CONSIDERATION OF WELD DISTORTION THROUGHOUT THE IDENTIFICATION OF FATIGUE CURVE PARAMETERS USING MEAN STRESS CORRECTION

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The effect of weld angular distortion on fatigue test specimens cut from butt welded plates is investigated by experimental and numerical methods. The weld specimens are made of a structural steel equivalent to BS 4360 grade 50D. The SN curve obtained from experimental data is used with the fatigue post-processor nCode DesignLife for fatigue life prediction. Mean stress correction is applied using the FKM approach to address the component of bending stress induced by clamping the distorted specimen, which is constant during the fatigue test. A parameter identification procedure for the SN curve and mean stress correction is proposed. The weld SN curve evaluated using the procedure is compared to the generic weld SN curves provided in the material database of nCode DesignLife and discussed.

INTRODUCTION

Fatigue test specimens cut from butt welded plates generally exhibit some degree of weld angular distortion, which may cause alignment problems when mounted in a standard test machine. In fatigue testing, it is good practice to minimize distortion effects by modifying the specimen or machine grips to minimize misalignment. Clamping a distorted specimen in a test machine induces bending stress in the specimen. When the distortion is significant, typically over 2°, the induced bending stress may be greater than the test membrane stress range. When it is not technically or contractually possible to fully counter specimen distortion, it is necessary to account for the effect of bending stress on fatigue life in the test procedure. This can be done by treating the clamp-induced bending stress as a constant or mean stress acting in addition to the varying membrane stress. In this way, the influence of bending stress can be represented by introducing a mean stress correction to the fatigue curve fitting procedure. This approach is proposed here for fatigue testing of a complex welded specimen, incorporating misalignment and thickness variation, for a target (minimum to maximum) stress ratio $R = 0$.

The welded test specimen geometry and dimensions are shown in Fig. 1. The specimen is cut from butt welded plates of different thickness, $t_1$ and $t_2 = 1.25 t_1$. The specimen shape conforms to ISO/TR 14345:2012 [1] and the weld to ASME B31.8-2014 [2], with eccentricity (distance between plate mid-surfaces) of $e_i = 0.125 t_1$. The specimen material is a moderately strong weldable structural steel, equivalent to BS 4360:1990 grade 50D [3], with yield stress 415 MPa and tensile strength 595 MPa.

Tests were performed at frequency 10 Hz for 17 samples (5 load levels – 3 samples each, plus 2 spare), with stress amplitude varying from 60 MPa to 110 MPa. The measured angular distortion of
the specimens ranged from 0.3° to 2.5°. Strain gauges were located on the specimen following PD 5500:2015 [4], as shown in Fig.2. The initial test arrangement is shown in Fig. 3a. Typical crack development and specimen separation are shown in Fig. 3b and Fig. 3c respectively.

The measurement procedure for strain gauges shown in Fig.3 consists of the following steps: 1) Strain gages installed; 2) Strain gages recorded un-gripped; 3) Strain gages recorded gripped; 4) Statically loaded from 0 kN to a maximum force twice & strain gages recorded; 5) Fatigue loaded from 1 kN to a maximum force – strain gages intermittently recorded.

Finite Element, FE, models incorporating individual measured distortion were created for all specimens tested, assuming an elastic material model and large deformation theory. The measured and calculated strains showed good agreement over the test range in all cases. The measured and calculated load-strain responses were found to be approximately linear, with some variation attributed to large deformation effects. Figure 4 shows the details on example of specimen 1 with comparison of strain gauges’ measurements to the results of linear FEA for the test case of $\Delta \sigma = 110$ MPa nominal stress range corresponding to 143 kN of the peak normal force. A plot of strain variation with time is shown in Fig.4a for all eight attached strain gauges. An illustration of the experimental specimen and numerical model in ANSYS Workbench is shown in Fig.4b. Readings from gauges 6, 7 and 8 for strain vs load in Fig.4c look quite linear. A comparison of experimental and predicted variation of strain with location for gauges 6, 7 and 8 at 100 kN of applied force is shown in Fig.4d.

**PROCEDURE FOR IDENTIFYING FATIGUE PARAMETERS**

The objective of the test programme is to develop an SN curve for use in fatigue analysis of complex structures using the fatigue postprocessor nCode DesignLife. This requires input of SN curves in the form of a power-law equation:

$$\Delta \sigma = B \cdot N^{-k}.$$  \hspace{1cm} (1)

This is an inverted form of Basquin’s equation [5], which has the form:

$$\Delta \sigma^m \cdot N = A.$$  \hspace{1cm} (2)

The Basquin model, in both forms (1) and (2), can be linearized by application of a decimal logarithm operator:

$$\log(\Delta \sigma) = \log(B) - k \cdot \log(N) \quad \text{and} \quad m \cdot \log(\Delta \sigma) + \log(N) = \log(A).$$  \hspace{1cm} (3)

These transformations make fitting corresponding fatigue parameters ($k$, $B$ or $m$, $A$) relatively simple and also helps to reduce the scatter of the experimental data to make the fitting procedure more effective.

