Strategic trade in pollution permits

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Abstract

Markets for pollution have become a popular regulatory instrument. In this article we investigate the implications of strategic trade in pollution permits. The permit market is developed as a strategic market game, where all firms are allowed to behave strategically and their roles as buyers or sellers of permits are determined endogenously with price-mediated trade. In a second stage, firms transact on a product market and we allow for a variety of market structures. We identify a unique equilibrium in permit exchange, investigate the properties of this equilibrium, and consider the effect of strategic behavior in the product market.

Key words: Pollution market, Market power, Strategic market game.

JEL classification: D43, Q52.

1 Introduction

Markets for pollution permits have emerged as a mainstream regulatory instrument. Since the early adoption of the US Acid Rain Program numerous schemes have been established to control pollution.[1] Behind this spirited regulatory response lies the economic rationale of least-cost pollution control: aggregate control costs are minimized when players trade pollution permits. This least-cost result relies on the existence of low transactions costs as well as players acting competitively.[2] Yet players' strategic behavior in these markets—and the resulting social losses—are a real concern (Hintermann, 2016a, 2017). The actions of large and influential players in the market have the potential to distort the equilibrium permit price, reduce the cost effectiveness of pollution control, and influence the product market equilibrium. Although the existence of market power and the associated losses may be significant, the fundamental aspects of this problem—the interactions between players in both the permit and product markets—are not well understood. In particular, little is known about the formation of equilibria in the permit market when all players behave strategically and the corresponding impact on the product market (Montero, 2009).

In this article we adopt a strategic market game to model trade in pollution permits and consider firms' subsequent decisions in the product market. In a strategic market game, signals are quantity-based and the market mechanism that determines the permit price is explicitly defined. All firms are allowed to behave strategically, and the role of firms as buyers or sellers of permits is determined endogenously. We demonstrate that there is a unique equilibrium in which trade in permits takes place, which from the perspective of an economic modeler is a very desirable facet of a market model. We study the conditions under which permit trade will occur and the cost effectiveness of the equilibrium, as well as studying the effect of increased demand in the product market, improvements in abatement technologies and altering initial permit endowments. In addition, we investigate the effects of different product market structures as we seek to understand the interplay between market power in the product market and the permit market equilibrium.

The idea that firms attempt to manipulate the permit market price has long been recognized.

[1]Examples include the European Union Emissions Trading Scheme (EU ETS), the Regional Greenhouse Gas Initiative (RGGI), and the California Cap-and-Trade Program. Markets are also commencing in South Korea, China, and India.

[2]In early schemes transaction costs appeared to be problematic, for example, in the Fox river (O'Neil et al., 1983) and RECLAIM (Foster and Hahn, 1995). Yet in most modern permit markets prohibitive transaction costs do not appear to be a significant problem. Aside from cost effectiveness, a whole host of explanations can be proposed for explaining inefficiency within schemes, such as the political economy aspects of regulation, compliance issues, and uncertainty.

A vast literature has followed the contribution by Hahn (1984).[3] In his study, Hahn developed a permit market model with a single large trader and a price-taking competitive fringe of small traders (this has since been extended to consider competition between oligopolists in the presence of a competitive fringe).[4] The competitive fringe framework, however, suffers from two distinct drawbacks: first, exogenous behavioral restrictions must be made in the model that relegate some firms to the role of price takers that have no market power; second, the model has the unfortunate consequence that as the competitive fringe shrinks, no trade will take place amongst large traders even if there are substantial gains from trade between them. These deficiencies render competitive fringe models inappropriate for studying permit exchange when there is strategic behavior by *all* firms.

To overcome the drawbacks of a competitive fringe framework, recent literature has focused on using the supply function approach of Klemperer and Meyer (1989) to model trade in permits (Malueg and Yates, 2009; Wirl, 2009; Lange, 2012; Haita, 2014).[5] In this framework firms are required to submit supply functions to an auctioneer who then determines the price to clear the market. Although the supply function approach can analyze strategic trade, a well-known drawback exists: there is a multiplicity of equilibria. While this approach is descriptively appealing, identifying a unique equilibrium is a key desiderata, particularly if one then wants to incorporate subsequent product market decisions in the analysis. To obtain a unique equilibrium, Malueg and Yates (2009) assume that each firm's supply function is similar apart from the intercept. But this assumption—essentially requiring all firms' marginal abatement costs to be linear and have the same slope—has the unfortunate consequence that, under complete information, the equilibrium price is the same for both strategic and competitive markets. Lange (2012) uses an equilibrium refinement that identifies a unique equilibrium if it is assumed that all firms' marginal abatement costs are linear (but not necessarily the same). Although Lange (2012)'s assumption of linear marginal abatement costs is not unreasonable with respect to fixed unrestricted emissions, it does become problematic when one wants to consider decisions in a subsequent product market. Assuming a linear relationship between permit holdings and marginal abatement costs will place restrictions on equilibrium production functions and will limit the scope and applicability of any insights that might be drawn from the analysis. Yet without this linearity assumption the uniqueness of the supply function equilibrium fails. Thus,

[3]See Montero (2009) and Reichenbach and Requate (2013) for comprehensive literature surveys on market power in pollution markets. Using frameworks that model exhaustible resources, market power in pollution markets has also been considered when pollution permits are storable (Liski and Montero, 2006, 2011).

[4]See, for example, Westskog (1996) and, more recently, Hagem (2013) that discusses the choice of strategic behavior.

[5]For a further discussion see Godal (2011).

to develop further analysis on the strategic trade in permits, a framework that can circumvent these problematic dichotomies is required.[6]

To model permit exchange, we propose a strategic market game (Shapley and Shubik, 1977), which requires only conventional assumptions (i.e., convex costs) and provides a unique equilibrium in which permits are traded. In addition, we incorporate firms' product market decisions following trade in permits. Our model is comprised of two stages. In the first stage, traders participate in a strategic permit market game, which is based on quantity signals à la Cournot and involves an explicit price formation mechanism. Traders in the permit market participate by submitting either an offer (of permits) or bid (of money). A trading post then aggregates the offers and bids and determines the price of permits in a way that clears the market. Trade is thus price mediated, and the sides of the market form endogenously: whether a trader wishes to buy or sell permits depends on their abatement technology and on their conjecture of the price in the market, which is determined by their beliefs about the market actions of other traders. In the second stage, once firms receive their final permit transfer from the permit market, they transact on the product market, and in our benchmark model we assume firms are price takers in the product market.

To provide a full equilibrium characterization in the permit market, we follow a three-step approach that exploits the aggregative properties of the permit market game and takes into account subsequent product market decisions. In Step 1, we hypothesize a permit price and consider whether firms would be (potential) buyers or sellers of permits. In Step 2, we consider the behavior of each side of the market separately at the hypothesized permit price, deducing the aggregate supply of, and demand for, permits at that price. Finally, in Step 3, we check whether the hypothesized permit price is consistent with aggregate demand and supply. If so, then we have identified a Nash equilibrium (given the subsequent product market activity). We thus identify the conditions under which an equilibrium with trade in permits exists, and demonstrate that such an equilibrium is unique.

This approach provides a comprehensive, realistic, and tractable structure to analyze strategic trade in permit markets (and the associated product market), and we use the framework to consider comparative static properties of equilibrium; investigating the effect of improvements in abatement technologies, changes in initial permit endowments, and changes in product market demand. On the basis that industries regulated by cap-and-trade markets are often highly

[6]We believe that it is important to allow for non-linear marginal abatement costs, particularly when subsequent product market decisions are considered where our model has a distinct advantage over the supply function approach of giving a unique prediction of equilibrium and associated comparative statics properties. Whether or not marginal abatement costs are non-linear could be subjected to empirical validation.

concentrated (regionally segregated) markets (Wolfram, 1999; Borenstein et al., 2002; Bushnell et al., 2008), we investigate the effects of market power in the product market by allowing for alternative product market structures.[7] We advance our analysis by first considering that firms act as independent monopolists on segregated markets to capture a pure market power effect, and then suppose that firms act as oligopolists competing in the same product market where strategic issues are of central importance. Using our framework, we highlight three pivotal avenues in the interaction between strategic permit and product markets: first, the acquisition of permits has a *direct* effect on production decisions due to the effect on the cost structure of the firm; second, there is an *indirect (strategic)* effect since a firm's permit transactions influence its own marginal cost that influences the equilibrium in the product market; and third there is a *changing rivals' costs* effect since any permits acquired by one firm cannot be acquired by another, influencing other firms' marginal costs which also influences the product market equilibrium.

Our contribution is twofold. First, we provide a novel framework that models strategic trade in pollution permits. This allows for the full characterization of the unique equilibrium in which trade in permits takes place when players are heterogeneous and have general abatement cost functions. Our approach, in a more general setting, eliminates the issue of multiple equilibria prevalent in the current literature. With such an approach, this provides a tractable foundation for the evaluation of contemporary cap-and-trade markets when strategic behavior exists for all market participants. Second, by combining our analysis with alternative product market structures, we also provide an encompassing model that incorporates many current regulatory market structures. Our general approach can be used to nest previous attempts to capture strategic behavior in the product market (e.g., Misiolek and Elder, 1989) as well as complementing the recent literature on strategic permit markets (Malueg and Yates, 2009; Wirl, 2009; Lange, 2012; Haita, 2014).

This article is structured as follows. Section 2 outlines the economic environment and determines the equilibrium characterization of a strategic market game in the permit market given that firms subsequently make product market decisions. Section 3 provides a discussion of the permit market equilibrium. Section 4 provides an illustrative example. Section 5 extends

[7]Recently, Fowlie et al. (2016) investigated the adoption of market-based instruments (without market power) on a highly concentrated product market (a regionally segregated cement industry), finding that the establishment of a market-based instrument coupled with the market-power distortions in the product market generate losses over-and-above any benefits associated with emissions mitigation. See also Ryan (2012). Earlier literature has also investigated the connectivity between the permit and product market, which has been framed through a traditional competitive fringe framework (e.g., Sartzetakis, 1997; Hintermann, 2011). For additional insights see De Feo et al. (2013).

the framework to include strategic behavior in the product market. We then conclude in Section 6.

2 The model

2.1 The economic environment

Consider an economic environment that is populated by an index set of firms $I = \{1, \ldots, N\}$, where firm $i \in I$ has an initial stock of money $m_i > 0$.[8] Firms operate in a product market where the production of goods generates pollution. This pollution is regulated by a cap-and-trade scheme, and the regulator has a pollution target given by E, which is assumed to be lower than the aggregate level of unconstrained emissions. Firms have the option to either hold a permit to cover emission liabilities, or reduce emissions by utilizing (costly) abatement technologies. Of the E permits the regulator brings into existence, a fraction $0 \leq \mu \leq 1$ of these may be retained and sold directly on the permit market[9], while the remaining $\Omega \equiv (1 - \mu)E$ permits are allocated to firms as a free initial endowment, with the endowment of firm i being denoted by $\omega_i \geq 0$. Firms have the opportunity to engage in permit trade where they can sell some or all of their initial endowment, or buy additional permits; we assume throughout that the regulator's fixed supply of permits to the market is modest in relation to the size of the market in the sense that it does not drive the permit price to zero. We consider a two-stage environment: in the first stage permit trade results in a distribution of permits among firms, which becomes common knowledge; in the second stage firms make production decisions in the product market.

Let $x_i \in \mathbb{R}$ denote firm i's permit transactions, which have been determined by trading on the permit market: $x_i > 0$ for purchases of permits and $x_i < 0$ for sales. Let $\mathbf{x}$ denote the vector of permit transactions for all firms, and $\mathbf{x}_{-i}$ the vector of all permit transactions excluding that of firm i. The total number of permits firm i has after trading is $\omega_i + x_i$ and we denote the price of permits by p; in our model this will be determined by firms' actions. In the product market firm i's output is denoted by z_i, and $Z = \sum_{i \in I} z_i$ is the aggregate supply of the good. While we allow for strategic behavior in trading permits, in our baseline model we assume firms act as price-takers in the product market so the price of the good is considered fixed and denoted

[8] We assume firms' initial money holdings are large enough so that their behavior is not constrained by a lack of money.

