A HOLISTIC ANALYSIS METHOD TO ASSESS THE CONTROLLABILITY OF COMMERCIAL BUILDINGS AND THEIR SYSTEMS

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ABSTRACT

This paper describes a novel design process for advanced MIMO (multiple inputs and multiple outputs) control system design and simulation for buildings. The paper describes the knowledge transfer from high technology disciplines such as aerospace flight control systems and the space industry to establish a three-step modelling and design process. In step 1, simplified, but holistic nonlinear and linearised dynamic models of the building and its systems is derived. This model is used to analyse the controllability of the building. In step 2, further synthesis of this model leads to the correct topology of the control system design. This is proved through the use of simulation using the simple building model. In step 3, the controller design is proved using a fully detailed building simulation such as ESP-r that acts as a type of virtual prototype of the building. The conclusions show that this design approach can help in the design of superior and more complex control systems especially for buildings designed with a Climate Adaptive Building (CAB) philosophy where many control inputs and outputs are used to control the building's temperature, concentration of CO_2, humidity and lighting levels.

Keywords: Buildings, Systems, Modelling, Controllability

1 NOMENCLATURE

MV = Mechanical Ventilation, NV = Natural ventilation

A - Surface Area (m^2) with subscripts: $w1$, $w2$ & $w3$ = walls, s = screed, c = concrete, win = window, r = roof, in = insulation, i = inlet and o = outlet openings in the zone.

C - CO_2 concentration level (kg/m^3) with subscripts: a = internal air and o = external air.

c_p - Specific heat capacity of (J/kg K) with subscripts: $w1$, $w2$ & $w3$ = walls, a = internal air, s = screed and c = concrete.

F - Radiation exchange factor (dimensionless).

g - Gravitational acceleration m/s^2.

H - Height between the two openings in zone.

k_s - Watts/m^2 per unit Lux.

k_f - Fraction of fan power converted to heat.

k_e - Proportion of light power converted to heat.

k_L - Lux/W Ratio for internal lighting.

L - Lux input from Sources with subscripts: i = internal lights, s = solar; (Lux).

L_o - External Lux level (Lux)

n - Air changes per second (s^{-1})

P_L - Electrical power into lights (W)

P_f - Fan power per volume air flow rateW/m^3/s

q_m - Mechanical Air flow rate (m^3/s)

$\dot{Q}$ - Internal Heat gain (W) with subscripts: oc = Occupancy and ap =Appliances.

S - Internal CO_2 gain (kg/s)

t - Time (s)

T - Temperature (K) with subscripts: a = internal air; 2, 4 & 6 = other zones; $w1$, $w2$ & $w3$ = walls, s = screed, c = concrete, r = roof and o = external air.

u_L - Control input for power into internal lights

U - Overall heat transfer coefficient (W/m^{2}K) with subscripts: $w1$, $w2$ & $w3$ = walls, s = screed, c = concrete, r = roof

v - Air flow speed (m/s) with subscripts: n = natural ventilation and o = external air

V_a - Volume of the air (m^3)

α - Transmissivity (dimensionless)

ρ - Density of (kg/m^3)

σ - Stefan-Boltzmann constant W/ (m^2.k^4)

2 INTRODUCTION

In recent times commercial buildings have utilised many different systems simultaneously, such as Solar Blinds, perimeter heating, mechanical ventilation, chilled beams and so on. Consequently advanced building management systems have been required to control them. In reality, building services engineers face huge problems in commissioning these complex building management systems to work reliably all year round. Often control systems of lighting, heating and ventilation become unstable as one system fights against another. This project bridges a gap by transferring systems engineering analysis and control theory knowledge developed for many years in the aerospace industry and applying it to assess the controllability of these advanced buildings. This knowledge enables the complex dynamic interactions between building services systems and the building to be better understood. The paper presents a case study of a school being designed in Scotland using the CAB philosophy.

