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Axion particle production in a laser-induced dynamical spacetime

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ABSTRACT

We consider the dynamics of a charged particle (e.g., an electron) oscillating in a laser field in flat spacetime and describe it in terms of the variable mass metric. By applying Einstein's equivalence principle, we show that, after representing the electron motion in a time-dependent manner, the variable mass metric takes the form of the Friedmann–Lemaître–Robertson–Walker metric. We quantize a pseudo-scalar field in this spacetime and derive the production rate of electrically neutral, spinless particles. We show that this approach can provide an alternative experimental method to axion searches.

It is well known that particle-production phenomena can occur in a curved or dynamic spacetime [1]. For example, thermal radiation can arise from particle production near the event horizon of a black hole, an effect commonly known as the Hawking radiation [2,3]. This is a quite general fact, not confined to black holes. As hypothesized by Davies, Unruh and Fulling [4–6], an observer in a uniformly accelerated frame experiences the surrounding vacuum as filled with thermal radiation with temperature $T_{DU} = \hbar a/2\pi k_B c = 4.05\times 10^{-23} a$ K, where a is the acceleration (in cm/s^2) and k_B is the Boltzmann constant. The expansion of the universe also gives rise to a curved metric called the Friedmann–Lemaître–Robertson–Walker (FLRW) metric: $ds^2 = dt^2 - h^2(t)d\mathbf{x}^2$. Here $h(t)$ is the scale factor which quantifies the relative expansion of the universe. In the FLRW metric, particles are spontaneously produced as a result of the expansion of the universe [7–10]. Of particular interest is the inflationary period, from 10^{-36} s until 10^{-32} s after the big bang. During this time, it is thought that the universe expanded exponentially, and spacetime was highly curved and dynamic. Understanding particle production during and after inflation [11–14] may help answer major questions like why the universe today is isotropic and flat, and why there is more matter than antimatter [15,16].

The latter is an example of a spontaneously broken symmetry that may require the existence of particles beyond the standard model. The axion is one of such particles, a pseudo Nambu–Goldstone boson which arises from the spontaneously broken Peccei–Quinn symmetry [17]. Both astrophysical bounds from stars and galaxies [18,19] as well as laboratory searches [20,21] have provided limits for the mass and coupling constants of these hypothetical particles. While experimental searches so far have not yet identified an axion candidate, the parameter space left to explore is still large and there is a need of more sensitive probes before the axion existence can be confidently ruled out.

Recent advancements in ultra-high intensity lasers [22] have stirred interest in the possibility of detecting both the Schwinger effect and dynamic spacetime phenomena [23–26]. Projects under development include the European Extreme Light Infrastructure [27], which will provide radiation beams of intensities exceeding 10^{23} W/cm^2; the X-ray free electron lasers (XFEL) based at DESY Hamburg, and the LCLS (Linac Coherent Light Source) facility at SLAC, where highly tunable x-ray pulses with narrow bandwidth and high intensity are already available. Over the last few years, a series of studies have been performed to assess the possibility of using collisions of high intensity lasers as a probe for axion-like-particles [28–32].

In the current letter, we propose a mechanism for pseudoscalar particle production in a laser field, whereby the variation of the metric around a charged particle oscillating in the laser field gives rise to spontaneous particle production. In fact, the model presented here is not restricted to accelerated charged observers, and it can be also applied to any accelerated frame. For example, neutral particles could be accelerated using radiation pressure from a laser beam [33], but the details of such mechanisms are outside the scope of the current paper.

