Hedge Fund Seeding via Fees-for-Seed Swaps under Idiosyncratic Risk

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Abstract

We develop a dynamic valuation model of the hedge fund seeding business by solving the consumption and portfolio-choice problem for a risk-averse manager who launches a hedge fund through a seeding vehicle. This vehicle, i.e. fees-for-seed swap, specifies that a strategic partner (seeder) provides a critical amount of capital in exchange for participation in the funds revenue. Our results indicate that the new swap not only solves the serious problem of widespread financing constraints for new and early-stage funds (ESFs) managers, but can be highly beneficial to both the manager and the seeder if structured properly.

G11, G12, G23

1 Introduction

There has been a significant increase in both the number of hedge fund seeders and the amount of capital available for hedge fund seeding since the aftermath of 2008's market upheaval.[1] However, there still remains a tremendous shortage of capital for new and early-stage funds (ESFs). This is mainly because most capital providers or institutional investors increasingly focus on larger established hedge funds whose assets under management (AUM) are usually larger than 1 billion and who are considered highly credible. Additionally a larger talent pool of ESFs managers is now competing for the scarce available seed capital. Worse still, barriers to entry for ESFs are much higher today than in the period before the 2008 financial crisis.[2]

Therefore, navigating the terrain to a successful launch of a hedge fund has become more difficult and the financing constraint faced by ESFs managers

[1] HFM-Week research reported in November 2011 that seeders had approximately \$4.6 billion in available capital, compared to approximately \$1 billion just one year earlier.

[2] The Dodd-Frank Wall Street Reform and Consumer Protection Act (Title IV) compels the U.S. Securities and Exchange Commission (SEC) to impose reporting requirements on all hedge funds as it deems necessary or appropriate in the public interest or for the assessment of systemic risk. According to "Launch bad; Hedge funds" (The Economist 20 Apr. 2013: 79), it is much harder now to break into the hedge-fund business than it used to be because of the rising expenses, more risk-averse investors and enhanced regulation.

nowadays is much more serious than before. In order to reach the initial AUM target and cover organizational expenses, more and more ESFs managers are likely to turn to seed investors for early stage of capital through a seeding vehicle. This is an arrangement to which we refer as fees-for-seed swap that specifies that a seed investor (or seeder) commonly commits to providing a remarkable amount of seed capital to an ESFs manager as an "anchor investor" in a new fund in exchange for a share of "enhanced economics" which is usually the fees that the ESFs manager generates from the entire pool of assets in the fund. If structured properly, the seeding approach can be highly beneficial to the ESFs manager and to investors who provide the seed capital. It is not uncommon that the hedge fund seeder receives a portion of the hedge fund's revenue stream to get greater return potential than an ordinary investor.

In general a seeder can expect about 1% of revenues for each $1 million of seed capital for seed transactions no larger than $50 million. However, seed arrangements can vary substantially based on factors such as the experience of the manager, the alpha record, the amount of seed capital provided, the withdrawal and lock-up period terms, and the relative negotiating power of each party.[3]

While the seed investor will often demand the flexibility to redeem her[4] investment as soon as possible, the manager needs (and should require) the seed capital to remain invested for a period sufficient to set its strategy, create a track record, and procure other investors. Generally, during the lock-up period, the seed investor should be prohibited from redeeming the investment if, in the reasonable judgment of the manager, doing so could adversely affect the interests of the other investors in the fund.[5]

The ordinary investors may withdraw capital if the fund shows poor performance. For simplicity , we assume that the withdrawal rate is constant. This is a common assumption in the hedge fund literature and has been employed by [Goetzmann, Ingersoll and Ross (2003)] and [Lan, Wang and Yang(2013)]. Also, depending on the terms of the deal, the seeder will generally commit to keep the investment in the fund for a defined lock-up period, typically two to four years. It makes sense to assume that during this initial phase, idiosyncratic risks take a more pronounced role as compared to later stages in the fund's life, possibly due to ordinary investors entering (or leaving) the fund and/or the fund manager experimenting with different asset classes in order to set up a successful strategy. As seed commitments expire, AUM will be divided among the ordinary investors, the seeder, and the ESFs manager according to a "waterfall" schedule.[6] After the initial seeding stage, the fund becomes more stable and

[3]Source form Larch Lane Advisors: Hedge Fund Seeding: A Compelling Alternative.

[4]Rather than using the gender neutral "she/he" and "her/his" we have chosen for simplicity to simply use the pronouns "she" and "her" throughout the manuscript.

[5]Infrequently, a seeder may identify certain conditions under which it will be permitted to redeem investment, prior to the lock-up period, such as in the event of poor fund performance, management attrition, sale or transfer of control of the management entity, criminal convictions or regulatory violations and material breaches of the investment parameters or provisions of the seed investment agreement.

[6]See details in the next Section.

in our idealized setup we assume that the ESFs manager no longer bears any idiosyncratic risk after the lock-up period has been completed. Therefore, we can apply Goetzmann-Ingersoll-Ross' ([Goetzmann, Ingersoll and Ross (2003)]) model to calculate the market value of the fund at termination of the lock-up period. While the manager's performance incentives during the lock-up period are implemented through a waterfall schedule, performance incentives after the lock-up period are provided by a high-water-mark (HWM) incentive, compare [Goetzmann, Ingersoll and Ross (2003)].

As there is no publicly available data on the historical performance of seeding strategies, there are only very few simple models in practice focusing on hedge fund seeding return, volatility and liquidity profile.[7] To our knowledge, this paper provides the first dynamic framework on valuation of the hedge fund seeding business by solving the portfolio-choice problem for a risk-averse manager.

Several other studies evaluate the performance of hedge funds focusing on different aspects. [Goetzmann, Ingersoll and Ross (2003)] provide the first quantitative inter-temporal valuation framework of investors' payoff and managers' fees in a setting where the fund's value follows a log-normal process and the fund managers have no discretion over the choice of portfolio. [Carpenter(2000)] shows that it is optimal for hedge fund managers who face no explicit downside risk to choose infinite volatility as asset value goes to zero. This behavior is referred to as risk-shifting. [Basak, Pavlova and Shapiro(2007)], [Hodder and Jackwerth (2007)] and [Aragon and Nanda(2012)] argue though that a manager's convex payoff structure does not necessarily induce risk shifting when the fund shows poor performance as long as the manager is exposed to downside risk, either through her ownership of fund share or through her annual fees. [Panageas and Westerfield(2009)], and [Lan, Wang and Yang(2013)] analyze the impact of management fees and high-water mark based incentive fee on leverage and valuation. None of these studies, however, model the hedge fund seeding innovation in the context of the ESFs manager's portfolio choice problem, and hence they do not assess the costs of illiquidity and unspanned risk of hedge fund seeding investments.

Our article also relates to the literature about valuation and portfolio choice with illiquid assets, such as restricted stocks, executive compensation, illiquid entrepreneurial businesses, and private equity (PE) investments. For example, [Kahl, Liu and Longstaff(2003)] analyze a continuous-time portfolio choice model with restricted stocks. Both [Chen, Miao and Wang(2010)] and [Wang, Wang and Yang(2012)] study entrepreneurial firms with unspanned idiosyncratic risks under incomplete markets. For PE investments, [Sorensen, Wang and Yang (2014)] develop a dynamic valuation model of PE investments by solving the portfolio-choice problem for a risk-averse investor, who invests in a private equity fund, managed by a general partner. We are unaware, though, of any existing models that capture the illiquidity, managerial skill (alpha), risk attitude and compensation of the hedge fund seeding business. Capturing these important features in a model that is sufficiently tractable to determine the subjective value of fees in

[7]Larch Lane Advisors LLC constructed a simple model to project returns and cash flows for a seeded fund featuring an innovative seeding strategy.

the hedge fund seeding business is one of the main contributions of this study.

The paper is organized as follows. Section 2 presents a dynamic valuation framework for modeling hedge fund seeding innovation and the impact of incentive contracts, managerial stake and hedge fund liquidation on a risk-averse ESFs manager's consumption and portfolio-choice behavior. A solution for this model is derived in Section 3. Section 4 and Section 5 discuss numerical results for breakeven alphas, seed costs and subjective value of management compensation. The main conclusions are summarized in Section 6. The appendix provides detailed computations relating to the market value of the hedge fund after the initial seeding stage.

