End-Point Control of a Flexible-Link Manipulator Using H∞ Nonlinear Control via a State-Dependent Riccati Equation

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Abstract. The problem of modeling and controlling the tip position of a single-link flexible manipulator is considered. In a flexible-link manipulator in general the effect of some parameters such as payload, friction amplitude and damping coefficients can not be exactly measured, One possibility is to consider these parameters including uncertainty. Recent results may then be applied on nonlinear robust regulators using a nonlinear H_∞ via state Dependent Ricatti Equation (SDRE) design method. Lagrangian mechanics and the assumed mode method have been used to derive a proposed dynamic model of a single-link flexible manipulator having a control joint. The full state feedback nonlinear H_∞ SDRE control law is derived to minimize a quadratic cost function that penalizes the states and the control input torques. Simulation results are presented for a single-link flexible manipulator to achieve the desired angular rotation of the link whilst simultaneously suppressing structural vibrations. The effect of payload on the system response and vibration frequencies is also investigated. The results are illustrated by a numerical example.

1. Introduction

Flexible manipulator systems offer several advantages in contrast to the traditional rigid manipulator. These include faster response, lower energy consumption, requiring relatively smaller actuators and lower overall mass and, in general, lower overall cost [4]. However, due to its flexible nature the control of the flexible system is to take into account both the rigid body degree of freedom, and elastic degrees of freedom. It is important to recognize the flexible nature of the manipulator and construct a mathematical model for the system that accounts for the interactions with actuators and payload. The efficiency of a single-link flexible manipulator moving at high speed and having a payload is highly dependent on its dynamic behavior. Lagrangian Mechanics and the Assumed Mode Method have been used to drive a proposed dynamic model of a single-link flexible manipulator having a revolute joint. The link has been considered as an Euler-Bernoulli beam subjected to large angular displacement. To establish a model of a single-link flexible manipulator, the kinematics of a single link flexible manipulator is described, here, based on the equivalent rigid link system and a transformation matrix method.

The overall motion of the flexible link manipulator consists of the rigid body motion, which is defined by the joint angle, and the elastic motion, which is defined by the first two modal coordinates. The application of Lagrangian equation yields two sets of equations. The first set is associated with the Rigid Body degrees of freedom, and the other set is associated with the Elastic degrees of freedom. These two sets of equations of motion, for a single-link flexible manipulator, are nonlinear time varying and represented by coupled second order ordinary differential equations. The flexible-link manipulator control problem is complicated by the fact the dynamics of the system are highly nonlinear and complex. As a result, theory has emerged for design according to a number of methods, including feedback linearization [11], variable structure

control [5], control Lyapunov functions [12], recursive backstepping and nonlinear H_∞ control [9]. Although H_∞ techniques were originally proposed for linear systems, the approach has also been studied for nonlinear systems. These techniques can provide a robustness property in the controller. The design of the nonlinear H_∞ via state Dependent Ricatti Equation (SDRE) technique based on the SDRE technique [13] for nonlinear control, which has recently appeared in the literature [1,6,7].

The main contribution of this paper is in adopting the full state feedback nonlinear H_∞ SDRE approach to the needs of the flexible manipulator system and then proving its value through tests on a fairly complex nonlinear simulation model. The outline of the paper is as follows: Section 2 provides a brief description of the dynamic model for a single-link flexible manipulator, and the effect of payload on the dynamic characteristics of the manipulator. Section 3 presents the nonlinear regulator problem. In section 4 the design of the nonlinear H_∞ SDRE controller for a class of nonlinear control systems is explained. Section 5 sets up the problem of applying the nonlinear regulator to a single-link flexible manipulator. Control of flexible manipulator in the presence of varying payloads is investigated in Section 6. In section 7 simulation results for a single-link flexible manipulator are presented. Concluding remarks are given in section 8.

