A two-step optimization model for quantifying the flexibility potential of power-to-heat systems in dwellings

Gbemi Oluleye⁎, John Allison, Graeme Hawker, Nick Kelly, Adam D. Hawkes

⁎ Corresponding author.
E-mail address: o.oluleye@imperial.ac.uk (G. Oluleye).

HIGHLIGHTS
- Flexibility quantification considers dimensions of time, energy and cost.
- Heat pumps integrated in dwellings to absorb surplus renewable electricity.
- Integrating thermal storage encourages surplus electricity absorption in summer.
- Prosumers analyzed heterogeneously.
- Number of active prosumers depend on the value placed on flexibility.

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ABSTRACT
Coupling the electricity and heat sectors is receiving interest as a potential source of flexibility to help absorb surplus renewable electricity. The flexibility afforded by power-to-heat systems in dwellings has yet to be quantified in terms of time, energy and costs, and especially in cases where homeowners are heterogeneous prosumers. Flexibility quantification whilst accounting for prosumer heterogeneity is non-trivial. Therefore in this work a novel two-step optimization framework is proposed to quantify the potential of prosumers to absorb surplus renewable electricity through the integration of air source heat pumps and thermal energy storage. The first step is formulated as a multi-period mixed integer linear programming problem to determine the optimal energy system, and the quantity of surplus electricity absorbed. The second step is formulated as a linear programming problem to determine the price a prosumer will accept for absorbing surplus electricity, and thus the number of active prosumers in the market.

A case study of 445 prosumers is presented to illustrate the approach. Results show that the number of active prosumers is affected by the quantity of absorbed electricity, frequency of requests, the price offered by aggregators and how prosumers determine the acceptable value of flexibility provided. This study is a step towards reducing the need for renewable curtailment and increasing pricing transparency in relation to demand-side response.

1. Introduction

1.1. Background

The UK is required to generate 15% of energy from renewable sources by 2020, under the EU Renewable Energy Directive 2009 [1]. To help meet this target, UK electricity suppliers need to increase contribution from renewable energy sources. Significant uptake of renewables can cause grid balancing issues, and high penetration of variable energy resources makes the electricity price more volatile due to difficulty of predicting power generation [2].

Difficulties in operating and balancing the electricity transmission system are predicted for when the installed capacity of solar PV feeding into the grid is above 10 GW in the UK [3]. Then with 22 GW of uncontrollable PV into the grid, summer peak PV generation together with anticipated baseline generation from nuclear could exceed demand [3]. Therefore, large amounts of curtailed renewable energy (especially from solar and wind) could exist [4]. For the low-voltage distribution network, fluctuating supply voltage levels and network overload are challenges of surplus renewable generation [5]. There is a need for new
In recent years, Germany is experiencing frequent and significant negative prices due to surplus renewable generation [6]. The effect of negative prices is that consumers will be offered money to use electricity during surplus periods. For example over the 2017 Christmas period, consumers were offered 4.4 p/kWh in Germany [6]. Extreme positive and negatives price spikes is observed in the arithmetic levy model presented in Fanone et al., [7] for German intra-day electricity prices. The negative prices occurred during periods of surplus electricity from renewable sources. A negative marginal effect in electricity price is also observed in the work of Martin De Lagarde and Lantz, [8]. They observed that renewable electricity has a significant impact on the frequency and expected duration of negative price periods. Previous research focused on shifting demand from peak to off-peaks period to address extreme positive price spikes [9]. To the best of our knowledge, researchers are yet to address providing flexibility during negative price spikes, especially in determining how much consumers are willing to accept for surplus electricity absorption.

Demand side management strategies, energy storage systems, flexible fossil fired generating units, and non-thermal units are main topics studied by researchers for enhancing the ability of the energy system to cope with large scale intermittent RES integration [10]. More recently advanced technologies for demand-side management such as power-to-gas and power-to-heat have been proposed as new flexibility providers for addressing surplus renewable generation [11]. Power-to-heat systems are considered promising because the costs of generating heat from electricity and the costs of heat storage are relatively low [11]. In these systems, flexibility could be provided when heat is generated by passing an electric current through a resistor, or using an electricity-powered heat pump, including the possibility of storing the heat for later use [12]. Power-to-heat systems can reduce surplus generation from 40% to 10% [13], and about 50% reduction in renewable curtailment is possible [9]. This means buildings energy systems can be an integral part of demand-side management in the future. KiviLuoma and Meibom, [14] find that power-to-heat options coupled with TES can be more cost-effective than the grid integration of electric vehicles. Power-to-heat systems in dwellings could significantly lower the costs of renewables integration [15]. A major barrier to deployment of power-to-heat systems is high capital costs [16]. Quantifying and placing value on the flexibility service offered by power-to-heat systems could serve as an intrinsic fiscal incentive.

Even though power-to-heat systems have received interest in recent years as a sink for surplus renewable generation, researchers are yet to quantify this form of flexibility in terms of energy, time and costs, especially where homeowners are heterogeneous prosumers actively participating in a flexibility market. There has been a tendency to group prosumers under aggregator agents to overcome market barriers, neglecting the heterogeneity of prosumers. The EU recommends putting consumers at the centre of energy markets ensuring they are better protected and empowered [17]. Therefore, market arrangements need to be modified to reward flexibility providers for benefits to the system [18].