2 Model Setup

2.1 Hedge fund seeding investment opportunities

We consider an infinitely-lived risk-averse ESFs manager who has the opportunity to launch a take-it-or-leave-it hedge fund at present time 0, which requires to raise the target AUM S_0. All sources of uncertainty arise from two independent standard Brownian motions B and Z defined on a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t : t \geq 0\}, P)$, where $F \equiv \{\mathcal{F}_t : t \geq 0\}$ describes the flow of information available to the seeder.

In addition to the opportunity of launching a fund, the manager has access to standard financial investment opportunities, see [Merton(1971)]. Let W_t denote the ESFs manager's liquid (financial) wealth process. At any time $t \geq 0$ the manager invests an amount of Π_t in a diversified market portfolio and the remaining amount $W_t - \Pi_t$ in the risk-free asset with a constant interest rate r. The return of the diversified market portfolio is denoted by R and satisfies

$$dR_t = \mu_M dt + \sigma_M dB_t, \tag{1}$$

where μ_M and $\sigma_M > 0$ are constants, and $\eta \equiv (\mu_M - r)/\sigma_M$ is the Sharpe ratio of the market portfolio.

We assume that AUM of the hedge fund $\{S_t : t \geq 0\}$ follows a geometric Brownian motion (GBM):

$$\frac{dS_t}{S_t} = (\mu - \omega - m)dt + \rho\sigma dB_t + \sqrt{1-\rho^2}\sigma dZ_t, \quad S_0 \equiv S \text{ given}, \tag{2}$$

where μ, ω, m and σ are constants; μ is the expected growth rate, ωS is the regular withdrawals or distribution among investors, mS is the management fee continuously occurring at the rate mS[8], σ is the total volatility of hedge fund

[8]Hedge fund managers normally receive 20% of the increase in fund value in excess of the last recorded maximum, i.e. high-water-mark (henceforth, HWM) as incentive fee in addition to 2% of AUM as annual fees, a compensation structure often referred to as two-twenty and considered as the industry standard. Several academic articles study the characteristics of hedge fund fees, such as [Fung and Hsieh(1997)], [Fung and Hsieh(1999)]), and[Aragon and Nanda(2012)].

growth and $\rho \in [-1, 1]$ is the correlation coefficient between the hedge fund and the return on the market portfolio given by (1).[9] The parameters $\xi \equiv \rho\sigma$ and $\epsilon \equiv \sqrt{1-\rho^2}\sigma$ are respectively the systematic and idiosyncratic volatility of the hedge fund. Similar to [Goetzmann, Ingersoll and Ross (2003)], we define $\alpha \equiv \mu - r - \beta(\mu_M - r) \equiv \mu - r - \rho\frac{\sigma}{\sigma_M}(\mu_M - r) \equiv \mu - r - \rho\sigma\eta$ as the premium return on ESFs, i.e. the managers' skills in CAPM context.[10]

The Brownian motions B and Z provide the sources of market risk (systematic) and idiosyncratic risk of the hedge fund, respectively. A higher absolute value $|\rho|$ of the correlation coefficient implies that the systematic volatility has a larger weight, *ceteris paribus*.

2.2 Seeding innovation with fees-for-seed swap

Since the aftermath of the 2008 financial crisis, there has been a tremendous shortage of capital available to ESFs. Investors have learnt their lessons from the financial crisis and become smarter and more cautious about their investments. This has tightened the financing constraints of hedge fund managers even further. According to the Seward & Kissel New Hedge Fund Study (2014), 65% of funds within the Study obtained some form of founders capital (significantly higher than the 43% in the 2013 study). Moreover, based on conversations with various industry participants, the study estimates, that within the entire hedge fund industry for the calendar year 2014, at least 40% of all launches greater than \$75 million (and an estimated 15% of all fund launches) had some form of seed capital.

In order to attract sufficient capital to cover organizational expenses and be considered credible, hedge fund managers may seek a strategic partner or a seeder who provides a critical amount of seed capital ϕS_0 in exchange for economic participation in the funds revenue, i.e. a proportion ψ of the manager's fees including both management fee and performance fees in the seeding stage.

2.3 Waterfall schedule upon the expiration of seed investments

At the end of the lock-up period at time T, AUM S_T will be divided among the ordinary investors, the seeder, and the manager according to a so-called "waterfall" schedule, similar as in [Sorensen, Wang and Yang (2014)]. More specifically, let y denote the hurdle rate during the lock-up period for the ordinary

[9] Unlike the standard literature which uses a HWM incentive structure throughout the whole lifetime of the fund, our model assumes a two-to-twenty rule under which the ESFs manager obtains incentive fees at the end of the lock-up period through a waterfall schedule only, and then for the remaining lifetime of the fund obtains incentive fees through a HWM rule.

[10] According to Hedge Fund Research (HFR) the 10-year period between 1994 and 2004 saw funds with less than a three-year track record outperform older funds by over 5% annually, with nearly identical volatility. Other studies, [Aiken, Clifford and Ellis (2013)] and [Aggarwal and Jorion(2010)] made a number of adjustments to raw performance data to mitigate survivorship or backfill biases.

investors, whose maximum payment at maturity T in consequence is:

$$Z_0 = (1-\phi)S_0 e^{yT}. \tag{3}$$

Any remaining proceeds after deducting the ordinary investors' share, Z_0, and returning the seed capital with preferred hurdle rate h, i.e. $\phi S_0 e^{hT}$, constitute the profits of the ESFs, given by:

$$S_T - Z_1, \tag{4}$$

where $Z_1 \equiv Z_0 + \phi S_0 e^{hT}$ is the upper boundary that equals to the sum of the maximum payment to the ordinary investors and the preferred return for the seeder. These profits are divided between the ordinary investors and the ESFs manager. The manager receives her carried interest, while the ordinary investor's share along with his maximum payments remain in AUM after the seeding stage. Therefore, there are three regions of the waterfall structure, depending on the amount of AUM at the end of the lock-up period.

Region 1: Hurdle rate for the ordinary investors ($S_T \leq Z_0$) In our model, the ordinary investors' payoffs is senior to the seeder's investment, thus the seeder and the manager receive nothing if S_T falls below the boundary of Z_0. The guaranteed payment to the ordinary investors is given by:

$$OP_1(A_T, T) = min\{S_T, Z_0\}. \tag{5}$$

Region 2: Preferred return ($Z_0 \leq S_T \leq Z_1$) At the upper boundary of this region, the seeder gets her seed capital back with a prescribed hurdle rate h, $\phi S_0 e^{hT}$, and the seeder's payoff in this region, at maturity T, is:

$$SP(S_T, T) = max\{S_T - Z_0, 0\} - max\{S_T - Z_1, 0\}. \tag{6}$$

Region 3: The ESFs manager's carried interest ($S_T > Z_1$) After deducting the guaranteed payment and preferred return of the seed capital, the ESFs manager claims her carried interest, the fraction k of the profits $S_T - Z_1$, given by:

$$GP(S_T, T) = k * max\{S_T - Z_1, 0\}. \tag{7}$$

Denote $OP_3(S_T, T)$ as the ordinary investors' share in this region which is given by:

$$OP_3(S_T, T) = (1-k) * max\{S_T - Z_1, 0\}. \tag{8}$$

One can easily compute the sum of the payoffs of all agents, at maturity T, which satisfies:

$$OP_1(A_T, T) + OP_3(S_T, T) + SP(S_T, T) + GP(S_T, T) = S_T. \tag{9}$$

After returning the seed capital, only the ordinary investors' payoff remains in the fund, and the adjusted AUM S_T^* is:

$$S_T^* \equiv OP(A_T, T) = OP_1(A_T, T) + OP_3(S_T, T). \tag{10}$$

Therefore, the ordinary investors' claim at maturity T, denoted by $OP^*(A_T, T)$, satisfies:

$$OP^*(A_T, T) = I(S_T^*, S_T^*), \tag{11}$$

where $I(S_T^*, S_T^*)$, the investors' claim in the Goetzmann-Ingersoll-Ross model, is determined by (A.11) in Appendix A. Appendix A presents more details about the market value of the ordinary investors' claim after the seeding stage.

