

## **Tuning of FOPID Controller for Meliorating the Performance of the Heating Furnace Using Conventional Tuning and Optimization Technique**

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### **Abstract**

The milieu of the paper is the melioration of the performance of heating furnace, which is done by the tuning of the fractional order proportional integral derivative (FOPID) controller using various tuning techniques and the optimization algorithms. These techniques help us to find out the standards of the integer order tuning parameters and also the fractional order parameters of the PID controller. The Astrom-Hagglund and the Chien-Hrones-Reswick methods of tuning are used for tuning the tuning parameters of the controller, where Nelder-Mead optimization technique is used for optimizing the values of differ-integrals. These standards of parameters obtained using mentioned tuning techniques help us to generate standardized differ-integral order of the FOPID controller. This helps to improve the performance of the heating furnace. The complete process has been executed after the approximate dynamic modelling of the heating furnace.

**Keywords:** FOPID controller, Nelder-Mead, Astrom-Hagglund, Chien-Hrone-reswick, Fractional calculus

## 1. INTRODUCTION

Models portray our convictions about how the world functions. In mathematical modeling we make an interpretation of those convictions into the dialect of mathematics [1]. This has numerous focal points, (i) Mathematics is an extremely exact dialect. This helps us to formulate thoughts and distinguish fundamental suppositions, (ii) Mathematics is a succinct dialect, with very much characterized rules for controls, (iii) every outcome that mathematicians have demonstrated over many years are available to us and (iv) computers can be utilized for performing the numerical computation [1].

There is an expensive component of trade off in mathematical modeling. The dominant part of interacting systems in this present reality is very muddled to model completely. Subsequently the first level of trade off is to distinguish the most critical parts of the framework. These will be incorporated into the model, the rest will be avoided. The second level of the trade off concerns the measure of mathematical manipulation which is beneficial. Despite the fact that the mathematics can possibly demonstrate the general results, these outcomes depend fundamentally on the type of comparisons utilized. Little changes in the structure of comparisons may require colossal changes in the mathematical methods. Utilizing computers to handle the model mathematical equations might never prompt exquisite results, however it is very much stout against modification [2].

A heating furnace is basically a thermal fenced in area and is utilized to process the unprocessed (raw) substances at high temperature both in fluid and solid state. A few commercial enterprises like iron and steel making non-ferrous metals creation, glass making, assembling, earthenware handling, calcination in generation of cement, et cetera utilize the heating furnace. The principle goals are, (a) to use the heat effectively with the goal that misfortunes are least and (b) to handle the distinctive stages (solid, fluid and gas) moving at diverse velocities for diverse time and temperatures such that wearing away and deterioration of the obstinate are least

The principle components of the heating furnace are the energy sources (fossil fuel, electric energy, and chemical energy), appropriate refractory material, exchanger of heat and the control and instrumentation. Heaters are utilized for wide assortment of handling of raw substances to completed items in a few commercial enterprises. Comprehensively they are utilized either for physical preparing or for concoction handling of raw substances. In the physical handling the condition of the reactants stays unaltered; though in the compound preparing condition of the reactants changes either to fluid or gas [3].

Now coming to the proportion integral derivative (PID) controller then it will be engrossing to note that ninety percent of the cutting edge controllers being utilized today are the proportion integral derivative (PID) controllers or the revised proportional integral derivative (PID) controller [4].

Proportional integral derivative (PID) controllers are of the integer order and it is being extensively used in the commercial or industry oriented tasks. This controller came into being in the year 1939 and starting now and into the foreseeable future it has

stayed critical in the perspective of its execution [5]. Since a large portion of the proportion integral derivative (PID) controllers are adjusted in adjacent, an extensive variety of sorts of tuning standards have been proposed in the writing. Using the tuning gauges, delicate and aligning of proportional integral derivative (PID) controllers can be made adjacent. In like manner, modified tuning methods have been made and a rate of the proportional integral derivative (PID) controller may have online customized tuning limits. Balanced sorts of proportional integral derivative (PID) control, for instance, I-PD control and multi degrees of flexible Proportional integral derivative (PID) control are at this time being utilized as a part of industry. Various sober minded frameworks for knock less changing (from manual operation to customized operation) and build booking are financially open. The handiness of proportional integral derivative (PID) controls laid in their general relevance to most control systems. In particular, when the exploratory model of the plant is not known and thus investigative layout techniques can't be used, proportional integral derivative (PID) controls winds up being generally supportive. In the field of procedural control systems, it is unquestionably comprehended that the central and the amended proportional integral derivative (PID) control techniques have shown their accommodation in giving attractive control, regardless of the way that in various given circumstances they may not give wonderful control.

