

## ROBUST CONTROL OF ROOM TEMPERATURE AND RELATIVE HUMIDITY USING ADVANCED NONLINEAR INVERSE DYNAMICS AND EVOLUTIONARY OPTIMISATION

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**Abstract.** A robust controller is developed, using advanced nonlinear inverse dynamics (NID) controller design and genetic algorithm optimisation, for room temperature control. The performance is evaluated through application to a single zone dynamic building model. The proposed controller produces superior performance when compared to the NID controller optimised with a simple optimisation algorithm, and classical PID control commonly used in the buildings industry. An improved level of thermal comfort is achieved, due to fast and accurate tracking of the setpoints, and energy consumption is shown to be reduced, which in turn means carbon emissions are reduced.

**Key words:** Temperature Control, Relative Humidity, MIMO, HVAC, BEMS, Genetic Algorithm, Inverse Dynamics, Robust Control

### NOMENCLATURE

$\dot{Q}_R$	=	Heat Transfer through Roof (W)
$\dot{Q}_W$	=	Heat Transfer through Windows (W)
$\dot{Q}_F$	=	Heat Transfer through Floor (W)
$\dot{Q}_{free}$	=	Heat Transfer from Free Heats (W)
$\dot{Q}_{si}$	=	Heat Transfer through internal structure (W)
$\dot{Q}_{se}$	=	Heat Transfer through external structure (W)
$\dot{Q}_{ft}$	=	Heat Transfer from furniture (W)
$T_o$	=	Outside Temperature (K)
$U_{ft}$	=	Furniture Heat Transfer Coefficient (W/m <sup>2</sup> K)
$A_s$	=	Area (structure) (m <sup>2</sup> )
$A_{ft}$	=	Area (furniture) (m <sup>2</sup> )
$M_a$	=	Mass (air) (Kg)
$M_{si}$	=	Mass (internal structure) (Kg)

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$M_{se}$	=	Mass (external structure) (Kg)
$M_{ft}$	=	Mass (furniture) (Kg)
$C_a$	=	Specific Heat Capacity (air) (J/KgK)
$C_s$	=	Specific Heat Capacity (structure) (J/KgK)
$C_{ft}$	=	Specific Heat Capacity (furniture) (J/KgK)
$\dot{m}_c$	=	Mass flow rate (mechanical ventilation) (Kg/s)
$\dot{m}_{nv}$	=	Mass flow rate (natural ventilation) (Kg/s)
$K_{si}$	=	Thermal Conductivity (internal structure) (W/mK)
$Th_{wall}$	=	Wall Thickness (m)
$\rho_a$	=	Density (air) (Kg/m <sup>3</sup> )
$n_{occ}$	=	Number of occupants
$P_{occ}$	=	Evaporation rate of occupants (Kg/h)

## 1 INTRODUCTION

In recent years the commitment to reducing carbon emissions has led to much interest in the development of energy efficient buildings designed with a climate adaptive philosophy. These buildings incorporate sophisticated designs, materials as well as advanced Heating, Ventilation and Air Conditioning (HVAC) systems. The dynamic and uncertain nature of buildings means that designing an effective Building Energy Management System (BEMS) to control these systems is by no means a trivial task. The control methods currently in use in the buildings industry are restricted in their design to Proportional-Integral-Derivative (PID) control, as in many other industrial applications. This strategy is commonly used in industry on account of its simplicity and ease of commissioning. Many variants of PID control have been applied to HVAC systems<sup>1,2</sup>. Generally an accurate model of the plant is required in the tuning process for a PID controller. HVAC systems however, are typically nonlinear time-variable multivariable systems which are subject to many disturbances and uncertainties. Consequently, obtaining an accurate model which is representative of the plant over a wide operating range is difficult<sup>3</sup>. The tuning process for traditional PID designs can be difficult, time consuming and consequently be an expensive process particularly if re-tuning is required, as is often the case in large HVAC systems<sup>2</sup>. Poorly tuned control systems can lead to poor energy management and consequently increased carbon emissions. They also result in poor thermal comfort and can even damage actuation systems. Many advanced self-tuning PID controllers have been proposed in attempts to alleviate the problems associated with tuning PID controllers<sup>4,5</sup>. These methods however, tend to require model identification as an initial step and model parameter identification in real time mode. Hence the methods are limited due to the difficulty involved in accurately identifying such a complex process which is subjected to disturbances<sup>3,6</sup>.