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We start with the Lagrangian density of a free, massive, minimally-coupled real pseudo-scalar field $\phi(x)$ under the FLRW metric $g_{\mu\nu} = h^2(\eta)\eta_{\mu\nu}$, where $\eta_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$ is the Minkowski metric and $h(\eta)$ is the scale factor [34]:

$$\mathscr{L} = \frac{1}{2}\sqrt{-\det g_{\mu\nu}}\left[g^{\mu\nu}(\partial_\mu\phi)(\partial_\nu\phi) - m^2\phi^2\right]. \quad (1)$$

Here $x^0 = \eta$ is the conformal time (not to be confused with the Minkowski metric $\eta_{\mu\nu}$) which is related to the physical time t by $dt/d\eta = h(\eta)$. Natural units ($\hbar = c = \epsilon_0 = 1$) are used in all the derivations, and conversion to physical units will be explicitly mentioned. We notice that while the field $\phi(x)$ is assumed to describe a pseudoscalar particle (that is, an axion or axion-like particle), the Lagrangian density (1) applies equally well to a scalar particle of mass m. This ambiguity means that, if both pseudoscalar and scalar particles of mass m are allowed by the underlying model (whatever theory beyond the standard model of particle physics may be), then the axion density in this dynamical spacetime is only a fraction $\mathscr{P} \leq 1$ of the total number of the created particles.

The equation of motion of the field is the one that extremises the action functional $S = \int d^4x\mathscr{L}$. The extremisation condition is equivalent to Euler–Lagrange equation, giving for our case the Klein–Gordon equation. The next step consists in the procedure of canonical quantisation of the field ϕ to provide a framework for particles to be created and annihilated. In FLRW spacetime, however, the vacuum states at different times, $|0\rangle_{\eta_0}$ and $|0\rangle_{\eta_1}$ are different, and a notion of particle number that is consistent at all times is unattainable. To circumvent the ambiguity about the vacuum state, we first assume the existence of a preferred particle model that provides time-independent creation/annihilation operators from which we can construct a reference vacuum state. Such conditions are fulfilled, for example, when looking at the solution of the Klein–Gordon equation at asymptotic times ($\eta \to \pm\infty$) [35–37]. These are used to define time-dependent creation and annihilation operators, related to the asymptotic ones by Bogoliubov transformation. The procedure outlined above corresponds to the kinetic approach to quantum field theory, leading to the so called quantum Vlasov equation [36,38]. We obtain (see *e.g.*, [37]),

$$\frac{d\mathscr{N}_{\mathbf{k}}}{d\eta}(\eta) = \frac{\dot\omega_k(\eta)}{2\omega_k(\eta)}\int_{\eta_0}^{\eta} d\eta'\,\frac{\dot\omega_k(\eta')}{\omega_k(\eta')}[1 + 2\mathscr{N}_{\mathbf{k}}(\eta')] \times \cos[2\Theta_k(\eta) - 2\Theta_k(\eta')], \quad (2)$$

where $\mathscr{N}_{\mathbf{k}}$ is the time-dependent number of pairs of spatial mode $\mathbf{k}$,

$$\omega_k^2 = \mathbf{k}^2 + m^2h^2 - \frac{\ddot h}{h}, \quad (3)$$

with the dot notation representing differentiation by η (i.e., $\ddot h = d^2h/d\eta^2$) and

$$\Theta_k(\eta) = \int^{\eta} d\eta'\,\omega_k(\eta'). \quad (4)$$

The time η_0 is defined such that $\mathscr{N}_{\mathbf{k}}(\eta_0) = 0$. As we will see later on, the time η_0 refers to the time the laser pulse starts. In principle, a non-zero population of cold axions may be already present due to vacuum realignment in the early universe [39]. Assuming that these axions are the main constituent of dark matter, the current upper limit on their comoving density is $\rho_{\mathrm{dm}} = 9.6 \times 10^{-12}$ eV^4 [40], which is orders of magnitude less than one axion on average in the four-volume of a laser pulse.

The quantum Vlasov equation, eq. (2), is formally similar to the one obtained by Kluger et al. [36] and Schmidt et al. [41] for bosonic pair production in flat spacetime under an oscillating electron field. However, in our case, it has been specialized such that there is no explicit presence of an electric field and the spacetime is more generally defined by the FLRW metric. This is in fact the case for field theories in background fields. Since the particle number operator does not, in general, commute with the interaction Hamiltonian, one must be cautious interpreting results at intermediate times. Different particle number definitions that coincide at asymptotic times, may disagree by orders of magnitude at intermediate times (this phenomenon has been recently studied using the superadiabatic basis to analyse the Schwinger effect [42]). We note the quantum Vlasov equation's non-Markovian character [43]: the term $1 + 2\mathscr{N}_{\mathbf{k}}(\eta')$ in the integral means that the equation is non-local in time, i.e., the production rate of pairs is dependent on the history of the system.