The FOPID controller which has been developed from the Proportional integral derivative (PID) controller by just renovating it into fractional order from the integer order which was first proposed in 1997 by Igor Podlubny [6]. In addition, the enticement behind the utilization of FOPID is that it is very trouble-free to design the controller for systems with higher order by using the practices of modeling based on regression and also because it has the iso-damping property which makes possible the variation over ample range of operating point for a particular controller [7]. There are numerous other motives which are accountable for the utilization of FOPID controller and they are the burliness from the high frequency noise as well as for the gain variation of the plant, the nonexistence of the steady state error and it holds both the phase and gain margin and also the gain and phase cross over frequency [8].

Optimization also termed as augmentation is the procedure of creating the things more unblemished, potent and dynamic in order to acquiesce the best result. The various techniques of optimization are Nelder-Mead, Active-Set, Interior-Point, SQP (sequential quadratic programming) et cetera. Numerically it can be explained as the procedure of expanding and shrinking of the endeavor capacity relying upon various conclusion variables under a deal of restrictions. The optimization technique has been used so as to discover and attain the finest results so as to design the most accurate FOPID controller that yields the finest output and assists the plant to augment its performance.

Since the classical calculus cannot decipher the equations with fractional order therefore the fractional calculus has been used extensively here. Basically the Fractional calculus is numerical computation that dissects the likelihood of accommodating the real and complex number powers of operator of differentiation.

## 2. PID CONTROLLER

PID stands for the proportional integral derivative which can be defined numerically using differential equation as,

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de}{dt} \quad (1)$$

On performing the Laplace transform of the equation (1) which is the PID controller equation is,

$$L(s) = K_p + \frac{K_i}{s} + K_d s \quad (2)$$

Where,  $K_p$  is the gain of proportionality,  $K_i$  is the gain of Integral,  $K_d$  is the gain of Derivative,  $e$  is the Error (SP-PV),  $t$  is the instantaneous time and  $\tau$  is the variable of integration that takes on the values from time 0 to the present  $t$ .

## 3. FOPID CONTROLLER

The FOPID controller can be defined numerically using differential equation as [6],

$$u(t) = K_p e(t) + K_i D_t^{-\lambda} e(t) + K_d D_t^\mu e(t) \quad (3)$$

FOPID stands for the fractional order proportional integral derivative. The equation of the FOPID in Laplace domain is [5],

$$L(s) = K_p + \frac{K_i}{s^\lambda} + K_d s^\mu \quad (4)$$

Where,  $K_p$  is the gain of proportionality,  $K_i$  is the gain of Integral,  $K_d$  is the gain of Derivative and  $\lambda$  and  $\mu$  are the differential-integral's order for FOPID controller.

## 4. A PRÉCIS ON FRACTIONAL ORDER CALCULUS

Fractional order calculus is a mathematical concept that has been in existence from 300 years ago [9]. It is the mathematical concept that has proved itself better as compared to the integer order methods.

According to Lacroix,

$$\frac{d^n}{dx^n} x^m = \frac{m!}{(m-n)!} (x)^{(m-n)} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} (x)^{(m-n)} \quad (5)$$

According to Liouville,

$$D^{\frac{-1}{2}} f = \frac{d^{\frac{-1}{2}} f}{(d(x-a))^{\frac{-1}{2}}} = \frac{1}{\Gamma(\frac{1}{2})} \int_{u=a}^{u=x} (x-u)^{\frac{-1}{2}} f(u) du = F^{\frac{-1}{2}}(x) \quad (6)$$

According to Riemann-Liouville [10],

$${}_aD_t^\alpha f(t) = \frac{1}{\Gamma(m-\alpha)} \left(\frac{d}{dt}\right)^m \int_a^t \frac{f(\tau)}{(t-\tau)^{1-(m-\alpha)}} dt \tag{7}$$

According to Grunwald-Letnikov, which is being used widely is,

$${}_aD_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{\frac{(t-a)}{h}} \left\{ \frac{\Gamma(\alpha+k)}{\Gamma(k+1)} \right\} f(t - kh) \tag{8}$$

Where,

$$\Gamma(t) = \int_0^\infty x^{t-1} e^{-x} dx \tag{9}$$

This is called the Euler’s gamma function.