2.4 The manager's problem

The ESFs manager's standard time separable preference is characterized by her initial wealth W_0 and a pure subjective discount rate δ, and her utility function $U(C)$, represented by:

$$\max_{c_s} E\left[\int_0^\infty \exp(-\delta s)\, U(c_s) ds\right]. \tag{12}$$

For tractability, we assume the manager has constant absolute risk aversion (CARA) utility preference, given by

$$U(c) = -\exp(-\gamma c)/\gamma, \tag{13}$$

where $\gamma > 0$ is the coefficient of absolute risk aversion. The overall time horizon $[0, \infty)$ entails the lock-up period $[0, T]$ during which the manager faces idiosyncratic risk (possibly due to assets in the fund being less liquid and proprietary) as well as the remaining period $[T, \infty)$, during which it is assumed that the underlying risks are fully spanned by public assets, as in [Sorensen, Wang and Yang (2014)] section 3.[11]

2.5 Manager's liquid wealth dynamics

During the lock-up period, the manager's financial wealth evolves according to,

$$dW_t = (rW_t + (1-\psi)mS - c_t)dt + \Pi_t((\mu_M - r)dt + \sigma_M dB_t), \quad 0 < t < T. \tag{14}$$

The first term in Equation (14) is the wealth accumulation when the manager fully invests in the risk free asset, plus the revenue of managing the ESFs net of her consumption. The second term is the excess return from the manager's investment in the market portfolio.

At the end of the lock-up period T, the manager's wealth (including current portfolio wealth and futures management fees) jumps from W_{T^-} to W_T, with

$$\begin{aligned} W_T &= W_{T^-} + G(S_T, T) \\ &= W_{T^-} + (1-\psi)[k * GP(S_T, T) + F(S_T^*, S_T^*)]. \end{aligned} \tag{15}$$

[11] This is an idealization of the fact that idiosyncratic risks in the start up of the fund and in particular during the lock-up period are significantly higher than when the fund has established itself.

The second term on the right hand side of Equation (15) represents the carried interest of the hedge fund seeding business. The term $F(S_T^*, S_T^*)$ consists of the market value of future fees paid to the manager after the lock-up period. Per assumption, the risk to which the fund is exposed after the lock-up period is fully spanned by public assets, as such the market value of the fees can be computed under the appropriate risk neutral measure as in [Goetzmann, Ingersoll and Ross (2003)]. An explicit expression for $F(S_T^*, S_T^*)$ is provided in Equation (A.12) in Appendix A. The dynamic programming principle and the fact that we have full spanning over the period $[T, \infty)$ implies that the solution of problem (12) can now been obtained from the solution of the corresponding problem starting at time T with wealth dynamics

$$dW_t = (rW_t + \Pi_t(\mu_M - r) - c_t)dt + \Pi_t \sigma_M dB_t, \tag{16}$$

and initial wealth W_T according to (15), and then by backward induction over the interval $[0, T]$ as in the following section.

3 Model Solution

In this section, we first derive seed costs with fees-for-seed swaps for the ESFs manager and the breakeven alpha for the ordinary investors. Then we analyze the manager's consumption and portfolio choice in a dynamic valuation model taking account of illiquidity, ESFs managers' value-adding skills (alpha), incentive compensation, and the fees-for-seed swap. However, the idiosyncratic risk which is present in the hedge funds seeding business invalidates the standard two-step complete-markets (Arrow-Debreu) analysis (first value maximization and then optimal consumption allocation)[12] due to the non-separability between value maximization and consumption smoothing. In order to derive the solution, we first solve the standard Merton consumption and portfolio choice problem faced by the manager after the expiration of the lock-up period, similar as in [Merton(1971)] and [Goetzmann, Ingersoll and Ross (2003)]. Following this, we solve an optimal control problem maximizing the ESFs manager's utility during the seeding stage.

3.1 Market value of total fees and call options on the seeding investment

We assume that the market prices cash-flows attached to liquid assets by using a risk neutral measure Q equivalent to the measure P when restricted to all $\mathcal{F}_t$ for any $t \geq 0$.[13] The corresponding state-price deflator π satisfies $d\pi = -r\pi dt - \eta\pi dZ$, $\pi_0 = 1$ and restricted to $(\Omega, \mathcal{F}_t)$, we have $\lambda_t = \frac{dQ}{dP}\Big|_{\mathcal{F}_t}$ and $\lambda_t = \exp(rt)\frac{\pi_t}{\pi_0}$, see [Duffie(2001)].

[12] See more details in [Cox and Huang (1989)]

[13] Such a measure may not be unique due to incompleteness, but we assume here that the market has chosen a risk neutral measure, which in consequence becomes the market measure.

Denote by $\nu \equiv \mu - \omega - m - \rho\sigma\eta \equiv \alpha + r - \omega - m$ the risk-adjusted drift rate of AUM, and B_t^Q a standard Brownian motion satisfying $dB_t^Q = dB_t + \eta dt$. Then under Q, the dynamics of AUM in (2) can be rewritten as

$$dS_t = \nu S_t dt + \rho\sigma S_t dB_t^Q + \epsilon S_t dZ_t. \tag{17}$$

Let $G^*(S_t, t)$ be the market value of the claim underlying S_t with a payment flow mS_s for $s \in [t, T]$ and a terminal payoff $G^*(S_T, T) \equiv k * \max\{S_T - Z_1, 0\} + F(S_T^*, S_T^*)$. According to the dynamic asset pricing theory ([Duffie(2001)]), it can be written as a conditional expectation under the risk-adjusted measure Q:

$$G^*(S_t, t) = E_t^Q \left[\int_t^T e^{-r(s-t)} mS_s ds + e^{-r(T-t)} G^*(S_T, T) \right]. \tag{18}$$

By using Ito's formula, $G^*(S_t, t)$ satisfies the follow PDE:

$$rG^*(S_t, t) = mS + G_t^* + \nu S_t G_S^*(S_t, t) + \frac{1}{2}\sigma^2 S_t^2 V G_{SS}^*(S_t, t), \tag{19}$$

with two boundary conditions

$$\begin{array}{rcl} G^*(0, t) & = & 0, \\ G^*(S_T, T) & = & k * \max\{S_T - Z_1, 0\} + F(S_T^*, S_T^*), \end{array} \tag{20}$$

where $F(S_T^*, S_T^*)$ is the market value of total fees generated after the lock-up period expires at time T, given by (A.12) in the Appendix A.

Similar to [Sorensen, Wang and Yang (2014)], the value at time t of a plain-vanilla European call option with strike price K and terminal payoff $\max\{S_T - K, 0\}$ at T, denoted by $Call(S_t, t, \alpha, K)$, is given by:

$$\begin{array}{rcl} Call(S_t, t, \alpha, K) & = & E_t^Q \left[e^{-r(T-t)} \max\{S_T - K, 0\} \right], \\ & = & e^{(\alpha-\omega-m)(T-t)} E_t^Q \left[e^{-\nu(T-t)} \max\{S_T - K, 0\} \right], \\ & = & e^{(\alpha-\omega-m)(T-t)} \left[S_t N(d_1) - K e^{-\nu(T-t)} N(d_2) \right], \\ & = & S_t e^{(\alpha-\omega-m)(T-t)} N(d_1) - K e^{-r(T-t)} N(d_2), \end{array} \tag{21}$$

where

$$\begin{array}{l} d_1 = d_2 + \sigma\sqrt{T-t}, \\ d_2 = \frac{ln(S_t/K) + (\nu - \sigma^2/2)(T-t)}{\sigma\sqrt{T-t}}. \end{array} \tag{22}$$

The pricing formula here is different from the classic Black-Scholes formula as we assume that the investment of the hedge fund seeding business could generate excess alpha.

3.2 Seed costs with fees-for-seed swaps

Under the arrangement of a fees-for-seed swap during the hedge fund seeding stage, the ESFs manager must give up a portion (ψ) of her fees in exchange for the seed capital (ϕS_0). The cost ψ of the seed capital with fees-for-seed swap, is determined endogenously within the model, in such a way that the value of the contract is zero at initiation.