Having obtained the particle production rate in an expanding spacetime, we now describe the dynamics of a particle in a laser field with an alternative metric that, as we shall see, bears many resemblances with the FLRW metric. In our approach we do not quantise the laser electromagnetic field nor the metric, which will be treated classically as existing in the background. Following closely the derivation by Crowley et al. [44,45], we consider the dynamics of a free particle of mass m_0 under the variable mass metric [46]. The name of the metric derives from the appearance of the "variable mass" hm_0 in the place of the rest mass m_0 in dynamical equations that are similar to flat spacetime equations. We have, for the variable mass metric, $g_{\mu\nu} = h^2(\mathbf{x})\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric and $h(\mathbf{x})$ is a spatial field. In general relativity, the dynamics of a free particle of mass m_0 in a spacetime with metric $g_{\mu\nu}$ is determined by its Lagrangian [47] $L = -(g_{\mu\nu}v^\mu v^\nu)^{1/2}m_0$, where $v^\mu = dx^\mu/dx^0$ is the 4-velocity. The Lagrangian for the variable mass metric is thus [44], $L = -hm_0/\gamma$, where $\gamma = (1 - v^2)^{-1/2}$. The canonical 3-momentum is given by $\mathbf{p} = \partial L/\partial\mathbf{v} = \gamma hm_0\mathbf{v}$, and the Hamiltonian is then $H = \mathbf{p}\cdot\mathbf{v} - L = \gamma hm_0$. Using Hamilton's equations one then obtains:

$$H = (\mathbf{p}^2 + h^2m_0^2)^{1/2}, \quad (5)$$

and

$$\mathbf{a} = \dot{\mathbf{v}} = -\frac{1}{\gamma^2}\frac{\partial \ln h}{\partial\mathbf{x}}, \quad (6)$$

where we have used the fact that the Hamiltonian has no explicit time dependence.

Let us now consider the dynamics of a charged particle, with charge q and mass m_0, oscillating, with frequency ν, in a laser pulse in flat spacetime, $\eta_{\mu\nu}$. The Lagrangian for this particle is $L = -m_0/\gamma + q\mathbf{v}\cdot\mathbf{A}$, which gives us the canonical momentum $\mathbf{p} = \gamma m_0\mathbf{v} + q\mathbf{A}$ and the Hamiltonian $H = [(\mathbf{p} - q\mathbf{A})^2 + m_0^2]^{1/2}$. We can decompose the momentum into parallel and perpendicular components with respect to $\mathbf{A}$, that is $\mathbf{p} = \mathbf{p}_\parallel + \mathbf{p}_\perp$. Thus if $\mathbf{v}_0$ is the velocity of the particle due to the influence of the laser field, and any remaining components are sufficiently small, we approximately have $(\mathbf{p}_\parallel - q\mathbf{A})^2 \approx \gamma_0^2m_0^2v_0^2$, with $\gamma_0 = (1 - v_0^2)^{-1/2}$. We notice that $|\mathbf{p}| \sim |\mathbf{p}_\perp| \approx \gamma_1\gamma_0m_0v_1$, where $\gamma_1 = (1 - v_1^2)^{-1/2}$ is calculated with respect to a particle velocity v_1 which is not associated to the motions induced by the laser field (that is, $\mathbf{v}_1 = \mathbf{v} - \mathbf{v}_0$, and $\mathbf{v}_1 \perp \mathbf{v}_0$). This holds under the condition that either $v_0 \ll 1$ or $v_1 \ll 1$. Hence,

$$H = (\mathbf{p}^2 + \gamma_0^2m_0^2)^{1/2}, \quad (7)$$

and

$$\mathbf{a} = -\frac{1}{\gamma_1^2}\frac{\partial \ln \gamma_0}{\partial \mathbf{x}}. \tag{8}$$