Some nonlinear controller designs have been developed for HVAC systems<sup>7-9</sup>. Serrano and Reyes<sup>7</sup> have shown that the nonlinear disturbance rejection controller is more effective at maintaining good thermal comfort levels owing to its ability to diminish the effects of thermal disturbances on the system.

Advanced Non-linear Inverse Dynamics (NID) control methods, typically used in the aerospace and automotive industries, tend to have robust designs meaning they can provide high performance control under uncertain or even adverse conditions<sup>10,11</sup>. However, these control systems generally require full knowledge of the system's physics and thus there still remains this model dependency which can significantly affect the performance of the control system. When attempting to control room

temperature with HVAC systems, the sensor placement in particular has been shown to have a major effect on the performance of the control system<sup>12,13</sup>.

This paper sets forth the development of a robust and high performance controller for room temperature control of a single zone with heating through mechanical ventilation. A state of the art NID control method using Robust Inverse Dynamics Estimation (RIDE)<sup>14</sup>, which has been successful in producing robust high performance control, is used as the foundation for the controller design described in this paper. A constrained optimisation scheme using a Genetic Algorithm (GA) is employed in order to further improve the robustness characteristics of the controller by finding a set of optimal nominal gains over a range of uncertainty. The NID-GA optimal control approach has the ability to achieve fast and accurate tracking without performance degradation over a range of parameter uncertainty.

## 2 BUILDING MODEL

The Building model used for controller analysis in this research is based on the dynamic model developed in<sup>15,16</sup>. The zone model consists of four state variables for temperature and two state variables for humidity. These are: zone air temperature ( $T_a$ ), internal wall structure temperature ( $T_{si}$ ), external wall structure temperature ( $T_{se}$ ), furniture temperature ( $T_{ft}$ ), zone humidity ( $W_a$ ) and relative humidity ( $W_{rel}$ ). The zone air is assumed to be fully mixed meaning the temperature distribution across the zone is uniform. The air density is also assumed to be constant and unaffected by changes in temperature and humidity of the zone. The differential equations that govern the zone temperature and humidity<sup>9</sup> are as follows:

$$M_a C_a \frac{dT_a}{dt} = \dot{Q}_H + \dot{Q}_{free} - \dot{Q}_{si} - \dot{Q}_F - \dot{Q}_R - \dot{Q}_W - \dot{m}_c C_a (T_a - T_o) - \dot{m}_{nv} C_a (T_a - T_o) - \dot{Q}_{ft} \quad (1)$$

$$M_{si} C_s \frac{dT_{si}}{dt} = \dot{Q}_{si} - \frac{K_{si}}{Th_{wall}} A_s (T_{si} - T_{se}) \quad (2)$$

$$M_{se} C_s \frac{dT_{se}}{dt} = \frac{K_{si}}{Th_{wall}} A_s (T_{si} - T_{se}) - \dot{Q}_{se} \quad (3)$$

$$M_{ft} C_{ft} \frac{dT_{ft}}{dt} = U_{ft} A_{ft} (T_a - T_{ft}) \quad (4)$$

$$\frac{M_a}{\rho_a} \frac{dW_a}{dt} = \frac{\dot{m}_c}{\rho_a} (W_s - W_a) + \frac{(n_{occ} P_{occ})}{\rho_a} - \frac{\dot{m}_{nv}}{\rho_a} (W_a - W_o) \quad (5)$$

