t_B = 0.01t_A$ (open circles)