3D printed small-scale acoustic metamaterials based on Helmholtz resonators with tuned overtones

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***Abstract*— Acoustic metamaterials have been extensively studied in recent decades due to their ability to control acoustic waves. In this paper, we present a prototype of a small-scale acoustic metamaterial based on Helmholtz resonators fabricated with additive manufacturing technology. The results confirm that 3D printed small-scale metamaterials can break the mass law by creating band gaps where the sound is deeply attenuated. We have also introduced a modification of the resonators whereby overtones are exploited and tuned in order to broaden the band gap. The output of this research could be used to provide passive filtering for transducers, to improve noise cancelling headphones, as well as in other smart acoustic sensors and IoT audio applications.**

Keywords—acoustic metamaterials; 3D printing; Helmholtz resonators; noise control; overtones; band gap.

I. Introduction

Acoustic metamaterials are capable of controlling and steering sound waves in unusual ways, as they have properties not found in conventional materials. The possibility of achieving negative effective bulk modulus, effective density, and refractive index in these materials has led to new developments in the field of acoustics, such as cloaking, super lenses and imaging [1]. When these parameters are negative, band gaps are formed where the sound is deeply attenuated. According to the mass law, to achieve a sound attenuation of 6 dB it is necessary to double the thickness of the absorbing material. By exploiting resonances, acoustic metamaterials can obtain deep band gaps without increasing the thickness and hence breaking the mass law. Similar results can be obtained with phononic crystals, periodic materials where the wavelength of the frequencies in the band gap is of the same order of the lattice parameter. However, phononic crystals can thus result in very large structures, especially when using audio frequencies, characterized by large wavelengths. Conversely, acoustic metamaterials are characterized by band gaps originating from the resonance of the unit cells, such as membranes and Helmholtz resonators. Hence, it is possible to work in the subwavelength domain, and so reduce the dimensions of the whole structure [2].

While research on acoustic metamaterials has seen many successful advancements, the results have remained mainly in laboratory environment and have rarely been converted to useful devices. New fabrication techniques such as 3D printing could result in valuable developments both from the point of view of research and production [1], [3].

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In this paper, we present small-scale acoustic metamaterials fabricated using additive manufacturing technology. In Section II we introduce the chosen unit cells, composed of Helmholtz resonators, which are extensively used in metamaterials research. Furthermore, we present a modified design that exploits the overtones of the resonators, which can be tuned to a selected frequency close to the fundamental resonance in order to enhance and broaden the band gap. This new design is easier to achieve thanks to the flexibility of CAD modelling and 3D printing. In Section III we describe in detail the experimental set-up and the results obtained in the lab. Finally, Section IV presents the main discussions and conclusions related to this research.

II. Metamaterial Design and Fabrication

A. The Helmholtz resonator

Helmholtz resonators consist of a cavity and an open neck, as shown in Fig. 1. They can be commonly found in nature, as for example in the sound-producing apparatus of some animals, especially insects [4]. They also form the basis of many musical instruments and common objects as bottles and cans. The modes of Helmholtz resonators have a monopolar nature and they influence the resonance of the bulk modulus, such that it can become negative since acoustic metamaterials act as effective continuous media [5], [6]. Helmholtz resonators such as soda cans have been used extensively as units in acoustic metamaterials for sound focusing, waveguiding [7] and in noise attenuation applications [8]. They act as simple harmonic oscillators, and are composed of a cavity that behaves as a spring and a neck that oscillates like a mass. The resonance frequency of the Helmholtz resonator is

$$f_0 = \frac{c}{2\pi}\sqrt{\frac{S}{L'V}} \quad (1)$$

where c is the speed of sound in the medium, S is the cross-sectional area of the neck, $L' = l + 1.7a$ is the extended length of the neck, l is the length of the neck, a is the radius of the neck and V is the volume of the cavity [9]. Since experiments exploiting soda cans have been successful, the dimensions of a soda can were scaled down by a factor of 20 in this work for an initial metamaterial design. The neck of the cylindrical Helmholtz resonator thus has an inner radius of 0.564 mm, and a length of 0.5 mm, while the cavity has an inner radius of 1.6 mm and a length of 5.8 mm. The thickness of the walls is 0.5 mm. The resonance of each resonator is at 6836 Hz.

B. Enhancing the band gap

While the origin of the fundamental frequency of Helmholtz resonators is due to oscillations of the mass of air contained in the neck, higher resonances are a byproduct of the oscillation of air in the cavity. For this reason, higher resonances are not integer multiples of the fundamental frequency and are classified as overtones instead of harmonics. According to Fletcher [4], in cylindrical Helmholtz resonators, if the cavity originating overtones is large it behaves as a pipe open at both ends, while, when volumes are smaller, the cavity acts as a pipe closed at one end. In the first case, the overtones will be:

$$f_n = (n-1)\frac{c}{2h'} \qquad (2)$$

(n = 2, 3, 4, ...)

