Complexity of Constrained Switching for Switched Nonlinear Systems with Average Dwell Time: Novel Characterizations

Georgi Dimirovski [1,2], Jiqiang Wang [3], Hong Yue [4]
[1]Dogus University of Istanbul, Istanbul 34722, Turkey
[2]St. Cyril and Methodius University, Skopje 1000, Macedonia
Email: gdimirovski@dogus.edu.tr
[3]Jiangsu Province Key Laboratory of Aerospace Power Systems, Nanjing University of Aeronautics & Astronautics, Nanjing 210016, China
Email: jiqiang.wang@nuaa.edu.cn
[4]Department of Electronic & Electrical Engineering, University of Strathclyde, Glasgow, UK
Email: hong.yue@strath.ac.uk

Abstract: In so far developed theory of switched systems is largely based on assuming certain small but finite time interval termed average dwell time, which represents a constraint even when extremely small. Thus currently most of it appears characterized by some slow switching condition with average dwell time satisfying a certain lower bound. However, in cases of nonlinear systems, when the switching seizes to be slow there may well appear non-expected complexity phenomena of particularly different nature. A fast switching condition with average dwell time satisfying an upper bound is explored and established. Thus the theory is extended by shading new light on the underlying, switching caused, system complexities. A comparison analysis of these innovated characterizations via slightly different overview yielded new results on the transient behaviour of switched nonlinear systems, while preserving the system stability. The multiple-Lyapunov functions approach is the analysis framework.

Keywords: arbitrary switching; average dwell time; lower bound condition; miltiple Lyapunov functions; switched nonlinear systems; stability; upper bound condition.

I. INTRODUCTION

It is well known that the behaviour of hybrid systems, among which switched systems also belong, may have remarkably different system dynamics from either of their components [4, 13, 19, 24, 28-30]. For example, one switched system can be stable although all its components are unstable, also, some inappropriate switching signal may destabilize the overall switched system even though all of its components are stable [4, 5-7, 13, 31]. Naturally, the analytical studies via the approach relying on Lyapunov stability theory and its extensions [1, 3, 12, 13, 22] were instrumental in the build-up of switched systems theory and switching based control.

The issue of system stability is the crucial one and also in the case of switched systems a rather delicate one [7, 16, 17, 22, 23, 27], most of the existing literature if focused on the problem of stability under arbitrary switching [7, 15, 23]. In due times, many important results have been obtained during a few of the past decades since the pioneering contributions of A. S. Morse (1996, 1997); for instance, see [8-12, 14, 15, 20, 21, 24, 25, 30-32]. In order to guarantee stability under arbitrary switching, the common

Lyapunov function method plays a rather important role (if not central role because of its conservatism). This is because the existence of a common Lyapunov function implies the global uniform asymptotic stability of the switched system. The importance of common Lyapunov function has been further consolidated by a converse theorem, due to Molchanov and Pyatnitskiy [16], that asserted if the switched system is globally uniformly asymptotically stable (GUAS), then all the subsystems ought to have a common Lyapunov function.

In time and more recently, in particular, the approach exploiting multiple Lyapunov functions [2, 4, 7, 11, 25] and the associated dwell time [17, 18] or average dwell time [6] are recognized as another rather efficient tool in stability studies of switched systems [5, 8, 10, 20, 21, 24, 27, 31, 32]. The concept of average dwell time switching, which was introduced by Hespanha and Morse (1999) in [6], appeared more general than the standard dwell time switching for both stability analysis and related control design and synthesis problems; for instance, see [8, 10, 13, 15, 21, 23, 29]. It does imply that the number of switching actions in a finite interval is bounded from above while the average time between two consecutive switching actions is not less than a constant [6, 9, 28]. It is believed the multiple Lyapunov function approach per-se reduces the inherent conservatism of the common Lyapunov function approach. In fact, when confining to linear dynamic systems only, some well-known design procedures for explicit construction of multiple Lyapunov functions have been developed among which the S-procedure and the LMI [1, 3] and the hysteresis switching action [8, 12, 24] have been particularly fruitful as most of references in this paper on control synthesis design and the references therein clearly demonstrated. In multiple Lyapunov functions approaches, it is generally assumed that each Lyapunov-like function associated for each subsystem is increasing (with the first time-derivative decreasing) with time as time elapses. For the first time, Ye and co-authors (1998) in their stability theory for hybrid systems [29] have also studied allowing for a Lyapunov-like function to rise to a limited extent and have established rather interesting property of a class of such functions called weak Lyapunov. Much more recently this finding was exploited in [9-11] and elsewhere. Work by Ye and co-authors (1998) was the point of departure in this study, which is aimed at extending the existing theory that shades new light on the underlying switching caused system complexities.