[9] This can, for example, reflect a regulator that auctions a proportion of permits, where the outcome of the auction is determined by the market mechanism that we subsequently describe. It is assumed μ is common knowledge among firms and, while it is interesting to consider the incentives of the regulator in setting μ, we take it to be exogenously given in our model.

ϕ; later we consider that firms' supply decisions in the product market influence the product price according to an inverse demand relationship $\Phi(Z)$. Production of the good generates pollution and, absent other considerations, the quantity of pollution emitted in producing z_i is given by $f_i(z_i)$. Any pollution that is not covered by a permit must be abated; accordingly, pollution abatement required by firm i is $a_i \equiv f_i(z_i) - [\omega_i + x_i]$. Firms undertaking production incur direct production costs and pollution abatement costs, so firm i's total cost of production is given by $C_i(z_i, a_i)$.

Assumption. *For each firm $i \in I$ the functions $f_i(\cdot)$ and $C_i(\cdot,\cdot)$ are twice continuously differentiable; $f_i', f_i'' \geq 0$; $C_i^z, C_i^a \geq 0$ with a strict inequality if $z_i > 0$; $C_i^{zz}, C_i^{aa} > 0$ and $C_i^{za} \geq 0$; $C_i^{zz}C_i^{aa} - [C_i^{za}]^2 > 0$; and finally $C_i^z + f_i' C_i^a = 0$ when $z_i = 0$, where superscripts z and a denote derivatives with respect to output and abatement, respectively.*

Firm i's payoff is comprised of any initial wealth m_i, revenue or costs associated with permit market activity $x_i p$, and, after accounting for all costs of production, the profit from productive activity:

$$V_i = m_i - x_i p + z_i \phi - C_i(z_i, f_i(z_i) - [\omega_i + x_i]).$$

Once the initial permit endowment has been set (which is common knowledge), firms have the opportunity to trade permits and the market mechanism will determine the permit transactions. To capture firms' strategic behavior in the market for pollution permits, we turn to a model of bilateral oligopoly with a market mechanism in which market actions are quantity-based and trade is price mediated; no price-taking assumptions are imposed *ex ante* (in the permit market) and the role of firms as buyers or sellers of permits is determined endogenously in the market. Such 'strategic market games' were introduced by Shapley and Shubik (1977) to model fully strategic behavior in general equilibrium settings, which we restrict to the case of two commodities—a good (permits) and money (see Dickson and Hartley, 2008).[10] Trade takes place by way of an *explicit* trading mechanism: there is a 'trading post' to which firms submit an *offer* of permits to be exchanged for money or a *bid* of money to be exchanged for permits, depending on whether they want to sell or buy permits.[11] The trading post aggre-

[10]Note that Dickson and Hartley (2008) provide a tractable analysis of a simplified version of the framework of Shapley and Shubik (1977) in which strategic behavior of both buyers and sellers in the market for a single good is considered. Here we extend this analysis to consider the interplay between two related markets—the permit market and subsequent product market—where the outcome in one determines the margins of decision-making in the other.

[11]This is in contrast to the existing literature on strategic trade in pollution permits (e.g., Hahn, 1984; Hintermann, 2011) that invariably assumes the presence of a 'competitive fringe' necessitating a 'black box' (auctioneer) approach to market clearing.

gates the offers and bids of all firms, augments the supply from firms with any fixed supply of permits from the regulator, and determines the price of permits as the *ratio* of the total amount of money bid to the total number of permits offered for sale. Exchanges are then determined according to the offers and bids made and the resulting market price. Trade is therefore price mediated, and each individual firm considers that their actions influence the price of permits. Whether a firm wishes to buy or sell permits will depend on any initial endowment of permits they received, their abatement technology and their belief about the price in the market.

Formally, firm i can make an *offer* of permits $0 \leq q_i \leq \omega_i$ to be exchanged for money, or make a *bid* of money $0 \leq b_i \leq m_i$ to be exchanged for permits.[12] We assume that firms only buy permits from their initial money holdings and we rule out firms making 'wash trades', i.e., contemporaneously buying *and* selling permits. The set of strategies available to firm $i \in I$ is therefore

$$\mathcal{S}_i = \{(b_i, q_i) : 0 \leq b_i \leq m_i, 0 \leq q_i \leq \omega_i, b_i \cdot q_i = 0\}.$$

If a firm has no initial endowment of permits so $\omega_i = 0$ then that firm can only act on the demand side of the market by making a bid for permits; if $\mu = 1$, so the regulator withholds all permits to sell directly on the market, then this is true for all firms.

The role of the trading post is to aggregate firms' offers and bids and determine trades. Let the aggregate bid be $B = \sum_{i \in I} b_i$ and the aggregate offer of permits be $Q = \sum_{i \in I} q_i$. If the regulator withheld permits to supply to the market then the total supply of permits is $\mu E + Q$. If $B = 0$ or, if $\mu = 0$, $Q = 0$ then the trading post is deemed closed and any offers or bids that are made are returned. So long as $B > 0$ and $\mu E + Q > 0$, the price of permits (denominated in units of money) is determined as $p = B/[\mu E + Q]$, and the number of permits allocated to firm i (in addition to their initial endowment) is given by

$$x_i = \begin{cases} b_i/p & \text{if } b_i > 0, q_i = 0 \text{ or} \\ -q_i & \text{if } q_i > 0, b_i = 0. \end{cases} \tag{1}$$

The change in firm i's money holdings is thus

$$-x_i p = \begin{cases} -b_i & \text{if } b_i > 0, q_i = 0 \text{ or} \\ q_i p & \text{if } q_i > 0, b_i = 0. \end{cases}$$

[12]Notice that in this trading environment firms submit a single offer or bid, rather than a schedule of net demands that depends on the price, as in supply function models. Also, throughout this article, it is assumed that a sufficiently large penalty can be levied on firms for offering more permits than are in their possession, or making bids that exceed their money holdings, that this will never constitute equilibrium behavior.

An intuitive interpretation of the mechanism is as follows: on one side, the market mechanism receives permits from those firms that want to sell, and on the other it receives money from those firms that want to buy; then, the total supply of permits is allocated among those firms that want to buy in proportion to their bids (so each firm that made a bid receives $[b_i/B][\mu E + Q]$ permits), and for each permit supplied, firms receive a price which is determined by the ratio of the total amount of money bid to the total number of permits offered for sale ($p = B/[\mu E + Q]$). Since traders' market signals are quantity-based, this market mechanism can be seen as an extension of Cournot competition to allow for strategic behavior on both sides of the market; as with the Cournot model itself, while it may not be a descriptively accurate account of market activity the model manages to sensibly account for strategic behavior with, as we will see, very plausible outcomes.

Once permit trading has taken place, permit transactions become common knowledge and firms engage in production decisions in the product market. In our baseline model we assume that firms behave as price-takers in the product market by modeling it as a perfectly competitive market in which the price is fixed at ϕ. Later in the article, we explore the implications of firms having market power in the product market.[13]

2.2 Product market decisions

Let ϕ denote the price within the perfectly competitive product market. Then the profit of a typical firm $i \in I$ from their product market activity is

$$\tilde{\pi}_i(z_i; x_i) = z_i\phi - C_i(z_i, f_i(z_i) - [\omega_i + x_i]). \tag{2}$$

Once the permit market has cleared and firm i has permit transactions x_i, the product market profit function $\tilde{\pi}_i(z_i; x_i)$ depends only on z_i. Firm $i \in I$ will seek to choose z_i to maximize $\tilde{\pi}_i(z_i; x_i)$, where the first-order condition is

$$\frac{\partial \tilde{\pi}_i(z_i; x_i)}{\partial z_i} \leq 0 \Leftrightarrow C_i^z(z_i, f_i(z_i) - [\omega_i + x_i]) + f_i'(z_i)C_i^a(z_i, f_i(z_i) - [\omega_i + x_i]) \geq \phi, \tag{3}$$

[13] To ensure tractability in the analysis, we abstract from some characteristics of cap-and-trade markets. As the permit market analysis presented here is in a static framework, the issues of permit banking and futures trading are not considered. To include such a system, an additional choice variable can be created in which the firms select the level of retained permits for future use (allowing for banking) or allow for additional bids and offers to be made in the futures market. Institutional restrictions of the market—such as the use of price collars and allowance reserves—can also be introduced by placing exogenous restrictions on the price level. Also, the extent to which changes in permit prices are passed on to consumers in the product market, while interesting, is not analyzed. Cost pass-through has seen recent empirical investigation and is commanding increasing attention in the literature—see, for example, Sijm et al. (2006), Chernyavs'ka and Gulli (2008), and Hintermann (2016b).

with equality if $z_i > 0$, so firms equate their 'overall marginal cost'—comprised of the marginal cost of production and abatement—to the price of the good.[14] Since we assume $C_i^z + f_i' C_i^a = 0$ when $z_i = 0$, the solution will always be interior where the first-order condition holds with equality, and we denote the solution to (3) by $\tilde{z}_i(x_i) > 0$.[15]

The relationship between a firm's actions in the permit market and their behavior in the product market is given by

$$\frac{\mathrm{d}\tilde{z}_i(x_i)}{\mathrm{d}s_i} = \frac{\partial \tilde{z}_i(x_i)}{\partial x_i}\frac{\partial x_i}{\partial s_i}, \text{ for } s = \{b, q\}, \tag{4}$$

which follows by virtue of firm i's product market strategy depending only on its own permit transactions. Using the implicit function theorem on (3) yields

$$\frac{\partial \tilde{z}_i(x_i)}{\partial x_i} = \frac{C_i^{za} + C_i^{aa} f_i'}{C_i^{zz} + 2C_i^{za} f_i' + C_i^{aa}[f_i']^2 + C_i^a f_i''} > 0 \tag{5}$$

under our assumptions. Intuitively, if a firm acquires more permits in the permit market then less abatement is required for a given level of output. This has two effects relevant for product market decisions: since $C_i^{aa} > 0$ the marginal cost of abatement falls; and since $C_i^{za} \geq 0$ the marginal cost of production falls. Both effects work to favor an increase in product market output when the firm is in possession of more permits.

To understand the effect of a change in permit transactions on a firm's profitability in the product market let us, with a slight abuse of notation, write the optimized profit function in the product market as

$$\tilde{\pi}_i(x_i) = \tilde{z}_i(x_i)\tilde{\phi} - C_i(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i + x_i]). \tag{6}$$

Since this is influenced only by x_i (and not the permit transactions of other firms), we can write

$$\frac{\mathrm{d}\tilde{\pi}_i(x_i)}{\mathrm{d}s_i} = \frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i}\frac{\partial x_i}{\partial s_i}, \tag{7}$$

where, by virtue of the envelope theorem applied to (2),

$$\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i + x_i]). \tag{8}$$

[14]This first-order condition is both necessary and sufficient under the assumptions stated in Subsection 2.1; the second-order condition is $-C_i^{zz} - C_i^{aa}[f_i']^2 - 2C_i^{za} f_i' - C_i^a f_i'' < 0$.

[15]Optimal product market output also depends on ω_i, which is suppressed for notational convenience.

Equations (7) and (8) show a direct link between the permit and product markets: this will be used to investigate firms' actions within the permit market.

2.3 Permit market equilibrium

Foreseeing the consequences of permit market activity on actions in the product market, each firm $i \in I$ can be seen as solving the problem

$$\max_{(b_i, q_i) \in \mathcal{S}_i} m_i - x_i p + \tilde{\pi}_i(x_i),$$

where $x_i = b_i/p - q_i$, $p = B/[\mu E + Q]$, and $\tilde{\pi}_i$ is defined in (6). This problem is concave in both b_i and q_i so the first-order conditions are both necessary and sufficient in identifying a best response.[16]

When engaging in permit market activity, a firm affects its product market profitability (according to (7)) and also its expenditure in the permit market. When choosing $s = \{b, q\} \in \mathcal{S}_i$ the firm will balance the marginal change in product market profitability with the marginal change in permit market expenditure, so that

$$\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} \frac{\partial x_i}{\partial s_i} \leq \frac{\partial x_i p}{\partial s_i}, \ s = \{b, q\},$$

where the inequality is replaced with an equality if $s_i > 0$.

