Electron trapping and acceleration on a downward density ramp: a two-stage approach

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\textbf{Abstract.} In a recent experiment at Lawrence Berkeley National Laboratory (Geddes \textit{et al} 2008 Phys. Rev. Lett. 100 215004), electron bunches with about 1 MeV mean energy and small absolute energy spread (about 0.3 MeV) were produced by plasma wave breaking on a downward density ramp. It was then speculated that such a bunch might be accelerated further in a plasma of low constant density, while mostly preserving its small absolute energy spread. This would then lead to a bunch with a high mean energy and very low relative energy spread. In this paper, trapping of a low-energy, low-spread electron bunch on a downward density ramp, followed by acceleration in a constant-density plasma, has been explored through particle-in-cell simulations. It has been found that the scheme works best when it is used as a separate injection stage for a laser-wakefield accelerator, where the injection and acceleration stages are separated by a vacuum gap.

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1. Introduction

Plasma-based electron acceleration [1] holds the promise to deliver electrons at multi-GeV energies while requiring only a short acceleration distance. This was demonstrated most vividly when the energy of a 42 GeV electron beam was doubled in a 0.85 m long plasma in a beam-driven wakefield experiment [2]. While operating at lower energies, laser-driven wakefield acceleration has the advantage of producing electron bunches with an energy spread below 5% [3–5]. To increase the output energy of a plasma accelerator, one has to decrease the density of the background plasma and increase the acceleration length. In this way, the mean energy of the accelerated electron bunches has steadily increased with the current record hovering around 1 GeV [6, 7]. However, while the mean electron energy has increased from 200 MeV to 1 GeV during the past five years, the relative energy spread has not yet dropped much below 5%. In addition, while a lower plasma density leads to a higher bunch energy, a higher plasma density leads to a higher bunch charge. In order to improve the energy spread and charge content of laser-accelerated electron bunches, acceleration schemes have been proposed in which the background plasma density is higher at the beginning of the acceleration trajectory than at the end. Suk et al [8] have performed numerical simulations to study electron trapping and acceleration near a sharp density transition, where electrons are injected into a laser-driven wakefield at high plasma density (to obtain a high bunch charge) and subsequently accelerated at a lower density (to obtain a high bunch energy) [9]. Sprangle et al [12] have demonstrated acceleration via a two-stage injection-acceleration scheme [13], using a high plasma density for the injection stage and a lower density for the acceleration stage. Tomassini et al [10] and Brantov et al [11] have performed numerical simulations to study electron trapping on a steep downramp (the length of the ramp is 10–50 laser wavelengths), and found that this facilitates trapping of electron bunches with a fairly high mean energy (up to 300 MeV) and energy spread (up to 40 MeV). Recently, Geddes et al [14] proposed a scheme in which electrons are trapped in a region of decreasing plasma density. This scheme was motivated by the observation that bunches of plasma electrons with low mean energy (∼1 MeV) and low absolute energy spread (∼170 keV) can be trapped on a fairly shallow downramp (the length of the ramp is 500–1000 laser wavelengths). It is expected that such bunches can be accelerated to much higher energies during a second acceleration stage, using a low background plasma density, while preserving the small absolute energy spread, leading to a much improved relative energy spread. Preliminary simulations indicate that, with a perfectly matched plasma channel following the density downramp, the electron bunch can be accelerated to beyond 25 MeV while its energy spread is preserved.

While Geddes et al [14] have demonstrated that the downramp acceleration scheme can indeed be used to produce electron bunches with low mean energy and very low absolute energy spread, they did not investigate whether the bunch parameters, in particular the low energy spread, can also be found for different density gradients or higher laser powers. Although they allude to acceleration of low-spread bunches to beyond 20 MeV, they do not show how this can be done, and which laser and plasma parameters would be needed to accomplish this. In this paper, we will investigate these issues by means of particle-in-cell (PIC) simulations of laser-driven electron trapping and acceleration on a downward plasma density ramp.