The first step towards modification is developing a robust method to quantify flexibility provided by power-to-heat systems integrated in dwellings. A robust quantification strategy can be useful to compare...
other systems capable of absorbing surplus electricity. Such a method should take the below into account (Fig. 1): (1) frequency of requests to absorb surplus renewable generation from the distribution system operator (DSO) via an aggregator, (2) the price an aggregator is offering to pay for flexibility service i.e. aggregator flexibility price, (3) the optimal quantity of renewable generation a prosumer can absorb, (4) the price a prosumer is willing to accept for flexibility provided (i.e. the prosumer acceptable flexibility price), and (5) the number of prosumers that can actively participate in the new market. In this work, the aggregator flexibility price is referred to as the aggregator price signal (APS) and the prosumer acceptable flexibility price is referred to as the Prosumer Price Signal (PPS). A typical low voltage distribution network has on average four feeders, each with 100 consumers [19], thus giving an idea of the possible number of prosumers that may participate in the flexibility market.

No existing studies have quantified the flexibility potential of power-to-heat systems taking the above into account [11]. Another novelty of this work is that the prosumer heterogeneity is maintained whilst determining the optimal surplus electricity absorbed, and the prosumer acceptable flexibility price. Accounting for prosumer heterogeneity increases the complexity of the problem, hence the novel quantification framework developed uses a two-step optimization framework. This novel framework can compare other forms of flexibility services for absorbing surplus renewable generation. Paying for flexibility provided by power-to-heat systems could incentivize integration of decentralized residential heat pumps, which are expected to provide 60% of residential heat in the UK in decarbonisation scenarios [20].

1.2. Literature review

Electrification of heat using heat pumps can deliver less carbon emissions when compared with conventional options [16]. However, a major barrier to implementation is cost. Previous research on power-to-heat systems focused on performance assessment, design and operational optimization [21]. Hong et al., [5] studied the potential of heat pumps to enhance demand flexibility by shifting the operating times from peak to off-peak periods. Patteeuw et al., [9] measured how much operational costs associated with electricity import can be reduced by widespread application of heat pump load shifting. Even though incentives such as direct load control and dynamic time-of-use pricing is applied, reduction in electricity import cost was not enough to cover majority of the heat pump capital investment. None of the aforementioned authors considered the flexibility of heat pumps during periods of negative prices associated with surplus electricity generation.

Coupling heat pumps with Thermal Energy Storage (TES) can increase their performance and reliability in meeting heat demand [21]. Integrating TES also reduces the operational cost of the heat pump system [22]. The use of TES to improve the reliability of electricity networks is not new. Grid coupled TES have been investigated to allow re-shaping demand to better match variable supply from renewable sources [23]. Vorushylo et al., [16] studied the use of TES in managing increases in peak electricity demand. The work of Rodriguez et al., [24] focus on achieving significant peak reductions on the grid by load shifting from peak to off-peak periods using a dynamic electricity pricing profile. Arteconi et al., [22] also coupled TES with heat pumps to shift electrical loads from high-peak to off-peak periods. Again, absorbing surplus electricity during periods of negative prices is not addressed. During period of negative prices, prosumers are offered money to absorb surplus electricity [6]. The money offered can incentivize heat pumps and TES. Therefore, there is need to determine how much a prosumer is willing to accept, and the number of prosumers participating in such flexibility market. The number of prosumers participating is useful for determining the market size of heat pumps.

The provision of flexibility to the distribution grid by distributed energy resource owners especially prosumers has received attention recently [25]. A two-step deterministic real-time economic zonal dispatch algorithm is proposed by Thatte and Xie, [26] to address flexibility provision from distributed energy resources. Again, addressing deficit generation is the focus of their work. Zhang and Kezunovic [27], propose using electric vehicles to provide flexibility in markets on the distribution grid. The indices proposed to quantify the potential are not adaptable to power-to-heat systems. A deterministic MILP cost minimization model is used in Roos et al., [28] to allocate prosumer flexibility among markets. The model considers grouping prosumers under a load aggregator participating both in the wholesale power market and tertiary regulation capacity market. Grouping prosumers neglects their
heterogeneous nature. Therefore, accounting for prosumer heterogeneity requires reviewing the main aggregator tasks, including consideration of local flexibility providers in local distribution grids, with the possibility of exporting to the rest of the system. Furthermore, the focus of their work is load reduction for electricity deficit, again surplus generation is not addressed.

Siebert et al., [29] propose the use of a MILP model to manage flexibility services from the point of view of the aggregator, for services provided by commercial and industrial sites with energy storage systems. The main decision variables were the storage activation times and duration. The cost dimension of flexibility is not addressed, and in terms of technology batteries are considered. Zhou et al., [30] propose a deterministic MILP problem for an integrated community energy system in a joint energy and ancillary service markets. The integrated community energy system aggregates diverse distributed energy resources, making the model unsuitable for individual prosumers. Furthermore, surplus renewable generation is not addressed.