2. Dynamic Model of Flexible Manipulator

The model of the flexible manipulator is obtained on basis of Lagrange's equations of motion [4], may be written as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial V}{\partial q_i} = Q_i \quad i=1,2,3 \qquad (1)$$

where T is the kinetic energy, V potential energy, q_i generalized coordinate, and Q_i generalized force. The application of Lagrange's yields two sets of equations. The first set is associated with the rigid body degree of freedom defined by θ, and the other set is associated with the elastic

degrees of freedom defined by δ_i . These two sets of equations of motion for a single-link flexible manipulator, are nonlinear time varying coupled, second order ordinary differential equations. The generalized coordinates are shown in Figure 1 for the single-link flexible manipulator. Under the Assumed Modes Method and retaining a finite number, $m=2$ of modes, the dynamic equations for the flexible-link manipulator are derived as:

$$M\begin{bmatrix}\ddot{\theta}\\\ddot{\delta}_1\\\ddot{\delta}_2\end{bmatrix}+\begin{bmatrix}h_1(\dot{\theta},\delta_1,\dot{\delta}_1,\delta_2,\dot{\delta}_2)+F(\dot{\theta})\\h_2(\dot{\theta},\delta_1,\dot{\delta}_2,\dot{\delta}_2)\\h_3(\dot{\theta},\delta_1,\delta_2,\dot{\delta}_1)\end{bmatrix}+\begin{bmatrix}0&0&0\\0&k_1&0\\0&0&k_2\end{bmatrix}\begin{bmatrix}\theta\\\delta_1\\\delta_2\end{bmatrix}+\begin{bmatrix}D_1&0&0\\0&D_2&0\\0&0&D_3\end{bmatrix}\begin{bmatrix}\dot{\theta}\\\dot{\delta}_1\\\dot{\delta}_2\end{bmatrix}=\begin{bmatrix}u\\0\\0\end{bmatrix}\quad(2)$$

where $\delta=[\delta_1,\delta_2]^T\in\Re^2$ is the deflection vector, $\theta\in\Re$ is the joint variable, M represents the inertia matrix, $h=[h_1 h_2 h_3]^T$ represents the vector of the Coriolis and centrifugal forces, F is the Coulomb friction, u is the control input torque, $D=[D_1\,0\,0;0\,D_2\,0;0\,0\,D_3]\in\Re^{(m+1)x(m+1)}$ represents the viscous structural damping, and $K=[0\,0\,0;0\,k_1\,0;0\,0\,k_2]\in\Re^{(m+1)x(m+1)}$ represents the stiffness matrix. Integer m is the number of flexible modes (or equivalently the number of mode shape functions), in our model $m=2$. Assuming that the beam deflection is small compared to the link length L , the normalized output may be written as.

$$y_t(t)=\theta+\frac{1}{l}\sum_{i=1}^{m}\phi_i(l)\delta_i\qquad(3)$$

where $\phi_i(l)$ represents the ith mode shape. For the purpose of design, simulation, and control the dynamic equations of flexible-link manipulator (2) can be represented in the state-space model form. A state vector is first defined $x(t)=[x_1(t)\cdots x_6(t)]^T$ where, $[x_1(t)\cdots x_6(t)]^T=[\theta\,\dot{\theta}\,\delta\,\dot{\delta}]^T$, Therefore, model (2) may be written as.

$$[\dot{x}_1;\dot{x}_2;\dot{x}_3;\dot{x}_4;\dot{x}_5;\dot{x}_6]=[x_2;L_1(x,u);x_3;L_2(x,u);x_5;L_3(x,u)]\quad(4)$$

$$\begin{bmatrix}L_1(x,u)\\L_2(x,u)\\L_3(x,u)\end{bmatrix}=[M]^{-1}\begin{bmatrix}-h_1(\dot{\theta},\delta,\dot{\delta})-D_1x_2-F(\dot{\theta})+u\\-h_2(\dot{\theta},\delta,\dot{\delta})-D_2x_4-K_1x_3\\-h_3(\dot{\theta},\delta,\dot{\delta})-D_3x_6-K_2x_5\end{bmatrix}$$
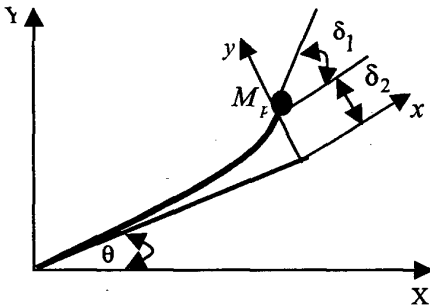