Recalling that B is the total money bid, Q is the total supply of permits, and $p = B/[\mu E + Q]$, for a buyer of permits for whom $s = b$, $x_i = b_i/p$, and so it follows that $\frac{\partial x_i}{\partial s_i} = [1 - b_i/B]p^{-1}$ and $\frac{\partial x_i p}{\partial s_i} = 1$; as such, the first-order condition for a buyer of permits is

$$\frac{\partial \tilde{\pi}_i(b_i/p)}{\partial x_i} \leq \left[1 - \frac{b_i}{B}\right]^{-1} p, \tag{9}$$

where the inequality is replaced with an equality if $b_i > 0$.

For a seller of permits for whom $s = q$, $x_i = -q_i$ and we have $\frac{\partial x_i}{\partial s_i} = -1$ and $\frac{\partial x_i p}{\partial s_i} =$

[16] This follows by noting that for $s = \{b, q\}$, the first derivative of the payoff function is $-\frac{\partial x_i p}{\partial s_i} + \frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} \frac{\partial x_i}{\partial s_i}$ and so the second derivative is $-\frac{\partial^2 x_i p}{\partial s_i^2} + \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} \left[\frac{\partial x_i}{\partial s_i}\right]^2 + \frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} \frac{\partial^2 x_i}{\partial s_i^2}$. When $s = b$: $x_i = \frac{b_i}{B}[\mu E + Q]$ so $\frac{\partial x_i}{\partial b_i} = \frac{B - b_i}{B^2}[\mu E + Q]$ and $\frac{\partial^2 x_i}{\partial s_i^2} = -\frac{2[B - b_i]}{B^3}[\mu E + Q]$; and $x_i p = b_i$ so $\frac{\partial^2 x_i p}{\partial b_i^2} = 0$. When $s = q$: $x_i = -q_i$ so $\frac{\partial^2 x_i}{\partial q_i^2} = 0$; and $x_i p = -\frac{q_i}{\mu E + Q} B$ so $\frac{\partial x_i p}{\partial q_i} = -\frac{\mu E + Q - q_i}{[\mu E + Q]^2} B$ and $\frac{\partial^2 x_i p}{\partial q_i^2} = \frac{2[\mu E + Q - q_i]}{[\mu E + Q]^3} B$. As noted, $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} > 0$ and we will subsequently show in Lemma 1 that $\frac{\partial^2 \tilde{\pi}_i}{\partial x_i^2} < 0$, which establishes the claim.

$-[1 - q_i/[\mu E + Q]]p$; consequently, the first-order condition is

$$\frac{\partial \tilde{\pi}_i(-q_i)}{\partial x_i} \geq \left[1 - \frac{q_i}{\mu E + Q}\right] p, \tag{10}$$

with equality if $q_i > 0$.

Since firms are heterogeneous in their cost structures, pursuing a standard best-response analysis of this game would be fruitless as the dimensionality of the problem makes it intractable. Rather than imposing additional assumptions to instil tractability (e.g., restricting firms to be one of two types), we follow an approach—first presented in Dickson and Hartley (2008) and later extended to the case of 'interior endowments', as is the case with permit exchange, by Dickson and Hartley (2013)—that exploits the fact that firms' payoffs depend on other firms' actions only through their aggregation in B and Q, which themselves influence the price p. Here we present the reasoning for permit exchange coupled with subsequent product market decisions. The method allows the construction of supply and demand functions in the permit market that account for strategic behavior and endogenous formation of the sides of the market, and can be used to identify a (non-autarkic) permit market equilibrium.[17] The method proceeds as follows.

Step 1: Hypothesize a permit price p, and consider which firms would act on each side of the permit market if there was a Nash equilibrium with this price. We define

$$\tilde{p}_i^*(\omega_i) \equiv C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i) \tag{11}$$

as firm i's marginal abatement cost at its initial endowment and will show (in Proposition 1) that firm i will be a buyer of permits only if $\tilde{p}_i^*(\omega_i) > p$ and a seller of permits only if $\tilde{p}_i^*(\omega_i) < p$ (and of course $\omega_i > 0$). When considering behavior consistent with a price p, this allows us to separate the set of firms into those that will potentially buy permits, and those that will potentially sell.

Step 2a: Hypothesize an aggregate supply of permits, Q, in addition to any supply μE from the regulator, and consider the individual supplies of those firms that might sell permits at price p that are consistent with a Nash equilibrium with this Q and p. Then ask

[17] A quirk of strategic market game models is that if there is no fixed supply to the market from the regulator autarky (in which no trader makes a bid or an offer) is always an equilibrium. This is clear by noting that if $\mu = 0$ and the bids and offers of all other firms are zero then there is no gain to any firm of making a positive bid or offer. The same is not true when $\mu > 0$ since if the bids of all other firms are zero any bid, no matter how small, will appropriate the permits supplied by the regulator. The analysis that follows focuses on identifying *non*-autarkic equilibria in which trade takes place (in the case where $\mu = 0$).

whether firms' individual supplies are consistent when aggregated, i.e., that individual supplies aggregate to Q. Let $\tilde{q}_i(p;Q)$ denote firm i's supply consistent with a Nash equilibrium in which the aggregate supply is Q and the price is p (which is given by the minimum of either the q_i that solves (10) or ω_i). Then we seek the value of Q such that $\sum_{\{i\in I:\omega_i>0,p_i^*(\omega_i)<p\}} \tilde{q}_i(p;Q) = Q$, which is the aggregate supply consistent with a Nash equilibrium in which the price is p. Note that if $\mu = 1$ then $\omega_i = 0$ for all $i \in I$ and this step is superfluous: supply of permits will be exogenously given as E whatever the resulting price.

Step 2b: Hypothesize an aggregate bid B from those firms that might buy permits at price p, and deduce individual bids consistent with this aggregate bid, which we denote $\tilde{b}_i(p;B)$ (this is given by the minimum of either the b_i that solves (9) or m_i). Seek consistency of the aggregate bid, i.e., find the value of B such that $\sum_{\{i\in I:p_i^*(\omega_i)>p\}} \tilde{b}_i(p;B) = B$.

Step 3: Seek a consistent price, i.e., a price p such that the consistent aggregate offer from Step 2a and bid from Step 2b satisfy $B/[\mu E + Q] = p$, which identifies a non-autarkic Nash equilibrium.

We begin by establishing Step 1. To do so, we first require the following lemma.

Lemma 1. *For each firm $i \in I$, $\frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} < 0$.*

Proof. Recall from (8) that $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i + x_i])$. As such,

$$\begin{aligned}\frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} &= C_i^{za}\frac{\partial \tilde{z}_i(x_i)}{\partial x_i} + C_i^{aa}\left[f_i'\frac{\partial \tilde{z}_i(x_i)}{\partial x_i} - 1\right] \\ &= \frac{\partial \tilde{z}_i(x_i)}{\partial x_i}[C_i^{za} + C_i^{aa}f_i'] - C_i^{aa}.\end{aligned}$$

In (5) we deduced that $\frac{\partial \tilde{z}_i(x_i)}{\partial x_i} = \frac{C_i^{za}+C_i^{aa}f_i'}{C_i^{zz}+2C_i^{za}f_i'+C_i^{aa}[f_i']^2+C_i^a f_i''}$, implying

$$\begin{aligned}\frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} &= \frac{[C_i^{za} + C_i^{aa}f_i']^2}{C_i^{zz} + 2C_i^{za}f_i' + C_i^{aa}[f_i']^2 + C_i^a f_i''} - C_i^{aa} \\ &= \frac{[C_i^{za}]^2 - C_i^{zz}C_i^{aa} - C_i^a C_i^{aa} f_i''}{C_i^{zz} + 2C_i^{za}f_i' + C_i^{aa}[f_i']^2 + C_i^a f_i''}\end{aligned}$$

which is negative as a result of our assumptions on cost and pollution generation functions. □

Lemma 1 confirms that if a firm obtains more permits, their marginal abatement cost will decline. With more permits a firm produces more, as observed in (5). Despite higher output directly raising the marginal abatement cost (because of a positive cross-partial), the level of abatement falls since the increase in pollution from this higher output is less than the increase in permit holdings: this provides an offsetting reduction in their marginal abatement cost (because of convexity of the cost function) that always dominates.

We are now in a position to complete Step 1. The following proposition allows us to understand, once a permit price has been hypothesized, how firms are determined as either buyers or sellers of permits.[18]

Proposition 1. *If there is a Nash equilibrium with price p then firm $i \in I$ will be a buyer of permits only if $\tilde{p}_i^*(\omega_i) > p$, and a seller of permits only if $\omega_i > 0$ and $\tilde{p}_i^*(\omega_i) < p$.*

Proof. Let $\tilde{p}_i^*(\omega_i) > p$ and assume, by contradiction, that i sells permits. Then $q_i > 0$ and $x_i = -q_i < 0$, so Lemma 1 implies $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} > \left.\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i}\right|_{x_i=0} \equiv \tilde{p}_i^*(\omega_i)$ (see (11)). But from the first-order condition for sellers, $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} = \left[1 - \frac{q_i}{\mu E + Q}\right] p < p$. As such, $\tilde{p}_i^*(\omega_i) < \frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} < p$, yielding a contradiction. Thus, if $\tilde{p}_i^*(\omega_i) > p$ for firm i then this firm will only buy permits in equilibrium. Demonstrating that if $\tilde{p}_i^*(\omega_i) < p$ then firm i will only sell permits (but can only do so if $\omega_i > 0$) is similar and so omitted. □

Operationally, the consistent behavior of firms is represented using *share functions*. Take a typical firm i. If $\omega_i > 0$ and $p > \tilde{p}_i^*(\omega_i)$ then we know that the firm will only be a seller of permits at such prices and we consider their behavior consistent with a Nash equilibrium in which the permit price is p and the aggregate supply of all 'potential sellers' (all firms $j \in I$ for whom $\omega_j > 0$ and $p > \tilde{p}_j^*(\omega_j)$) is $Q > 0$. Let $\sigma_i = q_i/Q$ be firm i's share of the total supply from all firms (not including any supply from the regulator); then using (10) we can deduce that firm i's optimal share of the total supply is given by its 'selling share function' $\tilde{s}_i^S(p; Q) = \min\{\sigma_i, \omega_i/Q\}$ where σ_i is the solution to

$$\tilde{l}_i^S(\sigma_i, Q, p) \equiv \frac{\partial \tilde{\pi}_i(-\sigma_i Q)}{\partial x_i} - \left[1 - \frac{\sigma_i Q}{\mu E + Q}\right] p \geq 0, \tag{12}$$

with equality if $\sigma_i > 0$.

It is useful to ascertain the properties of share functions. The share function $\tilde{s}_i^S(p; Q)$ is implicitly defined, and using the implicit function theorem on (12) reveals that it is decreasing in Q and non-decreasing in p.[19] If $\mu = 0$ the share function is defined for all $Q > 0$ and study of (12) reveals that $\lim_{Q \to 0} \tilde{s}_i^S(p; Q) = 1 - \frac{\tilde{p}_i^*(\omega_i)}{p}$. On the other hand, if $\mu > 0$ then there will

[18]This is similar to Dickson and Hartley (2013, Lemma 1); it is included here for the case of permit exchange for a self-contained treatment.

[19]Recall from Lemma 1 that $\frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} < 0$. As such, $\frac{\partial \tilde{l}_i^S(\sigma_i, Q, p)}{\partial \sigma_i} = -Q \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} + \frac{Q}{\mu E + Q} p > 0$ so there is at most one solution to $l_i(\sigma_i, Q, p) = 0$: $\tilde{s}_i^S(p; Q)$ is a function. Moreover, using the implicit function theorem on (12) gives

exist a $\underline{Q}_i(p) > 0$, defined such that $\tilde{l}_i^S(1, \underline{Q}_i(p), p) = 0$ at which $\tilde{s}_i^S(\underline{Q}_i(p), p) = 1$ and below which the share function is undefined.[20]

Consider now the case where $p < \tilde{p}_i^*(\omega_i)$, when firm i will only be a buyer of permits. The behavior of firm i consistent with a Nash equilibrium in which the price is p and the aggregate bid is $B > 0$ is represented by its 'buying share function' $\tilde{s}_i^B(p; B) = \min\{\sigma_i, m_i/B\}$ where, using (9), σ_i is the solution to

$$\tilde{l}_i^B(\sigma_i, B, p) \equiv \frac{\partial \tilde{\pi}_i(\sigma_i B/p)}{\partial x_i} - [1-\sigma_i]^{-1} p \leq 0, \tag{13}$$

with equality if $\sigma_i > 0$.