In Barton et al., [31] options for handling power surpluses are ranked based on heuristics. They are: storage charging, fuel replacement in industry, power-to-heat and hydrogen production. There is a need to formulate an optimal quantification criteria to rank all options systematically. Olivella-Rosell et al., [32] propose an MILP optimization model for scheduling flexible resources to meet distribution system operator requests. The flexibility resources were load, generators and batteries. Options considered for addressing surplus generation are shiftable loads, charging batteries, and disconnecting or reducing photovoltaic generators, power-to-heat is not considered. Furthermore, the price a distributed energy resource owner is willing to accept for flexibility offered is neglected as the aggregator fixes the price.

Finck et al., [33] considered a single office building integrated with a heat pump, an electric heater and thermal energy storage tanks. The flexibility dimension of energy is measured using the available storage capacity and storage efficiency, neglecting the optimal quantity of additional electricity absorbed. The flexibility dimension of costs is measured using an indicator that relates to electricity costs during operation. The price prosumers are willing to accept is not estimated, making the method unsuitable for quantification of flexibility from power-to-heat systems.

Stinner et al., [34] considered the flexibility potential of building energy systems with thermal energy storage. Again, flexibility is only analyzed in terms of time and energy related to the thermal energy storage unit. Furthermore, they focused on assessing the flexibility of coupling thermal energy storage to low carbon technologies. The available storage capacity, storage efficiency and power-shifting capability is applied in Reyniers et al., [35], neglecting the quantity of surplus electricity absorbed and the price a prosumer is willing to accept. Hedegaard et al., [36] analyzed the potential of heat pumps and thermal energy storage to absorb surplus electricity from wind power. However, optimal sizing of the heat pump is not considered and analysis is made for aggregated building stocks with an assumption of heat pumps already installed. The estimation of a prosumer acceptable flexibility price and the number of prosumers actively absorbing surplus electricity is not done.

In previous studies the heterogeneous nature of prosumers is not accounted for, as all dwellings are assumed to accept the same flexibility value when grouped under aggregator agents. Heterogeneity is accounted for in this work by each prosumer having its own Prosumer Price Signal (Fig. 1). Each unique PPS is then assessed against an aggregator price signal to determine the number of active prosumers.

Furthermore, even though coupling electricity with heat has received attention in recent years, researchers are yet to quantify this flexibility provided with respect to time (i.e. frequency of request to absorb surplus generation), energy (i.e. optimal quantity a prosumer can absorb) and costs (i.e. the fee a prosumer is willing to accept to absorb surplus generation). Flexibility quantification is a step towards a transparent flexibility value that compensates for demand response. This is required to enable prosumers to make informed investment decisions in order to participate in the market. This work begins with quantifying the potential of power-to-heat systems as an advanced technology for demand-side management, future work will consider other technologies such as electric vehicles and power-to-gas.

1.3. Contributions of this work

The main contribution of this paper is the development and application of a two-step optimization framework to quantify the flexibility potential of power-to-heat systems in dwellings. The three dimensions of flexibility (i.e. time, energy and costs) are considered in the

![Fig. 2. System Layout. Blue lines denote flow of electricity. Red lines denote flow of heat. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)](image-url)
framework. The method proposed is significant for improving the capacity of power systems to maintain balance when generation exceeds load. The two-step comprises the formulation of:

1. A multi-period MILP problem as the first level to address the time and energy dimension
2. A linear problem as the second level to address the costs dimension

Other innovations are:

1. Prosumer heterogeneity is considered.
2. Wide range of flexibility values considered through the analysis of 445 prosumers.
3. The quantification methodology allows a general comparison of other options for flexibility services to absorb surplus generation. For example, electric vehicles.
4. The methodology accounts for price offered by aggregators in estimating the number of active prosumers in the flexibility market.

2. Methodology

The mathematical formulation of the two-step optimization framework is presented in this section. A two-step optimization framework was chosen to reduce the complexity of the computational domain, since the prosumers heterogeneity is taken into account. The system under consideration is for a dwelling with natural gas boilers already installed for heating, and grid import to satisfy electricity demand. We propose integrating power-to-heat systems to absorb surplus electricity generation. The proposed energy system layout for each prosumer is provided in Fig. 2. Under a scenario of surplus electricity generation, a mono-energetic system is considered, where a mono-energetic air source heat pump system (including a back-up resistive heater) and thermal energy storage are integrated to absorb surplus electricity from the grid. The system boundaries are defined as shown in Fig. 2. The mathematical optimization is defined to determine the optimal system i.e. technology sizes and number, dispatch strategies, and associated costs, in particular the price a prosumer accepts to absorb surplus electricity (the prosumer acceptable flexibility price as shown in Fig. 1). Where there is more than one prosumer, as is the case where multiple homes are connected to a low voltage distribution network, the optimal system in Fig. 2 is determined for each prosumer.

The mathematical formulation of the two-step optimization framework is presented in Section 2.2. A description of the problem is presented in Section 2.1. The modelling in this work considers only the dwelling energy system. Modelling of the electricity generation system is beyond the scope of this work.