This extension is considerably interesting since it is appealing intuitively it may well contribute to reduce further the conservatism of stability results which may not be amenable to the usual multiple Lyapunov functions let alone to common Lyapunov function approaches. This observation forms the foundation of the paper, which has yielded an insight on how far it can proceed along the idea on reducing stability conservatisms. It is shown that both slow switching and fast switching can be studied within this framework. Then the implications are further worked out in producing important results in the stability theory of switched nonlinear systems.

Notation: The notation used in this paper is fairly standard, which may well be inferred from: $\mathbb{R}^n$ represents the n-dimensional Euclidean space; C^2 denotes the space of twice continuously differentiable functions; C^1 once differentiable piece-wise function. In addition, a function, $k:[0,\infty]\rightarrow[0,\infty]$ is called class K_∞ if it is continuous, strictly increasing and unbounded with value $k(0)=0$.

II. PRELIMINARIES AND NOVEL COMPLEXITY INSIGHT

In order to place this study into perspective some concepts, notions, and specifications about controlled nonlinear dynamic systems, in general, and about controlled switched dynamic systems, in particular, are recalled first. It is well known that, in general terms, a controlled nonlinear dynamic system can be represented by the state transition and output measurement equations as follows:

$$\left.\begin{cases} \dot{x}(t) = f(x(t),u(t)), \;\; x(t_0) = x_0 \\ y(t) = h(x(t)). \end{cases}\right\} \text{for } \forall t \in [t_0, +\infty) . \tag{1}$$

For such a system to have sustained operability functions its state transition mechanism must have the property $f\ (0,0) = 0$ and output measuring mechanism must have $h(0) = 0$ on the grounds of basic natural laws. System's quantities denote: $x \in X^n$ the state space, $u \in U^r$ the input space, $y \in Y^m$ the output space; $f : \mathbb{R}^n \times \mathbb{R}^r \to \mathbb{R}^n$; and $h : \mathbb{R}^n \to \mathbb{R}^m$. Upon the synthesis design of a certain controlling infrastructure then $u = u(t; t_0, u_0)$ for $\forall t \in [t_0, \infty)$. Notice that Figure 1 which depicts a relevant illustration, which is related to this class of systems and the quoted notions. For, it depicts controlled general nonlinear systems in accordance to the fundamental laws of physics on rigid-body energy, matter and momentum of motion, i.e. evolution in time.