Generally speaking, a seeder is usually a diversified investor who signs such contracts with a large number of ESFs and therefore the idiosyncratic risk of the hedge fund seeding business is well-diversified.[14] Thus, the seed capital provided by the seeder must be equal to the market value (equilibrium value) of the fees allocated to the seeder when the contract commences. That is

$$\begin{aligned} \phi S_0 &= \psi G^*(S_0,0), \\ \psi &= \tfrac{\phi S_0}{G^*(S_0,0)}. \end{aligned} \tag{23}$$

In addition to revenue sharing, seeding investments will be returned to seeders more or less by $SP(S_T,T)$ depending on the performance of the fund at the end of the lock-up period. Using the pricing formula defined in the last subsection, one can derive an explicit expression for $SP(S_t,t)$, the time t value of this claim, that is

$$SP(S_t,t) = Call(S_t,t,\alpha,Z_0) - Call(S_t,t,\alpha,Z_1). \tag{24}$$

Therefore, the time t value of the seeders' claim, denoted by $SP^*(S_t,t)$, is given by:

$$SP^*(S_t,t) = SP(S_t,t) + \psi * G^*(S_t,t). \tag{25}$$

3.3 Break-even alpha

In Section 2, we considered the terminal payoffs $(OP^*(S_T,T))$ for the ordinary investors at maturity T. Its present value, denoted by $I^*(S_t,t)$, satisfies:

$$I^*(S_t,t) = E_t^Q\left[e^{-r(T-t)}I(S_T^*,S_T^*)\right]. \tag{26}$$

By using Ito's formula, we obtain the following PDE:

$$rI^*(S_t,t) = \omega S_t + I_t^* + \nu S_t I_S^*(S_t,t) + \frac{1}{2}\sigma^2 S_t^2 V I_{SS}^*(S_t,t), \tag{27}$$

with the following two boundary conditions defined below:

$$\begin{aligned} I^*(S_T,T) &= I(S_T^*,S_T^*), \\ I^*(0,t) &= 0. \end{aligned} \tag{28}$$

In order to break-even at the start of the fund, the ordinary investors' claim has to be equal to their initial investment, i.e. $I^*(S_0,0) = (1-\phi)S_0$. Particularly, ordinary investors benefit (suffer the loss) from their investment in the ESFs if $I^*(S_0,0) \geq (\leq)(1-\phi)S_0$ when the contract commences. Using (27) to solve $I^*(S_0,0) = (1-\phi)S_0$ for α provides the minimum alpha that should be generated by the ESFs manager for the ordinary investors to break-even when the contract commences.

[14]For example, Larch Lane Advisors LLC ("Larch Lane") , one of the first dedicated providers of hedge fund seed capital, has seeded 25 hedge funds and continues to be an active capital provider for the hedge fund industry.

3.4 Consumption and portfolio choice after the lock-up period

As indicated in section 2.5, the manager's investment problem after the lock-up period has expired at time T is equivalent to the classical Merton problem studied in [Merton(1971)], where the initial wealth W_T consists of realized portfolio value prior to time T, waterfall schedule payoff at time T and the market value of future management and performance fees after time T. The optimal consumption and portfolio rule is therefore given by

$$c^*(W) = r\left(W + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right), \tag{29}$$

$$\Pi^* = \frac{\eta}{\gamma r \sigma_M}, \tag{30}$$

where W is the liquid wealth level.

The maximum of the expected utility of consumption after the seeding stage can be computed as

$$J^e(W) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(W + \frac{\eta^2}{2\gamma r^2} + \frac{\delta - r}{\gamma r^2}\right)\right]. \tag{31}$$

3.5 Manager's decisions and certainty equivalent valuation

Define $J^s(W, S, t)$ as the manager's value function before the the end of the seeding stage, i.e.

$$J^s(W, S, t) = \max_{(c_s, \Pi_s)} E\left[\int_t^T \exp\left(-\delta(s-t)\right) U(c_s) ds + e^{-\delta(T-t)} J^e(W_T) \middle| W_t = W, S_t = S\right], \tag{32}$$

where W is the manager's financial wealth process, and the function $J^e(\cdot)$ is given by (31). In light of section 2.5. the value function $J^s(W_0, S_0, 0)$ coincides with the value of the problem (12). During the lock-up period, the manager's financial wealth evolves according to,

$$dW_t = (rW_t + \Pi_t(\mu_M - r) + (1-\psi)mS - c_t)dt + \Pi_t \sigma_M dB_t, \quad 0 < t < T, \tag{33}$$

while as previously discussed, at the end of the lock-up period the wealth jumps to

$$\begin{aligned} W_T &= W_{T^-} + G(S_T, T) \\ &= W_{T^-} + (1-\psi)[k * GP(S_T, T) + F(S_T^*, S_T^*)]. \end{aligned} \tag{34}$$

Compared to the exogenously given fraction of management fees retained by the seeder in practice, the fraction ψ in our model is endogenously determined by the fees-for-seed swap. In this case, the manager's value function $J^s(W,S,t)$ satisfies the following Hamilton-Jacobi-Bellman equation:

$$\begin{aligned}\delta J^s(W,S,t) &= \sup_{c\geq 0,\Pi}\{U(c)+J^s_t+(rW+\Pi(\mu_M-r)+(1-\psi)mS-c)J^s_W \\ &+ \tfrac{1}{2}(\Pi\sigma_M)^2J^s_{WW}+\Pi\sigma_M\sigma\rho SJ^s_{WS}+\mu SJ^s_S+\tfrac{1}{2}\sigma^2S^2J^s_{SS}\}.\end{aligned} \quad (35)$$

The first-order conditions for the optimal consumption and portfolio choice are:

$$U'(c) = J^s_W(W,S,t), \quad (36)$$

$$\Pi(S,t) = \frac{-J^s_W}{J^s_{WW}}\left(\frac{\mu_M-r}{{\sigma_M}^2}\right)+\frac{-J^s_{WS}}{J^s_{WW}}\frac{\rho\sigma S}{\sigma_M}. \quad (37)$$

According to the utility indifference pricing principle, the utility indifference price [15] of the management fee owned by the ESFs manager in the seeding stage, denoted by $G(S,t)$ [16] , satisfies

$$J^s(W,S,t) = J^e(W+G(S,t)) = -\frac{1}{\gamma r}\exp\left[-\gamma r\left(W+G(S,t)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right)\right]. \quad (38)$$

Substituting (36), (37), and (38) into (35), we obtain the following theorem immediately: During the seeding stage for $t\in[0,T]$, the optimal consumption and portfolio rule is given by

$$c^*(W,S,t) = r\left[W+G(S,t)+\frac{\eta^2}{2\gamma r^2}+\frac{\delta-r}{\gamma r^2}\right], \quad (39)$$

$$\Pi^*(S,t) = \frac{\eta}{\gamma r\sigma_M}-\frac{\rho\sigma}{\sigma_M}SG_S(S,t), \quad (40)$$

where $G(S,t)$ is the utility indifference price of the fees owned by the manager, which is the solution of the following partial differential equation (PDE):

$$rG(S,t) = (1-\psi)mS+G_t+\nu SG_S(S,t)+\frac{1}{2}\sigma^2S^2G_{SS}(S,t)-\frac{\gamma r}{2}\epsilon^2S^2G_S(S,t)^2, \quad (41)$$

subject to the following two boundary conditions:

$$\begin{aligned}G(S_T,T) &= (1-\phi)[kGP(S_T,T)+F(S^*_T,S^*_T)] \quad (42)\\ G(0,t) &= 0, t\in[0,T]. \quad (43)\end{aligned}$$

The first boundary condition states that, at maturity T, the ESFs manager collects her carried interest plus the market fees paid by an outside manager. The second boundary condition reflects that the value of the manager's fees

[15] It is sometimes called certainty equivalent wealth, which is the risk-adjusted subjective value of managing the hedge fund seeding business.

[16] Thanks to the exponential utility assumption, the utility indifference price is independent of the wealth level of the fund manager.

falls down to zero as the underlying AUM converges to zero during the lock-up period.

Equations (39) and (40) indicate that the hedge fund manager will consume the implied value $G(S,t)$ and use the market portfolio to dynamically hedge the hedge fund seeding business risk. More specifically, equation (39) indicates that the manager's consumption is equal to the annuity value of the sum of financial wealth W and the implicit value of the hedge fund seeding business $G(S,t)$ plus two constant terms which appear in the classical Merton rule, see [Merton(1971)]. The portfolio-choice rule is given by equation (40) in which the second term represents the manager's hedging demand in the context of her hedge fund seeding business. Equation (41) implies that if the absolute risk-aversion index equals zero (i.e. the ESFs manager is risk-neutral towards the idiosyncratic risk), equation (41) becomes the standard equilibrium pricing equation. Therefore, the last term on the right side of equation (41) captures the idiosyncratic risk effect on the managers valuation of the hedge fund business.