We immediately notice that, if we make the substitutions $\gamma_0 \to h$ and $\gamma_1 \to \gamma$, these last two equations are the same as (5) and (6). Einstein's equivalence principle asserts that an observer cannot distinguish between his frame's acceleration in flat spacetime and a metric field whose geodesic has equal acceleration, *i.e.*, the physics is the same in both cases. Hence, the dynamics of the charged particle oscillating in the laser field in flat spacetime may be equivalently described by the variable mass metric Hamiltonian of a free particle.

The idea that electromagnetic acceleration can give rise to dynamics that can be equivalently described by the variable mass metric will now be used to represent h in a time-dependent functional form so that it becomes equivalent to the scale factor of the FLRW metric. This key result will allow us to employ the field quantisation formalism developed earlier and obtain the particle production rate with the quantum FLRW-Vlasov equation. In the frame of the charged particle the vacuum acquires a finite number of particles of mass m. As discussed in the introduction, if allowed by the underlying theory, the vacuum is filled by both scalar and pseudoscalar particles of the same mass. The fraction of axion particles is given by $\mathscr{P}$, a free parameter of the underlying theory. Let us assume for the time being $\mathscr{P} = 1$.

The fact that the vacuum is filled by pseudoscalars has an observable signature in the laboratory frame only if the vacuum in the accelerated frame couples with a detector [48]. To accomplish this, we assume that the accelerated motion occurs in the presence of an external magnetic field, $\mathbf{B}$, aligned with the velocity of the charge. We can then modify the Lagrangian density by an extra term which describes the coupling of an axion field with the photons, given by [18,49]

$$\mathscr{L}_a = \sqrt{-\det g_{\mu\nu}}\,\frac{1}{M}\,\mathbf{E}\cdot\mathbf{B}_{\text{total}}\,\phi, \tag{9}$$

where $1/M \equiv \alpha g_\gamma/\pi f_a$ is the coupling constant for QCD axions (while this is not required, assuming QCD axions allows us to perform numerical estimates), with α the fine structure constant, g_γ a coefficient of order unity which depends on the details of the axion model, and f_a the axion decay constant [18]. There are several interactions described by this term:

$$\begin{aligned}\mathbf{E}\cdot\mathbf{B}_{\text{total}} = \mathscr{E}\cdot\mathbf{B} + \mathscr{E}\cdot\mathbf{B}_{\text{Laser}} + \mathbf{E}_{\text{Laser}}\cdot\mathbf{B}\\ + \mathbf{E}_{e^-}\cdot\mathbf{B} + \mathbf{E}_{e^-}\cdot\mathbf{B}_{\text{Laser}},\end{aligned} \tag{10}$$

where $\mathscr{E}$ is the electric field of emitted photons, $\mathbf{B}_{\text{Laser}}$ is the magnetic field of the laser, and $\mathbf{E}_{e^-}$ the electric field of the accelerated electron. All other terms are zero because i) the laser is a plane wave; ii) for the constant magnetic field, $\mathbf{E} = 0$ or iii) $\mathbf{B}_{e^-}$ (the magnetic field of the accelerated electron) is negligible. This final point is shown by considering the Lenard–Wiechert potential of an accelerated electron [50], whose magnetic field is a factor β smaller than the electric field, and in our treatment $\beta \ll 1$ (where $\beta = v_0/c$). Furthermore, we will neglect the effect of the electric Coulomb field of the electron because we are in the perturbative regime of $\xi \ll 1$ (where ξ is the laser intensity parameter, see below), and the energies of the produced axions can be, at the most, of the order of eV, corresponding to a minimum Compton wavelength of the order of a micron. This means the axion wavefunction would sample regions of the Coulomb field that are mainly much less than the laser background field strength (at a distance of 1 μm, the electric field of an electron has a strength ≈ 10 V/cm). In addition, we will also neglect axion regeneration due to the laser background. To justify this we compare the two quantities:

$$C_B = \left(\frac{BL}{m_0}\right)^2; \qquad C_E = (\xi\Phi)^2,$$

which occur in expressions for regeneration in a magnetic field and a plane-wave laser respectively (here m_0 is the electron rest mass, and we use $\Phi = \nu\tau$, where ν is the laser frequency and τ its pulse duration). Then a 50 kG magnetic field of length 1 m gives $C_B \approx 10^8$ and a 10^{19} W/cm^2 optical laser of duration 100 fs gives $C_E \approx 4\times 10^5$. Neglecting the laser contribution is consistent with other approximations made throughout this work. On the other hand, we consider the acceleration of the electron to be entirely due to the laser pulse because the magnitude of the force due to the magnetic field is of the order βB. This is expected to be much less that the acceleration due to the laser field since: i) $\beta \ll 1$ and ii) the magnetic field to be employed for regeneration is much weaker than the electric field of the laser pulse.

Therefore we take $\mathbf{E}\cdot\mathbf{B}_{\text{total}} \approx \mathscr{E}\cdot\mathbf{B}$. In the accelerated frame, the pseudoscalar particles forming the vacuum couple with the external magnetic field to produce photons, which would be an observable signature. If we assume that the photon and the axion fields propagate with the same direction and phase, then the additional term in the Lagrangian density leads to a modified dispersion relation (3) (see, e.g. [51]),

$$\omega_k^2 = \mathbf{k}^2 + \left[m^2 + \frac{B^2}{M^2}\left(1+\frac{\mathbf{k}^2}{m^2}\right)\right]h^2 - \frac{\ddot{h}}{h}. \tag{11}$$

Only axions that interact with the external magnetic fields are the ones that are observed in the laboratory frame. The external magnetic field is the same both in the laboratory and accelerated frames.

We now describe the acceleration of a charge particle on mass m_0 in a strong laser pulse, and in presence of a much weaker, constant, external magnetic field $\mathbf{B}$ (see above). We thus assume that the motion of the charged particles is determined by the laser field only. We take a laser pulse of frequency ν, four-wavevector $\varkappa$, phase $\varphi = \varkappa\cdot x$, duration τ and intensity parameter ξ [52] to be represented at the focus by a vector potential

$$\mathbf{A} = \frac{m_0\xi}{q}\exp\left[-\left(\frac{\varphi}{\Phi}\right)^2\right]\cos\varphi\,\hat{\mathbf{z}}, \tag{12}$$

where $\Phi = \nu\tau$ and $\hat{\mathbf{z}}$ is the unit vector in the z-direction. We limit the analysis to non-relativistic electron motion ($\gamma_0 \approx 1$), by specifying that $\xi \ll 1$. Assuming the particle begins at the origin with zero momentum in the infinite past, the velocity component in the field direction is $\dot{\mathbf{x}}\cdot\hat{\mathbf{z}} = qA/m_0$, which gives:

$$h = \left(1 - v_0^2\right)^{-1/2} = \left[1 - (qA/m_0)^2\right]^{-1/2} \approx 1 + \frac{q^2A^2}{2m_0^2}.$$

We see that $h \geq 1$, meaning that space expands when the electric field is non-zero. This can be interpreted as the result of the increased energy density of free space due to the presence of an electric field. In other terms, the electron acquires an effective mass $m_{\text{eff}} = hm_0$. The idea of an effective mass to describe the motion of electrons in intense laser beams is not new, and it is associated with the frequency shift of the radiation emitted by a particle in an intense electromagnetic field [53].