$$\frac{dW_{rel}}{dt} = 5000.0 \dot{W}_a - 1.388 \dot{T}_a \quad (6)$$

When the control system is applied, the comfort temperature ( $T_c$ ) is tracked which is a combination of the air, internal structure and furniture temperatures. The comfort temperature is defined as follows:

$$T_c = 0.33T_a + 0.33T_{si} + 0.33T_{ft} \quad (7)$$

The actuation system model used to control the temperature and humidity of the zone in this case is a direct acting heater and mechanical ventilation. The dynamics of the

heater are characterised by a nonlinear first order transfer function which has a maximum heat output of 10kW. Mechanical ventilation is provided using a fan model which is also characterised by a nonlinear first order transfer function which can provide a maximum mass flow rate of 0.35kg/s.

### 3 CONTROLLER DESIGN

#### 3.1 Proportional and Integral Control

The PI controller is very commonly used in building control systems as well as many other industrial applications due to its simplistic design. For this reason, a PI controller tuned with a Nelder-Mead Simplex optimisation algorithm<sup>17</sup> is used in this paper as a representation of current best practice in industry. This serves as a reasonable benchmark against which the advanced control method presented in this research can be compared.

The PI control law is as follows:

$$uc(t) = K_p e(t) + K_I \int e(t) \quad (8)$$

The proportional and integral gains,  $K_p$  and  $K_I$  respectively, can be tuned in order to attain the best performance according to the design specifications of the system. The objective function for optimisation is taken as the root mean square of the error between the setpoint and the system response. Since there are two outputs i.e. two channels, the error is taken as the sum of the error on both channels. The objective function calculation is shown below:

$$obj = \sqrt{\frac{(E_1 + E_2 \dots + E_n)}{n}} \quad (9)$$

Where E is the sum of the error on both channels and n is length of the error vector.

#### 3.2 RIDE Control

The RIDE controller design has proven to be highly effective when applied to nonlinear systems<sup>18</sup>. An overview of the algorithm is given in this section in order to clarify the tuning problem. The algorithm is described in greater detail in<sup>14</sup>. The buildings differential equations can be represented in generalised state space format as shown in (10):

$$\dot{\vec{x}}(t) = A\vec{x}(t) + B\vec{u}(t) \quad (10)$$

$$\vec{y}(t) = C\vec{x}(t)$$

The RIDE control law is given by:

$$\vec{u}(t) = \vec{r} - K_p \vec{y}(t) + \hat{\vec{u}}_{eq}(t) \quad (11)$$

$$\dot{\vec{r}} = K_I \vec{e}(t) \quad (12)$$

$$\hat{\vec{u}}_{eq}(t) = -[CB]^{-1} \dot{\vec{y}}(t) + \vec{u}(t) \quad (13)$$

Where  $K_P$  and  $K_I$  are the proportional and integrals gains (which require tuning) respectively. The  $\widehat{u}_{eq}$  term (13) is an estimate of the equivalent control which is required to set rate of change of the output to zero. The equivalent control estimate uses dynamic inverse to diminish disturbances, cross-coupling and nonlinear plant dynamics. A diagram of the RIDE controller structure is shown in Fig.1