4 Breakeven Alphas and Seed Costs

We have derived the solution of the fees-for-seed swap portion ψ referred to as the seed cost and the subjective value of the ESFs manager's compensation in Section 3 In this section we provide some numerical results in order to develop more economic intuition. Baseline breakeven parameters are chosen according to [Sorensen, Wang and Yang (2014)]. Table 1 summarizes the parameter values used in our baseline breakeven case.

4.1 Breakeven alphas, compensation contracts, and seed capital involvement

How important is the managerial ability of producing superior performance (alpha)? In order to develop more economic intuition, Table 2 presents breakeven alphas in different compensation contracts for various levels of seed capitals. We first consider the case without any fees, $m = k = 0\%$. No positive alpha is then required by the ordinary investors for the case of no seed capitals. Interestingly, the ordinary investors can even bear some loss if the fund gets some seed capitals. For example, the ordinary investors' investments breakeven for a negative alpha, $\alpha = -0.77\%$, in the case that the seeder provides 15% of AUM seed capital as the anchor investor, which means the ordinary investors are more willing to invest in the fund by following the seeders.

Moreover, some typical compensation contracts are compared in Table 2. Naturally, increasing either m or k increases the breakeven alpha, for a given level of seed investments. More specifically, holding the seed capital ratio fixed, increasing the management fee rate m by 0.5 percentage-points results in an increase of the breakeven alpha by the same amount. However, increasing k by 10 % increases the breakeven alpha by 0.55% to 0.75%, depending on seed capital ratio.

Table 1: Summary of key parameters in baseline breakeven case

Parameter	Symbol	Value
Risk-free rate	r	5%
Expected return of market portfolio	μ_M	11%
Expected return of the ESFs	μ	
Volatility of market portfolio	σ_M	20%
Volatility of the ESFs	σ	25%
Market Sharpe ratio	η	30%
Guaranteed yield	y	5%
Hurdle rate	h	8%
Management fee	m	2%
Incentive fee	k	20%
Lock-up period	T	2
Managerial skills	α	
Subjective discount rate	δ	5%
Idiosyncratic risk	ϵ	23%
Seed capital ratio	ϕ	5%
Correlation coefficient	ρ	40%
Exogenous liquidation barrier	l	50%
Withdrawal rate, the liquidation parameter	$\omega + \lambda$	5%
Coefficient of absolute risk aversion	γ	2

Table 2 provides evidence that an ESFs manager would easily reach the target AUM level to launch a fund via a seeding vehicle as the ordinary investors benefit from the scheme. However, the ESFs manager backed up with seed capital has to give certain fraction (ψ) of her fees revenues as compensation to the seeder, and we will analyze these seed costs in the following subsection.

4.2 The seeder's value and seed costs

We refer to the last section for the seeder's value given by Equation (25). Not surprisingly, the seeder's value is significantly affected by the fund's performance as the left hand sub-figures in Figure 1 and Figure 2 indicate. For example, the seeder's value has more than tripled as the fund AUM is doubled (from $S_0 = 100$ to 200). On the other hand, the right subfigure in Figure 1 shows that the seeder's value is almost insensitive to time t, given AUM fixed at 100. The seeder's value changes only with time t near the maturity and AUM staying around Z_0 and Z_1. This is mainly because the seeder's positions in these options are in the money when AUM goes to the interval around Z_0 and Z_1.

Unlike an ad-hoc "rule of thumb" decision on the fees-for-seed ratio which is often common in practice, our paper provides a closed-form solution for the seed cost ψ which is informed by a number of factors such as the amount of the seed capital, the manager's alpha, the risk of the fund, etc. and takes account some key principles from Finance theory. Interestingly, the right hand subfigure

Table 2: The table gives breakeven alphas for different levels of seed capital ratio, ϕ, incentive fees k and management fees, m. Other parameters are $\beta = 0.5$, $h = 8\%$, $T = 2$ years, and $S_0 = 100$.

m	k	$\phi = 0$	$\phi = 5\%$	$\phi = 10\%$	$\phi = 15\%$
0.0%	0%	0.00%	-0.24%	-0.50%	-0.77%
1.5%	10%	2.14%	1.89%	1.62%	1.35%
1.5%	20%	2.79%	2.50%	2.22%	1.93%
1.5%	30%	3.54%	3.16%	2.83%	2.51%
2.0%	10%	2.65%	2.39%	2.12%	1.85%
2.0%[a]	20%	3.29%	3.00%	2.72%	2.43%
2.0%	30%	4.03%	3.66%	3.33%	3.01%
2.5%	10%	3.15%	2.89%	2.62%	2.35%
2.5%	20%	3.78%	3.50%	3.21%	2.93%
2.5%	30%	4.54%	4.16%	3.83%	3.51%

[a] Indicates the baseline breakeven case.

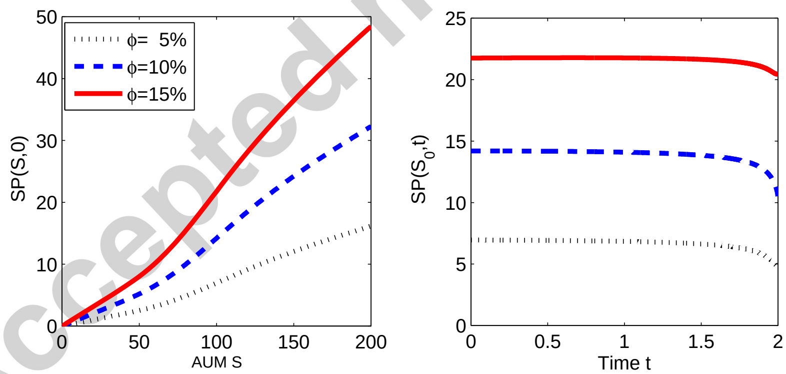

Figure 1: The figure provides comparative statics for the seeder's payoffs with respect to time t and AUM S for different levels of seed capital.

in Figure 2 shows that the fees-for-seed ratio ψ is indeed a linearly increasing function of the seed capital which could be interpreted as a rule of thumb in practice. However note that the slope of the function varies with parameters

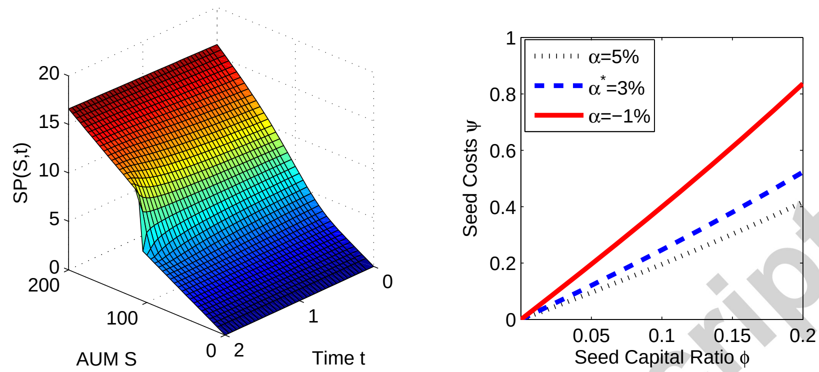

Figure 2: The figure presents comparative statics for the seeder's value with respect to Tt and S, and seed costs ψ with respect to seed capital ratio for three different levels of alpha.

such as the managerial skills. In details, the seeder demands more fund revenue share for a manager with negative alpha (-1%) than that for a more talented manager (5%) and the difference can be up to 40 % if providing 20 % of the initial AUM (S_0).

5 Seeding Investments with Lock-up Period, Subjective Value of Fees, and Idiosyncratic Risk

In this section, we further analyze the effects of the lock-up period, seeding investment, and idiosyncratic risk on management compensation, illiquidity discount, and the economic value of the fund.