2.1. Problem statement

A more specific definition of the design problem is:

- Given:
  - The energy demands of a dwelling: space heating, hot water and electricity provided in 5 min time slices for all the days in a year [23]. The electricity demand is for combined appliances, electronics and lighting.
  - Energy prices (electricity and fuel), electricity tariff structure, technology costs.
  - Technologies such as a boiler and heat pumps, and active thermal energy storage. A mono-energetic system is considered, where a heat pump is complemented by a back-up resistive heater.
  - Frequency of request to absorb surplus renewable from the DSO via an aggregator.
  - The aggregator flexibility price (Fig. 1), termed the Aggregator Price Signal (APS).
- Determine:
  - Optimal base case energy system design of a gas boiler satisfying the heat demand, with electricity import from the grid
  - Optimal energy system design for integration of power-to-heat systems, i.e. the ASHP and TES capacity, technology and system dispatch and technology contribution to heating
  - Techno-economic assessment of the base case energy system, and system with power-to-heat technologies
  - Optimal quantity of absorbed surplus renewable electricity
  - The prosumer acceptable flexibility price (Fig. 1), called the Prosumer Price Signal (PPS)
  - Number of prosumers that can actively participate in the market
    - Subject to:
      - Energy (both heat and electricity) balances
      - Technology capacity constraints
      - Constraints related to requests to absorb surplus generation
    - In order to:
      - Minimise the equivalent annual cost of meeting a dwelling energy demand (step 1)
      - Maximise revenue obtained from absorbing surplus renewable generation (step 2)

A detailed building model was developed in ESP-r to determine the space heating, hot water and electricity demands [23]. The demands are evaluated based on building geometry, thermal characteristics and weather forecasts. The hot water is point-of-use power based on a 50 °C supply temperature. ASHP are considered because of low cost of installation and the lack of a requirement for ground works.

2.2. Mathematical formulation for the two-step optimization model

The multi-period MILP model for the first step is presented in Eqs. (1)-(20). The model determines: (i) the optimal base case system consisting of a gas boiler, (ii) the optimal system where power-to-heat is integrated to absorb surplus electricity generation based on requests, and (iii) energy flows, economics and dispatch for (i) and (ii). The optimal design is selected based on minimising the Equivalent Annual Cost (EAC) in Eq. (1). The EAC is a sum of the annualised capital cost, fuel costs for operating the boiler, technology maintenance costs and electricity import cost to satisfy electricity demand.

Min: \[ ACC + FC + MC + W_{IMP,TOT}\]  \hspace{1cm} (1)

The degrees of freedom are the independent variables specified in the Nomenclature section.

Eqs. (2)-(5) are a breakdown of the objective function. The annualised capital costs (Eq. (2)) takes into account sizing of the air source heat pump \( j \).

\[
ACC = \left\{ \sum_{i} \left( \sum_{j} \left( \text{size}_i \times Y_{ij} \times IC_{j}\right) \right) + \sum_{j} \left( \text{size}_j \times Y_{ij} \times IC_{j}\right) \right\} \times AF \tag{2}
\]

\[
FC = \sum_{i} \sum_{r} \sum_{j} \left( \frac{Q_{ij,per} + Q_{ij,rel}}{perf} \right) \times t_{r} \times NGP \tag{3}
\]

The boiler fuel cost is estimated using Eq. (3). During period of surplus electricity, the boiler may be switched off, since heating demands can be satisfied using the heat pump.
MC = \sum_{l_r} \left\{ \sum_j \left( (Q_{1,l,r} + Q_{2,l,r}) \times t_{r} \times MC_l \right) \\
+ \sum_j \left( (Q_{1,l,r} + Q_{2,l,r}) \times t_{r} \times MC_l \right) \\
+ ((Q_{out,l,r} + Q_{out,l,r}) \times t_{r} \times MCTES) \right\} \\
W_{IMP,TOT} = \sum_{l_r} \left\{ \left( \sum_j \left( W_{IMP}^{(1)} + W_{IMP}^{(2)} \right) \times GIMP \times t_{r} \right) \right\} 

(4)

The total capital cost for the system lifetime is annualised using the annuity factor in Eq. (6). Design is made for each prosumer l considered, thus accounting for prosumer heterogeneity.

AF = \frac{IR \times (1 + IR)^n}{(1 + IR)^n - 1} 

(6)

The feasible region in which the model operates is described below: energy balance for hot water and space heating is provided in Eqs. (7) and (8) respectively over every time slice r for all prosumers l. Heat is produced to satisfy demand, and any surplus diverted to storage. Charging and discharging the store is limited by energy balance constraints.

\[ \sum_{j} Q_{1,l,r} + \sum_{j} Q_{2,l,r} + Q_{out,l,r} - Q_{in,l,r} = 0 \quad \forall \ r \in R, \ l \in L \]

(7)

\[ \sum_{j} Q_{2,l,r} + \sum_{j} Q_{2,l,r} + Q_{out,l,r} - Q_{in,l,r} = 0 \quad \forall \ r \in R, \ l \in L \]

(8)

The heat produced by the boiler and ASHP is limited by their capacity:

\[ Q_{1,l,r} + Q_{2,l,r} \leq SIZEL \quad \forall \ l \in L \]

(9)

\[ Q_{1,l,r} + Q_{2,l,r} \leq SIZEL \quad \forall \ j \in J, \ r \in R, \ l \in L \]

(10)