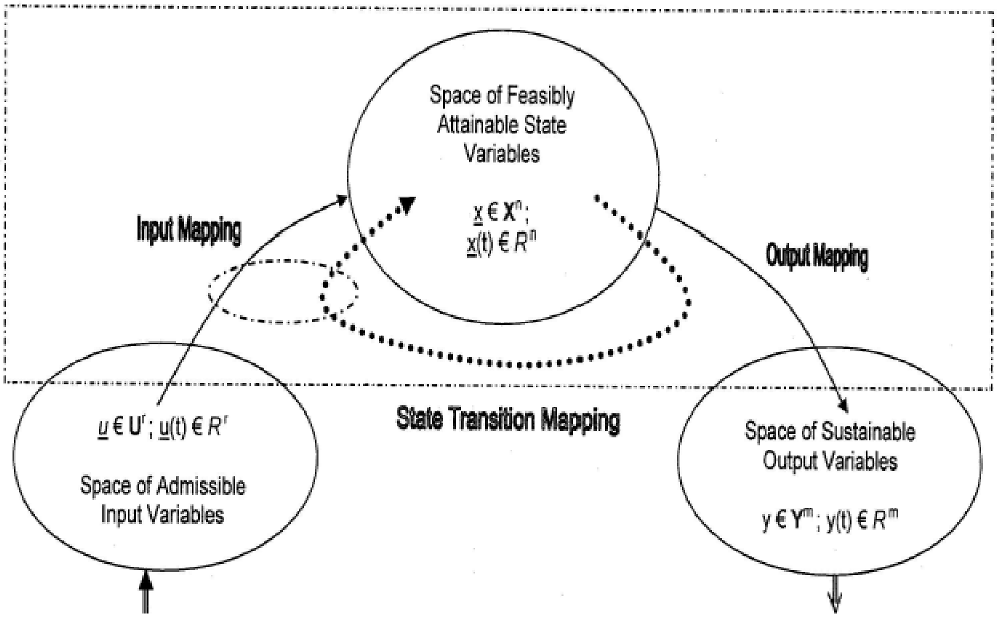

Fig. 1 An illustration of controlled general nonlinear systems in accordance to the fundamental laws of physics; although input, state and output spaces in terms of involved classes of functions can be mathematically defined by chosen measuring norm, at any fixed time instant all vector-valued variables become real-valued vectors that may be Euclidean ones.

Naturally, the system dynamics is captured by means of the state transition equation and the vector of outputs is merely an effect of the cause of the state transition which may be either the initial disturbance or the control input or both. If not control input is acting then the system is said to be in autonomous operating regime, the opposite being the non-autonomous or forced one. In this context, consequently, an acting external switching signal on an autonomous nonlinear system changes its operating regime.

Furthermore a controlled nonlinear dynamic system

$$\left.\begin{cases} \dot{x}(t) = f_\sigma(x(t),u(t)), \;\; x(t_0) = x_0 \\ y(t) = h_\sigma(x(t)). \end{cases}\right\} \text{for } \forall t \in [t_0, +\infty)\,, \qquad (2)$$

where subsystems $f_{\sigma(t)} = f_i, h_{\sigma(t)} = h_i$ with $i \in M$ in an index set M are fixed given models, and the control input $u = u(t)$ is also given (upon its synthesis) is called a switched system. An autonomous switched nonlinear dynamic system thus appears to be defined as follows:

$$\left.\begin{cases} \dot{x}(t) = f_\sigma(x(t)), \;\; x(t_0) = x_0 \\ y(t) = h_\sigma(x(t)). \end{cases}\right\} \text{for } \forall t \in [t_0, +\infty)\,. \qquad (3)$$

For a causal signal $\sigma : [t_0, +\infty),$ if it is

$$\sigma(t^+) = \Sigma([t_0,t], \sigma([t_0,t]), \{x([t_0,t]), y([t_0,t])\}), \forall t \in [t_0, +\infty) \qquad (4)$$

a piece-wise constant time-sequence function where $\sigma\left(t^+\right) = \lim_{\tau \downarrow t} \sigma(\tau)$ for $\tau \geq 0$ in continuous time and in discrete time $\sigma\left(t^+\right) = \sigma(t+1)$, is called switching signal.

A switching signal is said to be a switching path if it is defined as mapping of finite, semi-open time interval into the index set M such that $\sigma : [t_0, t_1) \to M$ for every $[t_0, t_1)$ with $t_0 < t_1 < +\infty$. A switching law is called a time-driven switching law if it depends only on time and its past value $\sigma\left(t^+\right) = \sum\left(t, \sigma(t)\right)$. A switching law is called a state-feedback switching law if it depends only on its past value an on the values of state variables at that time $\sigma\left(t^+\right) = \sum\left(\sigma(t), x(t)\right)$ for $\forall t \in [t_0, +\infty)$.