2. Theory

Electron trapping in a plasma wave propagating down a density ramp is mostly the result of secular behaviour of the oscillating electron fluid. This works as follows. A plasma oscillation having a position-dependent frequency is given by e.g. \( x(\bar{x}, \tau) = A \cos(\phi(\bar{x}, \tau)) = A \cos(k_0 \bar{x} - \omega(\bar{x}) \tau) \). The effective wave number \( k_{\text{eff}} \) is then derived from \( k_{\text{eff}} = \partial \phi / \partial \bar{x} = k_0 - (\partial \omega / \partial \bar{x}) \tau \).

Wave breaking will occur when \( |k_{\text{eff}} A| = 1 \), so even for \( k_0 A \ll 1 \) secular behaviour will cause the wave to break after a time of at most \( \tau_{\text{WB}} = 1/(A |\partial \omega / \partial \bar{x}|) \). Also, by defining the effective phase speed as \( v_{\phi,\text{eff}} = \omega / k_{\text{eff}} \) [15, 16], it follows that wave breaking occurs if the peak forward fluid speed \( v_{\text{max}} \) satisfies \( v_{\text{max}} = \omega A = v_{\phi,\text{eff}} k_{\text{eff}} A = v_{\phi,\text{eff}} \). As shown in [15, 16], secular behaviour will cause the wave’s phase speed to decrease until it equals the peak forward fluid speed, at which point the wave breaks. It is this very effect that has been exploited in the experiment by Geddes et al [14].

By exciting a wakefield in a plasma with a downward density ramp, background plasma electrons can be trapped easily because the wakefield has such a low phase speed. Of course the trapped electrons will dephase quickly in this wakefield, so the resulting bunch of accelerated electrons will have a low mean energy and a low absolute energy spread and transverse emittance (proportional to the mean energy). The goal is then to inject this bunch into a plasma channel with constant axial density extending from the bottom of the density ramp. Further acceleration in this channel should ideally preserve the low absolute energy spread and transverse emittance while increasing the mean energy, leading to an electron bunch with very low relative energy spread and transverse emittance.

When exciting a wakefield on a density downramp using a laser pulse, it is important that background electron trapping only occurs on the ramp itself. Bunches of electrons that get trapped in the high-density region preceding the top of the ramp will not have the desired low absolute energy spread and transverse emittance, as we shall show below. However, the laser pulse has to traverse this high-density region before it reaches the density ramp, and unwanted electron trapping may easily occur here. One solution, as employed by Geddes et al [14], is to use a tightly focused laser pulse, so that its Rayleigh length will not extend beyond the density ramp and its intensity before reaching focus will be too low to trigger electron trapping. However, this solution comes with its own issues, as the laser pulse defocuses rapidly after leaving the density ramp, and may be unfit to drive the wakefield that is needed to accelerate the trapped electron bunch further. A second laser pulse may therefore be needed, which renders the scheme more complicated.

In the next section, we present the results of two-dimensional PIC simulations exploring both the trapping of an electron bunch on a density downramp and the potential for further acceleration of this bunch.
3. Simulation results

The simulations were performed using the PIC code Osiris [17]. We performed three classes of simulation: those with a low-power pulse (10 TW), those with a high-power pulse (40 TW) and those with a gap between the density ramp and the channel (and a 10 TW pulse). In all simulations, cells of 60.5 by 242 nm are used, with 36 particles per cell, mobile electrons and static ions. A moving simulation window is used to follow the laser pulse, to reduce computational demands. Absorbing boundary conditions are used for both particles and fields. The plasma density profile consists of a density ramp, where the density would fall from $n_{cr}/100$ to $n_{cr}/400$ over a distance of 1 mm, followed by a region of constant density $n_{cr}/400$. The laser pulse is always focused at 0.8 mm down the ramp, i.e. near the bottom, and propagates in the $x$-direction. In some simulations, the ramp and the region of constant density are separated by a vacuum gap 1.5 mm long. The laser pulse has a carrier wavelength of 800 nm, a duration of 40 fs, and a Gaussian profile in both the longitudinal and transverse directions.