Implicit constraints for existence and operation of the ASHP are provided in Eqs. (11)-(13). The same equations apply for the boiler.

\[ Q_{1,l,r} + Q_{2,l,r} \leq L \times Z_{1,l,r} \quad \forall \ j \in J, \ r \in R, \ l \in L \]

(11)

\[ Q_{1,l,r} + Q_{2,l,r} \leq U \times Z_{1,l,r} \quad \forall \ j \in J, \ r \in R, \ l \in L \]

(12)

\[ Z_{1,l,r} \leq L \quad \forall \ j \in J, \ r \in R, \ l \in L \]

(13)

The use of integer variables (Z_{1,l,r}) for technology operation means a greater level of realism for technologies dispatch is achieved. The integer variable (Y_{r,l}) determines the existence of the air source heat pump. When there are no requests to absorb surplus renewable generation, the integer variable (Y_{r,l}) related to the existence of the ASHP is 0. For requests made in summer/winter, the integer variable related to the operation of the ASHP (Z_{1,l,r}) is 1 in summer/winter only.

Eqs. (14) and (15) define the operation of the TES and ensure charging/discharging the store do not occur simultaneously in any time slice r. Similar equations apply for space heating.

\[ Q_{out,l,r} \times U_{TES} \times YTES_{l,r} \leq 0 \quad \forall \ r \in R, \ l \in L \]

(14)

\[ Q_{in,l,r} \times U_{TES} \times (1 - YTES_{l,r}) \leq 0 \quad \forall \ r \in R, \ l \in L \]

(15)

Eq. (16) is formulated to trigger storage operation when the ASHP operates.

\[ YTES_{l,r} \times Z_{1,l,r} \leq 0 \quad \forall \ j \in J, \ r \in R, \ l \in L \]

(16)

In each time slice r, the energy content of the store is subject to constraints in Eqs. (17) and (18):

\[ 0 \leq \sum_{r} \left( \sum_{j} \left( Q_{in,l,r} \times n_{charge} \right) - \frac{Q_{out,l,r} \times \theta}{24} \times t_{r} \right) \right\} + (QTES_{l} \times ISH) \leq QTES_{l} \quad \forall \ r \in R, \ l \in L \]

(17)

\[ 0 \leq \sum_{r} \left( \sum_{j} \left( Q_{2,l,r} \times n_{charge} \right) - \frac{Q_{2,l,r} \times \theta}{24} \times t_{r} \right) \right\} + (QTES_{l} \times ISH) \leq QTES_{l} \times ISH + 0.1 \]

(19)

\[ (QTES_{l} \times ISH) - 0.1 \leq \sum_{r} \left( \sum_{j} \left( Q_{2,l,r} \times n_{charge} \right) - \frac{Q_{2,l,r} \times \theta}{24} \times t_{r} \right) \right\} + (QTES_{l} \times ISH) \leq QTES_{l} \times ISH + 0.1 \]

(20)

The MILP problem for the first optimization step is expressed in Eqs. (1)–(20) above.

The linear problem for the second step determines: (i) the prosumer acceptable flexibility price i.e. the Prosumer Price Signal (PPS), and (ii) the number of prosumers that can actively participate in the new market. The objective function for the second optimization step is the maximization of the revenue obtained from absorbing surplus generation (Eq. (21)). The surplus electricity absorbed by the power-to-heat system is determined in Step 1.

\[ \text{Max: } \sum_{l} \left[ \sum_{r} \left( \sum_{j} \left( W_{IMP}^{(1)} + W_{IMP}^{(2)} \right) \times t_{r} \right) \right] \times PPS_{l} \]

(21)

The PPS will depend on the investment in the power-to-heat system absorbing surplus renewable generation. A prosumer may want to fix a PPS such that all investment in the power-to-heat system is paid for less operational savings from not using a boiler. In this case, the savings from not using a boiler during period of requests is ploughed back. Thereby constraining the objective in Eq. (21). The constraint is presented below:

\[ \sum_{r} \left[ \sum_{j} \left( W_{IMP}^{(1)} + W_{IMP}^{(2)} \right) \times t_{r} \right] \times PPS_{l} \geq ACC_{j,l} + MC_{j,l} - FC_{j,l} \]

(22)

Solving Eqs. (21) and (22) sets the minimum acceptable price for absorbing surplus generation.

A prosumer may want to also fix a PPS such that all investment in the power-to-heat system is paid for. Where the savings from not using a boiler is not ploughed back. Thereby constraining the objective in Eq. (21). The constraint is presented below:
sectional outputs for the electricity absorbed is also determined. Eqs. (21)–(23) provide output for the techno-economic assessment model. The above model was formulated in GAMS 24.7.3, and solved using Lindo Global solver on a 64 bit 3.40 GHz Intel * Core™ i7-6700 CPU with 32 GB RAM.

3. Case study

The case study considers 445 prosumers. A data set of 445 prosumers will show the wide spread in flexibility quantification for a typical urban area in the UK. The prosumer demand profiles were obtained from detailed simulations using ESP-r tool. Each house model includes explicit details of hot water draw profiles, internal heat gains, heat control equipment and air tightness. Detailed description of the simulation model is found in Allison et al., [23]. The 445 prosumers represent a typical urban settlement in the UK with a spread of four UK housing types (i.e. detached, semi-detached, terrace and flat) with: (1) 1–3 bedrooms, (2) family sizes (with 1–2 adults and 1–2 children), (3) working age and retired adults, and (4) full time, part-time and non-working adults. The peak and total space heating, hot water and electricity demand are presented in Fig. 3 below.