. A switching law is called a output-feedback switching law if it depends only on its past value an on the values of output variables at that time $\sigma\left(t^+\right) = \sum\left(\sigma(t), y(t)\right)$ for $\forall t \in [t_0, +\infty)$. At present, no other concepts and notions about feasible switching signals matter.

The concept of average dwell time is given below and the respective rather important result is cited too.

Definition (Hespanha & Morse, 1999): For a switching signal σ and any $t_2 > t_1 > t_0$, let $N_\sigma(t_1, t_2)$ be the number of switching over the interval $[t_1, t_2)$. If the condition $N_\sigma(t_1, t_2) \leq N_0 + (t_2 - t_1)/\tau_a$ holds for $N_0 \geq 1$, $\tau_a > 0$, then N_0 and τ_a are called the average dwell time (ADT) and the chatter bound, respectively.

Theorem 1 (Hespanha & Morse, 1999): Consider the switched system (3), and let α and μ be given constants. Suppose that there exist smooth functions $V_{\sigma(t)} : \Re^N \to \Re$, $\sigma(t) \in \ell$, and two

K_∞ functions k_1 and k_2 such that for each $\sigma(t)=i$, the following conditions hold:

$k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|)$, $\dot{V}_i(x_t) \le -\alpha V_i(x_t)$, and for any $(i,j)\in \ell \times \ell$, $i \ne j$, $V_i(x_t) \le \mu V_j(x_t)$; then the system is globally uniformly asymptotically stable for any switching signal with ADT $\tau_a > \tau_a^* = \dfrac{\ln \mu}{\alpha}$.

Theorem 1 considers multiple Lyapunov functions with "jump" on switching boundary. An extension due to (Ye et al, 1998) in [29] and further exposed by Zhang and Gao (2010) in [31] allows the Lyapunov-like function to rise to a limited extent, in addition to the jump on switching boundary. This is the so-called week Lyapunov functions, and it allows both the jump on the switching boundary and the increase over any interval. Now consider $\sigma(t)=i$ and within the interval $[t_i,t_{i+1})$, denote the unions of scattered subintervals during which the week Lyapunov function is increasing and decreasing by $T_r(t_i,t_{i+1})$ and $T_d(t_i,t_{i+1})$, respectively. Hence $[t_i,t_{i+1}) = T_r(t_i,t_{i+1}) \bigcup T_d(t_i,t_{i+1})$. Further use $T_r(t_{i+1}-t_i)$ and $T_d(t_{i+1}-t_i)$ to represent the length of $T_r(t_i,t_{i+1})$ and $T_d(t_i,t_{i+1})$ correspondingly. Then the following important result can be obtained:

Then the following important result has been derived:

Theorem 2 (Ye et al, 1998; Zhang & Gao, 2010): Consider the switched system $\dot{x}_t = f_\sigma(x_t)$, and let $\alpha>0$, $\beta>0$ and $\mu>1$ are prescribed constants. If there exist smooth functions $V_{\sigma(t)}:\Re^n \to \Re$ and two K_∞ functions k_1 and k_2 such that for each $\sigma(t)=i$, the following conditions hold:

$$k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|)$$

$$\dot{V}_i(x_t) \le \begin{cases} -\alpha V_i(x_t) & over \quad t \in T_d(t_i,t_{i+1}) \\ \beta V_i(x_t) & over \quad t \in T_r(t_i,t_{i+1}) \end{cases}$$

$$V_i(x_t) \le \mu V_j(x_t) \qquad \forall\left(\sigma(t)=i \quad \& \quad \sigma(t^-)=j\right)$$

then the system is GUAS for any switching signal with ADT

$$\tau_a > \tau_a^s = \frac{(\alpha+\beta)T_{\max} + \ln^{\mu}}{\alpha}, \quad T_{\max} = \max T_r(t_{i-1},t_i), \qquad \forall i$$

It may well be seen that the result above actually includes Theorem 1 as a special case. Namely, $\beta = 0$ implies no increase over the interval and hence $T_{\max} = 0$, then the ADT condition reduces to the ADT condition $\tau_a > \tau_a^* = \dfrac{\ln \mu}{\alpha}$, (5) in Theorem 1. It is this generality of the weak Lyapunov functions that has given incentives to explore the alternative of deriving a fast switching rule, which is discussed in the sequel.