In the second case the overtones will be:

$$f_n = (2n-1)\frac{c}{4h'} \qquad (3)$$

(n = 1, 2, 3, …)

where c is the speed of sound in the host medium, $h' = h + 0.6r$ is the effective length of the cavity and r is its radius. Therefore, it is possible to change the volume of the cavity to obtain a second resonance close to the fundamental frequency, and to broaden the band gap without the need to use two different resonators. In this case to broaden the band gap a Helmholtz resonator with a neck radius of 1 mm and a length of 0.5 mm, and a cavity with a radius of 1.6 mm and a length of 10.8 mm was chosen. The fundamental frequency is 7202 Hz and the first overtone is at 7308 Hz.

C. 3D printing process

The metamaterials have been fabricated with the Asiga Freeform Pico Plus 27 3D printer. The material used to produce them is polymethyl methacrylate (PMMA) containing 0.15% of Sudan I (SI). Adding the SI resulted in a higher accuracy in the printing process in comparison to other materials and no compensation rate had to be adopted to obtain the modelled resonators. The arrays have been modelled with Autodesk Inventor 2018. The high Young's Modulus of PMMA allows us to model the walls as rigid structures that do not influence the resonance frequency of the resonators.

III. MEASUREMENTS AND RESULTS

A. Experimental set-up

As shown in Fig. 1, the set-up consists of a function generator connected to a speaker that emits a swept sine wave from 100 Hz to 20 kHz. A National Instruments (NI) data acquisition system is connected to a Bruel & Kjaer (B&K) 1/8-inch pressure field microphone and sends the acquired data to a laptop through a MATLAB session based interface. The microphone is connected to a pre-amplifier and positioned above the sample. A measurement is taken above each resonator in the array and a reference signal is acquired without the sample. The experiments are conducted in a semi-anechoic environment, nevertheless some noise coming from the building and from the lab are present in the recordings. By subtracting the reference signal from the signal recorded above the sample it is possible to obtain the normalized sound transmission loss (STL).

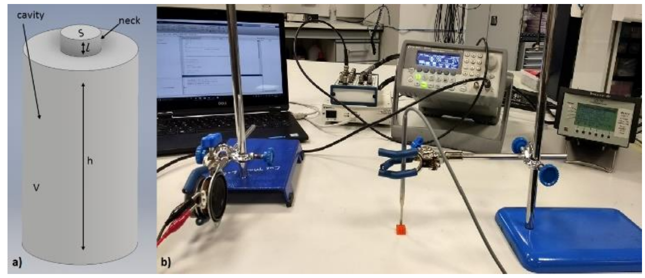

Fig. 1 Representation of a Helmholtz resonator composed of a cylindrical neck of length l, cross sectional area S and a cylindrical cavity of volume V and length h **(a)**. Experimental set-up: a speaker connected to a function generator emits a swept sine wave that impinges on the 3D printed sample. A microphone, connected to a pre-amplifier and positioned right above the metamaterials, records the signal controlled by NI-DAQ hardware and MATLAB code running on a laptop **(b)**.

B. Results

We first printed a 3X3 array of resonators, each of them having the dimensions (radius and height) of a soda can scaled down by a factor of 20, to show that the most important properties of metamaterials could be achieved at this small-scale using 3D printing technology. As illustrated in Fig. 2, the sound transmission loss (STL) above the central resonator is 10 dB. We then increased the number of resonators, obtaining an STL of 20 dB in a 3X6 array, and 30 dB in a 6X6 array.

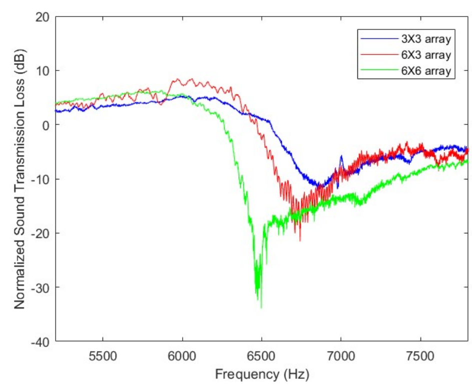

Fig. 2 By increasing the number of Helmholtz resonators, the band gap becomes deeper. Arrays containing 9, 18 and 36 resonators will result respectively in band gaps of 10 dB, 20 dB and 30 dB.