To deduce the properties of a buyer's share function, we note that if the aggregate bid is B and the price is p, the implied demand is B/p; thus, we write firm i's share function as $\tilde{s}_i^B(p; [B/p]p)$. Using the implicit function theorem on (13) reveals that the share function is strictly decreasing in $[B/p]$ for fixed p, strictly decreasing in p for fixed $[B/p]$, and has the property $\lim_{[B/p] \to 0} \tilde{s}_i^B(p; B) = 1 - \frac{p}{\tilde{p}_i^*}$.[21]

These share functions represent each firm's consistent behavior at a particular price, with particular aggregate bids or offers. We now seek consistency of these aggregates to complete Steps 2a and 2b above. Consistency of the aggregate offer from firms at price p requires the sum of the individual offers of all firms that wish to sell at price p to be equal to the aggregate offer of all firms, or, dividing both sides of this equation by Q, for the sum of the share functions to be equal to one. The aggregate selling share function is $\tilde{S}^S(p; Q) \equiv \sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p\}} \tilde{s}_i^S(p; Q)$: if $\mu = 0$ this is defined for all $Q > 0$; whereas if $\mu > 0$ it is defined only for $Q \geq \max_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p\}} \{\underline{Q}_i(p)\}$. Then at price p the *strategic supply*, denoted by

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$\frac{\partial \tilde{s}_i^S(p;Q)}{\partial Q} = -\frac{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial Q}}{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial \sigma_i}} = -\frac{-\sigma_i \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} + \frac{\sigma_i \mu E}{[\mu E+Q]^2}}{-Q \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} + \frac{Q}{\mu E+Q} p} < 0$ and $\frac{\partial \tilde{s}_i^S(p;Q)}{\partial p} = -\frac{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial p}}{\frac{\partial \tilde{l}_i^S(\sigma_i,Q,p)}{\partial \sigma_i}} = -\frac{-\left[1-\frac{\sigma_i Q}{\mu E+Q}\right]}{-Q \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} + \frac{Q}{\mu E+Q} p} > 0.$

[20]To understand this it helps to visualize the first-order condition. Plotted as functions of σ_i, the left-hand side ($\frac{\partial \tilde{\pi}_i(-\sigma_i Q)}{\partial x_i}$) is increasing in σ_i and the right-hand side ($\left[1 - \frac{\sigma_i Q}{\mu E+Q}\right] p$) is decreasing in σ_i. As Q decreases, the left-hand side decreases and (if $\mu > 0$) the right-hand side increases, giving an increase in the share value. As Q reduces further, since the left-hand side converges to $p_i^*(\omega_i)$ for all σ_i as $Q \to 0$ and the right-hand side to p, there will be a value of Q where the intersection is at $\sigma_i = 1$ and further reductions in Q would give an intersection at $\sigma_i > 1$, where share functions are undefined.

[21]The fact that $\frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} < 0$ (Lemma 1) is again important. With this in mind, note that $\frac{\partial \tilde{l}_i^B(\sigma_i,B,p)}{\partial \sigma_i} = [B/p] \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} - [1-\sigma_i]^{-2} p < 0$, so we are ensured $\tilde{s}_i^B(p; B)$ is a function. Using the implicit function theorem, $\frac{\partial \tilde{s}_i^B(p;[B/p]p)}{\partial [B/p]} = -\frac{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial [B/p]}}{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial \sigma_i}} = -\frac{\sigma_i \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2}}{B/p \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} - [1-\sigma_i]^{-2} p} < 0$. In addition, $\frac{\partial \tilde{s}_i^B(p;[B/p]p)}{\partial p} = -\frac{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial p}}{\frac{\partial \tilde{l}_i^B(\sigma_i,[B/p]p,p)}{\partial \sigma_i}} = \frac{[1-\sigma_i]^{-1}}{B/p \frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} - [1-\sigma_i]^{-2} p} < 0$. The limit is a consequence of taking limits in (13) as $[B/p] \to 0$.

$\tilde{\mathcal{Q}}(p)$, is identified as that level of Q where

$$\tilde{S}^S(p;Q) = 1. \tag{14}$$

This augments any fixed supply from the regulator μE to give the total supply of permits to the market.

The properties of the aggregate selling share function determine the nature of the strategic supply function. For a given p, all firms for whom $\omega_i > 0$ and $\tilde{p}_i^*(\omega_i) < p$ will be included in $\tilde{S}^S(p;Q)$ and since each $\tilde{s}_i^S(p;Q)$ is continuous and decreasing in Q, $\tilde{S}^S(p;Q)$ will inherit this property implying $\tilde{\mathcal{Q}}(p)$, where defined, is a function. When p changes, the share functions of those firms who remain sellers change in a smooth way, and those firms who become sellers as the price rises (or drop out of the set of sellers as the price falls) again do so in a smooth way, implying that $\tilde{S}^S(p;Q)$ is continuous in p and consequently $\tilde{\mathcal{Q}}(p)$ varies continuously in p. Moreover, consideration of the equation implicitly defining $\tilde{\mathcal{Q}}(p)$ reveals it is non-decreasing in p.[22] We will let $\tilde{P}^S$ denote the price above which strategic supply $\tilde{\mathcal{Q}}(p)$ is defined, the form of which depends on whether the regulator withheld permits to supply directly to the market (since this influences the properties of share functions, particularly what they look like when Q is small), and the value of which depends on firms' marginal abatement costs at their initial endowment. If $\mu > 0$ then $\tilde{P}^S = \min_{i \in I: \omega_i > 0}\{p_i^*(\omega_i)\}$, while if $\mu = 0$ then $\tilde{P}^S$ is uniquely defined by the equation

$$\sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < \tilde{P}^S\}} 1 - \frac{\tilde{p}_i^*(\omega_i)}{\tilde{P}^S} = 1. \tag{15}$$

The reason for this is that for $p > \tilde{P}^S$ (however it is defined) the aggregate share function $\tilde{S}^S(p;Q)$ exceeds one when Q is small enough and since it is continuous, decreasing in Q, and no larger than 1 when $Q = \Omega$, it is equal to one at exactly one value of Q: the strategic supply. Conversely, if $p \leq \tilde{P}^S$, then in the case where $\mu = 0$ it takes a value less than one when Q is close to zero and, since it is decreasing in Q, this is also true for larger values of Q, so it is never equal to one; and in the case where $\mu > 0$ no firm has a positive selling share function, so the aggregate is never equal to one.

On the buyers' side, we seek to find the consistent level of $[B/p]$, which is the aggre-

[22] Although $\sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p\}} \tilde{s}_i^S(p;Q)$ is continuous in p, it is not differentiable at values of p where firms enter or leave the set of sellers so implicit differentiation cannot be used. Rather, suppose by contradiction that for $p' > p$ we have $\tilde{\mathcal{Q}}(p') < \tilde{\mathcal{Q}}(p)$. Then the fact that share functions are decreasing in Q and non-decreasing in p implies

$$1 = \sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p\}} \tilde{s}_i^S(p; \tilde{\mathcal{Q}}(p)) \leq \sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p'\}} \tilde{s}_i^S(p; \tilde{\mathcal{Q}}(p)) < \sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p'\}} \tilde{s}_i^S(p'; \tilde{\mathcal{Q}}(p')) = 1,$$

a contradiction.

gate demand for permits. This requires that individual bids when aggregated exactly equal the aggregate bid B, or, in other words, the sum of share functions equals one. Defining $\tilde{S}^B(p;[B/p]p) \equiv \sum_{\{i\in I:\tilde{p}_i^*(\omega_i)>p\}} \tilde{s}_i^B(p;[B/p]p)$, the *strategic demand* for permits, denoted by $\tilde{\mathcal{D}}(p)$, is that level of $[B/p]$ which satisfies

$$\tilde{S}^B(p;[B/p]p) = 1. \tag{16}$$

Continuity of the strategic demand function follows by similar deductions to those made for strategic supply, and study of the condition implicitly defining strategic demand allows us to deduce that strategic demand is decreasing (strictly) in p.[23] The range of prices for which $\tilde{\mathcal{D}}(p)$ is defined is $p < \tilde{P}^B$, where $\tilde{P}^B$ is uniquely defined by the equation

$$\sum_{\{i\in I:\tilde{p}_i^*(\omega_i)>\tilde{P}^B\}} 1 - \frac{\tilde{P}^B}{\tilde{p}_i^*} = 1. \tag{17}$$

For reasons that are similar to those elucidated for strategic supply, if $p \geq \tilde{P}^B$ then the aggregate share function is less than one for all values of $[B/p]$ so for these prices strategic demand is undefined, whereas it takes positive values for $p < \tilde{P}^B$.

Turning finally to Step 3 and referring to Figure 1, a permit price p is consistent with a Nash equilibrium in which trade in permits takes place if and only if the strategic supply plus any supply from the regulator is equal to strategic demand at that price, for only then will the aggregate offer of permits and bid of money be consistent with the price. Thus, we seek a price $\hat{p}$ where $\mu E + \tilde{\mathcal{Q}}(\hat{p}) = \tilde{\mathcal{D}}(\hat{p})$. Since strategic demand is strictly decreasing in p and strategic supply is non-decreasing in p, if overall supply and demand cross they do so only once, implying that there is at most one Nash equilibrium in which trade in permits takes place: there is a unique non-autarkic Nash equilibrium. If $\mu = 0$ then strategic supply and demand will only cross if $\tilde{P}^S < \tilde{P}^B$ (as observed in Figure 1 Panel (a)); in a Nash equilibrium with permit exchange the equilibrium aggregate supply of permits to the market from firms is $\hat{Q} = \tilde{\mathcal{Q}}(\hat{p})$; the equilibrium aggregate bid of money is $\hat{B} = \hat{p}\hat{Q}$; the equilibrium supply of each firm for whom $\omega_i > 0$ and $\tilde{p}_i^*(\omega_i) < \hat{p}$ is $\hat{q}_i = \hat{Q}\tilde{s}_i^S(\hat{p};\hat{Q})$ and the equilibrium bid of each firm for whom $\tilde{p}_i^*(\omega_i) > \hat{p}$ is $\hat{b}_i = \hat{B}\tilde{s}_i^B(\hat{p};\hat{B})$. Conversely, if $\mu = 0$ and $\tilde{P}^S \geq \tilde{P}^B$ then there is no

[23] Suppose by contradiction that $p' > p$ and $\tilde{\mathcal{D}}(p') \geq \tilde{\mathcal{D}}(p)$. Then the facts previously deduced that the share function is *strictly* decreasing in p (and $[B/p]$) implies

$$1 = \sum_{\{i\in I:\tilde{p}_i^*(\omega_i)>p\}} \tilde{s}_i^B(p;\tilde{\mathcal{D}}(p)p) \leq \sum_{\{i\in I:\tilde{p}_i^*(\omega_i>p'\}} \tilde{s}_i^B(p;\tilde{\mathcal{D}}(p)p) < \sum_{\{i\in I:\tilde{p}_i^*(\omega_i)>p'\}} \tilde{s}_i^B(p';\tilde{\mathcal{D}}(p')p') = 1,$$

a contradiction.