Fatigue of welded joints is a complex and local phenomena, but there are however both local and global approaches to assess the fatigue life of weldments. Among the most famous local approaches are the hot-spot stress and notch stress methods. The most widely used method is a global method based on nominal stress, which indirectly accounts for local effects such as weld bead geometry. This approach is adopted here to characterise test results and adapt them for input to nCode DesignLife, where the nominal stress is the applied force $F$ divided by the minimum cross-sectional area of the specimen (in the thin plate).
Preparation of experimental data

Butt welded joints between plates or tubes are susceptible to misalignment and therefore transverse joints might experience secondary bending under applied axial loading. Referring to BS 7608:2014 [6], the design stress should include an allowance for the bending effects of any misalignment, i.e. the nominal distance between the centres of thickness of the two abutting components, eccentricity $e_t$, as illustrated in Fig.1. The nominal stress should be multiplied by the following stress magnification factor $k_m$:

$$k_m = 1 + 6 \cdot \frac{e_t}{t_1} \cdot \frac{t_1^3}{t_1^3 + t_2^3},$$

which gives a value of 1.254 for the specimen geometry shown in Fig.1.

The bending stress due to misalignment varies with the load applied to the specimen and must therefore be included in the nominal stress range. A component of bending stress can also arise in the specimen due to distortion but this can be considered to be constant throughout the test, as its variability is within 5%, and included in the mean stress. FE analysis of the specimens showed that the stress in the thick plate is more affected by misalignment than that in the thin plate. The maximum stress was found to occur in the weld toe of the thick plate, as shown in Fig. 5a. This finding is supported by the observation that the fatigue crack in the majority of tests most commonly initiated at the weld toe on the thick plate. Figure 5b shows an example of fatigue life assessment based on the nominal stress approach applied to the weld toe cross section.

The original vector of nominal stress range $\Delta \sigma_o$ (MPa) from experiments should therefore be multiplied by $k_m$ to account for the misalignment effect:

$$\Delta \sigma = \Delta \sigma_o \cdot k_m.$$  

(5)

The nominal stress range $\Delta \sigma$ (MPa) is then converted into the decimal logarithm form denoted as $\log N = \log(\Delta \sigma_o)$ to facilitate the fatigue parameters identification procedure. The rest of required experimental data comes in the form of separate vectors for number of cycles to failure, also presented in normal $N$ and decimal logarithm form as $\log N = \log(N)$, and bending stress $\sigma_b$. Here, four of 17 experiments ran out (didn't finish with failure) and were not included in the parameter identification procedure.

The bending stress $\sigma_b$ is a relatively constant component of stress during experiment. In several tests its value approached the value of stress range $\Delta \sigma$. The influence of bending stress is considered by introducing the mean stress effect into the parameter identification procedure. For this purpose, the vectors of stress ratios $R$ and mean stresses $\sigma_m$ are required. The stress ratio $R$ is estimated using its definition $R = \frac{\sigma_{\min}}{\sigma_{\max}}$, where $\sigma_{\min} = \sigma_b$ and $\sigma_{\max} = \sigma_b + \Delta \sigma$, as follows

$$R = \frac{\sigma_b}{\sigma_b + \Delta \sigma}.$$  

(6)

Using a similar approach, the mean stress is expressed as
\( \sigma_m = 0.5\left[ \sigma_a + (\sigma_b + \Delta\sigma) \right]. \) (7)

The available experimental data can be illustrated in 3D space \([x, y, z]\) for stress range \(\Delta\sigma\), logarithmic number of cycles to failure \(\log N\) and stress ratio \(R\) (or mean stress \(\sigma_m\)). Fig. 6a shows a 3D plot of the experimental data set, where the \(x\) axis is \(R\), \(y\) is \(\log N\) and \(z\) is stress range \(\Delta\sigma\). Figure 6b shows a 3D plot of the experimental data set, where the \(x\) axis is \(\sigma_m\), \(y\) is \(\log N\) and \(z\) is stress range \(\Delta\sigma\).