With this time-dependent form of h, the variable mass metric becomes equivalent to the FLRW metric. We can thus use the quantum field formalism developed earlier to estimate the particle production by integrating the FLRW quantum Vlasov equation (2). In doing so, we note that the field mass m appearing in equation (11) for the frequency ω_k is not necessarily the same as the mass m_0 of the oscillating particle. We assume that the test particle being accelerated by the laser field has mass m_0 and charge q (e.g., an electron for practical calculations), while the particles that are being produced as a result of the transformed metric have mass m and no charge.

Next, in the low density regime, that is, when the laser electric field is much smaller than the Schwinger's critical field, we make the approximation [54]

$$\mathcal{N}_{\mathbf{k}}(\eta_0,\eta) \approx \frac{1}{2}\int_{\eta_0}^{\eta}\int_{\eta_0}^{\eta''} d\eta'' d\eta' \frac{\dot\omega_k(\eta')}{\omega_k(\eta')}\frac{\dot\omega_k(\eta'')}{\omega_k(\eta'')}\times \cos[2\Theta_k(\eta')-2\Theta_k(\eta'')],$$

where we have assumed that $\mathcal{N}_{\mathbf{k}}(\eta_0)=0$. The integrand is symmetric with respect to the exchange $\eta' \leftrightarrow \eta''$, which means it is symmetric about the line $\eta'=\eta''$. Hence [55]

$$\mathcal{N}_{\mathbf{k}}(\eta_0,\eta) \approx \frac{1}{4}\left|\int_{\eta_0}^{\eta} d\eta' \frac{\dot\omega_k(\eta')}{\omega_k(\eta')}\exp[2i\Theta_k(\eta')]\right|^2, \tag{13}$$

where we have used the fact that the antisymmetry of the factor $\sin\{2\Theta_k(\eta')-2\Theta_k(\eta'')\}$ with respect to the exchange $\eta' \leftrightarrow \eta''$ has null contribution to the integral [54,55].

From (11) we then have:

$$\omega_k\dot\omega_k = \left[(\mathbf{k}^2+m^2)\frac{B^2}{M^2m^2}+m^2\right]h\dot h - \frac{h\dddot h - \dot h \ddot h}{2h^2} \tag{14}$$

The change in the metric perturbation depends only on the external field, so we have $h=h(\varphi)$, but on the other hand we wish to integrate over the conformal time, η. In general, the dependency $\eta(\varphi)$ can be complicated, but for a plane-wave background we can write $d/d\eta = \Omega^{-1}d/d\varphi$ where $\Omega = \varkappa\cdot p/m_0$ is the particle energy parameter [56]. As previously mentioned, of experimental relevance are the asymptotic values of observables for times long after the laser pulse has passed through the seed electrons [42]. For this reason, we integrate to finite phases, and in the final calculated observables, take the asymptotic limit. In this vein, $\mathcal{N}_{\mathbf{k}}(-\Omega R,\Omega R) \approx \frac{1}{4}|I_{\mathbf{k}}(R)|^2$ where:

$$I_k(R) = \frac{1}{\omega_k^2\Omega}\int_{-R}^{R}\left[\mathcal{M}^2 h\dot h - \frac{h\dddot h - \dot h\ddot h}{2h^2}\right]e^{2i\Theta_k}\,d\varphi, \tag{15}$$

where for brevity of notation we defined:

$$\mathcal{M}^2 = (\mathbf{k}^2+m^2)\frac{B^2}{M^2m^2}+m^2$$

Let us then define $\mathcal{N}_{\mathbf{k}} = \lim_{R\to\infty}\mathcal{N}_{\mathbf{k}}(-\Omega R,\Omega R)$. By assuming the hierarchy $m_0 \gg m_0\xi \gg \Omega \geq \mathcal{M}$ and expanding to lowest order in Φ^{-1}, terms in pre-exponents of order $O(\mathcal{M}^2\xi^4\Omega)$ and $O(\xi^4\Omega^3)$, $O(\mathcal{M}^2\xi^2\Omega/\Phi^2)$ were neglected in the integration. The leading-order terms were then:

$$\mathcal{N}_{\mathbf{k}} \approx \frac{\pi\xi^4\Phi^2(\mathcal{M}^2+2\Omega^2)^2}{2^7\omega_k^4}\left[e^{-\frac{\Phi^2}{\Omega^2}(\omega_k-\Omega)^2}+e^{-\frac{\Phi^2}{\Omega^2}(\omega_k+\Omega)^2}\right].$$