3 DESIGN PROCESS

The industry is currently using dynamic modelling and simulation in the design process to test the detail design of the building. At this stage control algorithms are also applied. If the building proves not to be controllable, then it is very difficult to identify the factors affecting the controllability as there are too many effects and parameters to be identified. Thus, it is crucial to address the issues of controllability of the building at the conceptual design stage [1]. In order to achieve this, a simplified mathematical model is required with enough detail to know which factors are affecting the controllability. After conducting extensive research and consultation, it has concluded that the building industry requires a method for assessing the controllability of buildings which may require advanced controllers using nonlinear elements and MIMO. This paper presents step 1 of the proposed modelling and design process consists of:

a) A simplified [1,2] dynamic model that provides answers to fundamental questions of controllability and also accurately predicts the dynamic and cross-coupled behaviour of the energy system.

b) An engineering science [13,14,15] developed for assessing the controllability of the building at the conceptual design stage by using the mathematical model of the building concept.

4 CASE STUDY

The part of school under analysis is the first floor of a two story block. The basic mathematical model is developed based on causes and effects illustrated in building concept design of the school (see figure 1).

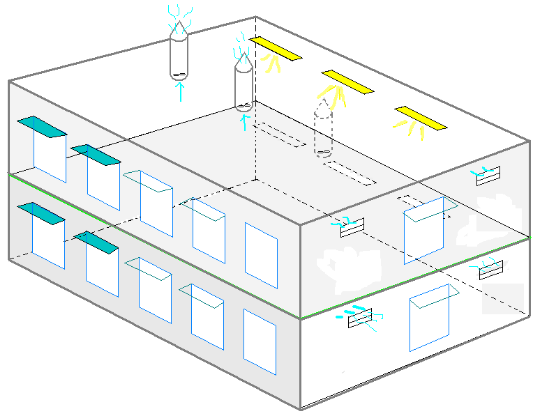

Figure [1]: Building concept

5 MATHEMATICAL MODEL

Lumped capacitance models of building envelopes with HVAC plant and control have been developed by Underwood *et al* as a test bed for analyzing control strategies. [3, 4, 5, 6]. The proposed model is specifically developed to test the controllability of a nonlinear multivariable system. The dynamic model describes the energy and mass balance of air in the building zone having under-floor heating, ventilation and lighting. The assumptions inherent in constructing this model are numerous. However, the purpose of the model is not to emulate future reality and base design decisions around it, as advanced integrated software packages, such as ESP-r already exist. The differential equations that govern temperature, lux and CO_2 levels are as follows:

5.1 Rate of change of indoor air temperature:

The simplified model assumes that the indoor zone air is fully mixed and the stack is stratified for natural ventilation. The roof and windows are considered to be in steady state. This leads to less complex equations, but are still detailed enough to analyse controllability.

The internal structures e.g. furniture, internal walls etc are considered to be at the same temperature as the indoor air [7, 8 and 9]. The Equation describes the indoor temperature T_a as follows:

$$\frac{dT_a}{dt} = \frac{1}{\rho_a V_a c_{pa}} \begin{bmatrix} \dot{Q}_{fan} + \dot{Q}_{sol} + \dot{Q}_L + \dot{Q}_{con} \\ +\dot{Q}_{rad} + \dot{Q}_{oc} + \dot{Q}_{ap} - \dot{Q}_{w1i} \\ -\dot{Q}_{w2i} - \dot{Q}_{w3i} - \dot{Q}_{win} \\ -\dot{Q}_r - \dot{Q}_{nv} - \dot{Q}_{ni} - \dot{Q}_{mv} \end{bmatrix} \quad (1)$$

where,

Mechanical Vent heat gain: $\dot{Q}_{fan} = q_m P_f \, k_f$,

Solar heat gain: $\dot{Q}_{sol} = L_o k_s A_{win}$,

Lighting heat gain: $\dot{Q}_L = k_e P_L$,

Convection: $\dot{Q}_{con} = A_s 2.13 \left| T_s - T_a \right|^{0.31} \left(T_s - T_a \right)$,