The fractional order derivatives and integrals properties are as follows [11],

- f(t) being a logical function of t then the fractional derivative of f(t) which is  ${}_0D_t^\alpha f(t)$  is an analytical function of z and  $\alpha$ .
- If  $\alpha = n$  (n is any integer) then  ${}_0D_t^\alpha f(t)$  produces the similar result as that of the traditional differentiation having order of n.
- If  $\alpha=0$  then  ${}_0D_t^\alpha f(t)$  is an identity operator  ${}_0D_t^\alpha f(t) = f(t)$  (10)
- The differentiation and integration of fractional order are said to be linear operations,

$${}_0D_t^\alpha f(t) + b_g(t) = a_0 D_t^\alpha f(t) b_0 D_t^\alpha f(t) \tag{11}$$

- The semi group property or the additive index law,

$${}_0D_t^\alpha f(t) {}_0D_t^\beta f(t) = {}_0D_t^\beta f(t) {}_0D_t^\alpha f(t) = {}_0D_t^{\alpha+\beta} f(t) \tag{12}$$

Which is being held under some sensible limitations on f(t). Derivatives which are of fractional order has the commutation with derivative of integer order which is as follows,

$$\frac{d^n}{dt^n} {}_aD_t^\alpha f(t) = {}_aD_t^\alpha \left( \frac{d^n f(t)}{dt^n} \right) = {}_aD_t^{\alpha+n} f(t) \tag{13}$$

where for  $t=a$ ,  $f^{(k)}(a)=0$  for  $k=\{0,1,\dots,n-1\}$ . The given equation shows that  $\frac{d^n}{dt^n}$  and  ${}_aD_t^\alpha$  are commuted.

## 5. DYNAMIC MODELING OF HEATING FURNACE

The approximate modeling of heating furnace includes quantity of input that varies with time and is actually the fuel mass gas flow rate and also the pressure inside the furnace which is the output value.

The dynamic modeling of heating furnace includes the mass, energy and the momentum balances. It also includes the transfer of heat from the hot flue hot gas to water, flue gas flow from the boiler model and steam model.

As we know for any physical system the total force is equal to the summation of individual forces exerted by mass (m), damping (b) and spring (k) element [12].

Mathematically we can state the same as,

$$F = ma + bv + kx \quad (14)$$

In the equation (14) acceleration is signified as a, velocity is signified as v and displacement is signified as x.

Therefore the differential equation of equation (14) is,

$$F = m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx \quad (15)$$

Note: For designing a network based PID the above equation or model is a rough process behavior description.

Therefore, the differential equation of the heating furnace using the above equation becomes [13],

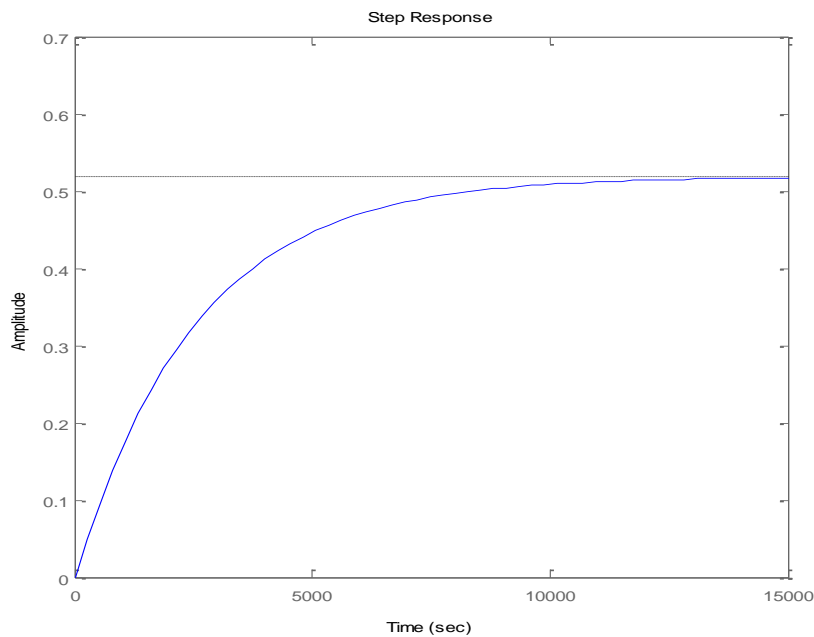
$$F = 73043 \frac{d^2x}{dt^2} + 4893 \frac{dx}{dt} + 1.93x \quad (16)$$

The Laplace transfer function of equation (16) which gives the Integer order model (IOM) as [14],

$$G_I(s) = \frac{1}{73043s^2 + 4893s + 1.93} \quad (17)$$

s is the Laplace operator

Where, the mass is denoted by 'm' is 73043, the damping denoted by 'd' is 4893 and the spring denoted by 'k' is 1.93.