Remark 1: Note that nothing is asserted about the CHB. In fact, the of chattering had thus become an issue of separate investigation.

III. RESULTS BASED ON THE NOVEL COMPLEXITY INSIGHT

It should be noted, Theorem 2 is a slow switching result in the sense that it is characterized by a lower bound on ADT. A would be fast switching result ought to provide for an upper bound on the average dwell time.

Proposition 1 (Theorem 3): Consider the switched system $\dot{x}_t = f_\sigma(x_t)$, and let $\alpha > 0, \beta > 0$ and $\mu > 1$ are prescribed constants. If there exist smooth functions $V_{\sigma(t)} : \Re^n \to \Re$ and two K_∞ functions k_1 and k_2 such that for each $\sigma(t) = i$, the following conditions hold:

$$k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|)$$

$$\dot{V}_i(x_t) \le \begin{cases} -\alpha V_i(x_t) & over \quad t \in T_d(t_i, t_{i+1}) \\ \beta V_i(x_t) & over \quad t \in T_r(t_i, t_{i+1}) \end{cases}$$

$$V_i(x_t) \le \mu V_j(x_t) \qquad \forall\left(\sigma(t) = i \quad \& \quad \sigma(t^-) = j\right)$$

then the system is GUAS for any switching signal with ADT

$$\tau_a < \tau_a^f = \frac{(\alpha+\beta)T_{\min} - \ln^\mu}{\beta}, \quad T_{\min} = \min T_d(t_{i-1}, t_i), \qquad \forall i$$

***A Proof Sketch*:** For $t \in [t_i, t_{i+1})$, we have:

$$\begin{aligned} V_i(x_t) &\le e^{-\alpha T_d(t_i,t) + \beta T_r(t_i,t)} V_i(x_{t^-}) \\ &\le e^{-\alpha T_d(t_i,t) - \beta T_d(t_i,t) + \beta T_d(t_i,t) + \beta T_r(t_i,t)} V_i(x_{t^-}) \\ &\le e^{\beta(t-t_i)} e^{-(\alpha+\beta)T_d(t_i,t)} V_i(x_{t^-}) \\ &\le e^{\beta(t-t_i)} e^{-(\alpha+\beta)T_{\min}} \mu V_{i-1}(x_{t^+}) \\ &\le e^{\beta(t-t_0)} \left(e^{-(\alpha+\beta)T_{\min}}\right)^{N_\sigma(t_0,t)} \mu^{N_\sigma(t_0,t)} V_0(x_{t_0}) \\ &\le e^{\left\{N_\sigma(t_0,t)\left[\ln^\mu - (\alpha+\beta)T_{\min}\right] + \beta(t-t_0)\right\}} V_0(x_{t_0}) \end{aligned}$$

where the definition $T_{\min} \equiv \min\limits_{l} T_d(t_i - t_{i-1})$ is made, that is the minimum decreasing interval over any switching sequence.

It is therefore that if $N_\sigma(t_0,t)\left[\ln^{\mu} - (\alpha+\beta)T_{\min}\right] + \beta(t-t_0) < 0$, then $V_i(x_t)$ will be decreasing and the system will achieve GUAS. Now the condition $N_\sigma(t_0,t)\left[\ln^{\mu} - (\alpha+\beta)T_{\min}\right] + \beta(t-t_0) < 0$ is exactly the average dwell time defined by $\tau_\alpha \equiv \dfrac{t-t_0}{N_\sigma(t_0,t)}$. This completes the proof. □

