**<u>Mathematics</u> [Chapter 37 in Bryce & Humes (2008) Scottish Education]**

***Effie Maclellan, University of Strathclyde***

## CURRENT POLICY

Since 1991 the mathematics curriculum in Scottish primary schools has been underpinned by the framework document, *Curriculum and Assessment in Scotland: National Guidelines on Mathematics 5-14*, (Edinburgh, Scottish Office Education Department, 1991). This framework delineates primary mathematics as being concerned with the use and application of number, handling information and problem solving, and the properties of shapes, position and movement. Although schools were never legally required to follow the guidelines, the advice by Local Authorities on how to implement the guidance, the use of national testing to confirm teachers' decision-making, and the publication of reports by Her Majesty's Inspectorate on the inspection of schools, resulted in a dominantly uniform interpretation and execution of the Mathematics Curriculum (Maclellan et al, 2003). There is no evidence to suggest that a consensual approach to mathematics education will change, given that there is no specialist mathematics teaching in primary schools with many primary teachers, for different reasons, feeling ill-equipped to deviate from school or local authority direction.

## INFLUENCES ON EVOLVING POLICY

The official conduit through which policy decisions are made is SEED which takes advice from HMIE, LT Scotland, SQA, local authorities, and (to a lesser extent, historically) from academics. The recent report, *Improving Achievement in Mathematics in Primary and Secondary Schools* (HMIE, Edinburgh, 2005) is proud of the strong start our youngest pupils get in mathematics but foregrounds issues that have pervaded mathematics education for years: seeing the relevance of mathematics in other subject areas, an improvement in mental calculation in *all* pupils and an ability to solve problems. The current review of the mathematics curriculum flowing from *A Curriculum for Excellence* recognises that Scotland needs both specialist mathematicians and a highly numerate population. To this end it proposes:

- essential numeracy skills, including arithmetical skills
- a secure understanding of the concepts, principles and processes of mathematics
- an understanding of the application of mathematics
- firm foundations for further specialist learning

as being key to improving mathematical performance in our pupils.

Currently, another major influence on policy is Assessment is for Learning (AiFL), a national initiative which has the intention of developing a streamlined and coherent system of assessment for Scottish schools that will support learning. This initiative and its evaluation is under the strategic management of the Assessment Action Group (AAG) comprising representatives from education authorities, schools, university faculties of education, parent groups, professional associations, the Scottish Qualifications Authority (SQA), Learning and Teaching Scotland, and the Scottish Executive Education Department (SEED). The AiFL initiative has been seminally influenced by the evidence for the improvements in achievement that can be attributed to formative assessment. The work of Black & Wiliam in reviewing research evidence for the effects of assessment has had wide-ranging influence on mathematics policy (Hodgen & Wiliam, 2006).

Although changes in policy have traditionally been rooted in the views of experienced practitioners (the basis on which the entire edifice of the *5-14 National Guidelines* were constructed), it is heartening to note that the current initiatives shaping policy are being informed by educational research findings. Fujita, Magill & Munn in their review of the mathematics research literature for *A Curriculum for Excellence* (Scottish Executive, Edinburgh, 2004) draw attention to the relevance of research into mathematical teaching practices (to enable the development of an informed understanding of practitioners whose engagement with the curriculum discourse will be crucial to its success); and into the learning process itself (to enable children's learning in maths to inform subsequent research).

**MATHEMATICAL PERFORMANCE**

Since 1991 National Testing has been a distinctive element but the results were never collated into a picture of mathematical competence in Scottish primary schools. Until 2004 there was an annual national survey of *5-14 Attainment in Publicly Funded Schools* by the Scottish Executive Education Department (SEED) reporting on the numbers in P2-7 and S1&2 performing at each of the attainment levels (A-E) specified in the framework document. As a result of SEED's consultation, *Assessment, Testing and Reporting 3-14* (2003) this survey was replaced in 2005 by the Scottish Survey of Achievement (SSA) to provide an overview of attainment levels at local authority and national levels. The SSA surveyed numeracy in 2006, although the resulting data are not yet available.

Other sources of information on performance in Mathematics are:

- Assessment of Achievement Programme (AAP)
- Programme for International Student Assessment (PISA)
- Trends in International Mathematics and Science Study (TIMSS)

The *Seventh AAP Survey of Mathematics* (SEED, 2004) reports positively on the achievement of almost 10,000 pupils, in terms of the levels in the National Guidelines for Mathematics 5-14.