Integrating over all modes $\mathbf{k}$ gives the total particle density:

$$\begin{aligned}N &= \int \frac{d^3k}{(2\pi)^3}\mathcal{N}_{\mathbf{k}}(\eta)\\ &= \frac{\Phi^2\xi^4m^3}{2^8\pi}\left[\frac{(m^2+2\nu^2)^2}{m^4}I_2(m,\nu,\tau)\right.\\ &\left.\quad + \frac{2B^2(m^2+2\nu^2)}{M^2m^4}I_4(m,\nu,\tau) + \left(\frac{B^2}{M^2m^2}\right)^2 I_6(m,\nu,\tau)\right],\end{aligned} \tag{16}$$

$$I_n(m,\nu,\tau) = \int_0^\infty \frac{dy\, y^n}{(y^2+1)^2}\left[e^{-\tau^2(\omega_k-\nu)^2}+e^{-\tau^2(\omega_k+\nu)^2}\right],$$

(recalling $\Phi=\nu\tau$) and we have approximated $\omega_k^2 = m^2(1+y^2)$ (where $y=k/m$) inside the integral, and we have assumed the particle starts at rest, so $\Omega=\nu$. If one assumes $\Phi \gg 1$, then the integrals I_{2n} can be approximated using the asymptotic Laplace method [57]. Let us take $B^2 \ll M^2m^3\tau$, meaning that lower powers of B contribute more to $N \equiv N_\phi$. The leading contribution to the axion number comes from the integral I_2, which in the regime $\Phi \gg 1$ gives:

$$N_\phi^{(1)} \approx \mathscr{P}\frac{\xi^4\nu^3\Phi}{2^6\pi^{1/2}}, \tag{17}$$

where we have reintroduced via the statistical factor $\mathscr{P}$ the possibility that not all particle of mass m in accelerated vacuum are pseudoscalars.

The dependency of the axion particle production on its mass (with $\mathscr{P}=1$) is given in Fig. 1.

We recall that only pseudoscalar particles that have interacted with the external magnetic field and converted into photons are the ones that are observed in the laboratory frame. Thus the leading-order contribution to N is from I_4. This would give a number density of observed photons in the laboratory frame:

$$N_\gamma^{(1)} \approx \mathscr{P}\frac{\xi^4\nu^3\Phi}{2^6\pi^{1/2}}\frac{B^2}{M^2m^2}. \tag{18}$$

A question which immediately arises is how this mechanism of axion particle production compares, for example, with the predicted axion flux from the Sun. We take the coupling coefficient to be $1/M = 2\times10^{-19}\,(m/\mathrm{eV})\ \mathrm{eV}^{-1}$ [59]. Consider axion-like particles produced incoherently by $\sim 10^{14}$ oscillating electrons confined in laser focal spot of radius $\mathrm{w}_0 \sim 0.5$ mm, conditions that are achievable in high-power laser experiments. Then let us define the number of detectable photon-converted axion per laser shot $\mathrm{N}_\gamma = N_\gamma \pi \mathrm{w}_0^2\tau$. Then we find:

$$\begin{aligned}\mathrm{N}_\gamma^{(1)} &\approx 10^{-4}\mathscr{P}\left(\frac{N_e}{10^{14}}\right)\left(\frac{\mathrm{w}_0}{0.5\ \mathrm{mm}}\right)^2\left(\frac{\tau}{100\ \mathrm{fs}}\right)^2\\ &\quad\times\left(\frac{B}{50\ \mathrm{kG}}\right)^2\left(\frac{I_L}{10^{19}\ \mathrm{W/cm^2}}\right)^2,\end{aligned} \tag{19}$$

where I_L is the laser intensity (in W/cm^2). As discussed earlier, $\mathscr{P}$ is a free parameter that cannot be determined *a-priori* from the theory discussed here. An experiment, on the other hand, could potentially be used to set a limit on this.