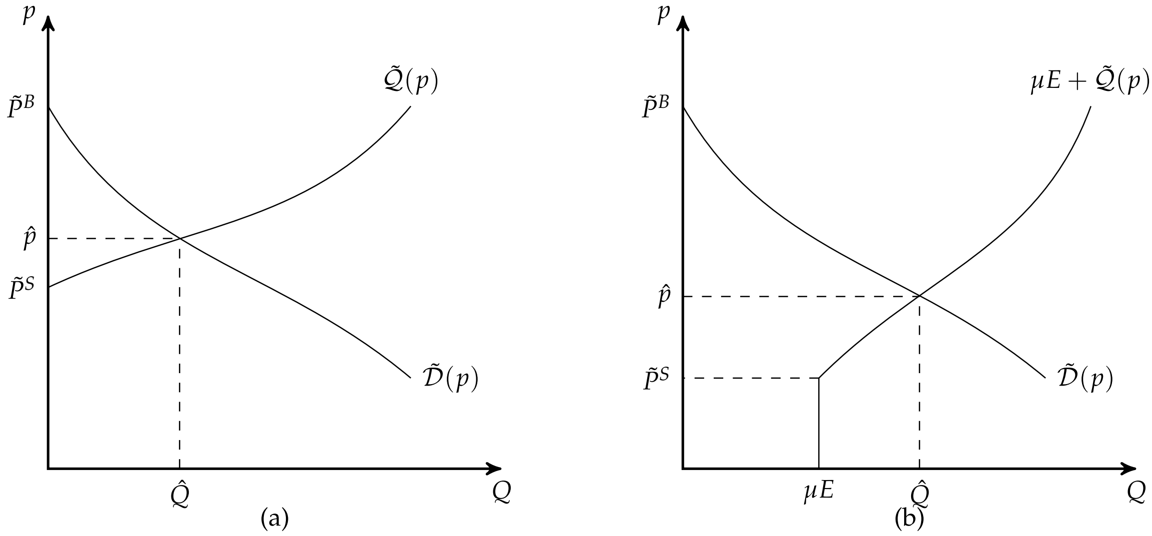

Figure 1: Identification of an equilibrium in bilateral oligopoly. In Panel (a) there is no fixed supply from the regulator; in Panel (b), where $\mu > 0$, the regulator provides a fixed supply.

Nash equilibrium in which trade in permits takes place; in such circumstances the only Nash equilibrium is autarky (which, as noted, is always an equilibrium in bilateral oligopoly when there is no fixed supply to the market) and each firm's final permit holdings are their initial endowment. If $\mu > 0$ then, recalling that we assume the regulator's fixed supply is modest in relation to demand so the permit price is not driven to zero, the presence of a fixed supply from the regulator means overall supply and strategic demand always cross (as observed in Figure 1 Panel (b)), and if $\hat{p} > \tilde{P}^S$ there will be some supply of permits from firms, not just the regulator, in this equilibrium.

3 Features of the permit market equilibrium

With our framework established in the previous section, it is pertinent to consider features of the permit market equilibrium and the consequences of strategic behavior. In particular within this section we will focus on the existence, structure, and comparative static properties of equilibrium, as well as its cost effectiveness in relation to a perfectly competitive outcome and the welfare losses due to strategic behavior.

3.1 Existence of equilibrium

As we noted previously, when there is no fixed supply from the regulator there is always an autarkic Nash equilibrium in which no trade takes place. If this exists alongside another non-autarkic equilibrium the no-trade equilibrium could be ruled out by an equilibrium se-

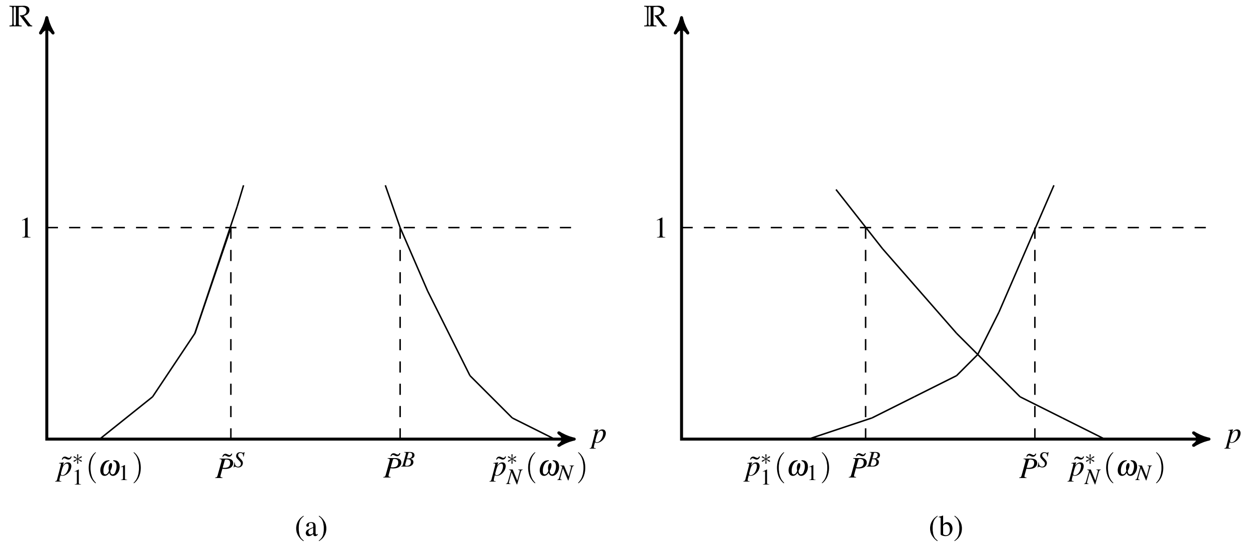

Figure 2: The construction of $\tilde{P}^S$ and $\tilde{P}^B$ when $\mu = 0$. The upward-sloping function is $\sum_{\{i \in I: \tilde{p}_i^*(\omega_i) < p\}} 1 - \frac{\tilde{p}_i^*(\omega_i)}{p}$, which identifies $\tilde{P}^S$, and the downward-sloping function, which identifies $\tilde{P}^B$, is $\sum_{\{i \in I: \tilde{p}_i^*(\omega_i) > p\}} 1 - \frac{p}{\tilde{p}_i^*(\omega_i)}$.

lection argument based on the non-autarkic equilibrium being Pareto superior. An interesting question, however, is whether autarky is the *only* equilibrium: it is important to understand whether firms' strategic behavior in manipulating the permit price leads to trade actually not taking place. Thus, we want to understand the conditions under which the only equilibrium is autarky when $\mu = 0$.

The existence of a *non-autarkic* Nash equilibrium in the market for permits in which $\mu = 0$ hinges on whether $\tilde{P}^S$ defined in (15) is less than $\tilde{P}^B$ defined in (17). To better understand the relationship between these two objects we next elucidate the details of their construction. Recall that $\tilde{p}_i^*(\omega_i) \equiv C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i)$ is firm i's marginal abatement cost with its initial endowment of permits. Given an initial distribution of permit endowments which we assume in this discussion is positive for each firm we can, without loss of generality, re-order firms according to the magnitude of their marginal abatement cost: $\tilde{p}_1^*(\omega_1) \leq \tilde{p}_2^*(\omega_1) \leq \cdots \leq \tilde{p}_N^*(\omega_N)$.

Now we construct two functions that each depend on p. The first function, that identifies $\tilde{P}^S$, is

$$\sum_{\{i \in I: \tilde{p}_i^*(\omega_i) < p\}} 1 - \frac{\tilde{p}_i^*(\omega_i)}{p}, \tag{18}$$

which is increasing in p. For $p \leq \tilde{p}_1^*(\omega_1)$ the function is undefined; for $\tilde{p}_1^*(\omega_1) < p \leq \tilde{p}_2^*(\omega_2)$ it takes the value $1 - \frac{\tilde{p}_1^*(\omega_1)}{p}$; for $\tilde{p}_2^*(\omega_2) < p \leq \tilde{p}_3^*(\omega_3)$ it takes the value $2 - \frac{\tilde{p}_1^*(\omega_1) + \tilde{p}_2^*(\omega_2)}{p}$; for $\tilde{p}_n^*(\omega_n) < p \leq \tilde{p}_{n+1}^*(\omega_{n+1})$ it takes the value $n - \frac{\sum_{i=1}^n \tilde{p}_i^*(\omega_i)}{p}$. The second function, that identifies

$\tilde{P}^B$, is

$$\sum_{\{i \in I : \tilde{p}_i^*(\omega_i) > p\}} 1 - \frac{p}{\tilde{p}_i^*(\omega_i)}, \tag{19}$$

which is decreasing in p and piecewise linear. Working from large values of p to smaller values, for $p \geq p_N^*(\omega_N)$ the function is undefined; for $\tilde{p}_{N-1}^*(\omega_{N-1}) \leq p < \tilde{p}_N^*(\omega_N)$ it takes the value $1 - \frac{p}{\tilde{p}_N^*(\omega_N)}$; for $\tilde{p}_{N-2}^*(\omega_{N-2}) \leq p < \tilde{p}_{N-1}^*(\omega_{N-1})$ it takes the value $2 - \frac{p}{\tilde{p}_N^*(\omega_N)} - \frac{p}{\tilde{p}_{N-1}^*(\omega_{N-1})}$; and for $\tilde{p}_{N-n}^*(\omega_{N-n}) \leq p < \tilde{p}_{N-n+1}^*(\omega_{N-n+1})$ it takes the value $n - \sum_{i=N-n+1}^{N} \frac{p}{\tilde{p}_i^*(\omega_i)}$.

$\tilde{P}^S$ is identified by the value of p where (18) is equal to one; $\tilde{P}^B$ is given by the value of p where (19) is equal to one. Figure 2 plots these functions for two different economies. From this it is clear that $\tilde{P}^S > \min_{i \in I}\{\tilde{p}_i^*(\omega_i)\}$ and $\tilde{P}^B < \max_{i \in I}\{\tilde{p}_i^*(\omega_i)\}$. In Panel (a) the $\tilde{p}_i^*(\omega_i)$s are widely dispersed and in this case $\tilde{P}^S < \tilde{P}^B$ and therefore a non-autarkic Nash equilibrium in which trade in permits takes place exists in this economy. In Panel (b), however, the $\tilde{p}_i^*(\omega_i)$s are less dispersed and in this case $\tilde{P}^S > \tilde{P}^B$, so the only equilibrium here involves no trade in permits.

The dispersion of the $\tilde{p}_i^*(\omega_i)$s measures the gains from trading permits: if they are all equal there are no gains from trade and as they become more dispersed the gains from trade increase. As our illustration makes clear, the existence of gains from trade is not sufficient to ensure trade will take place: $\tilde{p}_1^*(\omega_1) < \tilde{p}_N^*(\omega_N)$ does not imply $\tilde{P}^S < \tilde{P}^B$. Rather, for a non-autarkic permit market equilibrium to exist when $\mu = 0$ there must be 'sufficient' gains from trading permits. If the regulator identifies that there are insufficient gains from trade so that trade will not take place when firms are left to their own devices, they could withhold permits from the initial free allocation to supply directly to the market, since when $\mu > 0$ there is always an equilibrium in which permit exchange takes place; this can serve to 'prime' trade in the market.

3.2 Structure of the market

In the permit trading model developed in this article the sides of the market form endogenously: whether a firm becomes a seller or buyer of permits in equilibrium depends on their marginal abatement cost at their initial endowment in relation to the permit price, which depends on the actions of all firms. Since there is nothing in our model to suggest that the permit price will be the same with strategic behavior as with price-taking firms in a Walrasian model of permit exchange, *prima facie* it is unclear whether firms will take the same role as seller or buyer in these two market structures.