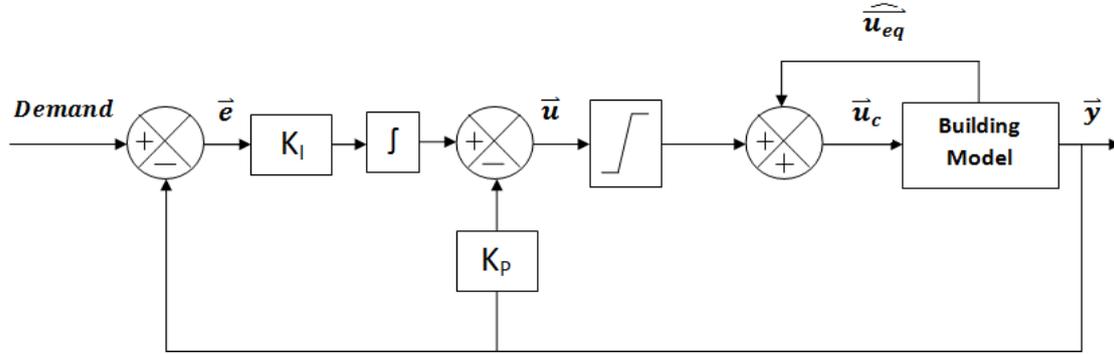


Figure 1: RIDE Controller Structure

The closed loop transfer function of the plant and control system is given by

$$[G(s)] = [s^2 I_m + s(K_P CB) + K_I CB]^{-1} K_I CB \quad (14)$$

Where  $I_m$  is an identity matrix. The proportional and integral gains can be selected such that they are expressed as follows:

$$K_P = [CB]^{-1} 2Z_d \Omega_n \quad (15)$$

$$K_I = [CB]^{-1} \Omega_n^2 \quad (16)$$

Where  $Z_d$  and  $\Omega_n$  are the designed system damping ratio and natural frequency respectively. By setting  $K_P$  and  $K_I$  as in (15) and (16), the system transfer function can be expressed as a diagonal matrix of second order transfer functions in generalised form as shown below:

$$[G(s)] = \begin{bmatrix} \frac{\Omega_n^2}{s^2 + 2Z_d \Omega_n s + \Omega_n^2} & 0 & \dots & 0 \\ 0 & \frac{\Omega_n^2}{s^2 + 2Z_d \Omega_n s + \Omega_n^2} & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \dots & 0 & \frac{\Omega_n^2}{s^2 + 2Z_d \Omega_n s + \Omega_n^2} \end{bmatrix} \quad (17)$$

The diagonal matrix  $[G(s)]$  has the dimensions  $m \times m$ , where  $m$  is the number of system inputs. The system response can be shaped by tuning  $K_P$  and  $K_I$  through  $Z_d$  and  $\Omega_n$ .

### 3.3 Genetic Algorithm Optimisation

The Genetic Algorithm is an evolutionary optimisation method based on Darwin's theory of evolution. The GA process is illustrated in the flowchart shown in Fig.2. A detailed explanation of Genetic Algorithms can be found in<sup>19</sup>.

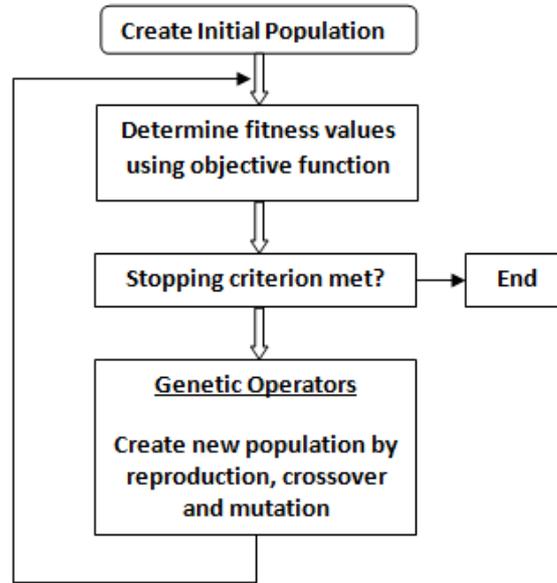


Figure 2: GA Process<sup>20</sup>

**Genetic Operations:-** These operations determine which individuals constitute the subsequent population. There are four operators used in the GA for this application; Elite Children, Selection, Crossover and Mutation. The settings for the GA used in the auto-tuning process are given in Table 1.