When a low laser power is used (10 TW focused onto a 7.5 $\mu$m spot), the coveted low-spread, low-emittance bunch is obtained at the bottom of the downramp, similar to the findings of Geddes et al [14]. Because the pulse is so tightly focused, its intensity before the start of the downramp is too low to trigger premature electron trapping. Electron trapping starts at the top of the ramp, and then the trapped electrons are injected into the second wakefield period behind the laser pulse when the wakefield slows down on the ramp. No trapping occurred in the first wakefield period behind the laser pulse in this simulation. It was found that, due to the strong focusing of the laser pulse near the bottom of the ramp ($a_0 = 3.0$ at focus), the resulting plasma wave is already very close to breaking at the moment it is excited. The subsequent decrease in phase speed is then enough to push it over the edge relatively quickly (even at the fairly shallow slope we used), leading to wave breaking, as observed in the simulations. To verify this, we conducted a simulation using a 10 TW laser pulse on a flat density profile at $n_{cr}/400$ (ramp removed, otherwise identical). As shown in figure 4, left frame, we observed the trapping of minute amounts of plasma electrons, too small to significantly change the structure of the plasma wave, but enough to prove that the wave is on the verge of breaking even without the ramp.

It should be noted at this point that the coveted low-spread, low-emittance electron bunch will only be produced in the second period behind the laser pulse, and only if there is no trapping in the first period. Trapping in the first period will produce a high-energy bunch with moderate energy spread that can also be produced without a density ramp [3]–[7]. Moreover, this bunch will perturb the wakefield behind it so much that the production of low-spread, low-emittance bunches in the second period is inhibited.

Although the production of low-spread electron bunches on a density downramp is fairly straightforward, their further acceleration proves to be more difficult. When the ramp goes over into a region of plasma with constant axial density, the wakefield wavelength increases suddenly. While the first wakefield period accelerates immediately when it reaches the constant-density region, the increase of the wakefield wavelength causes the second period to decelerate even further for a short while, before it speeds up as well. This sudden additional deceleration of the second period causes the trapped electron bunch inside to move from the back to the front of this period, where the electric fields are decelerating instead of accelerating. This causes the front of the electron bunch to decelerate, while the back of the bunch is accelerated. This leads to a very
large energy spread in the bunch (>50%), completely spoiling the effect that this downramp acceleration scheme was meant to yield. The simulation results for this case are displayed in figure 1.

This dephasing problem can be overcome by leaving a gap between the bottom of the downramp and the start of the region of constant plasma density. This works as follows. The trapping of the low-spread electron bunch needs to happen in the second period behind the laser pulse, while its subsequent acceleration needs to happen in the first period. Thus, the bunch needs to be moved from the second to the first period behind the laser pulse. Normally, the decelerating fields between the second and the first wakefield period would spoil the bunch properties. However, if a vacuum gap is used between the two regions of plasma, the wakefield amplitude drops to zero temporarily, and the bunch can safely be moved from the second to the first period without losing much quality. When the plasma density for the second (acceleration) stage is chosen sufficiently low, then the bunch will re-enter the laser pulse’s wakefield at the
Figure 2. Evolution of the low-energy, low-spread electron bunch when there is a vacuum gap between the downramp and the homogeneous plasma. Top: schematic view of the longitudinal density profile used. Bottom: snapshots of the \((x, p_x)\) electron phase space when the driving laser pulse is at 2.25, 2.5 and 2.75 mm. The shape of the electron bunch does not change significantly while it is crossing the vacuum gap, and it is injected cleanly into the wakefield in the homogeneous plasma beyond the gap. This shows the potential of electron trapping on a downramp as a separate injection stage for a laser wakefield accelerator.