Proposition 2. *Let p^W be the price of permits in a competitive market, and suppose that there is a permit* *market equilibrium with trade in which $\hat{p} < p^W$ and a firm i for whom $\hat{p} < \tilde{p}_i^*(\omega_i) < p^W$. Then in a competitive market firm i would be a seller, but when firms are modeled as behaving strategically the same firm, if active, is a buyer. Also, if $\hat{p} > p^W$ and $\hat{p} > p_i^*(\omega_i) > p^W$ then firm i would be a buyer in the competitive market, but when firms are modeled as behaving strategically the same firm, if active, is a seller.*

Proof. Let $\hat{p} < \tilde{p}_i^*(\omega_i) < p^W$. If firm i was a buyer in a competitive market then $x_i > 0$ and $C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i - x_i]) = p^W$. But the fact that $\frac{\partial^2 \tilde{\pi}_i(x_i)}{\partial x_i^2} < 0$ (Lemma 1) implies that $\tilde{p}_i^*(\omega_i) \equiv C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i) > C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i - x_i]) = p^W$, a contradiction. Thus, in a competitive market, firm i is a seller. In a strategic market, if firm i is also a seller then $x_i < 0$ and $C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i - x_i]) = \left[1 - \frac{\hat{\sigma}_i \hat{Q}}{\mu E + \hat{Q}}\right] \hat{p}$. But then Lemma 1 again implies $C_i^a(\tilde{z}_i(0), f_i(\tilde{z}_i(0)) - \omega_i) < C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i - x_i])$ so we have the inequality $\tilde{p}_i^*(\omega_i) < \left[1 - \frac{\hat{\sigma}_i \hat{Q}}{\mu E + \hat{Q}}\right] \hat{p} < \hat{p}$, a contradiction. The proof of the case $\hat{p} > \tilde{p}_i^*(\omega_i) > p^W$ is similar and so omitted. □

Malueg and Yates (2009) present a competing model of fully strategic trade in permits that relies on the supply function approach of Klemperer and Meyer (1989). Although their focus is on the role of private information in permit markets, to ensure tractability of the model they must restrict supply functions to be linear and have the same slope. This has the consequence that, under the assumption of complete information, the equilibrium price will be equivalent to the competitive permit price regardless of the distribution of market power (their Proposition 1).[24] The equivalence of the equilibrium price between a strategic framework and a perfectly competitive framework, regardless of the distribution of market power, is a rather unrealistic feature of the supply function approach. In our bilateral oligopoly framework, the equilibrium price under strategic behavior is only equal to the competitive price if there is a perfect balance in strategic manipulation between both sides of the market which, generically, will not be the case.

3.3 Comparative statics

As observed throughout this article, a number of fundamentals determine how firms trade permits: firms' initial endowments; their production (and abatement) technologies; as well as the demand in the goods market that influences its price. We now consider the influence of these features on the permit market equilibrium. Throughout our discussion we suppose the equilibrium involves at least some supply of permits from firms, not just from the regulator;

[24]This price equivalence may not hold when there exists private information.

if the equilibrium involves only supply from the regulator overall supply does not change in the neighborhood of the equilibrium when the economic environment changes, with straightforward implications.

Recall that the equilibrium in the permit market is identified by the intersection of the overall supply and strategic demand functions, the construction of which relies on aggregating firms' share functions defined in (12) and (13). A merit of the approach is that the properties of these share functions are relatively straightforward to deduce, allowing a comparative static analysis of equilibrium.

A firm's 'selling share function' is determined by the first-order condition $\frac{\partial \tilde{\pi}_i(-\sigma_i Q)}{\partial x_i} - \left[1 - \frac{\sigma_i Q}{\mu E + Q}\right] p = 0$ (see (12)), the left-hand side of which is increasing in σ_i (as we noted in Footnote 19). As such, anything that increases [decreases] $\frac{\partial \tilde{\pi}_i(-\sigma_i Q)}{\partial x_i}$ will decrease [increase] the share function. Also note that strategic supply is determined by $\sum_{\{i \in I: \omega_i > 0, \tilde{p}_i^*(\omega_i) < p\}} s_i^S(p; Q) = 1$, the left-hand side of which is decreasing in Q. Consequently, if a firm's selling share function decreases [increases] then, other things equal, strategic supply will decrease [increase], for the range of prices where this firm would be a seller.

A similar rationale can be made for buyers' share functions. A firm's 'buying share function' is determined by $\frac{\partial \tilde{\pi}_i(\sigma_i B/p)}{\partial x_i} - [1 - \sigma_i]^{-1} p = 0$ (see (13)), the left-hand side of which is decreasing in σ_i. Thus anything that increases [decreases] $\frac{\partial \tilde{\pi}_i(\sigma_i B/p)}{\partial x_i}$ will increase [decrease] the share function. Again recall that strategic demand is determined by $\sum_{\{i \in I: \tilde{p}_i^*(\omega_i) > p\}} s_i^B(p, [B/p]p) = 1$, the left-hand side of which is decreasing in $[B/p]$. It follows that if a firm's buying share function increases [decreases] then strategic demand will increase [decrease], over the range of prices where this firm would be a buyer.

Now, from (8) we know that $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i + x_i])$. Our assumptions on firms' cost functions then imply that $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i}$ will increase [decrease] if (a) there is an increase [decrease] in demand in the product market that results in an increase in the price, resulting in $\tilde{z}_i(x_i)$ increasing [decreasing] for all x_i; (b) the pollution generated from a given level of production increases [decreases], where a reduction may be due to, for example, improvements in abatement technology; and (c) the firm's initial permit endowment decreases [increases].

Consider a situation, then, where demand increases in the product market leading to an increase in the product price, which influences all firms. Each firm's selling share function will decrease, which decreases the strategic supply of permits, and each firm's buying share function will increase, which increases the strategic demand for permits (recall that strategic supply is an increasing function of p, and strategic demand is strictly decreasing in p). Consequently, an increase in demand in the product market increases the equilibrium price of permits. The effect on the equilibrium volume of permits traded is unclear since, while supply has contracted, the permit price has increased.

Consider next a situation where abatement technologies become more efficient so less pollution is generated from the production of goods and suppose this influences all firms equally. Then selling share functions will increase, which will result in an increase in the strategic supply of permits, and buying share functions will decrease resulting in a reduction in strategic demand for permits. The effect of more efficient abatement technologies is to reduce the equilibrium price of permits, but the effect on the quantity of permits traded is unclear.

If the regulator wishes to reduce total emissions then the effect is to decrease all firms' selling share functions which reduces strategic supply, and increase their buying share functions which increases strategic demand. The consequence will be upward pressure on the equilibrium price of permits. Note, however, that changes in permit endowments are often not undertaken in a uniform way. For example, we may consider a situation where a regulator changes policy from an equitable distribution of permits to a distribution where more highly polluting firms receive more permits. Suppose that with an equitable distribution of permits the equilibrium price is $\hat{p}$ and suppose further that the regulator increases the endowment of permits to those who are buyers (i.e., for whom $\tilde{p}_i^*(\omega_i) > \hat{p}$) and reduces the endowment of permits to sellers (i.e., those firms for whom $\tilde{p}_i^*(\omega_i) < \hat{p}$). For those firms that received a greater [smaller] endowment, their buying share function reduces [increases] and their selling share function increases [reduces], with the necessary implication that for all $p \geq \hat{p}$ strategic demand reduces and, likewise, for all $p \leq \hat{p}$ strategic supply also reduces. Consequently, the equilibrium quantity of permits traded will decline under the new regulation. In fact it is even possible that under an equitable distribution of permits where $\tilde{P}^S < \tilde{P}^B$, a change to the initial endowment towards a 'grandfathered' distribution of permits contracts both the strategic supply and demand enough to make $\tilde{P}^S \geq \tilde{P}^B$, so no trade in permits takes place: referring back to Figure 2, grandfathering may shift the economy from a situation depicted in Panel (a), to that depicted in Panel (b).[25] The effect on the equilibrium permit price when there remains a non-autarkic equilibrium is unclear, and even if aggregate emissions decline it does not necessarily follow that the permit price will increase.

[25]Note that if a firm's $\tilde{p}_i^*(\omega_i)$ under an egalitarian distribution of permits is low then it will increase under grandfathering, whereas if $\tilde{p}_i^*(\omega_i)$ is high it will decrease under grandfathering, thus reducing the gains from trade.

3.4 Cost efficiency of equilibrium

If trade in permits does take place (either in the presence of an external supply from the regulator or if $\tilde{P}^S < \tilde{P}^B$) will this equilibrium reduce pollution levels to E in a cost-effective way? If we were willing to assume that firms act as price-takers then the standard Walrasian equilibrium of the permit market would be used to describe equilibrium. Well-known results tell us that at the Walrasian equilibrium marginal abatement costs will be equalized; thus, whenever gains from trade in permits exist trade will take place, and emission reductions will be achieved in a cost-effective manner (Montgomery, 1972). In our model, consider two firms i and j that are active in a non-autarkic equilibrium with permit price $\hat{p}$, where i is a seller of permits ($\tilde{p}_i^*(\omega_i) < \hat{p}$) and j is a buyer of permits ($\tilde{p}_j^*(\omega_j) > \hat{p}$). Then it follows from (9) and (10) that

$$\left[1 - \frac{\hat{\sigma}_i \hat{Q}}{\mu E + \hat{Q}}\right]^{-1} C_i^a(\tilde{z}_i(\hat{x}_i), f_i(\tilde{z}_i(\hat{x}_i)) - [\omega_i + \hat{x}_i]) = \hat{p} = [1 - \hat{\sigma}_j] C_j^a(\tilde{z}_j(\hat{x}_j), f_j(\tilde{z}_j(\hat{x}_j)) - [\omega_j + \hat{x}_j]). \tag{20}$$

From (20), the following proposition is immediate.

Proposition 3. *In any permit market equilibrium in which trade takes place there exist $i, j \in I$ for whom*

$$C_i^a(\tilde{z}_i(\hat{x}_i), f_i(\tilde{z}_i(\hat{x}_i)) - [\omega_i + \hat{x}_i]) < C_j^a(\tilde{z}_j(\hat{x}_j), f_j(\tilde{z}_j(\hat{x}_j)) - [\omega_j + \hat{x}_j]),$$

so emissions reductions are not achieved in a cost-effective manner, unless all firms are negligible (so $\hat{\sigma}_i \approx 0$ for all $i \in I$).

This implies that between any buyer and seller (with non-negligible market share), further cost reductions are possible by transferring more permits from the seller to the buyer. All firms in bilateral oligopoly behave strategically; those that sell permits will restrict supply to try to increase the price, those that buy will restrict their bids to put downward pressure on the price. These strategic tensions combine to result in generic inefficiencies in the final determination of permit transactions.

We can explore the welfare losses of permit exchange in more depth by considering the dead-weight loss associated with strategic trade in permits. To do this, we will construct competitive supply and demand functions assuming firms in the permit market are price takers. As noted, the equilibrium in this case exhausts all gains from trade and, since we consider traders as firms and their payoffs are quasi-linear, welfare losses from a restriction in trade can be measured by the area between the supply and demand curves between the volume of trade that takes place and the volume of trade that would take place in a competitive market. For each of these permits that are not traded the difference between demand and supply gives the difference in the marginal abatement costs that remain because of a reduced volume of trade.

Consider then that the permit market is perfectly competitive: firms choose their permit transactions and a Walrasian auctioneer sets the permit price p to clear the market. Constraining each firm not to sell more permits than their initial endowment, firms can be seen as solving the problem

$$\max_{\{x_i : -\min\{x_i, 0\} \leq \omega_i\}} m_i - x_i p + \tilde{\pi}_i(x_i)$$

where $\tilde{\pi}_i(x_i)$ is the usual product market profit, defined in (6). The first-order condition is

$$\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} = p \tag{21}$$

where $\frac{\partial \tilde{\pi}_i(x_i)}{\partial x_i} = C_i^a(\tilde{z}_i(x_i), f_i(\tilde{z}_i(x_i)) - [\omega_i + x_i])$, as we have previously noted. The solution to this problem will be denoted $x_i^W(p)$, and inspection of the first-order condition reveals that if $p < p_i^*(\omega_i)$ then $x_i^W(p) > 0$ and firm i will be a buyer of permits, while if $p > p_i^*(\omega_i)$ then $x_i^W(p) < 0$ and i will be a seller of permits (so long as $\omega_i > 0$).