Parameter	Value
Population size	10
Elite count	3
Crossover fraction	0.7
Mutation	0.15
Selection method	roulette

Table 1: GA Parameters

#### 3.3.1 Objective Function

Due to the large parameter uncertainty in buildings, problems may arise when implementing building control systems in practice. The proposed method in this research attempts to alleviate the problems associated with parameter uncertainty by tuning the controller parameters over a range of uncertainty. This makes the controller more robust against discrepancies between the building model and the real building. This is achieved through the design of the objective function for optimisation. For the case presented in this paper, the controller is optimised for a range of uncertainty in the heat transfer coefficient of the furniture ( $U_{ft}$ ) only. The principle however, can be extended so as to include a number of other parameters. It is considered that  $U_{ft}$  can vary by  $\pm 60\%$ . In order to optimise the controller parameters over this range, the objective function was designed such that it calculates the root mean square of the error between the setpoint and the system response over three simulations: one at the normal operating condition ( $U_{ft} = 2.0\text{W/m}^2\text{K}$ ) and two others at the extremes of the uncertainty

range ( $U_{ft} = 0.8\text{W/m}^2\text{K}$  and  $U_{ft} = 3.2\text{W/m}^2\text{K}$ ).

#### 4 RESULTS

The control systems discussed above were all applied to the building model and simulated over a three month winter/spring period with weather data from January to March. Their performance was evaluated over three different operating conditions across the range of uncertainty in the heat transfer coefficient of the furniture. This was done in order to demonstrate each controller's ability to cope with parameter uncertainty which is very common in buildings. The different operating conditions were:  $U_{ft} = 0.8\text{W/m}^2\text{K}$ ,  $U_{ft} = 2.0\text{W/m}^2\text{K}$  and  $U_{ft} = 3.2\text{W/m}^2\text{K}$ , where  $U_{ft} = 2.0\text{W/m}^2\text{K}$  is the 'normal' operating condition.

The PI controller was tuned using the aforementioned Nelder-Mead Simplex optimisation algorithm so as to provide a representation of the control systems currently in use in the buildings industry. The controller was tuned at the lower setting for the furniture heat transfer coefficient as achieving good control at this condition was found to be the most difficult. Simulation results for the RIDE controller tuned using both the simplex algorithm and the GA are also presented in order to provide a direct comparison of the efficacy of both methods. The objective function described in section 3.3.1 was used for both tuning algorithms when tuning the RIDE controller. The tuning results for all three controller setups are detailed below in Table 2 and Table 3.

<b>Tuning algorithm</b>	Simplex
<b>Time taken</b>	56m 44s
<b><math>K_P</math> (Temperature)</b>	64.037
<b><math>K_I</math> (Temperature)</b>	0.598
<b><math>K_P</math> (Humidity)</b>	64.2
<b><math>K_I</math> (Humidity)</b>	$1.238 \times 10^{-5}$

Table 2: PI Auto-Tuning Results

<b>Tuning algorithm</b>	Simplex	GA
<b>Time taken</b>	2h 14m 04s	42m 24s
<b><math>\zeta</math></b>	0.7315	0.81
<b><math>\omega</math></b>	0.000301	0.00062

Table 3: RIDE Auto-Tuning Results

From the tuning results above, it is clear that the GA is more efficient than the simplex as the time taken for it to auto-tune the RIDE controller was much shorter than the simplex algorithm. Fig.3 shows plots of the comfort temperature and external temperature over four days at normal operating conditions. It can be seen that the PI controller does not track the comfort temperature setpoint ( $21^\circ\text{C}$ ) accurately, with large overshoot occurring. The PI controller can be seen to push the heater on the limit and cause integrator wind up which results in the large overshoot (approx.  $6^\circ\text{C}$ ). The GA tuned RIDE controller achieves a quick response as well as accurate tracking of the setpoint. It can be seen that when the heater reaches its limit, no integrator wind up occurs as no overshoot can be seen. The simplex tuned RIDE controller can also be seen to produce an acceptable response. It is evident however, that the simplex tuning algorithm resulted in sub optimal gains for the controller as the response is much slower to reach the setpoint.