**Mean stress correction**

Mean stress correction of SN curves used in fatigue analysis in nCode DesignLife of welds is based on the FKM approach [7], which has the following form:

\[
\sigma_a = \sigma_{ak} - M \cdot \sigma_m \iff \sigma_{ak} = \sigma_a + M \cdot \sigma_m,
\] (8)

where \(\sigma_a\) is the stress amplitude applied at the non-zero mean stress and resulting in fatigue life of \(N\) cycles; \(\sigma_{ak}\) is the fully reversed stress amplitude applied at zero mean stress resulting in the same fatigue life of \(N\) cycles, and \(M\) is a correction factor, which defines the sensitivity to mean stress. The FKM approach can be presented in a form similar to conventional methods such as Gerber, Goodman, Soderberg and Morrow methods, which are based on ultimate strength \(\sigma_u\), yield strength \(\sigma_y\) or true stress at fracture \(\sigma_f\) as limiting values of the mean stress:

\[
\frac{\sigma_u}{\sigma_{ak}} + \frac{\sigma_m}{\sigma_{ak}/M} = 1,
\] (9)

where \((\sigma_{ak}/M)\) is replaced with \(\sigma_u\), \(\sigma_y\) or \(\sigma_f\) in classical approaches.

The method proposed here is based on the slope of the line \(M\) in coordinates of mean stress \(\sigma_m\) and stress amplitude \(\sigma_a\) (or stress range \(\Delta\sigma = 2\sigma_a\)). \(M\) is not related to basic material properties but characterises structural properties and manufacturing quality.

The available experimental data can be fitted by a surface defined by a function for \(\Delta\sigma\) that combines the Basquin equation and FKM correction, resulting in a non-linear dependence on \(N\) and linear on \(\sigma_m\). Application of FKM mean stress correction (8) to the Basquin equation (1) results in the following function dependent on two variables:

\[
\Delta\sigma(N, \sigma_m) = B \cdot N^{-k} - 2 \cdot M \cdot \sigma_m,
\] (10)

where the fatigue parameters \(B\), \(k\) and \(M\) are to be identified.

**Surface fitting of experimental data**

In the first step of the parameter identification procedure, the mean stress correction is applied to the available experimental data. To find an optimal value of \(M\), the range of values from 0.01 to 0.06 is examined for step size 0.0005, resulting in 101 discrete values for the vector \(M_i\):
\[ M_i = 0.01 + 0.0005 \cdot i, \quad \text{with } i = 0, 1, \ldots, 100. \]  

This gives 101 full vectors of corrected stress range:

\[ \Delta \sigma_i'' = \Delta \sigma + 2 \cdot M_i \cdot \sigma_m. \]

In the second step of the identification procedure, the optimal value of \( M \) is determined by fitting the power function to the available experimental data with 101 variants of \( \Delta \sigma_i'' \). Application of the Mathcad’s Genfit function is an effective approach to fit data with a power law function. The fitting function is considered in the form (1), where \( B \) and \( k \) are unknown fatigue parameters. Thereby, 101 values for the fatigue parameters \( (B, k) \) are obtained with corresponding values of \( M \).

An optimal value of \( M \) corresponds to the minimum difference between the experimental vector of stress range \( \Delta \sigma \) and its 3D function fit \( \Delta \sigma(N, \sigma_m) \) in the form of (10) as follows:

\[ \Delta_i = \Delta \sigma - \left( B \cdot N^{-k} - 2 \cdot M_i \cdot \sigma_m \right). \]

Thus 101 discrepancy vectors are obtained having both positive and negative values. In order to conclude about the accuracy for each of 101 variants of fatigue parameters, all these vectors are compressed into a corresponding single value characterising a total error of each fitting. The vector containing all 101 normalised total errors is obtained by summation of the squared components of the vectors \( \Delta_i \):

\[ \Delta \sigma_i'' = \sqrt{\sum_{j=0}^{\text{rows}(\Delta_i)-1} \left( (\Delta_i)_j \right)^2}. \]

Using the method of least squares, the minimum value in the vector \( \Delta \sigma_i'' \) corresponds to the optimal set of fatigue parameters. The smallest component of the vector \( \Delta \sigma_i'' \) has the value of \( \min \{ \Delta \sigma_i'' \} = 76.487 \) MPa corresponding to the index \( i_{\text{min}} = 52 \). The value of factor \( M \) corresponding to this index is 0.036 as can be confirmed graphically in Fig. 7.