Competitive supply and demand functions can then be constructed by aggregating individual decisions, also accounting for any fixed supply from the regulator:

$$\mathcal{S}^W(p) = -\sum_{i \in I} \min\{0, x_i^W(p)\},$$
$$\mathcal{D}^W(p) = \sum_{i \in I} \max\{0, x_i^W(p)\}.$$

In equilibrium the competitive price of permits p^W will satisfy $\mu E + \mathcal{S}^W(p^W) = \mathcal{D}^W(p^W)$ and the aggregate volume of permits traded is denoted Q^W. Assuming that all firms with no endowment of permits have $p_i^*(\omega_i) \geq p^W$, marginal abatement costs are equalized with this volume of permit exchange, as is readily deduced from the first-order conditions that will hold for each firm.[26]

When we consider that firms may behave strategically in permit exchange, we analyze their behavior using strategic versions of supply and demand. This facilitates simple comparison with price-taking behavior that is represented by competitive supply and demand functions. The relationship between competitive and strategic supply and demand in bilateral oligopoly

[26]If this were not the case, there would be firms in equilibrium that would wish to sell permits but cannot, in which case their marginal abatement cost would be less than the equilibrium price; if we accounted for this we should consider the constrained minimized marginal abatement cost difference, which for brevity we do not pursue.

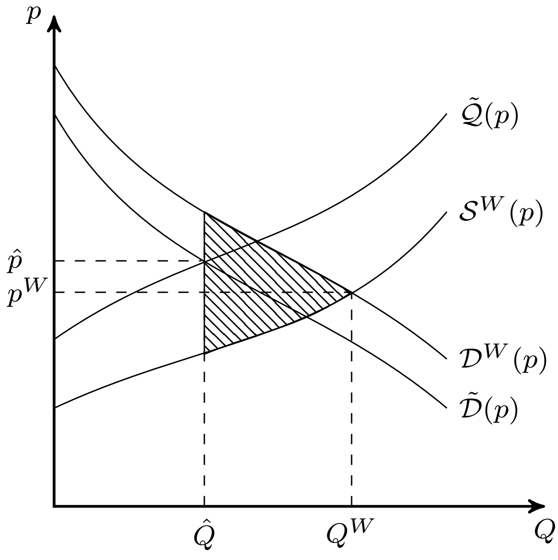

Figure 3: Strategic supply and demand compared with their competitive counterparts (assuming $\mu = 0$), and identification of the welfare loss resulting from the exercise of market power.

(where traders roles are exogenously determined by their endowments) was investigated in Dickson and Hartley (2008) who demonstrated that strategic versions of supply and demand lie to the left of their competitive counterparts (Corollaries 7.2 and 7.4). The same results can be derived in this setting where the side of the market on which a trader acts is determined endogenously with market outcomes (we omit the detail of this argument). This necessarily implies, as can be seen in Figure 3, that the volume of trade in permits when firms exercise market power will be less than if firms were to behave as price takers in the permit market. Although the effect on the volume of trade is unambiguous, the effect on the permit price is not: whether the permit price with strategic behavior is higher or lower than with assumed price-taking behavior depends on the balance of market power between buyers and sellers of permits in equilibrium. The exception to this is where the regulator supplies permits to the market and the equilibrium with strategic behavior involves no further supply of permits from firms, in which case (since strategic demand lies to the left of competitive demand) the permit price will be unambiguously lower when strategic behavior is considered.

With strategic permit exchange, marginal abatement costs are not equalized due to the restricted volume of trade in permits between firms: the maximum disparity between marginal abatement costs is given by the difference between competitive demand and supply at $\hat{Q}$. Since payoffs are quasi-linear, competitive supply and demand functions can be used to measure the dead-weight loss associated with trade in permits being restricted relative to the volume of trade that exhausts all gains from trade, which is given by the area between competitive supply and demand between $\hat{Q}$ and Q^W (the hatched area in Figure 3). If we considered replicating the economy then we could show, as was considered in Dickson and Hartley (2008), that strategic versions of supply and demand converge to their competitive counterparts as the number of replications increases without bound. As such, as the number of firms increases the volume of trade in the Nash equilibrium that accounts for strategic behavior will converge to the competitive volume of trade that exhausts all gains from trade, and the welfare loss due to strategic behavior will shrink to zero.

4 Illustrative example

In this section we use an example to demonstrate the method of analysis and some of the ideas we have presented in the general model. Throughout we assume $\mu = 0$, so the regulator does not withhold any permits to sell directly on the market. While the general model allows for all firms to be heterogeneous, in the example we restrict attention to two types of firms: There are n_1 firms of type 1 that have endowment $\omega_1 = \frac{1}{\theta}\omega$ and n_2 firms of type 2 that have endowment $\omega_2 = \theta\omega$, where $\theta \geq 0$. All firms are assumed to have the bi-quadratic cost function $C_i(z_i, a_i) = \frac{1}{2}z_i^2 + \frac{1}{2}a_i^2$ and pollution generation function $f_i(z_i) = z_i$, so $a_i \equiv z_i - [\omega_i + x_i]$.

Given permit transactions of x_i, the first-order condition governing optimal behavior in the product market with fixed product market price ϕ, as given in (3), is

$$\begin{aligned} 2z_i - [\omega_i + x_i] = \phi \Rightarrow \\ \tilde{z}_i(x_i) = \frac{\phi + \omega_i + x_i}{2}. \end{aligned}$$

As such, $p_i^*(\omega_i) = \frac{\phi - \omega_i}{2}$, and since $\omega_1 < \omega_2$, we have $p_1^*(\omega_1) > p_2^*(\omega_2)$.

In a permit market where all firms are allowed to behave strategically, their behavior consistent with a Nash equilibrium is captured by share functions. Recall that a firm will only be on the supply side of the market if $\tilde{p}_i^*(\omega_i) < p$, and will only be on the demand side if $\tilde{p}_i^*(\omega_i) > p$. As such, given that $p_1^*(\omega_1) > p_2^*(\omega_2)$ if $p \geq p_1^*(\omega_1)$ no firms will be on the demand side, and if $p \leq p_2^*(\omega_2)$ no firms will be on the supply side, so we seek an equilibrium in which the price is between $p_2^*(\omega_2)$ and $p_1^*(\omega_1)$.

For $p > \tilde{p}_i^*(\omega_i)$ firm i will be on the supply side of the permit market and their share function is given by (12), so selling share functions are governed by the first-order condition

$$\frac{\phi - \omega_i + \sigma_i Q}{2} = [1 - \sigma_i]p \Rightarrow$$
$$\tilde{s}_i^S(p; Q) = \frac{2p - \phi + \omega_i}{2p + Q}.$$

For $p < \tilde{p}_i^*(\omega_i)$ firm i will be on the demand side of the market and their share function is given by (13), so buying share functions are governed by

$$\frac{\phi - \omega_i - \frac{\sigma_i B}{p}}{2} = \frac{1}{1 - \sigma_i} p \Rightarrow$$
$$\tilde{s}_i^B(p; [B/p]) = \frac{\phi - \omega_i + \frac{B}{p} \pm \sqrt{[\phi - \omega_i + \frac{B}{p}]^2 - 4\frac{B}{p}[\phi - \omega_i - 2p]}}{2\frac{B}{p}}.$$

Now we suppose $p_2^*(\omega_2) < p < p_1^*(\omega_1)$ and construct strategic supply and demand functions where type 1 firms are buyers of permits, and type 2 firms are sellers of permits. For $p < p_1^*(\omega_1)$ the strategic supply function must satisfy (14), and therefore

$$n_2 \frac{2p - \phi + \theta\omega}{2p + Q} = 1 \Rightarrow$$
$$\tilde{\mathcal{Q}}(p) = n_2[\theta\omega - \phi] + 2[n_2 - 1]p$$
$$= n_2[2p + \theta\omega - \phi] - 2p.$$

Notice that the price above which strategic supply is defined is $\tilde{P}^S = \frac{n_2}{n_2 - 1} p_2^*(\omega_2)$. Turning next to strategic demand, for $p > p_2^*(\omega_2)$ the strategic demand function must satisfy (16), and therefore

$$n_1 \left[\frac{\phi - \frac{1}{\theta}\omega + \frac{B}{p} \pm \sqrt{[\phi - \frac{1}{\theta}\omega + \frac{B}{p}]^2 - 4\frac{B}{p}[\phi - \frac{1}{\theta}\omega - 2p]}}{2\frac{B}{p}} \right] = 1 \Rightarrow$$
$$\tilde{\mathcal{D}}(p) = n_1 \left[\phi - \frac{1}{\theta}\omega - 2\frac{n_1}{n_1 - 1} p \right]$$

regardless of which root is chosen for the buying share function. The price below which strategic demand is defined is $\tilde{P}^B = \frac{n_1 - 1}{n_1} \tilde{p}_1^*(\omega_1)$. Notice that, as with competitive supply and demand with quadratic costs, the details of which we elucidate below, strategic supply and demand are linear functions of the permit price.

Assuming the economy is such that $\tilde{P}^S < \tilde{P}^B$, algebraic manipulation of the equilibrium condition reveals the equilibrium price to be

$$\hat{p} = \frac{n_1 + n_2}{\frac{n_1}{n_1 - 1} n_1 + \frac{n_2 - 1}{n_2} n_2} \left[\frac{\phi}{2} - \frac{n_1 \frac{1}{\theta}\omega + n_2 \theta \omega}{2[n_1 + n_2]} \right],$$

and the equilibrium volume of trade is

$$\hat{Q} = \frac{n_1 n_2 \omega [\frac{n_1}{n_1 - 1}\theta - \frac{n_2 - 1}{n_2}\frac{1}{\theta}]}{\frac{n_1}{n_1 - 1} n_1 + \frac{n_2 - 1}{n_2} n_2} - \frac{n_1 n_2 [\frac{n_1}{n_1 - 1} - \frac{n_2 - 1}{n_2}]}{[n_1 + n_2] + \frac{1}{n_1 - 1}} \phi. \tag{22}$$

If we assume firms act as price-takers in the permit market, the first-order condition governing their behavior given in (21) is

$$\begin{aligned} \tilde{z}_i(x_i) - [\omega_i + x_i] = p \Rightarrow \\ x_i^W(p) = \phi - \omega_i - 2p, \end{aligned}$$

which is positive (so i is a buyer) if $p < \tilde{p}_i^*(\omega_i)$, and negative (so i is a seller) if $p > \tilde{p}_i^*(\omega_i)$. A competitive equilibrium can occur only at prices between $\tilde{p}_2^*(\omega_2)$ and $\tilde{p}_1^*(\omega_1)$ and for this range of prices, the competitive supply function takes the form

$$\mathcal{S}^W(p) = n_2[2p + \theta\omega - \phi]$$

and the competitive demand function takes the form

$$\mathcal{D}^W(p) = n_1 \left[\phi - \frac{1}{\theta}\omega - 2p \right].$$

The competitive equilibrium occurs at the intersection of these two functions, where the equilibrium price is

$$p^W = \frac{\phi}{2} - \frac{n_1 \frac{1}{\theta}\omega + n_2 \theta \omega}{2[n_1 + n_2]}$$

and the equilibrium volume of trade is

$$Q^W = \frac{n_1 n_2 \omega [\theta - \frac{1}{\theta}]}{n_1 + n_2}.$$

We see that strategic supply and demand lie to the left of their competitive counterparts, and note that even when $\theta > 0$ so $\tilde{p}_1^*(\omega_1) < \tilde{p}_2^*(\omega_2)$—which implies there are gains from trade—it may be the case that $\tilde{P}^S \geq \tilde{P}^B$ so there is no Nash equilibrium with permit exchange: for this not to be the case n_1 and n_2 need to be large enough. The welfare loss attributable to strategic behavior is given by the triangle created between the competitive supply and demand functions between $\hat{Q}$ and Q^W. The reduction in trade from the efficient level that is due to strategic behavior is given by $Q^W - \hat{Q}$ (the analytical expression of which is cumbersome). Inverse competitive demand is given by $\mathcal{P}^{DW}(Q) = \frac{1}{2}[\phi - \frac{1}{\theta}\omega - \frac{Q}{n_1}]$ and therefore the difference in inverse demand at $\hat{Q}$ and the competitive equilibrium price is $\frac{1}{2n_1}[Q^W - \hat{Q}]$, meaning the loss in buyers' surplus due to strategic behavior is $\frac{1}{4n_1}[Q^W - \hat{Q}]^2$. Likewise, inverse competitive supply is $\mathcal{P}^{SW}(Q) = \frac{1}{2}[\phi - \theta\omega + \frac{Q}{n_2}]$, and therefore the difference between the competitive equilibrium price and inverse supply at $\hat{Q}$ is $\frac{1}{2n_2}[Q^W - \hat{Q}]$, meaning the loss in sellers' surplus due to strategic behavior is $\frac{1}{4n_2}[Q^W - \hat{Q}]^2$. The loss in total surplus, therefore, is $[\frac{1}{4n_1} + \frac{1}{4n_2}][Q^W - \hat{Q}]^2$. Inspection of the expression for $\hat{Q}$ in (22) reveals that as both n_1 and n_2 increase without bound, the second term in the expression converges to zero, and the first term becomes equivalent to the expression for Q^W. As such, as the number of firms increases without bound, welfare losses due to strategic behavior shrink to zero.