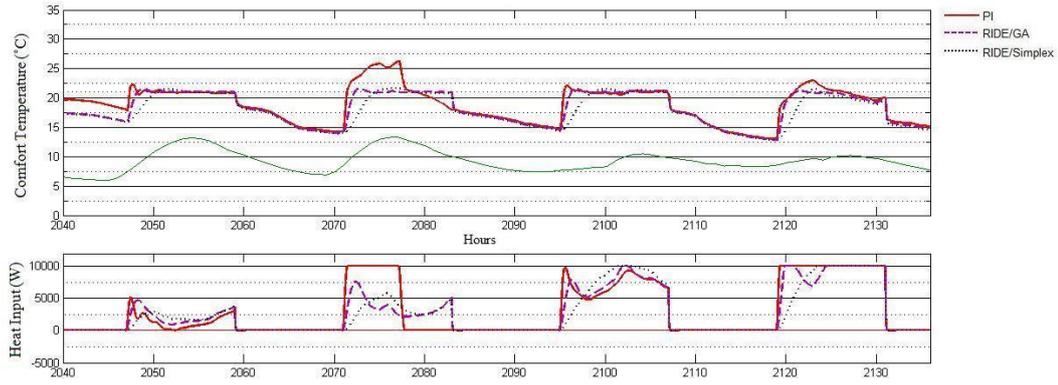


Figure 3: Comfort temperature and heat input ( $U_{ft} = 2.0W/m^2K$ )

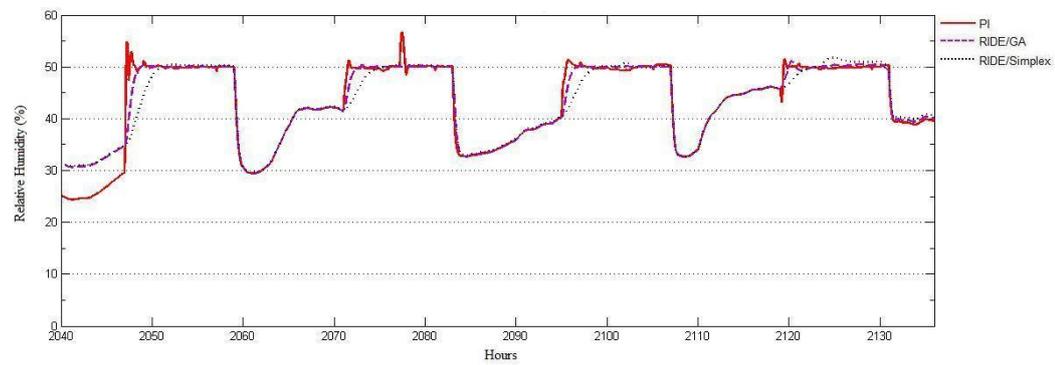


Figure 4: Relative humidity ( $U_{ft} = 2.0W/m^2K$ )

Fig. 4 shows that all three controllers track the relative humidity ratio setpoint (50%) accurately. The PI controller however can still be seen to produce some overshoot. The GA tuned RIDE controller again achieves a quick and accurate response whilst the simplex tuned RIDE controller shows a significantly slower response.

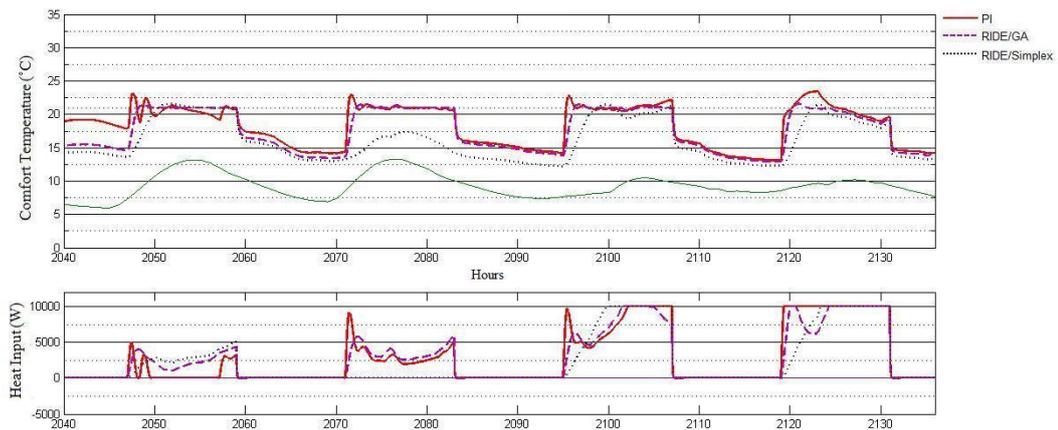


Figure 5: Comfort temperature and heat input ( $U_{ft} = 0.8W/m^2K$ )