The FKM method [7] as implemented in nCode DesignLife uses 4 factors, \( M_{1-4} \), which define the sensitivity to mean stress \( \sigma_m \) in 4 regimes: I) \( R > 1 \), II) \( -\infty \leq R < 0 \), III) \( 0 \leq R \leq 0.5 \), IV) \( 0.5 \leq R < 1 \). The method determines the equivalent stress amplitude \( \sigma_{eq} \) at a particular material \( R \)-ratio, and it is illustrated in the form of a constant life or Haigh diagram in Fig. 8. Due to the location of tests, the obtained value of \( M \) corresponds to the regime III as \( M_3 = 0.036 \). In relation to fatigue of welds, \( M_2 = 3 \cdot M_3 = 0.108 \), \( M_1 = 0 \) and \( M_4 = M_3 = 0.036 \) are recommended in [7].

The optimal parameters of the SN curve \( (k \text{ and } B_{-\text{III}}) \) corresponding to the index \( i_{\text{min}} = 52 \) are also easily identified. It should be noted that the identified value of parameter \( B_{-\text{III}} \) is a virtual stress range intercept, because it describes the interception of the stress amplitude axis by the Haigh diagram considering regime III and \( M_3 \). However, in reality the stress amplitude axis is intercepted by the Haigh diagram in regime II using slope \( M_2 \). Nevertheless, the identified value
of \( B_{-I(III)} \) corresponds to specific fatigue conditions: 50% of probability of failure and reference thickness \( t_1 \) (\( t_1 < t_2 \)) for welded plates. It needs to be converted to more general fatigue conditions for its practical application in fatigue assessments.

Before doing this, a basic verification of the developed non-linear fitting procedure is required. For this purpose, the same experimental data is fitted with a linear function presented visually by a 3D plane (not surface). Linear regression on the experimental data is performed using the Mathcad’s Regress function with the fitting function having the following form:

\[
F_{plm}(\log N, \sigma_m) = p_1 \cdot 10^{\log N} + p_2 \cdot \sigma_m + p_3, \tag{15}
\]

where the fitting parameters \( p_1, p_2 \) and \( p_3 \) are identified by the linear regression.

To enable visualisation with Eq. (15) on a single plot, the equation for the non-linear surface (10) is modified to the following form:

\[
F_{sur}(\log N, \sigma_m) = B_{-I(III)} \cdot (10^{\log N})^k - 2 \cdot M_3 \cdot \sigma_m. \tag{16}
\]

The fitting plane (blue mesh), the fitting surface (green mesh) and experimental data set (red dots) are shown on 3D graph in Fig. 9. Both plane and surface are located very close to each other, having similar angles of inclination relatively to standard planes. Based on visual comparison in Fig. 9, the result of fitting for the surface can be characterised as accurate. This is confirmed by the value of the parameter \( p_2 \), which defines inclination in the plane \([\Delta \sigma, \sigma_m]\), and it is exactly \( p_2 = 2 \cdot M_3 \), demonstrating the same dependence on the mean stress.

**Standard error of fitting**

To have access to different levels of probability of fatigue failure (not only 50%), the standard fitting error is identified for the suggested fatigue surface function (10). The function of form \( \Delta \sigma(N, \sigma_m) \) is converted to \( N(\Delta \sigma, \sigma_m) \) as

\[
N(\Delta \sigma, \sigma_m) = \left( B_{-I(III)} \cdot [\Delta \sigma + 2 \cdot M_3 \cdot \sigma_m]^{-1} \right)^{1/2}. \tag{17}
\]

Using its conventional form, the standard error of \( \log(N) \) is usually presented as

\[
SE = \sqrt{\frac{1}{n_{exp}} \cdot \sum_{i=0}^{n_{exp}-1} \left[ (\log N_i) - \log(N) \right]^2}, \tag{18}
\]

where \( n_{exp} \) is a number of considered experiments, \((\log N)_i\) is a vector of experimental values, while \((\log N_i)\) is a vector of fatigue life predictions with a suggested model using Eq. (17). For the available input, the standard error of \( \log(N) \) using Eq. (18) takes the value of 0.815, which characterises the scatter of material data, and used as an input in nCode DesignLife.
**DISCUSSION**

For practical application in fatigue assessment, Parameter $B_{-1(III)}$ must be converted to more general fatigue conditions. The parameter $B$ is defined for the any values of $R$ within the FKM regime III ($0 \leq R < 0.5$), as shown in Fig. 8 [8]:

$$ B_{III}(R) = B_{-1(III)} \cdot \left(1 + M_3 \cdot \frac{1+R}{1-R}\right)^{-1}, $$

(19)

where the values of $B_{-1(III)}$ and $M_3$ are obtained in parameters identification procedure. Therefore, the values of $B_{0(III)}$ ($R = 0$) and $B_{0.5(III)}$ ($R = 0.5$) can be easily identified with Eq. (19). The parameter $B$ in the FKM regimes II ($-\infty \leq R < 0$) is defined [8] using the equation similar to (19)

$$ B_{II}(R) = B_{1} \cdot \left(1 + M_2 \cdot \frac{1+R}{1-R}\right)^{-1}, $$

(20)

Since in Eq. (20) parameter $M_2$ is know from the FKM guideline [7] and parameter $B_{0(II)} = B_{0(III)}$ for $R = 0$ has the same value in both regimes II and III, then

$$ B_{1} = B_{0(II)} \cdot (1 + M_2), $$

(21)

where the value of $B_{0(III)}$ is defined by Eq. (19).