Radiation: $\dot{Q}_{rad} = A_s \sigma F_r \left(T_s^4 - T_a^4 \right)$,

Wall 1 heat transfer: $\dot{Q}_{w1i} = U_{w1} A_{w1} (T_a - T_{w1})$,

Wall 2 heat transfer: $\dot{Q}_{w2i} = U_{w2} A_{w2} (T_a - T_{w2})$,

Wall 3 heat transfer: $\dot{Q}_{w3i} = U_{w3} A_{w3} (T_a - T_{w3})$,

Window heat transfer: $\dot{Q}_{win} = U_{win} A_{win} (T_a - T_o)$,

Roof heat transfer: $\dot{Q}_r = U_r A_r (T_a - T_r)$,

Natural ventilation: $\dot{Q}_{nv} = v_n A_n \rho_a c_{pa} (T_a - T_o)$,

Natural infiltration: $\dot{Q}_{ni} = V_1 n \rho_a c_{pa} (T_a - T_o)$,

Mechanical ventilation: $\dot{Q}_{mv} = q_m \rho_a c_{pa} (T_a - T_o)$

and the area of the fresh air infiltration openings A_n is given by [10]:

$$A_n = \frac{A_i A_o}{\sqrt{A_i^2 + A_o^2}}$$

The air speed in inlet and outlet openings of the zone v_n, (m/s) due to stack and wind pressure is given by [11]:

$$v_n = C_d \sqrt{\frac{gH\Delta T}{T_i} + v_o^2 \frac{\Delta C_p}{2}} \quad (2)$$

5.2 Rate of change of wall temperatures:

Thermal corner effects are neglected so that internal and external wall areas can be assumed to be the same. U-Values (overall thermal transmittance coefficient) are used to model the heat transfer through the building fabric. While the thermal resistances and thermal capacities can be calculated, a weighted average of these resistances and capacities was used for a single capacity equivalent of a multi-layer wall construction to simplify the model for controllability analysis. The wall temperatures T_{w1}, T_{w2} and T_{w3} are given by the following:

$$wall\ 1 \quad \frac{dT_{w1}}{dt} = \frac{1}{\rho_{w1} V_{w1} c_{pw1}} \left[\dot{Q}_{w1i} - \dot{Q}_{w1o} \right] \quad (3)$$

where heat leaving is: $\dot{Q}_{w1o} = U_{w1} A_{w1} (T_{w1} - T_o)$.

$$wall\ 2 \quad \frac{dT_{w2}}{dt} = \frac{1}{\rho_{w2} V_{w2} c_{pw2}} \left[\dot{Q}_{w2i} - \dot{Q}_{w2o} \right] \quad (4)$$

where heat leaving is: $\dot{Q}_{w2o} = U_{w2} A_{w2} (T_{w2} - T_6)$.

$$wall\ 3 \quad \frac{dT_{w3}}{dt} = \frac{1}{\rho_{w3} V_{w3} c_{pw3}} \left[\dot{Q}_{w3i} - \dot{Q}_{w3o} \right] \quad (5)$$

where heat leaving is: $\dot{Q}_{w3o} = U_{w3} A_{w3} (T_{w3} - T_4)$.

5.3 Rate of change of floor temperatures:

The floor has three sections: screed, insulation and concrete. Since the floor is heated, each section of the floor is modelled separately for heat transfer. The radiant interchange is assumed to be between the floor and the air [8]. Thus, the temperature of the screed is given by,

$$\frac{dT_s}{dt} = \frac{1}{\rho_s V_s c_{ps}} \left[\dot{Q}_{in} + \dot{Q}_h - \dot{Q}_{con} - \dot{Q}_{rad} \right] \quad (6)$$

Where, the heat transfer through the insulation is: $\dot{Q}_{in} = U_{in} A_{in} (T_c - T_s)$.

The temperature of the floor's concrete is given by:

$$\frac{dT_c}{dt} = \frac{1}{\rho_c V_c c_{pc}} \left[\dot{Q}_{ci} - \dot{Q}_{in} \right] \quad (7)$$

where heat entering is: $\dot{Q}_{ci} = U_c A_c (T_2 - T_c)$.

Note, the floor's insulation layer is assumed to be in a steady state condition.