One can compare this with the number of invisible axions produced every seconds by the Primakoff process in the Sun is given by [18]

$$\mathrm{N}_{\mathrm{Sun}} \approx 8.7\times10^{42}\left(\frac{m}{\mathrm{eV}}\right), \tag{20}$$

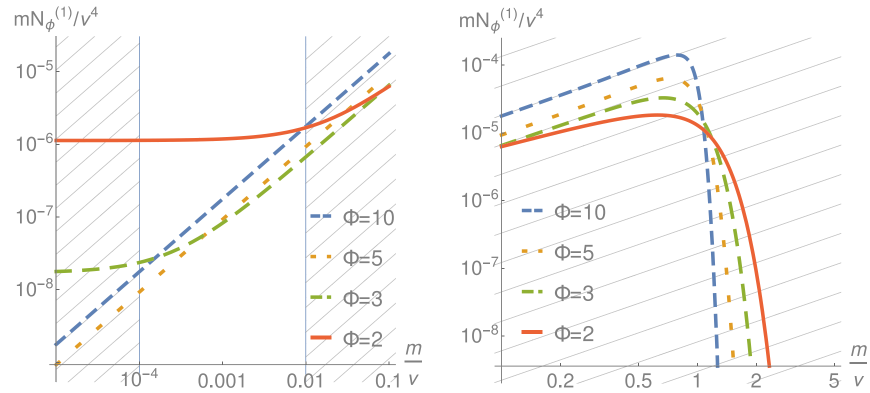

Fig. 1. Mass dependency of $mN_\phi^{(1)}/v^4$ for fixed $\xi = 0.1$. Shaded regions indicate pseudoscalar particle masses not ruled out by cosmological and astrophysical bounds on axion masses [58]. Left: as Φ is reduced, I_1 becomes flat, indicating a dependency $1/m$. Right: As Φ is increased, the heaviest particle mass to which the method is sensitive, decreases.

which is much larger that N_γ. However, suppose axions are emitted isotropically, a detector on Earth of area A_d would receive (the helioscope experiment, CAST, recently published new limits on the axion coupling and axion mass [59])

$$N_{\text{helioscope}} \approx 3 \times 10^{16} \left(\frac{A_d}{\text{m}^2}\right)\left(\frac{m}{\text{meV}}\right). \tag{21}$$

Of those, only a tiny fraction will be regenerated into photons (through the $1/M$ coupling). This is because axion-like-particle masses are predicted to lie in the sub-eV range, and we take the range to be $0.1\text{ meV} < m < 100\text{ meV}$ based upon current cosmological and astrophysical limits [58] (although note the recent limits predicted from a calculation in QCD of $50\,\mu\text{eV} < m < 1.5\text{ meV}$ [60]). While in the above estimates we have taken the axion coupling, $1/M$, to be set at the QCD scale, other coupling mechanisms are also possible for axion-like particles. One example would be to take the leading-order interaction between the pseudoscalar particle and the magnetic field in the dispersion relation Eq. (10), which gives $N_\phi \propto \mathscr{P}(B/Mm)^2$ and $N_\gamma \propto \mathscr{P}(B/Mm)^4$ in line with other light-shining-through-the-wall experiments [61]. Another example is the production of axions via the coupling between the laser electric field and the constant magnetic field. Also in this case, due to the extra axion interaction vertex, the mechanism is suppressed with a factor of the coupling squared, giving a dependency $N_\gamma \propto \mathscr{P}(B/Mm)^4 \ll N_\gamma^{(1)}$. The effective number of measurable invisible axions that the laser-based set-up produces is potentially superior to sun-based searches. Moreover, if the laser repetition rate is significantly higher than a few Hz (as feasible in the foreseeable future), then an axion search of the type proposed here could become competitive against other possible laser-based approaches [62,61,63].

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