The energy demand profiles for a year is shown in Fig. 4 for all prosumers.

The two-step methodology will be applied to quantify the potential to absorb surplus electricity in year 2018. The quantification approach takes into account dimensions of: (i) time, related to the requests to absorb surplus generation, (ii) energy, related to the optimal quantity of surplus electricity absorbed, and (iii) cost, related to the price a prosumer is willing to accept for absorbing surplus generation. Therefore, four design cases are considered to reflect the frequency of requests for surplus electricity absorption: (1) Base case for a business as usual system using a boiler for heating and electricity imports to satisfy demand, here no requests are made, (2) Case A for surplus electricity absorption requests in summer, (3) Case B for surplus electricity absorption in winter, and (4) Case C, an ideal case that assumes surplus electricity is available throughout the year. These cases are selected to determine the impact of request frequency and quantity of absorbed electricity on the Prosumer Price Signal (PPS), and number of active participants. Assumptions on technology prices and the energy market are provided in the Appendix.

4. Results and discussion

The technology contribution to heating and economics of the design cases considered are in Section 4.1. Results in Section 4.1 were obtained by solving the multi-period MILP problem defined in Eqs. (1)–(20). Section 4.2 contains results related to the optimal quantity of surplus electricity absorbed, the minimum and maximum PPS, and the number of active prosumers. The optimal quantity of surplus electricity absorbed was estimated using the multi-period MILP problem in Eqs. (1)–(20), while the PPS and number of active prosumers was obtained...
by solving the linear problem in Eqs. (21)–(23).

4.1. Techno-economic results

The energy flows through the boiler, ASHP and TES is a degree of freedom in the systematic framework. This is used to estimate the contribution to heating from the three options. For the base case, all the heat is from the boiler, since there are no requests to absorb surplus electricity.

The contribution from the power-to-heat system consisting of the ASHP and back-up resistive heater increases from Case A to Case C (Fig. 5). In case A, requests to absorb surplus electricity is received during the summer months, but demand for space heating during summer is low (Fig. 4). The integration of TES allows for surplus electricity absorption during periods of low demand.

In case B, requests to absorb surplus electricity occurs during winter. Since the total space heating and hot water demand is higher in winter, the proportion of heating from the power-to-heat system is greater in Case B (compared to case A). Case C is an ideal case were the requests to absorb surplus electricity occur throughout the year, hence Case C has the highest proportion of heating from the power-to-heat system. In this ideal case, the gas boiler is turned off.

The Equivalent Annual Cost (EAC) for operating both the boiler and ASHP for all prosumers is shown in Fig. 6. In general, prosumers with high demands for heating and electricity, have a higher EAC, except where their peak demands are low. In

Fig. 3 some prosumers with low total energy demand have a higher peak demand compared to prosumers with a higher total energy demand. In Section 4.2 the value of electricity absorbed will be determined using the second optimization step, in order to reduce investment in the ASHP.

4.2. Flexibility quantification

Surplus electricity absorbed by the power-to-heat system for a typical day in summer and winter for a house with full time working individuals is shown in Figs. 7 and 8 respectively. In case B, absorption is allowed in winter only and Case A; only in summer. The additional electricity absorbed satisfies the peak thermal demand. The peaks in surplus electricity absorption (Fig. 7) coincide with heat demand peaks. Even though surplus electricity is not absorbed in every hour in the day, the quantity absorbed when converted to heat is able to satisfy the daily heat demand, due to the integration of TES.

The total electricity absorbed by all houses increases from Case A to Case C (Fig. 9). Case C is an ideal case that assumes surplus renewable electricity is available throughout the year; hence the peaks in Fig. 9 (case C).

The optimal quantity of surplus electricity absorbed in summer, winter and throughout the year represents the energy dimension of flexibility. The cost dimension of flexibility is quantified using the prosumer acceptable flexibility price (Fig. 1). The Prosumer Price Signal (PPS) is the most important criterion to determine the number of
active prosumers in a flexibility market. The minimum and maximum PPS for all prosumers are determined in this work using Eqs. (21) and (22), and Eqs. (21) and (23) respectively. A prosumer may only absorb surplus electricity if the price offered can offset investment in the power-to-heat system.

The minimum PPS is determined for a scenario where the prosumer ploughs back savings in fuel during periods of requests to absorb surplus electricity since the boiler will be switched off. The savings in fuel is dependent on the total heating demand during period of requests. Thus fuel savings increases from Case A to Case C. The minimum PPS

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**Fig. 5.** Proportion of heat from the technologies and thermal energy storage.

**Fig. 6.** Energy system economic output.
Electricity absorbed also reduces as the quantity of surplus electricity absorbed increases (Fig. 10a–c). The graphs in Fig. 10 were zoomed in by limiting the minimum PPS to 50 p/kWh (y-axis) and the surplus electricity absorbed to 2000 kWh/y (x-axis).