**Fig. 1** Step response of the equation (17) or IOM

### 6. NELDER-MEAD OPTIMIZATION TECHNIQUE

Nelder-Mead optimization method is also called the Downhill simplex method or the amoeba method which is used to find the minimum and maximum of an objective function in various dimensional spaces. The Nelder-Mead method is a technique which is a heuristic search method that can coincide to non-stationary points. However, it is easy to use and will coincide for a large class of problems. The Nelder-Mead optimization method was proposed by John Nelder & Roger Mead in year 1965. The procedure uses the concept of a simplex (postulation of notion of triangle or tetrahedron to arbitrary dimensions) which is a special polytope (geometric objects having flat sides) type with  $N + 1$  vertices at  $n$  dimensions. Illustrations of simplices are, a tetrahedron in three-dimensional space, a triangle on a plane, a line segment on a line, et cetera.

The different operations in Nelder-Mead optimization method are,

Taking a function  $f(x)$ ,  $x \in R^n$  which is to be minimized in which the current points are  $x_1, x_2, \dots, x_{n+1}$ .

- i. Order : On the basis of values at the vertices,  $f(x_1) \leq f(x_2) \leq \dots \leq f(x_{n+1})$ .
- ii. Calculate the centroid of all points ( $x_0$ ) except  $x_{n+1}$ .

- iii. Reflection: Calculate  $x_r = x_0 + \alpha(x_0 - x_{n+1})$ . If the reflected point is not better than the best and is better than the second worst, that is,  $f(x_1) \leq f(x_r) < f(x_n)$ . After this by replacing the worst point  $x_{n+1}$  with reflected point  $x_r$  to get a new simplex and go to the first step.
- iv. Expansion: If we have the best reflected part then  $f(x_r) < f(x_1)$ , then solve the expanded point  $x_e = x_0 + \gamma(x_0 - x_{n+1})$ . If the reflected point is not better than expanded point, that is,  $[f(x_e) < f(x_r)]$  then either by substituting the worst point  $x_{n+1}$  by expanded point  $x_e$  to get new simplex and then go to the first step or by replacing the worst point  $x_{n+1}$  by reflected point  $x_r$  to obtain or get a new simplex and then go back to the first step.

Else if the reflected point is not better than second worst then move to the fifth step.

- v. Contraction: Here we know that  $f(x_r) \geq f(x_n)$ , contracted point is to be calculated,  $x_c = x_0 + \rho(x_0 - x_{n+1})$ , if  $f(x_c) < f(x_{n+1})$  that is the contracted point is better than the worst point then by substituting the worst point  $x_{n+1}$  with contracted point  $x_c$  to procure a new simplex and then go to first step or proceed to sixth step [15].
- vi. Reduction: substitute the point with  $x_i = x_1 + \sigma(x_i - x_1)$  for all  $i \in \{2, \dots, n+1\}$ , then go to the first step.

Note: Standard values for  $\alpha$ ,  $\sigma$ ,  $\rho$ ,  $\gamma$  are 1,  $\frac{1}{2}$ ,  $-\frac{1}{2}$ , 2 respectively. In reflection the highest valued vertex is  $x_{n+1}$  at the reflection of which a lower value can be found in the opposite face which is formed by all vertices  $x_i$  except  $x_{i+1}$ . In expansion we can find fascinating values along the direction from  $x_0$  to  $x_r$  only if the  $x_r$  which is the reflection point is new nadir along vertices. In contraction it can be expected that a superior value will be inside the simplex which is being formed by the vertices  $x_i$  only if  $f(x_r) > f(x_n)$ . In reduction to find a simpler landscape we contract towards the lowest point when the case of contracting away from the largest point increases  $f$  occurs and which for a non-singular minimum cannot happen properly. Indeed initial simplex is important as the Nelder-Mead can get easily stuck as too small inceptive simplex can escort to local search, therefore the simplex should be dependent on the type or nature of problem [15].