Figure 1. Geometric and Generalized Coordinates of a flexible link

3. Nonlinear Regulator Control Problem

In the nonlinear quadratic regulator problem the aim is to minimize the infinite horizon cost function [14] of the form:

$$J=\frac{1}{2}\int_{t_o}^{\infty}[x^T Q(x)x+u^T R(x)u]dt\qquad(5)$$

With respect to the state x and control u subject to the nonlinear constraint.

$$\dot{x}=a(x)+b(x)u\qquad(6)$$

Given state $x\in R^n$ and control $u\in R^m$, with $a,b,R,Q\in C^k$, $k\geq1$, where $Q(x)=H^T(x)H(x)\geq0$, and $R(x)>0$ for all x. It is assumed that $a(0)=0$ so that the origin is an equilibrium point of the open loop system. We seek a stabilizing solution in the form $u=L(x)x$ where the nonlinear feedback gain L is a matrix function of the state vector (x). The above formulation is analogous to linear quadratic regulator (LQR) theory [10] except that the matrices Q,R and L all have elements that are allowed to be functions of the state x. The SDRE method hinges on being able to write the constraint dynamics (6) in a point-wise linear structure, having a state-dependent coefficient (SDC) form.

$$\dot{x}=A(x)x+B(x)u\qquad(7)$$

So that $a(x)=A(x)x$ and $b(x)=B(x)$, it is also known that there are an infinite number of ways to bring the nonlinear system to SDC form. Associated with the SDC form the following definitions apply:

- $\{H(x),A(x)\}$ is an observable detectable parameterization of the nonlinear system [in a region Ω] if the pair $\{H(x),A(x)\}$ is point-wise observable (detectable) in the linear sense for all $[x\in\Omega]$.

- $\{A(x),B(x)\}$ is controllable stabilizable parameterization of the nonlinear system [in a region Ω] if the pair $\{A(x),B(x)\}$ is point-wise controllable (stabilizable) in the linear sense for all $[x\in\Omega]$.

4. Nonlinear H_∞ Control Via SDRE Method

In this section, the proposed methods for the nonlinear H_∞ suboptimal control problem based on the SDRE technique [13] are introduced. Solution approaches for input-affine systems under both state and output feedback are proposed in [6].

4.1 Output Feedback

Consider the general nonlinear system:

$$\dot{x}=f(x)+B_1(x)w+B_2(x)u\qquad(8)$$
$$z=c_1(x)+D_{12}(x)u\qquad(9)$$
$$y=c_2(x)+D_{21}(x)w\qquad(10)$$

where $x\in\Re^n$, $u\in\Re^{m_2}$, $w\in\Re^{m_1}$ all of the functions are smooth (i.e., C^1), $D_{12}(x)$, and $D_{21}(x)$ have full rank, $D_{12}^T(x)D_{12}(x)=\Re_u$ and $D_{21}(x)D_{21}^T(x)=\Re_w$, and $f(0)=0$, $c_2(0)=0$ and $B_2(x)\neq0$ for all x, z is the controlled output and y is the measured output. The exogenous input signal w may include tracking commands and/or disturbances. It is

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desired to bound the L_2-gain of the system. For $\gamma \geq 0$, the system (8)-(10) has L_2-gain less than or equal to γ if:

$$\int_0^T \|z(t)\|^2 dt \leq \gamma^2 \int_0^T \|w(t)\|^2 dt \qquad (11)$$

for all $T \geq 0$ and all $w \in L_2(0,T)$. If a controller can be found such that the closed loop system is internally stable and such that the inequality (11) is satisfied, the exogenous signals will be locally attenuated by γ. The inequality (11) can be satisfied by solving the nonlinear max-min differential game problem.

$$\max_{w \in L_{2+}} \min_{u \in L_{2+}} \frac{1}{2} \int_0^\infty \|z(t)\|^2 - \gamma^2 \|w(t)\|^2 dt \qquad (12)$$

subject to the constraints (8)-(10). The SDRE approach for obtaining an approximate solution of the nonlinear H_∞ problem is:

4.1.1 Use direct parameterization to bring the nonlinear dynamic to the SDC form:

$$\dot{x} = A(x)x + B_1(x)w + B_2(x)u \qquad (13)$$
$$z = C_1(x)x + D_{12}(x)u \qquad (14)$$
$$y = C_2(x)x + D_{21}(x)w \qquad (15)$$

Assumption: $(A,B_1),(A,B_2)$ and $(C_1,A),(C_2,A)$ are pointwise stabilizable and detectable in the linear sense, respectively, for $x \in \Omega$, where Ω is the region of interest which may be the entire space.