The total quantity depends on the frequency of absorption requests by the DSO via an aggregator (Fig. 1). During summer, where the surplus electricity absorbed is limited by heat demand; the minimum PPS ranges from 0 to 1800 p/kWh, the upper limit reduces to 150 p/kWh for winter absorption (Case B), and 6 p/kWh for an ideal case (Case C). The number of active prosumers will depend on the aggregator price signal (APS). In 2018, 35 p/kWh was offered for surplus electricity [37]. Therefore in this study, an APS from 0 p/kWh to 50 p/kWh (Fig. 11) is used to determine the number of active prosumers. Determination of active prosumers is significant for determining early adopters of power-to-heat systems.

A prosumer will participate in the flexibility market if the PPS is less than or equal to the price offered by aggregator (i.e. APS). The number of active prosumers participating in the flexibility market increases with the APS (Fig. 11), and the frequency of requests. Case B has a higher number of active prosumers because more electricity is absorbed in winter due to a higher heat demand, resulting in a better use of the power-to-heat system. Note that the investment cost for the power-to-heat system is the same for all cases. About 53 and 105 active prosumers in Cases A and B respectively, are willing to absorb surplus electricity at an APS of 0 p/kWh (Figs. 10 and 11). For these prosumers, the fuel saved by not using natural gas boilers during periods of requests to absorb surplus generation is greater than the annualised capital investment in power-to-heat systems. The number of active prosumers in Case B reduces from 136 (based on minimum PPS in Fig. 11) to 44 (based on maximum PPS in Table 2). The surplus electricity absorbed reduces by 77%. For the same APS (i.e. 2 p/kWh), the number of active prosumers in Case B reduces from 136 (based on minimum PPS in Fig. 11) to 44 (based on maximum PPS in Table 3). The surplus electricity absorbed reduces by 56%. Again, the TES capacity required increases with the total absorbed electricity (Table 3).

There are more opportunities to absorb surplus electricity in winter due to a high demand for heat; however, integrating TES makes absorption in summer possible, reflected by the number of active prosumers (Tables 2 and 3).

The number of active prosumers is limited by the frequency of requests, the value of the APS, and how a prosumer determines the PPS (i.e. maximum or minimum). In general more electricity is absorbed when requests are made more frequently in winter (Fig. 13). The number of active prosumers determined based on the maximum PPS is lower compared to the minimum PPS, hence the absorbed electricity is low (Fig. 13).

Based on these results, the flexibility offered by power-to-heat systems in dwellings can reduce renewable curtailment. Furthermore, paying for the flexibility by distribution system operators can intrinsically incentivise electrification of heat and thermal energy storage in dwellings. The methodology proposed for quantifying the flexibility of power-to-heat systems integrated in dwellings can be applied to quantify other forms of flexibility (for example electric vehicles and power-to-gas) especially when they address surplus electricity.
absorption. The methodology can also be applied with different demand profiles. New demand can be included in Eqs. (7) and (8). For future housing stock, insulated to passive house standards, improved insulation levels will reduce demand, however integration of TES ensure power-to-heat system flexibility is still possible.

In 2018, Octopus Energy offered to pay 35 p/kWh to consumers to turn up demand during periods of surplus electricity [37]. This can be seen as an opportunity to integrate power-to-heat systems in dwellings for surplus electricity absorption. If the method in this work is applied to the Octopus Energy case, where there are 445 prosumers connected to a low voltage distribution network, only 207 will actively participate if the offer is made in summer and 428 in winter. The proposed strategy can be implemented by installing a local controller in each participant dwelling in order to receive and apply requests and price signals from

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Fig. 9. Annual electricity absorption profile for all prosumers.

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Fig. 10. Minimum Prosumer Price Signal (PPS).
an aggregator (or service provider), such as a smart meter. However, the extent to which existing "smart" meters can support the required communications to property implement power-to-heat systems as flexibility providers is out of the scope of this article.

The findings of this work need to be viewed alongside the assumptions on energy demand, investment costs in the power-to-heat systems, and energy prices. The energy demand data set consist of 445 prosumers, resulting in a wide spread in flexibility quantification. Even though the assumptions have been taken from reputable sources, they are subject to variations. However, detailed uncertainty quantification of the parameters and model used is out of the scope of this article.

5. Conclusions and future works

The capability to maintain balance between load and generation, especially when generation exceeds load becomes more challenging with increasing penetration of variable energy resources in the electricity system. Absorption of surplus electricity by power-to-heat systems can offer flexibility to the electricity system. The present work has determined the price a prosumer is willing to accept to absorb surplus renewable electricity using a novel two-step optimization framework. Additionally, prosumer heterogeneity is accounted for.

The novel methodology is applied to a case study consisting of 445 prosumers. The time dimension of the flexibility offered by power-to-heat systems is accounted for through the use of scenarios to depict requests for surplus electricity absorption in summer, winter and throughout the year. The energy dimension is accounted for by determining the optimal quantity of surplus electricity absorbed. The cost dimension is accounted for by estimating the price a prosumer will accept to absorb surplus electricity using a novel two-step optimization framework. Additionally, prosumer heterogeneity is accounted for.