## 7. CHIEN-HRONE-RESWICK METHOD OF TUNING

The modified method of the Ziegler-Nichols method is the Chien-Hrone-Reswick method. There are basically two forms of CHR which are Chien-Hrone-Reswick (set point regulation) also known as CHR-1 and the Chien-Hrone-Reswick (disturbance rejection). The development of this tuning was done in the year 1952 by Chien-Hrone-Reswick. For process control application this method provides a better way of selecting the compensator [16]. On the basis of this method the controller parameters are often tuned in the industrial processes. The parameters of the controller for the method for 0% and 20% overshoot is summarized in table 4 and table 5



**Table 1** CHR 1 method of calculating  $K_p$ ,  $K_i$  and  $K_d$  [16]

Overshoot	0%			20%		
Controller	$K_p$	$K_i$	$K_d$	$K_p$	$K_i$	$K_d$
PID	0.6/a	T	0.5L	0.95/a	1.4T	0.47L
PI	0.35/a	1.2T	-	0.6/a	T	-
P	0.3/a	-	-	0.7/a	-	-

**Table 2** CHR 2 method of calculating  $K_p$ ,  $K_i$  and  $K_d$  [16]

Overshoot	0%			20%		
Controller	$K_p$	$K_i$	$K_d$	$K_p$	$K_i$	$K_d$
PID	0.95/a	2.4L	0.42L	1.2/a	2L	0.42L
PI	0.6/a	4L	-	0.7/a	2.3L	-
P	0.3/a	-	-	0.7/a	-	-

## 8. ASTROM-HAGGLUND OR AMIGO METHOD OF TUNING

The Astrom-Hagglund method is the approximate that completes the processing a very simple way. The other name for this tuning method is AMIGO which stands for approximate M-constrained integral gain optimization method for tuning. The procedure of the tuning method is almost similar to the Ziegler-Nichols method of tuning. The tuning procedure of the AMIGO is as follows [17],

$$a. \quad K_p = \frac{1}{K} (0.2 + 0.45 \frac{T}{L}) \quad (18)$$

$$b. \quad K_i = (\frac{0.4L+0.8T}{L+0.1T}) L \quad (19)$$

$$c. \quad K_d = \frac{0.5LT}{0.3L+T} \quad (20)$$

## 9. DESIGNING AND TUNING OF FOPID CONTROLLER FOR HEATING FURNACE

The integer order model (IOM) of heating furnace, using Laplace transform, which is a second order transfer function, which is given as [18],

$$G_I(s) = \frac{1}{73043s^2 + 4893s + 1.93} \quad (21)$$

Now, by using the Grunwald-Letnikov equation (8) for fractional calculus which is being given as,

$${}_a D_t^\alpha f(t) = \lim_{h \rightarrow 0} \frac{1}{\Gamma(\alpha)h^\alpha} \sum_{k=0}^{\frac{(t-a)}{h}} \left\{ \frac{\Gamma(\alpha+k)}{\Gamma(k+1)} \right\} f(t - kh)$$

When the equation (16) is being solved using the Grunwald-Letnikov equation given above then we get the fractional order model (FOM) of heating furnace which comes out to be [19],

$$G_F(s) = \frac{1}{14494s^{1.31} + 6009.5s^{0.97} + 1.69} \quad (22)$$

The equation for FOPDT (first order plus dead time) is given as [20],

$$G_{FOPDT}(s) = \frac{K}{(1+Ts)} e^{-Ls} \quad (23)$$

Where, K is referred to as the gain, L is referred as time delay and T is referred as the time constant.

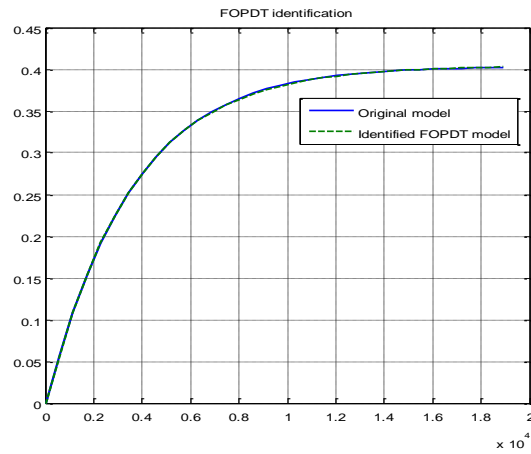
Then, by finding out the step response of the transfer function of the plant (heating furnace) we find out the value of K, L and T.

$$\text{Where, } T = \frac{3(T_2 - T_1)}{2}, L = (T_2 - T_1) \text{ and } a = \frac{KL}{T}$$

Where,  $T_1$  and  $T_2$  are the time instances in seconds taken from the step response obtained having a particular steady state gain.