The proportions demonstrating 'secure' knowledge (65% to 79% accuracy) or 'considerable strengths' (80% + accuracy) at the expected level for their stage progressively increased from 32% at P3 (Level B) through 40% at P5 (Level C) to 46% at P7 (Level D). At S2, where the expectation is Level E, the proportion of pupils achieving at least 'secure' status is only 37%. This is a concerning figure for S2, where as many as 41% are recorded as demonstrating less than 'basic' skills (between 50% and 64% accuracy) at Level E. As well as highlighting the need to increase the proportion of P3 and P5 pupils who are at least 'secure' in their grasp of mathematics at an earlier point in the expected time interval, there is a disquieting dip in 'secure' or 'strong' performance from P7 to S2. Findings in the 2004 survey were not statistically significant from those in the previous survey.

Beyond the Scottish context there are two international programmes of assessment in mathematics. While these different programmes may appear to have significant similarities, such as the age of pupils or content areas studied, each is designed to serve a different purpose and each is based on a separate and unique framework and set of items. Scotland took part in *Trends in International Mathematics and Science Study (TIMSS),* the international comparative study, in 2003 and will participate again in 2007. TIMSS assesses the performance of P5 and S2 pupils in mathematics and investigates the factors influencing their attainment. One of the key findings in 2003 was a decrease in mathematical achievement from 1999 amongst Scottish primary pupils. Outwith the primary school sphere, PISA studies offer participating countries comprehensive internationally comparable data about their educational systems, using 15-year olds as the target population. PISA studies emphasise the mastery of processes, the understanding of concepts, and the application of knowledge and functioning in various situations; all of which they describe as mathematical literacy. PISA 2000 ranked United Kingdom pupils eighth in Mathematics. PISA 2003 unfortunately does not report on UK performance because the participation rate was so low as to render comparisons problematic.

**WHAT COUNTS AS CURRICULUM CONTENT**

Historically, the curriculum was exclusively concerned with numerical computation, its associated notational language and the rules for formal processing. Although the inception of the *Primary Memorandum* (Scottish Education Department, Edinburgh 1965) laid the foundations for a mathematics (rather than just an arithmetic) curriculum that would develop a range of mathematical competencies in real-world contexts (such as shopping, travelling, cooking, and dealing with finance), it continues to be of concern that the attempted widening of the curriculum has not resulted in the mathematical understanding which allows skills application in situations where the directions are not so clear, where the context is not tightly structured and where individual decision-making about relevant knowledge and its application is central. In spite of the rich research literature which points to effective mathematical problem solving as a critical mechanism through which individuals can construct mathematical meaning, the primary mathematics curriculum in Scotland has been conceptualised very narrowly, as a utilitarian tool for accurate and speedy computation.

SEED's acknowledgement of the importance of mathematical understanding for full and equitable participation in a technologically advanced society is reflected in its aspirations, noted above, in its review of the curriculum. What is being proposed does privilege numeracy but this should not mean the dominance of rapid paper-and -pencil calculation skills. Rather there is a need to use numeracy as a site in which pupils develop methods for solving problems and gaining understanding about number systems and operations within number systems. To realise this, two criteria seem important in the determination of content. One is that the task problematises mathematics, not to meaningless difficulty but to the extent that the task invites pupils to explore the relationships that characterise mathematics. Another criterion in the determination of content is that the task should allow pupils to work out *for themselves* the completion method(s). Without such melding of conceptual understanding and procedures, pupils may forget procedures, learn flawed procedures or apply procedures rigidly to cause ineffective problem-solving. In essence, curriculum content should be determined on the basis of the mathematics of the situation rather than extraneous features which have no mathematical salience.

**WHAT IT MEANS TO TEACH AND LEARN MATHEMATICS**

Different teaching styles in mathematics are in an almost constant state of flux with successive generations seeking to alter the education system to address perceived deficiencies. In recent years the advice to teachers in *Improving Mathematics Education 5-14* (HMI, Edinburgh 1997) has been to:

- Move from mixed ability grouping to some form of setting by ability,
- Adopt more teacher-led whole class activity,
- Allow calculators only for well-defined purposes,
- Increase facility in mental calculation.

While such advice is given in the belief that it is emulating the teaching in high performing TIMSS countries, its specification necessarily ignores other potentially important aspects. Any advice on mathematics teaching should seek to preserve the inherent complexity both of learning and of the subject's structure; otherwise mathematics is conceptualised as sets of algorithmic procedures which, if not tied to the mathematical concept, result in meaningless learning.