5.4 Rate of change of CO_2 concentration:

The air is assumed to be of constant pressure and volume inside the zone, consequently the CO_2 concentration is given by [12]:

$$\frac{dC_a}{dt} = \frac{1}{V_a} \left[S - \dot{C}_{mv} - \dot{C}_{nv} - \dot{C}_{ni} \right] \quad (8)$$

where, CO_2 transferred via mechanical ventilation (MV) is $\dot{C}_{mv} = q_m (C_a - C_o)$,

CO_2 transferred via natural-ventilation (NV) is $\dot{C}_{nv} = v_n A_n (C_a - C_o)$ and the CO_2 transferred via infiltration is: $\dot{C}_{ni} = V_a n (C_a - C_o)$.

5.5 Rate of change of lighting power:

Lighting systems are constrained to limiting the frequency at which their lux levels can power on and off. Consequently, it is sensible to control the rate-of-change of power delivered to the lighting system such that,

$$\frac{dP_L}{dt} = u_L \tag{9}$$

This enables simple limits to the rate-of-change of lighting lux to be easily controlled.

5.6 Sensors for the Controller

A feedback control system, can only control (i.e. track) what it feeds back as measured system outputs. Thus, to analyse the controllability of these measurements, they must be defined and are as follows:

Measured Comfort CO_2 is $C_{cm} = C_a$, (10)

Measured Comfort lux level is given by:

$$L_{cm} = L_i + L_s = k_L P_L + \alpha L_o, \tag{11}$$

and the *Comfort Temperature level is given by:*

$$T_{cm} = T_a \tag{12}$$

In order to apply the aerospace controllability science [13,14,15], the equations (1-12) must be linearised. The results of this linearisation enable the total system to be represented in the state-space form [16]:

$$\dot{x}(t) = Ax(t) + Bu(t) + Fd(t) \tag{13}$$

$$y(t) = Cx(t) + Du(t) + Ed(t) \tag{14}$$

This linear model describes the dynamic behaviour of the building and its systems for a small amplitude perturbation δ about a steady state equilibrium condition. Where *y(t)* is the measured output vector, *x(t)* is a vector of state variables, *u(t)* is a vector of system inputs (i.e. controller outputs) and *d(t)* is a vector of disturbances. A, B, C, D, E and F are time invariant matrices consisting of constants which can be readily derived using symbolic mathematical software tools. The vectors associated with these vectors are given as follows:

$$x = [\delta T_a, \delta T_{w1}, \delta T_{w2}, \delta T_{w3}, \delta T_s, \delta T_c, \delta C_a, \delta P_L]^T$$

$$u = [\delta q_m, \delta u_L, \delta \dot{Q}_h]^T$$

$$y = [\delta C_{cm}, \delta L_{cm}, \delta T_{cm}]^T \tag{15}$$

$$d = \begin{bmatrix} \delta L_o, \delta \dot{Q}_{oc}, \delta \dot{Q}_{ap}, \delta T_o, \delta T_r, \delta T_6, \delta T_4, \\ \delta T_2, \delta C_o, \delta S, \delta v_o, \delta n \end{bmatrix}^T$$

6 CONTROLLABILITY ANALYSIS

For specific operating conditions, it is important to know whether the corresponding inputs i.e. δq_m, δu_L and $\delta \dot{Q}_h$ are practically attainable. Poor controllability will result in unstable behaviour around the desired operating conditions. In state space format, the controllability of the building model is assessed by investigation of two fundamental MIMO controller design principles: the diagonal dominance of the product of the C and B matrices and the values of the Transmission zeros [13, 15].