4.1.2 With γ sufficiently large so that the solutions $P(\hat{x}) \geq 0, Q(\hat{x}) \geq 0$ exist with $\lambda_{max}[P(\hat{x})Q(\hat{x})] < \gamma^2$, solve the state-dependent Riccati equations which are given below in terms of their state-dependent Hamiltonian matrices:

$$\begin{bmatrix} A - B_2 R_u^{-1} D_{12}^T C_1 & \gamma^2 B_1 B_1^T - B_2 R_u^{-1} B_2^T \\ -\hat{C}_1^T \hat{C}_1 & -(A - B_2 R_u^{-1} D_{12}^T C_1)^T \end{bmatrix} \qquad (16)$$

$$\begin{bmatrix} (A - B_1 D_{21}^T R_w^{-1} C_2)^T & \gamma^{-2} C_1^T C_1 - C_2^T R_w^{-1} C_2 \\ -\hat{B}_1 \hat{B}_1^T & -(A - B_1 D_{21}^T R_w^{-1} C_2) \end{bmatrix} \qquad (17)$$

$$\hat{B}_1 = B_1(I - D_{21}^T R_w^{-1} D_{21}) \quad \hat{C}_1 = (I - D_{12} R_u^{-1} D_{12}^T)C_1$$

4.1.3 Construct the SDRE nonlinear H_∞ feedback controller via:

$$\dot{\hat{x}} = A_0(\hat{x})\hat{x} + B_0(\hat{x})y \qquad (18)$$
$$u = F(\hat{x})\hat{x} \qquad (19)$$
$$A_0 = A + B_2 F + \gamma^{-2} B_1 B_1^T P + ZL(C_2 + \gamma^{-2} D_{21} B_1^T P)$$

$$B_0 = -ZL \quad Z = (I - \gamma^{-2} QP)^{-1}$$
$$F = -R_u^{-1}(B_2^T P + D_{12}^T C_1) \quad L = -(QC_2^T + B_1 D_{21}^T)R_w^{-1}$$

4.2 Full State Information
In this case of full state information, equations (10) and (17) disappear along with the observer equation (18) and the

static nonlinear controller is given by equation (19) where $\hat{x} = x$.

4.2.1 Parametrize (8) in SDC form equations (13)-(15).
4.2.2 Solve the H_∞ SDRE.

$$A^T P + PA - P\left[B_2 B_2^T - \frac{1}{\gamma^2} GG^T\right]P + C_1^T C_1 = 0 \qquad (20)$$

All matrices here are functions of the state x but it is omitted here for simplicity. The γ is assumed sufficiently large so that the stability and complementarily properties hold in order to obtain $P(x) > 0 \, \forall \, x$.

4.2.3 Construct the nonlinear H_∞ feedback control via:

$$u(x) = -B_2^T(x)P(x)x \qquad (21)$$

The local stability of the closed loop system resulting from using the SDRE nonlinear regulator technique is determined by the following theorems from [6].

Theorem Consider (8) and assume $z \in \Re^s, C_1(0) = 0$. Also assume that all mappings in (8-10) are C^∞ and that $\{C_1(0),A(0)\}$ is detectable and $\{C_1(0),B(0)\}$ is stabilizable. Then the state feedback SDRE design procedure given by (20) yields a local solution to the nonlinear H_∞ control problem for (8).

5. Application to a Single-Link Flexible Manipulator
In this section the application of the nonlinear H_∞ via SDRE control algorithm to design a robust regulator for flexible-link manipulator is discussed. In this paper the method is applied to a single-link flexible manipulator. For the purpose of design, first augment the dynamics of the exosystem with the manipulator dynamics and then define the error as the difference between the regulated output and the exosystem output. Note that the design technique used in this paper involves state feedback. Consequently, for simplicity of design, the first state is taken as the output, i.e., the hub position. The effect of rotary inertia and shear deformation is ignored by assuming that the cross-sectional area of the link is small in comparison with its length. For the purpose of design, the dynamic equations of flexible-link manipulator (2) can be represented in the state-space model form. By choosing $[\theta_1, \theta_2, \delta_1, \delta_2, \delta_3, \delta_4]$ as the state vector and the tip position as the output, we may write the dynamics of the manipulator augmented with the exosystem in the form:

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_1(x_1,x_2,x_3,x_4,x_5,x_6,w) + g_1(x_3,x_5)u \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= f_2(x_1,x_2,x_3,x_4,x_5,x_6,w) + g_2(x_3,x_5)u \\
\dot{x}_5 &= x_6 \\
\dot{x}_6 &= f_3(x_1,x_2,x_3,x_4,x_5,x_6,w) + g_3(x_3,x_5)u \\
e &= x_1 - x_{ref} \\
z &= [e,u]^T
\end{aligned} \qquad (22)$$

and the tip position given by:

$$y = x_1 + k_{tip}(C_1 x_3 + C_2 x_5)$$

where K_{tip}, C_1, C_2 are constants, depending on the arm characteristics, and w is the vector of uncertainties that

represent the deviations of parameters from their nominal values. For instance, the inertia matrix is a function of the load mass M_P . Therefore, in deriving the state-space equations, we need to take into account that the uncertainty on the load mass does not propagate throughout the system dynamics. To consider this uncertainty, one may assume that [3].

$$M_P = M_{P0}(1 + w_1) \qquad (23)$$

where M_{P0} is the nominal value of the load mass and w_1 is an L_2 bounded disturbance acting on it. Note that here are several parameters that may have uncertain values. The amplitude of a sigmoidal function that models the Coulomb friction, the value of hub damping for each joint and/or the value of structural damping due to link flexibility are a few examples.

The design approach used is the nonlinear H_∞ via SDRE technique where the objective is to attenuate the disturbances on the controlled output so that the exogenous input are selected to have bounded energy. Therefore, any bounded signal with a compact support can enter the system as a disturbance. Consequently, in the non-affine model (22) with respect to the exogenous input w, all deviations must be L_2 bounded. Based on the results developed in the preceding sections, a nonlinear controller was designed to attenuate the effect of disturbances on the controller output for the flexible-link manipulator described above. The procedure for designing a nonlinear robust regulator for a flexible-link manipulator is as follows:

Step 1. Construct the state space model as in (22).
Step 2. Parameterize (22) in SDC form (13).
Step 3. Solve the state feedback nonlinear H_∞ SDRE algebraic Riccati equation (20) for P.
Step 4. Construct the nonlinear H_∞ feedback control via (21).

6. Simulations and Discussion
As mentioned earlier, the main objective is to control the tip position of a single-link flexible manipulator robustly. Simulations were performed in Matlab/Simulink using Runge-Kutta, fourth-order numerical integration to implement and design the nonlinear controller. The purpose of the simulation is to demonstrate the performance of the developed model and controller algorithm in analyzing the effects of manipulator flexibility, and payload on the dynamic behavior of the system. A simulated example is described in this section. To implement this, consider that the flexible link rotates on the horizontal plane *i.e.*, the axis of rotation is vertical, the geometric and mass properties of the flexible manipulator are: length $L=1m$, mass density $\rho=7842$ kg/m^3, Young Modulus $E=2x10^{11}$ Nm, area moment of inertia $I=20x10^{-11}$ m^4, and cross-sectional area $A=9x10^{-9}$ m^2.

Before developing the control design, we study the open loop response of the flexible manipulator system. The

flexible manipulator is excited with a bang-bang input torque profile of amplitude 1 [Nm], shown in Figure 2. This torque was applied at the hub of the manipulator. The system variables considered here are: the joint angle θ, the tip deflection v, and the tip position y with no payload as shown in Figure 2. In the stability analysis, consider the unforced system (*i.e.*, *u=0*):

$$\dot{x} = f(x) \qquad (24)$$

The stability of this system may be examined around the origin via the techniques of linearization and first integrals [10], by the so-called Principle of Stability in the First Approximation. If it is assumed that $f(x)$ is at least twice continuously differentiable and that the equilibrium of interest is the origin, then the following statements can be made about the local asymptotic stability of (23) based on the linear approximation of $f(x)$ at $x = 0$:

$$J_o = \left[\frac{\partial f}{\partial x}\right]_{x=0} \qquad (25)$$

Denote the Jacobian matrix of $f(x)$ at $x = 0$. Then if

- All the eigenvalues of J_o have negative real parts, the origin is a locally asymptotically stable equilibrium of (22),
- At least one eigenvalue of J_o has a positive real part; the origin is an unstable equilibrium of (22).