5.1 Lock-up period effects, illiquidity discount and management compensation

Table 3 presents valuations for the ordinary investors, the seeder, and the ESFs manager for various levels of alpha. Panel A of Table 3, with a short lock-up period, shows that the ordinary investors breakeven with $\alpha = 3.13\%$, while the seeder's values for various alphas outperform her initial seeding investment $\phi \cdot S_0 = 5$. This is straightforward because the seeder has an extra option value in addition to the breakeven swap when the seeding contract commences. As the alpha decreases from 5% to -1% the seed cost ψ increases from 9.98% to 19.94 %, which makes sure the seeder breakeven at the beginning of the contract. Due to the unspanned risk of the seeding business, the ESFs manager bears the illiquidity costs. Similar to [Wang, Wang and Yang(2012)], the illiquidity discount for the manager is defined as the difference of the market value and the certainty-equivalent value of fees, which is given by $ID = G^*(S_0, 0) - G(S_0, 0)$. This discount is the amount the manager would be willing to pay for not bearing the idiosyncratic risk during the initial seeding stage. As we can see from Table 3, the amount of ID increases by 1.8 to 2.44 as increasing alpha from -1% to 5%.

In Table 3, Panel B, with a relative long lock-up period $T = 4$ years, shows the illiquidity discount is much greater than that in Panel A. For the case of $\alpha = 5\%$, the discount for $T = 4$ around is six times of that in Panel A. This means the manager bears more illiquidity discount for longer lock-up periods. Moreover, the seeder also suffers a loss for a long lock-up period. For example, fixing $\alpha = -1\%$, the seeder suffers losses of 1.03 (around 20% of her initial investment) by extending the lock-up period from 6 months to 4 years.

Table 3: The table presents valuations of different agents' claims with non-spanned risk for various levels of alpha. The columns refer to: the ordinary investors' interest (I^*), the seeders' payoffs (SP^*), the market value of fees (G^*), the economic value of the ESFs ($V = I^* + SP^* + G^*$), the ESFs manager's certain-equivalent valuation (G), the illiquidity discount ($ID = G^* - G$), and the seed costs (ψ). Parameter values are $\gamma = 2$, $S_0 = 100$, $m = 2\%$, $k = 20\%$, $\phi = 5\%$, $\beta = 0.5$, and $h = 8\%$. Panel A and B report the results for the case with the lock-up periods $T = 0.5$ and $T = 4$, respectively.

α(%)	I^*	SP^*	G^*	V	G	ID	ψ(%)
Panel A: $T = 0.5$							
-1.0	64.53	7.19	20.07	91.79	19.63	0.44	19.94
0.00	69.96	7.25	22.79	100.00	21.19	0.65	17.99
2.00	84.38	7.36	30.19	121.93	29.11	1.08	14.21
3.13 [a]	95.00	7.42	35.91	138.33	34.42	1.49	12.22
5.00	106.38	7.53	45.09	159.01	42.86	2.24	9.98
Panel B: $T = 4$							
-1.0	64.04	6.16	20.54	90.74	18.43	2.11	19.58
0.00	70.15	6.29	23.56	100.00	20.04	2.30	17.51
2.00	86.36	6.58	31.77	124.71	25.60	6.17	13.60
2.83 [a]	95.00	6.70	36.31	138.02	28.44	7.87	12.10
5.00	114.48	7.06	49.53	171.06	35.77	13.76	9.17

[a] Indicates baseline breakeven case.

5.2 Seed capitals effects, economic values of the fund

Table 4 presents results of economic values of the fund and subjective values of fees with or without seeding vehicle (shown in Panel A and Panel B respectively) for various levels of alpha. More specifically, the economic value of the fund with seed capital is less (more) than a fund without seed capital for a positive (negative) alpha, holding alpha fixed. This is because the seed capital will be returned at a hurdle rate and no longer earns the premium alpha. However, if we take the illiquidity discount into consideration, the adjusted economic value, denoted by $V^* = V - ID$, of the fund with seed capital is always greater than that without seed capital. Take the case $alpha = 5\%$ for example, the adjusted economic value of the ESFs with 5% of seed capital is 156.12, compared to 154.44 for a fund without seed capital. The main reason is that ψ percent of the ESFs manager revenue is transferred to the outside investors via fees-for-seed swap, thus generating diversification benefits for the fund.

One may note that the ESFs manager may be better off without any seeding investments. This is true only when the ESFs manager can reach the target AUM level for a successful launch. If it is hard for the manager to achieve the target, she may turn to the seeder and give up some fraction of her fees revenue due to lack of bargaining power. For example, Panel B of Table 4 illustrates that the manager should give up 12.08% of her fees in exchange of 5% of AUM from the seeder.

For an unskilled ESFs manager, $\alpha = 0$, both Table 3 and Table 4 show that the economic value of the fund equals to the initial investment $S_0 = 100$, which means the ordinary investor bears the loss to pay fees to the manager. As alpha increases, both the economic value of the fund and the ordinary investors' payoffs grow as we expected.

5.3 Idiosyncratic risk effect, risk aversion and fees

It is obvious that the (subjective) value for the ESFs manager is generally an increasing function of AUM and time t. Unlike the risk-neutral case, in the case of risk aversion the subjective values are concave functions of AUM. This is mainly because of the nonlinear terms in our pricing PDE (41). As shown in Figure 3, the more risk-averse the manager is, the greater the illiquidity discount the manager has to bear. More specifically, the illiquidity discount is very small when AUM stays at a very low level (e.g. below 40), while it increases quickly and reaches its peak at 33 (for the case $\gamma = 4$) as AUM increase to 200. Moreover, the right hand subfigure in Figure 3 indicates that the gap of the illiquidity discount between the case $\gamma = 4$ and the case $\gamma = 2$ converges to zero as t is approaching the end of the lock-up period T. Once the lock-up period expires, the ESFs manager is out of the seeding business and bears no idiosyncratic risk any more.

Finally, Figure 4 shows the effect of the correlation coefficient ρ between the market and the fund on the subjective value of the management compensation. Interestingly, the management compensation is a not a monotonous function. It

Table 4: The table presents valuations of different agents' claims with non-spanned risk for various levels of alpha. Panel A and B report the results for the case with $\phi = 0$ and $\phi = 5\%$, respectively.

$\alpha(\%)$	I^*	SP^*	G^*	V	G	ID	ψ (%)
Panel A: the seed capital ratio $\phi = 0$							
-1.0	65.28	0.00	25.90	91.18	22.46	3.45	0.00
0.00	71.16	0.00	28.84	100.00	24.72	4.11	0.00
2.00	86.76	0.00	36.80	123.56	30.70	6.10	0.00
3.29 [a]	100.00	0.00	43.95	143.95	35.88	8.08	0.00
5.00	112.09	0.00	53.36	165.45	42.37	10.99	0.00
Panel B: the seed capital ratio $\phi = 5\%$							
-1.0	64.27	6.53	20.51	91.31	19.08	1.43	19.60
0.00	70.00	6.63	23.37	100.00	20.45	1.60	17.62
2.00	85.17	6.85	31.14	123.15	27.38	3.76	13.84
3.00 [a]	95.00	6.96	36.39	138.35	31.34	5.05	12.08
5.00	109.73	7.19	47.25	164.16	39.21	8.04	9.57

[a] Indicates baseline breakeven case.