The values of parameter $B$ are obtained for different mean stress levels ($R = -1, 0$ and 0.5) and corresponding SN curves are compared to experimental data on a 2D plot ignoring the mean stress values in Fig. 10a. The illustrated SN curves correspond to 50% probability, minimum plate thickness $t_1$ and pure tension loading (no bending). They can be interpreted as 2D cross-sections of the fitting 3D surface in Fig. 9 corresponding to different $R$ ratios.

In order to consider the thickness and bending effects, the stress range intercept parameters $B$ are modified according to the British Standard BS7608:2014 [6] using a correction factor as

$$ k_{ib} = \left(\frac{t_{ref}}{t}\right)^n \cdot (1 + 0.18 \cdot \Omega^{1.4}) \Rightarrow \Delta\sigma(N) = k_{ib} \cdot B \cdot N^{-k}, $$

(22)

where $t$ is the thickness of the welded components, $t_{ref}$ is the reference thickness, $n$ is the thickness exponent, and $\Omega$ is the bending ratio. In this study, $t_{ref} = t_1$, the thickness of the thinner welded plate (see Fig. 1), and $n = 0.16667$ is a standard thickness exponent for generic weld seam SN curves from the nCode DesignLife materials database [8]. Equation (22) is used to convert an SN curve to any thickness and bending ratio as shown in Fig. 10b for $t_{ref} = 1$ mm and $\Omega = 1$ and 0.

**CONCLUSIONS**

The weld SN curve (for example at $R = 0$) obtained using the proposed procedure is incorporated into nCode DesignLife and can be used for fatigue life predictions using the stress input from ANSYS Workbench or any other structural FE-code. The curve can be applied to fatigue analysis at any fatigue conditions as it accounts for probability, mean stress and thickness.
effects automatically through the weld fatigue analysis engine available in nCode DesignLife for solids and shells.

The comparison of obtained SN curves to the available experimental data using 2D presentation in Fig.8a suggests a major visual discrepancy but this is significantly reduced when 3D presentation of data points and the fitting surface of Fig.9 is used. This observation shows the importance of mean stress correction when processing experimental data for welds with significant angular distortion.

The weld SN curve, normalised to 1 mm thickness, is compared to other SN curves available for fatigue analysis of structural steel welds in Fig.10b [8]. The weld SN curve from this study looks rather flat in contrast to other curves, because of insufficient experimental points to define a more curved shape. With more experiments conducted at different stress levels and wider stress range, the shape of SN will a more typical power law distribution.

REFERENCE LIST

FIGURE 2  Arrangement of strain gauges, following PD 5500:2015 [4]: a) schematic, b) in situ top, c) in situ bottom.

FIGURE 3  Fatigue test arrangement: a) start, b) crack growth in specimen, c) separation of specimen.
FIGURE 4  Comparison of strain gauges’ measurements to the results of linear FEA for the test case of $\Delta \sigma = 110$ MPa nominal stress range corresponding to 143 kN of the peak normal force: a) applied load and readings from all eight attached strain gauges vs time; b) specimen vs model; c) readings from gauges 6, 7 and 8 for strain vs load; d) comparison of experimental and predicted variation of strain with location for gauges 6, 7 and 8 at 100 kN of applied force.

FIGURE 5  Results of FEA showing (a) the location of maximum equivalent stress and (b) assessment of fatigue life based on the nominal stress approach at the weld toe cross section.
FIGURE 6 3D plots of the test points in coordinates of a) $[R, \log N, \Delta \sigma]$ and b) $[\sigma_m, \log N, \Delta \sigma]$.

FIGURE 7  Finding an optimal value of the mean stress correction factor $M$.

FIGURE 8  Representation of the FKM mean stress correction [7] and location of experiments.
FIGURE 9  3D graph of experiments (red dots), plane (blue mesh) and surface (green mesh).

FIGURE 10  Comparison of the obtained SN curves: a) at different mean stress levels at $t_{ref} = t_1$ and b) with other available SN curves for welds [8] normalised to $t_{ref} = 1$ mm and $R = 0$. 