So, the FOPDT model for the plant which is the heating furnace comes out to be,

$$G_{FOPDT}(s) = \frac{0.404272}{1 + 3421.93s} e^{-72.464s} \quad (24)$$



**Fig. 2** FOPDT identification, comparison between the original and the identified one which appears to be perfect.

Now, on applying Astrom-Hagglund method, Chien-Hrone-Reswick 1 and Chien-Hrone-Reswick 2 method,

**Table 3** Calculated values  $K_p$ ,  $K_i$  and  $K_d$

	Astrom-Hagglund	CHR-1	CHR-2
$K_p$	53.0586	111.005	140.217
$K_i$	0.109746	0.0231707	0.967806
$K_d$	1910.28	3779.4	4266.11

The value of  $\lambda$  and  $\mu$  is being calculated by the Nelder-Mead optimization algorithm separately for both the methods with phase margin =  $60^\circ$  and gain margin = 10dB,

**Table 4** Calculated values of  $\lambda$  and  $\mu$

	Astrom-Hagglund	CHR-1	CHR-2
$\lambda$	0.10596	0.49282	0.53323
$\mu$	0.010011	0.015107	0.19204

Now, the FOPID ( $PI^\lambda D^\mu$ ) model using the values obtained from and for the Astrom-Hagglund method is,

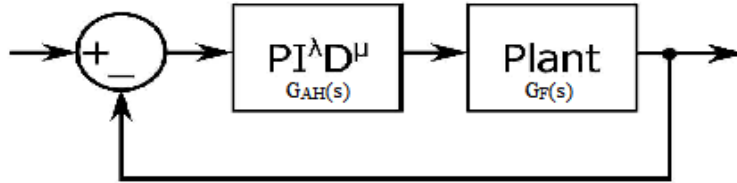
$$G_{AH}(s) = 53.0586 + \frac{0.109746}{s^{0.10596}} + 1910.28s^{0.010011} \quad (25)$$

and, the FOPID ( $PI^\lambda D^\mu$ ) model using the values obtained from and for the CHR-1 and CHR-2 methods are,

$$G_{CHR1}(s) = 11.005 + \frac{0.0231707}{s^{0.49282}} + 3779.4s^{0.015107} \quad (26)$$

$$G_{CHR2}(s) = 140.217 + \frac{0.967806}{s^{0.53323}} + 4266.11s^{0.19204} \quad (27)$$

Feeding the equation obtained from Astrom-Hagglund method in the closed loop shown in Fig.3 [21], then,

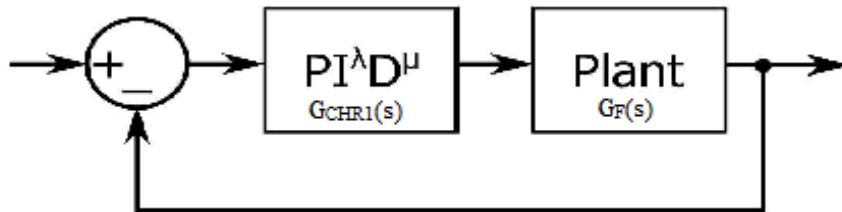


**Fig. 3** Closed loop with the plant  $G_f(s)$  and FOPID  $G_{AH}(s)$

The output obtained after solving the Fig.3 is, that is the value of  $G_{AHO}(s)$  is,

$$G_{AHO}(s) = \frac{1910.3s^{0.11597} + 53.059s^{0.10596} + 0.10975}{14494s^{1.416} + 6009.5s^{1.076} + 1910.3s^{0.11597} + 54.749s^{0.10596} + 0.10975} \quad (28)$$

Feeding the equation obtained from CHR-1 method in the closed loop shown in Fig.4, then,

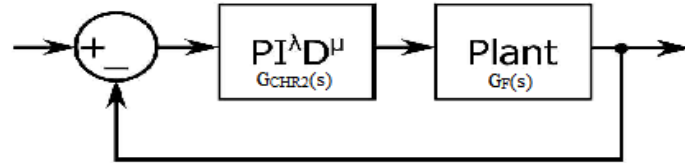


**Fig. 4** Closed loop with the plant  $G_f(s)$  and FOPID  $G_{CHR1}(s)$

The output obtained after solving the Fig.4 is, that is the value of  $G_{CHR1O}(s)$  is,

$$G_{CHR1O}(s) = \frac{3779.4s^{0.50793} + 111.01s^{0.49282} + 0.023171}{14994s^{1.8028} + 6009.5s^{1.4628} + 3779.4s^{0.50793} + 112.69s^{0.49282} + 0.023171} \quad (29)$$