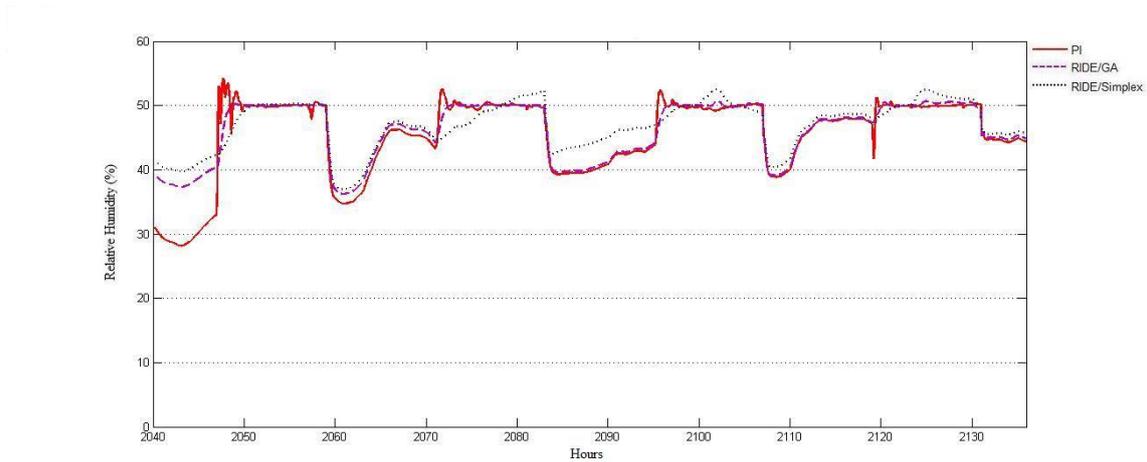
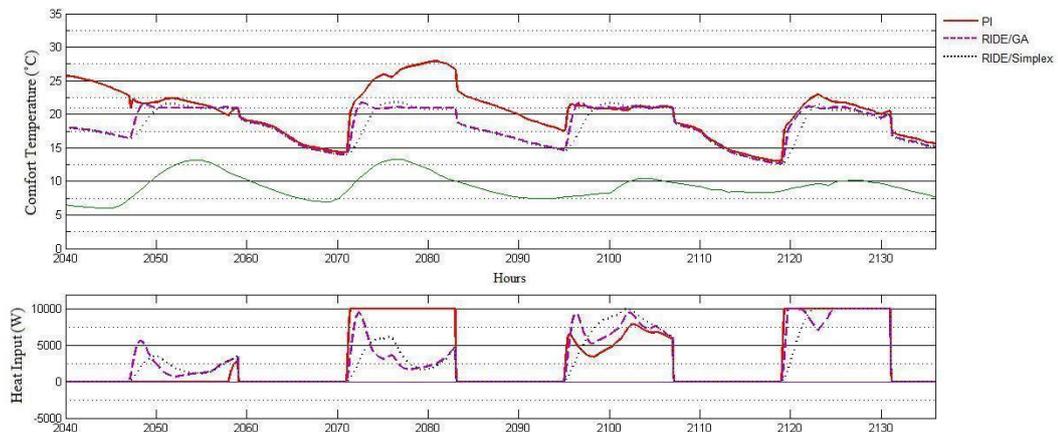
Figure 6: Relative humidity ( $U_{ft} = 0.8W/m^2K$ )