For mathematical understanding to be more than an espoused aim, a fundamental change in our perceptions of teaching is needed. It is a common belief that the best way to teach mathematics is to explain important concepts (such as place-value) and thereafter have pupils practise. But meaningful mathematical relationships can only develop when pupils have to use the knowledge they have in the contexts of specific tasks. This means that the task (out of which the particular concept can emerge) is the mechanism through which pupils build understandings of mathematical relationships (Maclellan et al, 2003). Furthermore, effective teaching necessitates a consistent expectation that pupils will explain their thinking processes (Askew et al, 1997). In order that pupils can be assisted to think in complex ways, teachers need to structure discussions so that their questions cause reflection on mathematical mental activity (such as comparing, correcting, verifying, exemplifying, and reversing).

**WHAT IS PROBLEMATIC**

The essential problem for a sizeable proportion of pupils in primary education is low levels of mathematical literacy: the ability to understand and apply mathematical ideas proficiently. Not only are such pupils limited in the extent to which they can combine algorithmic procedures with mathematical ideas, they have considerable difficulty in making connections and relationships between the mathematical ideas themselves so that, for example, they do not appreciate the relationships among fractions, decimals, percentages, ratio and proportion. Low levels of mathematical literacy are recognised by SEED as a serious obstacle to our socio-economic survival and progress. Much of the research (and indeed common sense) supports the idea that traditional, transmission models of teaching, in which particular numerical concepts are taught 'discretely', discourage pupils from making the connections and relationships which are a major goal of mathematics education.

However, awareness that effective teaching depends fundamentally on understanding how pupils come to know mathematical ideas and on the corollary of giving pupils opportunities both to explain their thinking and to compare their thinking with that of peers/teachers, creates the potential for pedagogical practices to better realise the aims of mathematics education. Lest it be assumed, however, that pedagogical practice will change automatically, two caveats are in order. Firstly, while the historical emphasis on the procedural mathematics emphasised speed and accuracy, it is now being better appreciated that errors and misconceptions in mathematical learning are valuable in that they provide a useful insight into a pupil's thinking and understanding, providing the basis for formative assessment. Therefore, rather than teaching to *avoid* errors and misconceptions, greater value needs to be given to 'learning from mistakes', particularly when pupils are beginning to grasp complex, abstract ideas in an intuitive or concrete way. Pupils will only take risks and share their mathematical ideas when errors are acknowledged to be integral to learning: not when' right-answerism' is the dominant value position. So learning is not a matter of what is 'right' and what is 'wrong' but a matter of 'shifting' one's thinking or seeing things differently. This leads to the second caveat.

HMIE have repeatedly reported that pupils' excellent start in mathematics is not maintained from the middle of the primary school onwards; evidenced by lack of understanding of common and decimal fractions, of ratio and proportion, of percentages and of the relationships between all these numerical representations. This deterioration is typically noted with some regret and an exhortation to improve pedagogical practice. However, pedagogical practice is unlikely to improve if attention is not paid to the qualitative shift in thinking from additive to multiplicative reasoning that is a function of psychological maturity. A sophisticated understanding of number involves a development from natural number to rational number since it is this that underpins the mathematical concept of ratio, and others that subsume ratio. Although early success is attributable to robust, *intuitive* understanding of additive reasoning (which begins to develop long before children enter school), it would seem that multiplicative reasoning is more dependent on structured learning experiences (Howe et al,.2004) than is the more limited additive reasoning. Furthermore teachers' own understandings of multiplicative reasoning are key (Orton & Frobisher, 2005) since these are powerful mediators in their own practice.

In conclusion, mathematics education in Scotland is at an important stage in its development. It is beginning to be realised that improvements in pedagogical practice to achieve major, mathematical goals need to be informed by an evidence-base. At the same time teachers experience a reality of external requirement to cover mathematical content in order to fulfil the demands for public accountability. If radical improvement is to be achieved, teachers and policy makers must build on the recent inceptions of applied educational research which is specifically targeted at improving teaching and learning.

**REFERENCES**

Askew, M., Brown, M., Rhodes, V., Johnson, D. & Wiliam, D. (1997) *Effective Teachers of Numeracy*. Final Report. London: King's College.

Hodgen, J. & Wiliam, D. (2006) *Mathematics inside the Black Box*. London: nfer Nelson

Howe, C., Jafri, S., Nunes, T. & Bryant, P. (2004) Intensive quantities: why they matter to Mathematics education. Paper presented at *TLRP Annual Conference,* Cardiff, November 2004.

Maclellan, E., Munn, P. & Quinn, V. (2003) *Thinking about Maths: A Review of Issues in Teaching Number from 5 to 14 years*. Dundee: Learning & Teaching Scotland.

Orton, A. & Frobisher, L. (2005) *Insights into Teaching Mathematics*. London: Continuum