Feeding the equation obtained from CHR-2 method in the closed loop shown in Fig.5, then,



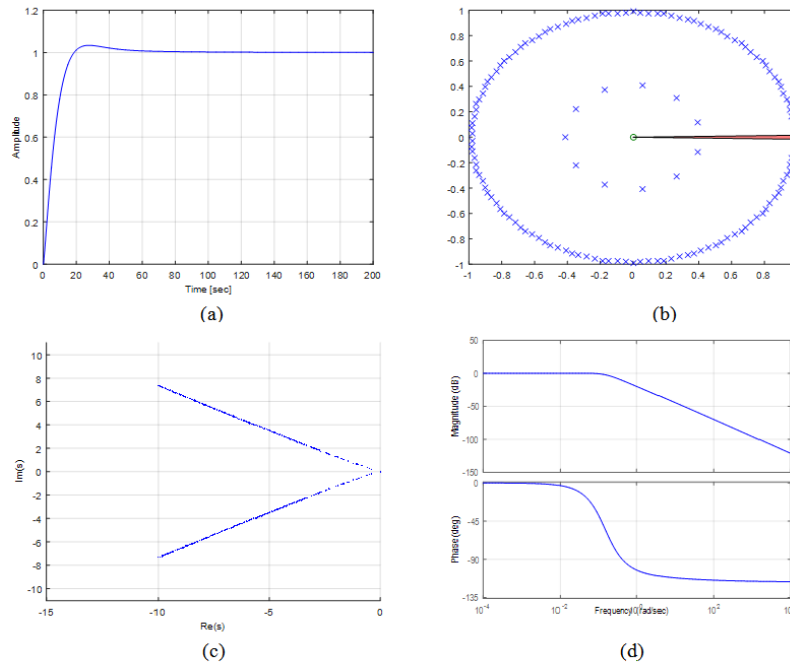
**Fig. 5** Closed loop with the plant  $G_f(s)$  and FOPID  $G_{CHR2}(s)$

The output obtained after solving the Fig.5 is, that is the value of  $G_{CHR2O}(s)$  is,

$$G_{CHR2O}(s) = \frac{4266.1s^{0.72527} + 140.22s^{0.53323} + 0.96781}{14994s^{1.8432} + 6009.5s^{1.5032} + 4266.1s^{0.72527} + 141.91s^{0.53323} + 0.96781} \quad (30)$$

## 10. RESULT

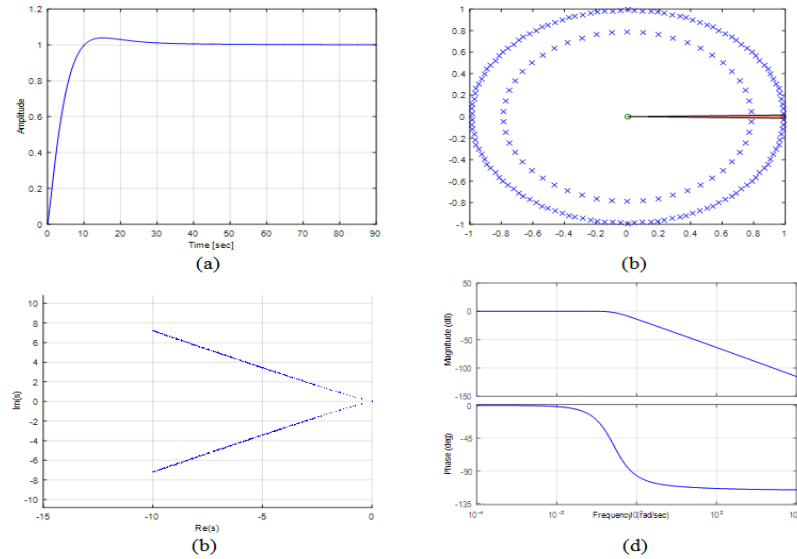
### 10.1 FOR ASTROM-HAGGLUND METHOD



**Fig. 6** Response plots of equation (28)

(a) Step response of equation (28), (b) Stability of the system which appears to be stable, (c) Root locus plot of equation (28), (d) Bode plot graph for equation (28)