It appeared that Theorem 3 is of considerable interest since it can be combined with Theorem 2 in order to prove a another theorem stating that the average dwell time condition is "if and only if ", which is further progress in the stability theory of switched nonlinear systems. Basically, the idea comes from the following rather simple observation: A system will achieve arbitrary switching induced stability if the upper bound for fast switching is larger than the lower bound for slow switching. Yet before this conclusive result is being discussed as appropriate, it is necessary first work out the implications of the fast and the slow switching conditions. Then the proof may well become self-evident. There were found three cases to be closely examined, namely: when α and β remain free positive real-valued while μ=1 is fixed; when α remains free positive real-valued but β = 0 and μ =1 are fixed; and when α = 0 is fixed while μ and β remain free positive real-valued. Obviously, the word is about exploring Theorem 2 on the respective boundary edges of the assumed constants α , β and μ .

Case 1: $\mu = 1$

Then the fast and slow switching conditions reduce to:

$$\tau_a > \tau_a^s = \frac{(\alpha+\beta)T_{\max}}{\alpha}, \quad T_{\max} = \max T_r(t_{i-1}, t_i), \qquad \forall i$$

and

$$\tau_a < \tau_a^f = \frac{(\alpha+\beta)T_{\min}}{\beta}, \quad T_{\min} = \min T_d(t_i - t_{i-1}), \qquad \forall i \in \ell$$

Hence if the value of the fast switching is greater than that of the slow switching, that is: $\tau_a^s \le \tau_a^f$, or: $\dfrac{T_{\max}}{T_{\min}} \le \dfrac{\alpha}{\beta}$, then the system will be GUAS under any switching signals. The situation here is that the minimum decreasing should be larger than the maximum increasing of the energy function over any interval. In the extreme case, that is $\beta = 0$, which also implies $T_{\max} = 0$, then the slow switching becomes $\tau_a^s = 0$ while the fast switching is $\tau_a^f = \infty$. The system is GUAS under arbitrary switching! This actually becomes the case of usual multiple Lyapunov functions with exact matching on the switching boundary. To summarize, we have the following result:

Proposition 2 (Theorem 4): Consider the switched system $\dot{x}_t = f_\sigma(x_t)$, and let $\alpha > 0, \beta > 0$ are prescribed constants. If there exist smooth functions $V_{\sigma(t)} : \Re^n \to \Re$ and two K_∞ functions k_1 and k_2 such that for each $\sigma(t) = i$, the following conditions hold:

$$k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|)$$

$$\dot{V}_i(x_t) \le \begin{cases} -\alpha V_i(x_t) & over \quad t \in T_d(t_i, t_{i+1}) \\ \beta V_i(x_t) & over \quad t \in T_r(t_i, t_{i+1}) \end{cases}$$

$$V_i(x_t) = V_j(x_t) \qquad \forall\left(\sigma(t) = i \quad \& \quad \sigma(t^-) = j\right)$$

then the system is GUAS for arbitrary switching signal if the following condition are fulfilled:

$\dfrac{T_{\max}}{T_{\min}} \le \dfrac{\alpha}{\beta}$, where $T_{\max} \equiv \max T_r(t_{i-1}, t_i)$ and $T_{\min} \equiv \min T_d(t_{i-1}, t_i), \quad \forall i$.

***A Proof Sketch*:** The correctness of this result is, in fact, proved by collecting all the above arguments in discussing both the previous and current results.

Case 2: $\beta = 0$

The case $\beta = 0$ means no increasing over any interval and hence $T_{\max} = 0$. Then the slow and fast switching conditions become:

$$\tau_a > \tau_a^s = \frac{\ln^\mu}{\alpha}$$

and

$$\tau_a^f = \infty \,\&\, T_{\min} \ge \frac{\ln^\mu}{\alpha}, \quad T_{\min} = \min T_d(t_i - t_{i-1}), \qquad \forall i \in \ell$$

That is, the slow switching condition reduces to the ADT condition by Hespanha and Morse (1999) in [6], while the fast switching rule implies that the minimum dwell time required for arbitrary switching stability with ADT is exactly the limit $\dfrac{\ln^\mu}{\alpha}$ for the average dwell time.