The Principle of Stability in the First Approximation obviously does not cover all cases of interest. In particular, it provides no information when all of the real parts of the eigenvalues of J_o are nonpositive, and at least one eigenvalues has a zero real part. When this is the case, "Center Manifold Theory" may often be used to draw conclusions regarding the local stability properties of an equilibrium point for a time-invariant system. We also know that for any SDC dynamic parameterization $f(x)=A(x)x$ of (24), $A(0)$ must equal the Jacobian of f evaluated at zero, so that this necessary condition becomes that the pair $\{J_o, B(0)\}$ is stabilizable. Computing the Jacobian J_o of (22) at the origin obtains:

$$J_o = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 100087 & 0 & -41984.7 & 0 & 9249.11 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -452323 & 0 & 188890 & 0 & -49233.3 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The eigenvalues of J_o are $\{0, 0, 0 \pm 60.45i, 0 \pm 295.9i\}$. We can see that all eigenvalues have zero real part, but in fact the damping effects on a flexible manipulator push the eigenvalues of J_o, to be negative real parts. $A(0)$ is equal to the Jacobian of f evaluated at zero. So the origin is a locally asymptotically stable equilibrium of (22). Based on the results obtained in the preceding sections, and the simulation results from the dynamic model of the

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single-link flexible manipulator, the nonlinear H_∞ SDRE controller technique was designed and implemented in Matlab/Simulink to control the output of the single-link flexible manipulator without the addition of a payload at the free end. For comparison purposes we fix the attenuation factor, $\gamma=10$, which is one of the controlling parameters affecting the performance of the closed-loop system, and varying the state weighting matrices Q. We report here some simulation results obtained for the single-link flexible arm described above via nonlinear H_∞ SDRE controller technique.

Figure 3 shows the closed loop output response of the tip position, and tip deflection for a step input with amplitude of 1.0. As can be observed, a considerable good tracking, and smallest settling time of the tip position for the step input is achieved. The tip deflection is completely damped after 0.65 sec. Indeed, we see that the nonlinear H_∞ SDRE regulator is an effective way of direct handling of unstable non-minimum phase systems, the simple way to adjust the control and state weighting matrices, and also offers significant design flexibility while yielding closed loop stability. As in other optimal control algorithms the controlled output may be weighted with respect to the disturbance for obtaining a faster response. Since the cost function is of quadratic type, increasing the weighting on the output state result in a more damped response, and more emphasis on rise time, decreasing the weighting on the output state result in a more overshoot response, and lower rise time.

7. Control of Flexible Manipulator in the Presence of Varying Payloads

In previous sections control of the flexible manipulator was set without the effect of payload at the free end. However, the payload is a very important parameter for the design and control of a flexible manipulator. Changes in payload mass result in changes in the dynamic performance of the arm, an important objective of the manipulator mechanical and control design is to increase its payload. It is anticipated, however, that the increase of payload will be accompanied by an increase of the elastic displacement and the residual vibration after performing a maneuver. In this section the nonlinear H_∞ SDRE controller is applied to control the flexible manipulator system for three-different ratios, of the payload mass to the mass of the arm, $m_p/m=0,0.25,0.5$. where m_P mass of the payload, and m mass of the flexible link.

For the purpose of comparison, we use the same attenuation factor γ, and the state weighting Q respectively: as for case without payload in previous section. But we now change the fixed payload mass at the free end of the flexible manipulator. To facilitate comparison between cases, we start the simulations from the same initial conditions, we again sample at $T=0.005$ seconds, and we use $A(x)$ parameterization given by (7) for all simulation cases. As a

final basis of comparison, we show the plots for the three cases in Figure 4. In this figure, the solid lines represents the case without payload, dashed lines represent the case, $m_p/m=0.25$, and doted lines represent case, $m_p/m=0.5$. As expected, all of the outputs of the hub displacement, and tip displacement of the three cases are asymptotically approaching the value of one as desired.

Two things are immediately apparent from the figure. Note first of all that increasing the mass ratio, m_p/m, increases the settling time. The second thing is the increase in the amplitude of overshoot as the ratio, m_p/m increase. However, overall, the increase in the payload was handled sufficiently well by the nonlinear H_∞ SDRE controller.