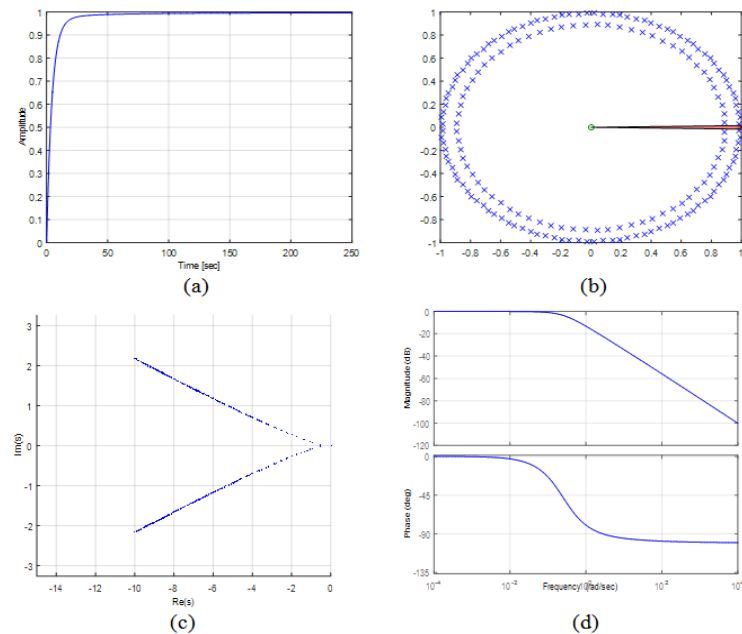
## 10.2 FOR CHIEN-HRONE-RESWICK-1 METHOD



**Fig. 7** Response plots of equation (29)

(a) Step response of equation (29), (b) Stability of the system which appears to be stable, (c) Root locus plot of equation (29), (d) Bode plot graph of equation (29)

## 10.3 FOR CHIEN-HRONE-RESWICK-2 METHOD



**Fig. 8** Response plots of equation (30)

(a) Step Response of equation (30) , (b) Stability plot of the system that appears to be stable, (c) Root locus plot of equation (30), (d) Bode plot graph of equation (30)

## **11. DISCUSSION**

The Integer Order Model transfer function of heating furnace exhibits very deprived response with a steady state error of more than 50%. Therefore this PID is designed based on the Fractional Order Model of Transfer function. The tuning methods used in above are AMIGO, CHR1 & CHR2. In all these three cases  $\lambda$  &  $\mu$  are optimized using Nelder-Mead Algorithm. When AMIGO method along with Nelder Mead optimization Algorithm was applied to FOM the final system developed to be stable with a revealed overshoot of merely 3%, where as the settling time also diminished drastically up to 95secs. When CHR1 along with Nelder Meid algorithm was exercised then also system yielded a low overshoot of 4%, and a very swift response with settling time less than 50secs. The system with FOPID tuned by CHR2 & Nelder Meid algorithm exhibited an excellent but slightly lethargic response with nil overshoot, zero steady state error & settling time of 250secs.

## **12. CONCLUSION**

Thus, we successfully designed the PID controllers for heating furnace with fractional elements, i.e. FOPID controllers using various new algorithms & Nelder Meid optimization. The plots of time response characteristics made it very clear that the integer order model of furnace gave a very terrible response by using traditional tuning methods also. From the results it is obvious that Ziegler-Nichols method yields a very high Overshoot where as Cohen-Coon method exhibits a very sluggish response with a very high settling time. The overshoot in furnace may create sudden high pressure and endanger the life of workers and properties. Similarly due to high settling time in Cohen Coon method the furnace takes a very long time to maintain the steady state hence resulting in a huge amount of heat loss. Therefore here the above mentioned techniques are used. By using the AMIGO method to FOM for tuning the PID along with Nelder Meid algorithm the overshoot reduced significantly with a quick response. Correspondingly the system exhibited a very low overshoot and also a comparatively low settling time when CHR1 was applied with Nelder Meid optimization. Then CHR2 method was applied which is slightly modified form of CHR1 with a newly added disturbance rejection algorithm. CHR2 along with Nelder Meid algorithm produced a disturbance less output without any steady state error, Overshoots & undershoots, although the settling time was high as compared to previous two methods. But with zero overshoot the controller designed with CHR2 technique had a enhanced settling time contrast to Controllers designed using traditional Ziegler Nicholas & Cohen Coon method. Thus it can be concluded that the controllers designed using fractional elements with the above mentioned efficient tuning techniques meliorated the performance of Heating Furnace as a result of providing almost zero standards of overshoot and rapid response.

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