This simple observation leads to the following important result:

Proposition 3 (Theorem 5): Consider the switched system $\dot{x}_t = f_\sigma(x_t)$ and let $\alpha > 0$, $\mu > 1$ be given constants. Suppose that there exist C^1 functions $V_{\sigma(t)} : \Re^N \to \Re$, $\sigma(t) \in \ell$, and two K_∞ functions k_1 and k_2 such that $\forall \sigma(t) = i$, $k_1(|x_t|) \le V_i(x_t) \le k_2(|x_t|)$, $\dot{V}_i(x_t) \le -\alpha V_i(x_t)$, and $\forall (i, j) \in \ell \times \ell$, $i \ne j$, $V_i(x_t) \le \mu V_j(x_t)$; then the system is GUAS for any switching signal ***if and only if*** the ADT satisfies the condition $\tau_a > \tau_a^* = \dfrac{\ln \mu}{\alpha}$.

A Proof Sketch: The sufficiency part has already been shown in Theorem 1 and we show the necessity part below. We do this by first considering the fast switching rule in theorem 3. Then the above discussion has shown that the ***minimum dwell time*** among all the switching sequences is exactly $\tau_a^* = \dfrac{\ln \mu}{\alpha}$. This implies that the ADT condition $\tau_a > \tau_a^*$ is in fact tight. That is, to guarantee the system to be GUAS for any switching signal, the ADT has to satisfy $\tau_a > \tau_a^*$, thus the necessity of the ADT condition is proved.

To explain further this conclusive result without referring to the special cases as discussed above, let proceed by means of another look-up angle onto the discussed problem task.

Consider $\beta = 0$, that is, over any interval $[t_i, t_{i+1})$ the Lyapunov-like function $V_i(x_t)$ is non-increasing, then Theorem 3 tells that the only requirement for fast switching stability is the nominator $(\alpha + \beta)T_{\min} - \ln^{\mu} \geq 0$, that is: $T_{\min} \geq \dfrac{\ln^{\mu}}{\alpha}$. It can then deduce from the definition $T_{\min} \equiv \min T_d(t_i - t_{i-1}), \forall i \in \ell$ that the minimum decreasing duration over any interval should satisfy $T_{\min} \geq \tau^* = \dfrac{\ln^{\mu}}{\alpha}$. This is equivalently to say that the minimum dwell time over any switching sequence should be $\tau^* = \dfrac{\ln^{\mu}}{\alpha}$ at least.

Now the case $\beta = 0$ is exactly the Lyapunov-like functions defined in Theorem 5. To recap, the sufficiency part says that the system is GUAS for any switching signal ***if*** the ADT satisfies the condition $\tau_a > \tau_a^* = \dfrac{\ln \mu}{\alpha}$; the analysis here shows that to guarantee GUAS, the minimum dwell time over any switching sequence should be at least $\tau^* = \dfrac{\ln^{\mu}}{\alpha}$. That is the estimation $\tau_a > \tau_a^* = \dfrac{\ln \mu}{\alpha}$ is actually tight, demonstrating the necessity of the ADT condition. □

Case 3: $\alpha = 0$

The case $\alpha = 0$ implies no decreasing over any interval, hence $T_{\min} = 0$. The two switching inequalities now reduce to $\tau_a > \infty$ and $\tau_a < 0$, which is obviously trivial. Hence no switching sequence exists to stabilize the system at all, conforming with the intuitive result.

IV. CONCLUDING REMARKS

This paper has contributed a novel characterization of nonlinear switched systems via adopting constrained switching through slow switching and fast switching. A fast switching rule to guarantee GUAS has been derived and the condition for arbitrary switching stability of switched nonlinear system has also been obtained. Then this note has proved that the usual ADT condition associated with multiple Lyapunov functions, in fact, appears to be an if and only if condition. These three results extend the existing stability theory of switched nonlinear systems.

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