5 Market power in the product market

We now turn to consider non-competitive product market structures and the effect of accounting for market power on the permit market equilibrium. In the previous framework it was assumed that firms were price-takers in the product market; yet it is possible that some element of market power may exist. This is, in fact, quite likely as many industries regulated by a cap-and-trade scheme are highly concentrated, such as the electricity (generation) market (Wolfram, 1999; Borenstein et al., 2002; Bushnell et al., 2008) and the cement industry (Ryan, 2012; Fowlie et al., 2016).[27]

The manifestation of market power in a product market is the restriction of supply to increase the price. A simple way to isolate this market power effect is to assume that firms are independent monopolists in the product market, which would be the case if the output of their production process was sufficiently differentiated or firms served regional markets. We deduced in our comparative statics exercise that there is a positive relationship between firms' supply in the product market and their net demand for permits, and therefore with the equilibrium permit price. As such, if firms have market power in the product market and the supply of goods to the market reduces, the 'market-power effect' will suppress net demand for permits and put downward pressure on permit prices relative to the situation where firms

[27] Although highly concentrated electricity markets are common, the issue of market power is not guaranteed. For example, Hintermann (2016b) has recently found that the German electricity market showed no signs of market power.

are assumed to be price takers. Our focus in this section, however, is to consider the issues associated with imperfect competition in a product market, where the strategic importance of a firm's cost function (in relation to its competitors) provides a rich link between the product and permit markets.

The effect of trading permits changes the firm's cost function for the product market and, importantly, influences the marginal cost of production. If the market is perfectly competitive (or if firms serve independent monopolies), this 'direct effect' of permit market activity is the only effect that influences firms' optimal output. If strategic behavior is considered in the product market, however, the outcome from engaging in Cournot competition hinges crucially on the firm's marginal cost in relation to those of its competitors. This raises two additional effects of permit market activity: an 'indirect effect' that results from the change in the product market equilibrium attributable to a change in the firm's own marginal cost; and, since the total number of permits is fixed, a 'changing rivals' costs' effect that results from a change in the product market equilibrium attributable to the change in other firms' marginal costs. These effects provide an additional incentive to acquire permits, thereby (at least) partially mitigating the suppressed net demand for permits that occurs due to the existence of product market power.

To consider the effect of strategic interaction in the product market, suppose that the firms participating in the permit market then go on to supply the same product market in which they compete à la Cournot. Since there is now strategic interaction in the product market, we consider the sub-game perfect equilibrium of this two-stage game. The price in the goods market will be determined as $\Phi(Z)$, which depends on the aggregate supply of all firms $Z = \sum_{i=1}^{n} z_i$. Consider a product market subgame in which the vector of permit transactions is $\mathbf{x} = \{x_i\}_{i=1}^{n}$. In this subgame, we want to deduce the Cournot equilibrium. The payoff function of firm i in this subgame takes the form

$$V_i = m_i - x_i p + \pi_i(z_i, Z, x_i) \text{ where}$$
$$\pi_i(z_i, Z, x_i) = z_i \Phi(Z) - C_i(z_i, f(z_i) - [\omega_i + x_i]).$$

When engaging in Cournot competition firms can be seen as maximizing their payoff with respect to z_i, taking the actions of other traders as given, which implies

$$C_i^z + f_i' C_i^a \geq \Phi(Z) + z_i \Phi'(Z),$$

with equality if $z_i > 0$. Since we assume $C_i^z + f_i' C_i^a = 0$ when $z_i = 0$ each firm will be active in a Cournot equilibrium, and we denote by $\hat{z}_i(Z; x_i)$ the output of firm i consistent with a Nash equilibrium in which the aggregate supply of all firms is Z, which satisfies the above first-order condition with equality. A Nash equilibrium in the subgame requires that these consistent individual supplies are also consistent with the aggregate supply. As such, the aggregate supply at the Cournot equilibrium in the subgame in which the vector of permit transactions is $\mathbf{x}$ is given by $\hat{Z}(\mathbf{x})$, defined as that level of Z where

$$\sum_{i=1}^{n} \hat{z}_i(Z; x_i) - Z = 0. \tag{23}$$

The equilibrium supply of firm i is then written $\hat{z}_i(\hat{Z}(\mathbf{x}); x_i)$. Notice that this depends on the entire vector of permit transactions. Our assumptions on demand and cost functions imply that individual 'replacement functions' $\hat{z}_i(Z; x_i)$ are decreasing in Z and therefore that $\sum_{i=1}^{n} \hat{z}_i(Z; x_i)$ is decreasing in Z so there is a unique fixed point and so a unique Cournot equilibrium.[28]

Returning now to first-stage decisions in the permit market, the reduced-form payoff function for firm i is

$$V_i = m_i - x_i p + \pi_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), \hat{Z}(\mathbf{x}), x_i) \text{ where}$$
$$\pi_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), \hat{Z}(\mathbf{x}), x_i) = \hat{z}_i(\hat{Z}(\mathbf{x}); x_i)\phi(\hat{Z}(\mathbf{x})) - C_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), f(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i)) - [\omega_i + x_i]).$$

When considering its optimal action in the permit market, a firm needs to consider the marginal effect on its permit transactions and the benefits (or costs) that this brings in terms of product market profitability. With $s = \{b, q\}$, the first-order condition governing optimal behavior in the permit market requires

$$\frac{\mathrm{d}\pi_i(\hat{z}_i(\hat{Z}(\mathbf{x}); x_i), \hat{Z}(\mathbf{x}), x_i)}{\mathrm{d}s_i} \leq \frac{\partial x_i p}{\partial s_i}.$$

The right-hand side of this first-order condition is the same as when we assumed the firm is a price-taker in the product market. The left-hand side, however, is different as it accounts not only for the direct effect of permit market activity on product market profitability, but also the indirect and changing rivals' cost effects. Since this depends on the entire vector of permit transactions we cannot, without further restrictions, derive strategic supply and demand functions, nor clearly deduce the properties of the permit market equilibrium. We can, however,

[28]This method was first used in the analysis of Cournot equilibrium by Novshek (1985).

consider the influence of strategic behavior in the product market on the incentives firms face in the permit market.

Decomposing the effect of permit market activity on product market profitability, we find

$$\frac{\mathrm{d}\pi_i}{\mathrm{d}s_i} = \frac{\partial \pi_i}{\partial x_i}\frac{\partial x_i}{\partial s_i} + \frac{\partial \pi_i}{\partial z_i}\frac{\mathrm{d}\hat{z}_i}{\mathrm{d}s_i} + \frac{\partial \pi_i}{\partial Z}\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i}. \tag{24}$$

Now, in the second term,

$$\frac{\mathrm{d}\hat{z}_i}{\mathrm{d}s_i} = \frac{\partial \hat{z}_i}{\partial x_i}\frac{\partial x_i}{\partial s_i} + \frac{\partial \hat{z}_i}{\partial Z}\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i}. \tag{25}$$

In both (24) and (25) the effect on the equilibrium aggregate output $\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i}$ can be decomposed into the direct effect from firm i's permit market strategy, and the indirect effect that comes through firm i's strategy influencing the permit transactions of other firms:

$$\frac{\mathrm{d}\hat{Z}}{\mathrm{d}s_i} = \frac{\partial \hat{Z}}{\partial x_i}\frac{\partial x_i}{\partial s_i} + \sum_{j\neq i}\frac{\partial Z}{\partial x_j}\frac{\partial x_j}{\partial s_i}. \tag{26}$$

Inserting (26) and (25) into the initial decomposition (24) and re-arranging yields the following proposition.

Proposition 4. *For firm i, the effect of permit market activity on product market profitability is:*

$$\begin{aligned}\frac{\mathrm{d}\pi_i}{\mathrm{d}s_i} =& \frac{\partial \pi_i}{\partial x_i}\frac{\partial x_i}{\partial s_i} + \frac{\mathrm{d}\pi_i}{\mathrm{d}z_i}\frac{\partial \hat{z}_i}{\partial x_i}\frac{\partial x_i}{\partial s_i} \\ &+ \left[\frac{\partial \pi_i}{\partial z_i}\frac{\partial \hat{z}_i}{\partial Z} + \frac{\partial \pi_i}{\partial Z}\right]\frac{\partial \hat{Z}}{\partial x_i}\frac{\partial x_i}{\partial s_i} \\ &+ \left[\frac{\partial \pi_i}{\partial z_i}\frac{\partial \hat{z}_i}{\partial Z} + \frac{\partial \pi_i}{\partial Z}\right]\sum_{j\neq i}\frac{\partial \hat{Z}}{\partial x_j}\frac{\partial x_j}{\partial s_i}.\end{aligned} \tag{27}$$

The first line of (27) captures the direct effect of permit market activity on profit that comes about from a change in optimal supply; the second line captures the indirect effect of permit market activity that comes from the change in firm i's permit transactions influencing the equilibrium in the product market; and the third line captures the changing rivals' cost effect that changes the product market equilibrium indirectly through the effect of firm i's actions on the permit transactions of others. Note that in the final term $\frac{\partial x_j}{\partial s_i} = 0$ for those traders $j \neq i$ that are sellers of permits, since their permit transactions are unilaterally decided by $x_j = -q_j$, so the changing rivals' cost effect only materializes for firms on the demand side of the permit market.

The overall effect of accounting for strategic behavior in the product market is ambiguous. Consider, for example, the decision of a permit buyer that engages in a strategic product market compared to a buyer in a competitive product market. Their output in a strategic market will be less than in a competitive market which will serve to reduce their demand for permits. However, by acquiring permits the firm lowers its marginal cost relative to others: purchasing permits reduces its own marginal cost and simultaneously increases the marginal costs of other permit buyers since any permits acquired by the firm in question cannot be acquired by other firms. These strategic considerations serve to increase the demand for permits. Which of these effects dominates depends on a multitude of factors, not least the competitiveness of the product market and the firm's market power in that market; developing a full understanding of the exact nature of this is left for future research.

6 Conclusion

The purpose of this article is to investigate the implications of strategic trade in pollution markets. By establishing a strategic market game where firms' roles as buyers or sellers are determined endogenously, we create a two-stage framework, where in the first stage firms participate in a price-mediated permit market and, in the second stage, firms select their level of production.

In the permit market, we use a strategic market game to identify firms' roles as buyers or sellers of permits and allow for price-mediated trade. We show that the unique equilibrium in which permit trade takes place, if one exists, is generically inefficient. Our framework also shows that strategic trade can alter the structure of the market, as the role of firms (buyers or sellers) and the equilibrium price are now endogenously determined: buyers (sellers) in a competitive market can switch their role in a market with strategic trade. Thus we show the use of strategic trade via a price-mediated strategic market game has fundamental consequences for the cost efficiency, level of exchange, equilibrium permit price, and structure of the market.

As cap-and-trade markets are now frequently implemented to control major pollution problems, it is important to identify how, in the presence of non-competitive behavior, the market equilibrium is established, and, of course, the associated cost inefficiencies. Our approach, by focusing on endogenous market formation and a price-mediated solution, has identified links between strategic behavior, cost inefficiency, market formation, and the nature of the equilibrium. Further analysis can take this core framework and analyze market formation in specific cap-and-trade markets, for example, by the inclusion permit banking (dynamic aspects), price collars, and allowance reserves.

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