8. Conclusions

The Lagrange mechanics and the assumed mode method have been used to derive a proposed dynamic model of a single-link flexible manipulator having a revolute joint. The model is valid for an arbitrary number of deflection modes. The model may be used to investigate the motion of the manipulator in the horizontal and vertical planes. The proposed model has been used to investigate the effect of two main design parameters, the payload, and the open loop control torque profile. The results of the investigation show that as long as the rest-to-rest rotational maneuver is considered, the payload has a dominant effect on the elastic deflection of the manipulator. In general, in a flexible-link manipulator, the system parameters may not be known exactly a priori. Consequently, this will introduce significant uncertainties in the robot's dynamic model.

The uncertainties considered in this paper are the deviations of parameters from their nominal values. The focus was on providing a theoretical basis for the control of nonlinear systems via the state feedback nonlinear H_∞ via SDRE techniques, which, have proven quite successful in a number of simulated applications, including the control of single-link flexible manipulator. The proposed control methodology is based on minimizing the effect of the disturbance on the tip position.

Extra design degrees of freedom arising from the non-uniqueness of the SDC parameterization can be utilized to enhance controller performance and the nonlinear H_∞ via SDRE method does not cancel beneficial nonlinearities. It was shown that the proposed model and controller, under certain relatively mild conditions, renders the origin a locally asymptotically stable equilibrium point. Additional results in the paper show that the regulator is near optimal. Throughout this paper, it was assumed that all the states of the plant were available for measurement. Obviously some of these states are available via standard sensors (such as hub angle, hub velocity and tip position). Other states may require more sophisticated sensors or the introduction of observers.
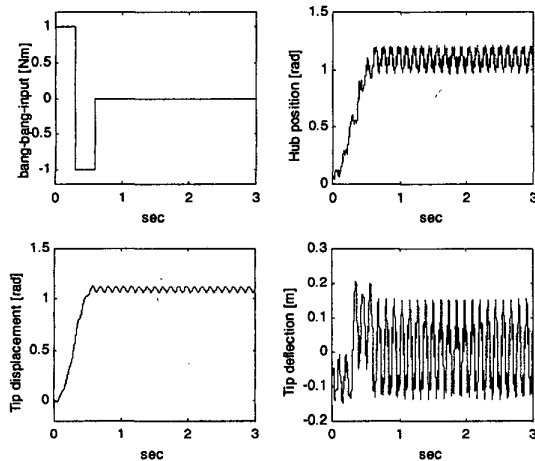
Figure 2. bang-bang input torque, hub and tip displacement, and tip deflection without payload
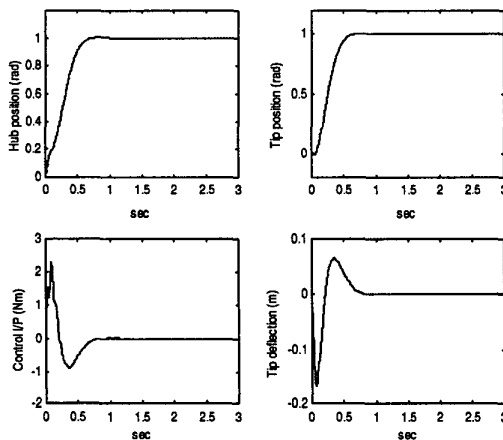

Figure 3. Step response of the flexible manipulator using nonlinear H_∞ via SDRE controller without payload
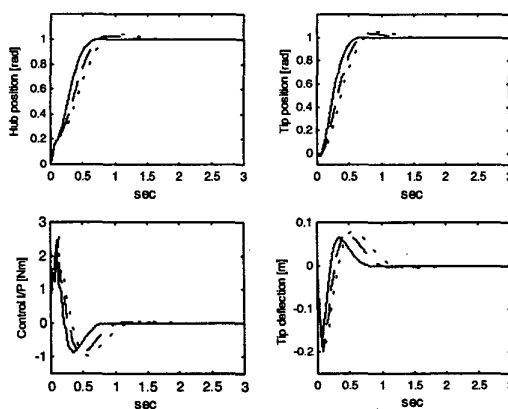

Figure 4. Step response of the flexible manipulator using nonlinear H_∞ SDRE controller with $m_p/m=0,0.25,0.5$ payload

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