The band gap is a byproduct of interferences between the sound wave originated from the speaker and the resonances inside the metamaterials. A Helmholtz resonator can be approximated to a mass-spring system, where the air inside the cavity corresponds to the spring and the air inside the neck oscillates like a mass [4]. Since a speaker emitting a swept sine wave is used to excite the metamaterials, our system can be described by the equation of motion of the driven damped harmonic oscillator:

$$\ddot{x} + \gamma\dot{x} + {\omega_0}^2 x = F(t) \qquad (4)$$

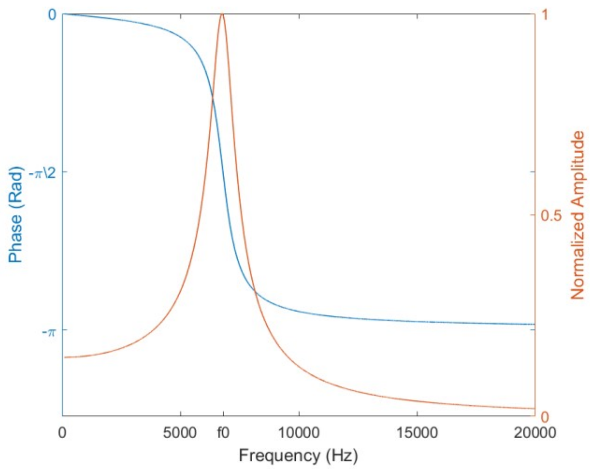

Fig. 3 Amplitude and phase lag of a driven Helmholtz resonator. When the frequency of driving force corresponds to f_0, the mass of air in the neck oscillates at a high amplitude, but in antiphase with the impinging wave.

where γ is the damping parameter, ω_0 is the fundamental angular frequency of the resonator and $F(t)$ is the driving force. When the frequency of the swept sine wave matches the resonance frequency of the resonator, the mass of air inside the neck begins to oscillate with a higher amplitude, but in antiphase with the original wave (See Fig. 3). Therefore, the sum of the two waves with opposite phase results in a narrow frequency band where the sound is deeply attenuated.

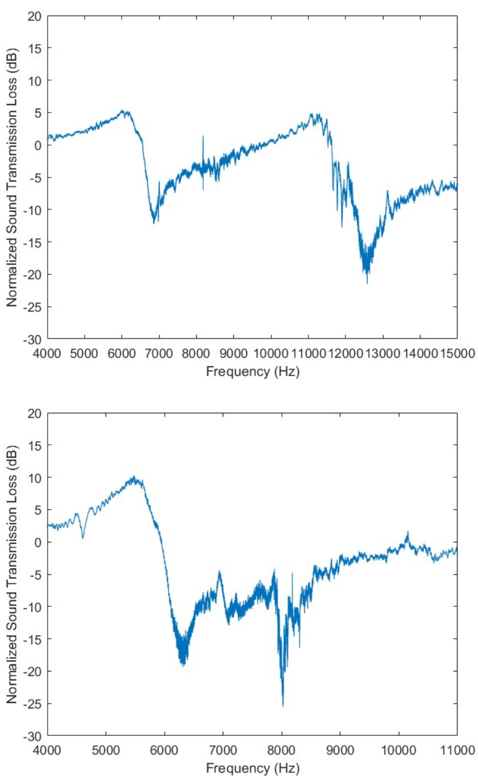

Fig. 4 The second band gap is positioned 6000 Hz from the first one **(top)**. By changing the cavity length, it is possible to tune the first overtone to be close to the fundamental frequency and to achieve a broader band gap **(bottom)**.

It is important to notice that the sound attenuation is generated by interferences and not by coupling between the resonators. When many resonators are employed to form an acoustic metamaterial, the first line of the array acts as an antiphase source destructively interfering with the incoming sound wave and in the further lines the band gap becomes deeper, as the impinging wave is already attenuated. Since the band gap originating from Helmholtz resonators is narrow, further arrays were printed with different cavity lengths, to tune the overtones to be closer to the fundamental frequency of the resonators. Fig. 4 illustrates that when the first overtone was 6000 Hz from the fundamental, the first band gap was very narrow. When the overtone was set much closer, i.e. 100 Hz from the fundamental, the band gap was broader. Hence, by tuning the overtones to a selected frequency, it is possible to control and enhance the band gaps in acoustic metamaterials.

IV. CONCLUSIONS

An acoustic metamaterial based on Helmholtz resonators has been successfully fabricated at a small-scale using additive manufacturing technology. Experimental measurements have shown that the metamaterial can break the mass law as predicted in the theory and previously demonstrated in other research using larger resonators. Moreover, it has been shown that by tuning the overtones of the resonators it is possible to obtain much deeper and broader band gaps. The results produced in this paper could be useful for sensing applications such as passive microphone filtering without the need of signal processing, wearable smart sensors and noise control. The metamaterials will be further modified in future work to include active components.

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