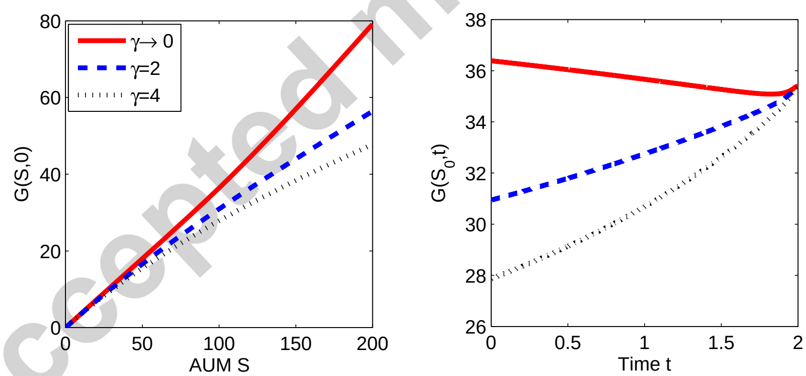

Figure 3: Comparative statics for management compensation with respect to AUM and time t for various levels of risk aversion.

first increases with ρ to its maximum point and then decreases afterward. On the one hand, managerial skills alpha drops ($\alpha = \mu - r - \rho\sigma\eta$)) as ρ increases, which causes a negative effect on the subjective value G. On the other hand, the idiosyncratic risk decreases as ρ increases (keeping total fund volatility σ

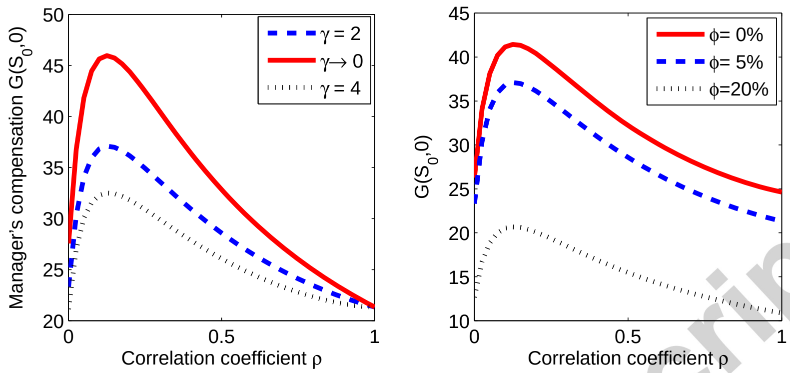

Figure 4: Comparative statics for management compensation with respect to correlation coefficients ρ.

unchanged), which is a positive effect on management compensation. Therefore, the ESFs manager faces a trade off between the correlation coefficient and idiosyncratic risk. Particularly, management compensation is an increasing function as the positive effect of the idiosyncratic risk dominates the negative effect of diminishing alpha for ρ less than around 20 %. As ρ continues to increase, the negative effect dominates, and the function turns into a decreasing function.

6 Conclusion

In this article, we developed a dynamic valuation model for the hedge fund seeding business by solving the consumption and portfolio-choice problem of a risk-averse manager who launches a hedge fund through a seeding vehicle. As traditional approaches to attract the initial AUM and covering of organizational expenses becomes much harder for ESFs managers in a much tighter financial landscape, nowadays more and more ESFs managers are likely to turn to seed investors for early stages of capital by offering a certain proportion of their fees through a seeding vehicle. The new swap specifies that a seeder commonly commits to providing a remarkable amount of seed capital to an ESFs manager as an "anchor investor" in a new fund in exchange for a share of "enhanced economics" which is usually a proportion of the fees that the ESFs manager generates from the entire pool of assets in the fund. Our results indicate that the new swap not only solves the serious problems of widespread financing constraints for ESFs managers, but can also be highly beneficial to both the manager and the seeder if structured properly.

Moreover, we derived a closed-form solution for the fees-for-seed ratio, i.e. the seed cost, as well as the manager's value attached to the hedge fund seeding business. In addition we presented a detailed numerical analysis in which we discussed sensitivity effects of various model parameters such as the risk aversion coefficient as well as the skills factor α on our results. Our analysis showed that, as we would expect, the greater the seed capital obtained, the more fees the manager should give up. The fees-for-seed ratio is a linear increasing function of the amount of the seed capital. However, more interestingly, unlike an uninformed "rule of thumb", the slope term in this linear relationship depends on factors such as fund volatility and managerial skills. Therefore, the closed-form solution of the seed costs in our model is much more informed and can be regarded as a theoretical guide to the design of a seeding vehicle contract.

Our model assumes the ESFs manager is risk averse towards the hedge fund seeding business, thus she suffers the illiquidity discount for her valuation due to unspanned idiosyncratic risk. Once the ESFs is out of the seeding stage and enters into normal stage, the manager bears no idiosyncratic risk. We found that the more risk-averse the manager is, the greater the illiquidity discount the manager has to bear, thus the lower her subjective value is. In addition, the manager bears more idiosyncratic risk for longer duration of the lock-up period, i.e. the hedge fund seeding stage. Given fixed expected return and volatility of the fund, the ESFs manager faces a trade off between the correlation coefficient and idiosyncratic risk. As the correlation coefficient increases, the positive effect of decreasing idiosyncratic risk at first dominates the negative effect of diminishing alpha for low level of ρ, but then is dominated by the negative effect of diminishing alpha.

More importantly, the ordinary investors are more willing to invest in an ESFs backed up by seeders via a fees-for-seed swap. As our numerical results illustrate, the more seed capital the fund gets the smaller breakeven alphas the ordinary investors demand for their investment. Therefore, the seeding vehicle helps the ESFs attract more investors and get sufficient capital for a successful launch. Moreover, our results show that the adjusted economic value, denoted by $V^* = V - ID$, for a fund with seed capital is always greater than that without any seed capitals due to ψ percent of the ESFs manager revenue is transferred to the outside investors via fees-for-seed swaps.

There are various ways to extend this research. First, dynamic leverage could be incorporated into our model to further analyze ESFs managers' risk shifting behavior, inspired by [Lan, Wang and Yang(2013)]. Second, we could extend our model to feature risk aversion among the hedge fund seeding investors to capture the illiquidity of hedge fund seeding investment, see [Sorensen, Wang and Yang (2014)], [Wang, Wang and Yang(2012)] among others. Finally, it would be worth to capture the feature of partial information about managerial skills in our model.

Appendices

Appendix A Market Value of the Hedge Fund

After the lock-up period, we assume that the fund enters into a normal stage and the manager no longer bears the idiosyncratic risk, which is quite similar to the case discussed in [Goetzmann, Ingersoll and Ross (2003)]. In the normal stage, the manager is paid via both management and performance fees. The management fee is specified as a constant fraction m of the net asset value while the incentive fee is commonly accompanied by a high water mark (HWM) provision. Intuitively, the HWM H_t is the running maximum of net asset value S when $g \equiv \omega + c^{'}$, i.e. $H_t = max\{S_u; u \in [0,t]\}$. In a more general setting, for $S_t < H_t$, the HWM H_t evolves deterministically as

$$dH_t = (g - \omega - c^{'})H_t dt, \tag{A.1}$$

where g is the contractual growth rate at which H changes (generally zero or r) and $c^{'}$ is the cost or fees allocated to reducing the HWM.

At any time $t \geq 0$, we can compute the value of the total fees $F(S,H,t)$ and the ordinary investors' value $I(S,H)$ respectively as follows:

$$F(S,H,t) = E_t^Q \left[\int_t^{\tau} e^{-r(s-t)}[mS + k(dH_s - (g - \omega - c^{'})H_s ds)]ds \right], \tag{A.2}$$

$$I(S,H,t) = E_t^Q \left[\int_t^{\tau} e^{-r(s-t)}\omega S + e^{-r(s-t)}S_\tau ds \right], \tag{A.3}$$

where τ, the stochastic liquidation time, is defined as $\tau = \min\{\tau_1, \tau_2\}$ where τ_1 is the exogenous liquidation time and $\tau_2 \equiv inf\{t; S_t/H_t = l\}$ is the endogenous liquidation time.

Using the same valuation approach as in subsection 3.1, the market values of the contingent claims defined above satisfy the following ODE[17]

$$(\alpha + r - \omega - m)SV_S + \frac{1}{2}\sigma^2 S^2 V_{SS} + (g - \omega - c^{'})HV_H + \lambda V - rV + f(S,t) = 0, \tag{A.4}$$

where $f(S,t)$ represents any payment made to the claims to be valued for the two different cases, and two boundary conditions apply as stated below.

Further, it is clear that the underlying economics of the problem implies that $V(y)$ is homogeneous of degree one in S and H, hence the solution has the form $V(S,H) = HQ(x)$, where $x \equiv S/H$. Substituting this expression as well as its derivatives into Equation (A.4) gives an ODE

$$\frac{1}{2}\sigma^2 x^2 Q_{xx} + (\alpha + r + c^{'} - g - m)xQ_x - (r + c^{'} - g + \omega + \lambda)Q + \theta x = 0, \tag{A.5}$$

[17] Similar to [Goetzmann, Ingersoll and Ross (2003)], the values of the contingent claims are independent of time in our setup, i.e. $V_t \equiv 0$

where $\theta = m$ for the case of management fee or total fees, and $\theta = \omega + \lambda$ for the case of payoffs allocated to the ordinary investors.