6.1 CB Matrix

As the feedback gain of a control system is varied from zero to infinity the closed-loop poles can be traced out on a root-locus [16] and a number of these poles equal to the number of inputs/outputs approach the asymptotes. On the root-locus, the structure of the CB matrix determines the direction of these asymptotes. If the matrix is found to be invertible and diagonal, then it means that the asymptotes can be aligned with the negative real axis of the complex plane, thus greatly assisting high gain and high performance controls to be successfully deployed. It also means that classical single input single output (SISO) controllers such as Proportional plus Integral control (PI), could be sufficient. In general, in order to align these asymptotes a MIMO controller is required using a pre-filter matrix given by $(CB)^{-1}$to re-align the asymptotes along the negative real axis and allow high gain control to be implemented in this more complex situation [13,14,15]. It was noted that when air temperature T_a was controlled for achieving the required comfort temperature i.e. $T_{cm} = T_a$, the product (CB) is not of full rank (i.e. not invertible) as shown in (16). This indicates that temperature is not easily controlled using high gain PI controllers and thus extra measurements from the sensors are needed for control system stability. Examples of this are rate of measured output feedback sensors which are successfully deployed in the aerospace [15] and the electric motor based systems [17].

$$CB = \begin{pmatrix} b_{71}c_{17} & 0 & 0 \\ 0 & b_{82}c_{28} & 0 \\ b_{11}c_{31} & 0 & 0 \end{pmatrix} \qquad (16)$$

To make the CB invertible, rate of change of measured temperature was feedback and tracked i.e. $T_{cm} = \dot{T}_a$. With this new output variable, the following CB matrix is produced:

$$(CB) = \begin{pmatrix} b_{71}c_{17} & 0 & 0 \\ 0 & b_{82}c_{28} & 0 \\ b_{11}c_{31} & b_{82}c_{38} & b_{53}c_{35} \end{pmatrix} \qquad (17)$$

The constants shown in the matrixes above are functions of building parameters and operating conditions as follows:

$b_{71} = \frac{-(\bar{C}_a - \bar{C}_o)}{V_a}$, $c_{17} = 1$, $b_{82} = 1$, $c_{28} = k_L$,

$b_{11} = \frac{1}{\rho_a V_a c_{pa}}\left[P_f k_f - \rho_a c_{pa}(\bar{T}_a - \bar{T}_o)\right]$,

$d_{31} = \frac{1}{\rho_a V_a c_{pa}}\left[P_f k_f - \rho_a c_{pa}(\bar{T}_a - \bar{T}_o)\right]$,

$c_{35} = \frac{1}{\rho_a V_a c_{pa}}\left[A_s 2.13\left|\bar{T}_s - \bar{T}_a\right|^{0.31} + A_s \sigma F \bar{T}_s^3\right]$,

$c_{38} = \frac{k_e}{\rho_a V_a c_{pa}}$, $b_{53} = \frac{1}{\rho_s V_s c_{ps}}$ and

$$c_{31} = \frac{1}{\rho_a V_a c_{pa}}\begin{bmatrix} -A_s 2.13\left|\bar{T}_s - \bar{T}_a\right|^{0.31} - A_s \sigma F \bar{T}_a^3 \\ -U_{w1}A_{w1} - U_{w2}A_{w2} - U_{w3}A_{w3} \\ -U_{win}A_{win} - U_r A_r - (x\Delta\bar{T} + y\bar{v}_o)A_n \rho_a c_{pa} \\ -V_a \bar{n} \rho_a c_{pa} \end{bmatrix}$$

The cross coupling between the inputs and outputs in the CB matrix is indicated as follows:

$$\begin{matrix} & q_{mv} & P_L & \dot{Q}_h & \\ (CB) = & b_{71}c_{17} & 0 & 0 & CO_2 \\ & 0 & b_{82}c_{28} & 0 & Light \\ & b_{11}c_{31} & b_{82}c_{38} & b_{53}c_{35} & Temp \end{matrix} \qquad (18)$$