**Figure 2.** Evolution of the low-energy, low-spread electron bunch when there is a vacuum gap between the downramp and the homogeneous plasma. Top: schematic view of the longitudinal density profile used. Bottom: snapshots of the \((x, p_x)\) electron phase space when the driving laser pulse is at 2.25, 2.5 and 2.75 mm. The shape of the electron bunch does not change significantly while it is crossing the vacuum gap, and it is injected cleanly into the wakefield in the homogeneous plasma beyond the gap. This shows the potential of electron trapping on a downramp as a separate injection stage for a laser wakefield accelerator.

back of the first wakefield period, i.e. the optimal position for acceleration. However, an intense laser pulse is needed to drive a suitable wakefield when the density is low, and the ‘injection’ laser pulse may have diverged too much already at this point. So a second ‘acceleration’ pulse may be needed (which is less tightly focused and should not pass through the plasma where the trapping occurs) to drive the wakefield in the channel. While this introduces an additional complication, it also offers fresh opportunities. Since the timing of the electron bunch is tied to the first laser pulse, but not to the second, the distance at which the electron bunch follows the second pulse can be regulated by proper timing of the second pulse. This allows one more control over the second (acceleration) stage, and more freedom to choose the plasma density for this stage.

The simulation results for the case of two-stage trapping and acceleration separated by a vacuum gap are displayed in figure 2. A vacuum gap of 1.5 mm length was used. While a much shorter gap would have sufficed to prevent an increase in energy spread, we used a gap of this length to show that the electron bunch does not suffer much from its journey through vacuum, so the two stages can be clearly separated spatially without any undesired side effects. We find that the low-energy, low-spread bunch retains its shape when crossing the vacuum gap from one
region of plasma to the next. By choosing the appropriate plasma density after the gap, so the length of the first wakefield period is slightly larger than the distance between the laser pulse and the electron bunch, the electron bunch can be injected at the back of the first period, an ideal position for further acceleration. However, the laser pulse has already defocused significantly at this stage, and is no longer able to excite a wakefield with a sufficiently large amplitude for proper electron acceleration. Since increasing the power of the ‘injector’ laser pulse does not appear to be a good idea (see below), one may well need a second laser pulse having a wider focal spot to drive the wakefield during the second (acceleration) stage, as discussed above.

It should be noted that the above dephasing problem is caused in part by the sudden transition from the density ramp to a constant axial density. If a much smoother transition is used, then it might be possible to have the wavelength of the wakefield increase so gradually that the trapped electrons will never move ahead of the centre of the second bucket behind the laser pulse, and thus not dephase. However, this will require more control over the density profile than is generally available in experiments. Also, it is better to have the electron bunch sit in the first bucket behind the laser pulse during the acceleration stage, as the field amplitude (and thus the final energy) is generally higher there. So for practical reasons, it is better to leave a vacuum gap between the end of the density ramp (trapping stage) and the section with constant axial density (acceleration stage).

Increasing the power of the ‘injection’ pulse does not help at all, surprisingly. The injection of an electron bunch with a low absolute energy spread and emittance on a density downramp turns out to be a rather delicate process. Too many electrons will be injected when the ‘injection’ pulse is too powerful, and the entire effect will be spoiled. We performed a simulation using a 40 TW laser pulse focused onto a 15 µm wide spot, in an attempt to trap more electrons. However, in such a configuration the laser pulse is too powerful: rather than seeing some gentle electron injection at the bottom of the ramp only, wave breaking and electron trapping happen over the entire length of the ramp. Large quantities of electrons are injected into the wakefield, obscuring the coveted low-energy, low-spread, low-emittance bunch. The simulation results for this case are displayed in figure 3. We also conducted simulations with 20 and 40 TW pulses focused on a 7.5 µm wide spot (not shown), with similar results. These results show that the true strength of the downramp scheme lies in the production of low-spread, low-emittance electron bunches by a laser pulse of moderate power (10 TW), and not in the production of high-charge, high-energy pulses by high-power laser pulses.