Fig.5 and Fig.6 show the response of the controllers at the lower extreme operating condition,  $U_{ft} = 0.8W/m^2K$ . The PI controller shows an improvement in tracking accuracy over the response shown in Fig.3 however, a significant level of overshoot still remains. This performance improvement can be partly attributed to the fact that the PI controller was tuned for optimum performance at this operating condition. The GA tuned RIDE controller shows a very similar response to that seen under the normal operating condition. The simplex tuned RIDE controller does however show significant performance degradation in the tracking of comfort temperature and relative humidity. This is clearly down to poor tuning on the simplex algorithms behalf since the performance of the GA tuned RIDE controller is unaffected. This highlights the efficacy of the GA for auto-tuning as well as elucidating the benefit of auto-tuning over a range of parameter uncertainty.

Figure 7: Comfort temperature and heat input ( $U_{ft} = 3.2W/m^2K$ )

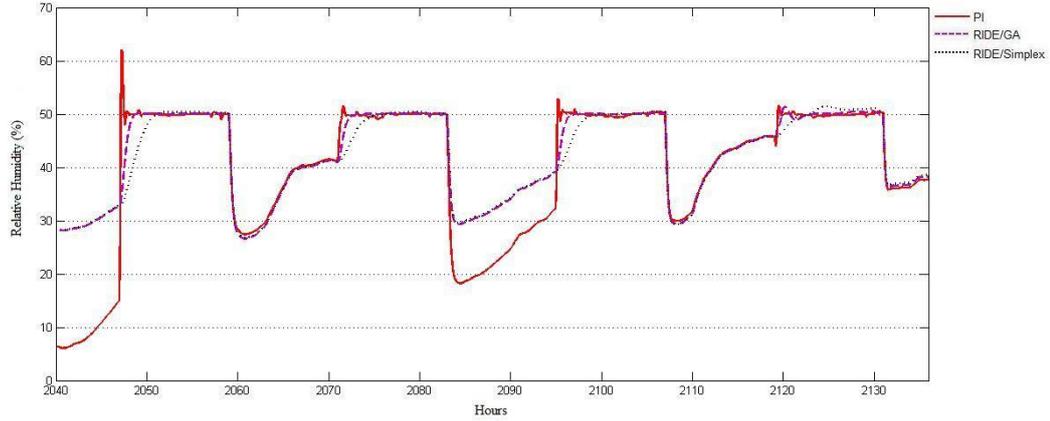


Figure 8: Relative humidity ( $U_{ft} = 3.2\text{W/m}^2\text{K}$ )

Fig.7 and Fig.8 corroborate the results observed above. The PI controller shows a severe performance degradation with very large overshoots occurring in the comfort temperature. The GA tuned RIDE controller again shows quick and accurate tracking of the setpoint in both cases whilst the simplex tuned RIDE controller has a significantly slower response time.

The total energy usage over three months under all three operating conditions for the controller setups is shown in Table 4.

$U_{ft}$ ( $\text{W/m}^2\text{K}$ )	RIDE/GA Energy used (W)	RIDE/Simplex Energy used (W)	PI/Simplex Energy used (W)
0.8	$3.1368 \times 10^8$	$2.7396 \times 10^8$	$3.7125 \times 10^8$
2.0	$3.1078 \times 10^8$	$2.8915 \times 10^8$	$3.5992 \times 10^8$
3.2	$3.0663 \times 10^8$	$2.8726 \times 10^8$	$3.6397 \times 10^8$

Table 4: Total Energy Usage

The simplex tuned RIDE controller clearly uses less energy than the other two setups; however it produces an unsatisfactory system response. The PI controller uses substantially more energy than both RIDE controller setups. The GA tuned RIDE controller has substantially lower energy usage than the PI controller whilst maintaining very good performance under all three operating conditions.

## 9 CONCLUSION

It was shown in the simulation results that the RIDE control method with GA optimisation produced superior performance over the other methods tested. High performance control was achieved under all three operating conditions meaning that, in practice, a good level of thermal comfort for building occupants would be achieved as well as a reduced level of carbon emissions.

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