The CB matrix (18) is not diagonal and this means that some of the asymptotes are pointing at angles to the negative real axis of the root-locus. This means that when high gain is used, which is desirable to reject disturbances such as changes in solar gains and outside temperature, the system is more likely to become oscillatory in its response. It is clear that there is cross coupling between temperature and CO_2 controls via the MV. Also there is coupling between lighting and temperature modes through the heat emitted by the lighting power. However, as can be seen, the coupling between the temperature and CO_2 controls is only one way, i.e. rate of change of temperature control has no effect on CO_2 control. Thus, the heating system cannot respond faster to the change in ventilation. This is the reason for the non-diagonal terms in CB matrix. The CB matrix gives an indication of which properties of the building such as U values, wall area etc. embedded in the constants could be changed to give as near a diagonal matrix as physically possible and thus allowing simple PI controllers to be successfully utilised. As can be seen from the CB matrix that asymptote directions are affected by coupling of constants $b_{11}c_{31}$ and $b_{82}c_{38}$. These constants correspond to the building parameters as well as operating points. This shows that the stability is dependent on the operating internal & external temperatures, CO_2 levels, floor temperature, external wind speed and air exchange rate. The direction of asymptotes is also a function of the temperature difference between the internal and external operating temperatures ΔT. This indicates that as ΔT changes with the seasons, the stability of the system will be affected. In winter lighting power will be at its greatest and will have a greater impact on temperature control stability than in the summer season. It is also interesting to note that the thermal mass of the construction has NO impact on the asymptote direction, and thus no impact on the potential for high performance control of the system.

6.2 Transmission Zeros

The transmission zeros determine how well the system can be stabilised when using high gain feedback control [15]. The linear time-invariant state-space model equations (13) and (14) were used for the calculation of transmission zeros and their stability analysis over the operating range. The transmission zero locations are determined by the determinant (19) and can be readily solved using control system analysis software.

$$\begin{vmatrix} A - sI & B \\ C & D \end{vmatrix} = 0 \qquad (19)$$

There are in this case study five transmission zeros which if it is assumed that all walls have the same U value, density, thermal capacitance and thickness are given by:

$s_1 = 0, s_2 = a_{33}, s_3 = a_{33}, s_4 = a_{33}, s_5 = a_{66}$, where,

$$a_{33} = \frac{-2U_w A_w}{\rho_w V_w c_{pw}} = -5.33754 \times 10^{-6}, \quad (20)$$

$$a_{66} = \frac{-U_c A_c - U_{in} A_{in}}{\rho_c V_c c_{pc}} = -7.65 \times 10^{-6} \quad (21)$$

The transmission zeros are stable as they are always negative i.e. on the left half plane of the root-locus. In this case they are not a function of operating conditions such as outside temperature, but are a function of thermal properties; area, mass and thermal capacitance of the wall, concrete and insulation. This shows that the transmission zeros will always remain negative and therefore stable since the building parameters will never have negative values. There is a transmission zero at the origin i.e. s =0. This is due to rate of change of temperature being a function of temperature and thus we can only track temperature and not rate of change of temperature. To eliminate the effect of this undesirable transmission zero, a cascade control solution is required where temperature feedback is utilised to produce a rate of change of temperature command for the rate of change of temperature control system previously analysed. This control strategy is widely used with great effect in electric motor based systems [17].

7 CONCLUSIONS & FURTHER WORK

a) The CB matrix of the zone takes into account all the inputs and feedbacks of the control system. The matrix is rank defective due to temperature feedback and the thermal mass of the under-floor heating system. With the rate of change of temperature feedback the CB matrix is full rank. The CB matrix is not aligned diagonally which will make fast and accurate control very difficult with independent SISO controllers for temperature, light and CO_2. This undesirable coupling changes seasonally and throughout the day as the root-locus asymptote directions are a function of the operating conditions. By assessing the state-space model CB matrix, the suitability of the system for high performance control has been proven to be seasonal and not a function of the thermal mass of the construction.

b) Transmission zeros in the case study presented are all negative, thus stable except the zero at the origin. Its effect can be eliminated through a cascade control feedback loop to ensure stability, good tracking and robustness. The transmission zero locations will allow a high performance high gain control to be successfully utilised when simultaneously controlling, CO_2, lux and rate of change of room temperature with a MIMO controller.

c) The controllability analysis method described in this paper is sufficiently described for alternative zone formats to be easily analysed.

d) Currently work is being done to improve the model to include the effect of internal thermal mass, long-wave radiation, moisture and higher order representations of the multilayer walls and roofs.

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