Table 1

<table>
<thead>
<tr>
<th>APS (p/kWh)</th>
<th>Total TES capacity (kWh)</th>
<th>Total TES capacity (kWh)</th>
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<tbody>
<tr>
<td>0</td>
<td>26</td>
<td>190</td>
</tr>
<tr>
<td>2</td>
<td>39</td>
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<td>5</td>
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<td>289</td>
</tr>
<tr>
<td>11.6</td>
<td>68</td>
<td>513</td>
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<td>136</td>
<td>681</td>
</tr>
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<td>50</td>
<td>151</td>
<td>690</td>
</tr>
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</table>

Table 2

<table>
<thead>
<tr>
<th>APS (p/kWh)</th>
<th>Number of active prosumers</th>
<th>Total electricity absorbed (kWh/y)</th>
<th>Total TES capacity (kWh)</th>
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<tbody>
<tr>
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<td>18</td>
<td>1852</td>
<td>6</td>
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<tr>
<td>2</td>
<td>34</td>
<td>16,410</td>
<td>16</td>
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<td>5</td>
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</tr>
<tr>
<td>35</td>
<td>174</td>
<td>107,970</td>
<td>110</td>
</tr>
<tr>
<td>50</td>
<td>219</td>
<td>114,781</td>
<td>142</td>
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</table>

Table 3

<table>
<thead>
<tr>
<th>APS signal (p/kWh)</th>
<th>Number of active prosumers</th>
<th>Total electricity absorbed (kWh/y)</th>
<th>Total TES capacity (kWh)</th>
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</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
<td>310</td>
<td>17</td>
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<tr>
<td>2</td>
<td>44</td>
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<td>5</td>
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<td>496</td>
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<td>35</td>
<td>413</td>
<td>338,370</td>
<td>667</td>
</tr>
<tr>
<td>50</td>
<td>426</td>
<td>340,475</td>
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</table>
accept for absorbing surplus electricity (i.e. the prosumer price signal), and the number of active prosumers in the flexibility market. Results show that the Prosumer Price Signal (PPS) depends on the frequency of requests to absorb surplus electricity, and quantity of surplus electricity absorbed. The PPS ranges from 2000 p/kWh to 6 p/kWh. An Aggregator Price Signal (APS) from 0 to 50 p/kWh was used to determine active prosumers, and the impact on absorbed electricity. For a prosumer to participate in the flexibility market, the PPS must be less than or equal to the APS. Out of the 445 prosumers considered, 207 participated based on an APS of 35 p/kWh in summer, and 428 based on the same price signal in winter. The associated absorbed electricity was on average 120 MWh/y in summer, 340 MWh/y in winter. The average absorption by active prosumers in summer and winter for the same APS are 580 kWh/y/prosumer and 810 kWh/y/prosumer respectively. Key findings of this work are: (i) the investment in the power-to-heat system and savings in boiler fuel during periods of surplus electricity absorption determines the PPS, (ii) the PPS reduces with increase in requests and the quantity of surplus electricity, (iii) the thermal storage capacity increases with the quantity of electricity absorbed, (iv) prosumers whose savings in boiler fuel is greater than the annualised investment in the power-to-heat system always participate in the flexibility market, and (v) The flexibility offered and subsequent value could help incentivise electrification of heat via air source heat pumps and integration of thermal energy storage in dwellings in a key limitation to prosumers willingness and ability to participate will be the frequency of request, quantity of electricity absorbed and the aggregator price signal. The quantification framework developed allows different options to be compared.

Future work will consider comparing power-to-heat in dwellings with power-to-heat in industry, and electric vehicles in order to provide complementary insights. Main barriers to the procurement of flexibility services from prosumers are lack of suitable smart metering and lack of suitable market arrangements. Roll-out of smart meters is the first step to contracting flexibility services from prosumers, but the smart meters in question should be sufficiently enabled and flexible to accommodate possible developments in markets for flexibility to ensure this resource is accessible.

Acknowledgements

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Appendix A

See Table A1.

Table A1

<table>
<thead>
<tr>
<th>Technical attributes</th>
<th>Natural gas boiler</th>
<th>ASHP (2 kW)</th>
<th>ASHP (5 kW)</th>
<th>ASHP (8 kW)</th>
<th>ASHP (14 kW)</th>
<th>Hot water tank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turnkey cost (£/kW)</td>
<td>163</td>
<td>1100</td>
<td>1000</td>
<td>900</td>
<td>700</td>
<td>20</td>
</tr>
<tr>
<td>Maintenance cost (£/kW)</td>
<td>0.001</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.01</td>
<td>0.001</td>
</tr>
<tr>
<td>Performance</td>
<td>0.895</td>
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<td>2.75</td>
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<td>Charge efficiency (%)</td>
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<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>90</td>
</tr>
<tr>
<td>Discharge efficiency (%)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>90</td>
</tr>
<tr>
<td>Initial store heat (%)</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>100</td>
</tr>
</tbody>
</table>

The design year is 2018. Off-peak and peak electricity import tariffs are 6.139 and 17.081 p/kWh, average value is 11.61 p/kWh, and fuel price is 4.209 p/kWh [38]. Technology lifetime is 15 years and discount rate 3.5%.

References