The solution to Equation (A.5) is given by

$$Q(x) = \frac{\gamma x}{m + \omega + \lambda - \alpha} + Ax^{\vartheta_1} + Bx^{\vartheta_2}, \tag{A.6}$$

where A and B are constants of integration and the two real roots, denoted by ϑ_1 and ϑ_2 solve the following quadratic equation:

$$\frac{1}{2}\sigma^2\vartheta(\vartheta - 1) + (\alpha + r + c' - m - g)\vartheta - (r + c' - g + \omega + \lambda) = 0. \tag{A.7}$$

Solving the above equation and imposing the no bubble conditions $m + \omega + \lambda \geq \alpha$ [18]leads to:

$$\vartheta_{1,2} = \frac{-(\varpi - \sigma^2/2) \mp \sqrt{(\varpi - \sigma^2/2)^2 + 2\sigma^2(\varpi + m - \alpha + \omega + \lambda)}}{\sigma^2}, \tag{A.8}$$

where$\varpi \equiv \alpha + r + c' - m - g$ obviously $\vartheta_1 < 1 < \vartheta_2$.

In order to solve the ODE, two boundary conditions are required. One boundary condition is determined as the asset value falls to the liquidation barrier, $\underline{x} \equiv l$:

$$\begin{aligned} i(l) &\equiv I(lH, H)/H = l, \\ f(l) &\equiv F(lH, H)/H = 0. \end{aligned} \tag{A.9}$$

The other condition applies along the boundary $\bar{x} \equiv 1$ when the HWM is reset to $H + \varepsilon$, while the net asset value exceeds the HWM at the level of $H + \varepsilon$ and then the manager obtains a performance fee of $k\varepsilon$, reducing the asset value to $H + \varepsilon(1 - k)$. For $\varepsilon \to 0$, we have

$$\begin{aligned} i(1) &= (k + 1)i'(1), \\ f(1) &= (k + 1)f'(1) - k. \end{aligned} \tag{A.10}$$

One can identify the solution $V(S, H)$ by specifying the general solution of the homogeneous ODE with the two boundary conditions defined in equations (A.9) and (A.10).

Solving the above equation and applying the no bubble conditions $m+\omega+\lambda \geq \alpha$ provides us with:

$$\begin{aligned} I(S,H) &= \frac{\omega+\lambda}{m+\omega+\lambda-\alpha}S - \frac{(\omega+\lambda)k+[\vartheta_1(1+k)-1](m-\alpha)l^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_2}S^{\vartheta_2} \\ &+ \frac{l^{\vartheta_2-\vartheta_1}(\omega+\lambda)k+[\vartheta_2(1+k)-1](m-\alpha)l^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_1}S^{\vartheta_1}, \end{aligned} \tag{A.11}$$

$$\begin{aligned} F(S,H) &= \frac{m}{m+\omega+\lambda-\alpha}S + \frac{(\omega+\lambda-\alpha)k+[\vartheta_1(1+k)-1]ml^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_2}S^{\vartheta_2} \\ &- \frac{l^{\vartheta_2-\vartheta_1}(\omega+\lambda-\alpha)k+[\vartheta_2(1+k)-1]ml^{1-\vartheta_1}}{(m+\omega+\lambda-\alpha)\{\vartheta_2(1+k)-1-l^{\vartheta_2-\vartheta_1}[\vartheta_1(1+k)-1]\}}H^{1-\vartheta_1}S^{\vartheta_1}. \end{aligned} \tag{A.12}$$

[18] Similar to [Goetzmann, Ingersoll and Ross (2003)] who state that, "the total withdrawals from the assets, $m + \omega + \lambda$, must exceed the superior performance, α; otherwise, the fund will have a residual value at infinity whose present value is infinite."

References

[Agarwal, Daniel and Naik(2009)] Agarwal,V., Daniel, N.D. and Naik, N.Y., 2009. Role of managerial incentives and discretion in hedge fund performance. J. Finance 64(5), 2221-2256.

[Agarwal, Daniel and Naik(2011)] Agarwal,V., Daniel, N.D., Naik, N.Y., 2011. Do hedge funds manage their reported returns? Rev. Financ. Stud. 24(10), 3281-3320.

[Aggarwal and Jorion(2010)] Aggarwal, R.K., Jorion, P., 2010. The performance of emerging hedge funds and managers. J. Financ. Econ. 96(2), 238-256.

[Aiken, Clifford and Ellis (2013)] Aiken, A.L., Clifford, C.P., Ellis, J., 2013. Out of the dark: hedge fund reporting biases and commercial databases. Rev. Financ. Stud. 26(1), 208-243.

[Aragon and Nanda(2012)] Aragon, G.O., Nanda, V., 2012. Tournament behavior in hedge funds: high-water marks, fund liquidation, and managerial stake. Rev. Financ. Stud. 25(3), 937-974.

[Basak, Pavlova and Shapiro(2007)] Basak, S., Pavlova, A., Shapiro, A., 2007. Optimal asset allocation and risk shifting in money management. Rev. Financ. Stud. 20(5), 1583-621.

[Carpenter(2000)] Carpenter, J., 2000. Does option compensation increase managerial risk appetite? J. Finance 55(5), 2311-2331.

[Chen, Miao and Wang(2010)] Chen, H., Miao, J.J., Wang, N., 2010. Entrepreneurial finance and non-diversifiable risk. Rev. Financ. Stud. 23(12), 4348-4388.

[Cox and Huang (1989)] Cox, J. C., Huang, C., 1989. Optimum consumption and portfolio policies when asset prices follow a diffusion process. J. Econ. Theory 49(1), 33-83.

[Duffie(2001)] Duffie, D., 2001. Dynamic asset pricing theory, 3rd ed. Princeton, NJ: University Press.

[Fung and Hsieh(1997)] Fung, W., Hsieh, D., 1997. Empirical characteristics of dynamic trading strategies: the case of hedge funds. Rev. Finanv. Stud. 10(2), 275-302.

[Fung and Hsieh(1999)] Fung, W., Hsieh, D., 1999. A primer on hedge funds. J. Empirical Finance 6(3), 309-331.

[Goetzmann, Ingersoll and Ross (2003)] Goetzmann, W.N., Ingersoll, J., Ross, S.A., 2003. High-water marks and hedge fund management contracts. J. Finance 58(4), 1685-1717.

[Grossman and Zhou(1993)] Grossman, S.J., Zhou, Z., 1993. Optimal investment strategies for controlling drawdowns. Math. Finance 3(3), 241-276.

[Hodder and Jackwerth (2007)] Hodder, J.E., Jackwerth, J.C., 2007. Incentive contracts and hedge fund management. J. Financ. Quant. Anal. 42(4), 811-826.

[Ingersoll(2006)] Ingersoll, J.E., 2006. The subjective and objective evaluation of incentive stock options. J. Bus. 79(2), 453-487.

[Kahl, Liu and Longstaff(2003)] Kahl, M., Liu, J., Longstaff, F.A., 2003. Paper millionaires: how valuable is stock to a stockholder who is restricted from selling it? J. Financ. Econ. 67(3), 385-410.

[Lan, Wang and Yang(2013)] Lan, Y., Wang, N., Yang, J.Q., 2013. The economics of hedge funds. J. Financ. Econ. 110(2), 300-323.

[Merton(1971)] Merton, R. C., 1971. Optimum consumption and portfolio rules in a continuous-time model. J. Econ. Theory 3(4), 373-414.

[Merton(1976)] Merton, R. C., 1976. Option pricing when underlying stock returns are discontinuous. J. Financ. Econ. 3(1-2), 125-144.

[Panageas and Westerfield(2009)] Panageas, S., Westerfield, M.M., 2009. High-water marks: high risk appetite? Convex compensation, long horizons, and portfolio choice. J. Finance 64(1), 1-36.

[Sorensen, Wang and Yang (2014)] Sorensen, M., Wang, N., Yang, J.Q., 2014. Value private equity. Rev. Financ. Stud. 27(7), 1977-2021.

[Wang, Wang and Yang(2012)] Wang, C., Wang, N., Yang, J.Q., 2012. A unified model of entrepreneurship dynamics. J. Financ. Econ. 106(1), 1-23.