More generally, the trapping of electron bunches having low energy, energy spread and emittance on a density downramp does not prove to be too amenable to a change in laser or plasma parameters, a fact that is not sufficiently recognized in the original paper by Geddes et al [14]. This is because the scheme depends so critically on electron trapping in the second wakefield period and the absence of trapping in the first period. Changing any of the laser or plasma parameters, e.g. a higher laser power or a steeper density gradient, will soon trigger trapping in the first wakefield period. For 40 TW laser pulses, this has already been demonstrated. Further confirmation was found when we ran a simulation with a 10 TW pulse focused on a 7.5 µm spot, but with a steeper density ramp: the density drops from \( n_{\text{cr}}/100 \) to \( n_{\text{cr}}/400 \) over 0.5 mm rather than 1 mm, and the pulse was focused near the bottom of this new ramp. The most important result of this simulation is shown in figure 4, right frame. Once again, a low-spread, low-energy bunch has been trapped in the second wake period behind the laser pulse, but the effect is somewhat spoiled by an electron bunch in the first wake period with higher energy and higher spread. This bunch obscures the coveted low-spread bunch in the
Figure 3. \((x, p_x)\) electron phase space when a 40 TW laser pulse and a 15 \(\mu\text{m}\) spot diameter are used. The left frame has been taken from a simulation with a downramp, with the laser pulse at 1.0 mm. The position of the low-energy, low-spread electron bunch has been highlighted, but the picture is dominated by other electron bunches that do not have a low energy spread. The right frame has been taken from a simulation without a downramp, only homogeneous plasma at \(n_{cr}/400\), with the laser pulse at 3.0 mm. As the quality of the electron bunches is comparable between these cases, it is obvious that the presence of a density ramp does not add anything when a 40 TW laser pulse is used.

energy spectrum, and the beam loading of the bunch in the first wake period will disturb the second wake period, and thus the properties of the second bunch. Further acceleration of both bunches would increase their energy difference, allowing them to be separated, but this will also decelerate the second bunch and increase its energy spread, via the same ‘dephasing’ process as seen in our earlier simulations. As in the case of the 40 TW pulse on a 1 mm long ramp, electron trapping in the first wake period will at best lead to an energetic bunch with a moderate energy spread, and at worst to trapping of multiple electron bunches and a very large energy spread. For this reason, a parameter scan will yield fewer results and be less useful than might be expected. It is therefore better to concentrate on getting the scheme to work using those laser and plasma parameters for which it has been proven than to attempt to make it work for parameters that are well removed from those originally used in [14].

4. Conclusions

We studied electron trapping in a laser-driving wakefield that was propagating down a density ramp. In particular, we investigated the possibility of accelerating the trapped electron bunch in a secondary stage, with the aim of increasing its energy while preserving its absolute energy spread and transverse emittance. We found that, as first demonstrated by Geddes et al [4], it is indeed possible to obtain an electron bunch with a small absolute energy spread and small transverse emittance using the downramp scheme. However, we also found that it is not at all straightforward to ease this bunch into a wakefield at constant axial background density. The sudden increase in the wakefield period causes the electron bunch to dephase, increasing its energy spread and spoiling the effect.
The most promising results are obtained when a gap is left between the bottom of the ramp and the start of the region of homogeneous plasma. The electron bunch remains more or less intact while it crosses this gap, and the wakefield beyond the gap can be excited in such a way that it accepts the electron bunch without causing it to dephase or receive a large energy spread. However, for the wakefield period to be large enough so that the electron bunch ends up at the back of the first period behind the laser pulse, the channel’s axial density needs to be quite low, and the laser pulse needs to be quite intense to drive a proper wakefield. The ‘injection’ pulse needs to be very tightly focused and will have defocused significantly once it reaches the channel, so this pulse will not be intense enough at this point. A secondary ‘acceleration’ pulse may be needed to drive the wakefield in the channel.

Increasing the intensity of the ‘injection’ pulse does not help at all, surprisingly. A more intense ‘injection’ pulse causes the injection of too many particles at all energies, leading to a large energy spread and spoiling the effect that one is aiming for. Therefore, we conclude that the scheme works best for the production of bunches with modest charge, and when it is used as an injector for a separate wakefield accelerator.

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