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An Inverse-Problem Based Stochastic Approach to Model the Cumulative Damage Evolution of Composites

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Abstract

Fibre-reinforced composites are often selected for high-responsibility structural applications due to their high specific stiffness and strength, and corrosion resistance. These materials have a rather good rating as regards to life time in fatigue, but the same does not occur with the number of cycles to initial damage or with the evolution of damage.

Due to material heterogeneity, random nature of fatigue loading and random environment conditions, the damage phenomenon is a cumulative evolutionary stochastic process in essence, characterized by changes in laminate compliance with time. As a consequence, it is more appropriately examined within a probabilistic framework.

An approach that considers stochastic damage accumulation by means of discrete time Markov chains is proposed in this work to analyze damage evolution of composite laminates. Additionally a *unitary time transformation* by means of monotonic cubic Hermite splines has been introduced to take into account the nonstationary effects with a reduced set of model parameters. An inverse procedure is proposed to find the optimal stochastic model parameters together with the time transformation parameters.

A numerical comparative study of the above-proposed technique is presented.

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Selection

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1. Introduction

Due to the increase of applications of composite materials for engineering structures, effective methods to determine their long term response and reliability under damage conditions are needed. Deterministic (and phenomenological) methods are strongly dependent on experimental input and therefore they are difficult to extend outside laboratory conditions (Degriek and Paepegen 2001). Even under highly controlled laboratory conditions, a considerable amount of scatter can be observed in fatigue data due to material random properties, random spatial distribution of defects, and imperfections within the material structure. (Bogdanoff 1978; Madsen et al. 1986; Sobczyk 1987; Sobczyk and Spencer 1992)

For this reason a stochastic cumulative damage approach is more appropriate to understand the actual phenomena of damage accumulation at macroscopic level by means of Markov chains given that the damage accumulation assumes a Markovian type of evolution (Bogdanoff and Kozin 1985). This means that the *future* of the process only depends on its *present* state, which is independent of the *past*. This approach allows us to make predictions of the full-scale system based on the specimen's tests as well as to carry out theoretical (stochastic) considerations about the behaviour of the damaged composite material.

2. Methodology

2.1. Cumulative damage theory using Markov Chains

Damage accumulation is treated as an evolutionary stochastic process characterized by changes in the components of laminate compliance with time. The change in the k^{th} compliance component can be discretized into a finite number of states $i = 1, 2, \dots, b^{(k)}$ with the magnitude of change increasing with the index of the state. State $b^{(k)}$ represents the *absorbing* or ultimate failure state. The accumulation of damage after a given number of X duty cycles (or time units) is then modeled as a Markov chain as follows:

$$p_X^{(k)} = p_0^{(k)} \prod_{i=1}^X P_i^{(k)} \quad (1)$$

where $P_i^{(k)}$ are $b^{(k)} \times b^{(k)}$ *unit-jump* probability transition matrices (PTM) and $p_0^{(k)}$ and $p_x^{(k)}$ are $1 \times b^{(k)}$ row vectors denoting the probability mass functions of compliance changes at the starting and after X duty cycles respectively.

Assuming that duty cycles are sufficiently small so that changes in compliance increase in one state during one duty cycle, the PTM at duty cycle x can be written as

$$P_x^{(k)} = \begin{pmatrix} p_1 & q_1 & & & & \\ & p_2 & q_2 & & & \\ & & \ddots & \ddots & & \\ & & & & p_{b-1} & q_{b-1} \\ & & & & & 1 \end{pmatrix} \quad (2)$$

As can be observed the PTM has $b-1$ dependent transition states and only one absorbing state. Additionally PTM satisfies the Chapman-Kolmogorov identity (Isaacson and Madsen 1976), so that:

$$p_j + q_j = 1, \text{ for } j = 1, 2, \dots, b-1 \quad (3)$$

and

$$r_j = p_j / q_j \tag{4}$$

This cumulative process can be non-stationary when different damage accumulation rates take place with increasing time. In that case, the above formulation is also valid but the elements of PTM change as time increases. Thus, it would be necessary to know the time-dependent law $r_j : r_j(x)$ to take into account such nonstationary effects, which is not easy to find. There exists an efficient and easier method called the *time transformation-condensation method* (Bogdanoff and Kozin 1985) consisting on the introduction of a nonlinear polynomial transformation of time in order to convert the CDF $F_X(x)$ modeled by a stationary process to the CDF $F_Y(y)$ of a nonstationary process (Bogdanoff and Kozin 1985; Spanos and Rowatt 1995). An alternative method to take into account nonstationary effects within the inverse problem (IP) is proposed in this work. The main goal of this method is to introduce a *unitary time transformation* by means of monotonic cubic-Hermite interpolation within a unitary time window. Two interpolating points (t_1, s_2) and (t_2, s_1) between $[0, 1]$ are left as degrees of freedom to be determined by the inverse problem together with (b, r) . This formulation allows us to reduce the set of model parameters ($r_j = r$) because the unitary time transformation accounts for the variable duty-cycle severity as damage increases.

2.2. Inverse problem reconstruction

The number of parameters necessary to define the stochastic model when confronting data depends on the complexity of the CD process under consideration. In the simplest case, where the PTM elements are time-independent and state-independent ($r_j = \text{constant}$) with the initial distribution of damage p_0 known, two parameters have to be estimated: the ratio of the *stay to jump* probabilities r , and the size of the PTM b . For this simple case the Method of Moments (MM) can be used to obtain an estimation of the aforementioned parameters (b, r) due its simplicity and efficiency (Bogdanoff and Kozin 1985).

Unfortunately composite materials subjected to real damage conditions often require more than two model parameters, since a state-dependent, time-dependent CD process is habitually observed and also p_0 is frequently unknown. Consequently statistical superior methods or numerical search techniques are required.

In the inverse procedure proposed herein (Figure 1), values of model parameters (p) , where

$p = [b, r, t_1, t_2, s_1, s_2]$, are iteratively proposed to best fit the experimental measurements using a strategy based on minimizing the discrepancy between experimental data and model data, denoted by s^x and $s(p)$ respectively. That discrepancy is represented by a feature vector, called residue $r(p)$, defined as:

$$r(p) = s^x - s(p) \tag{5}$$

Since two vectors cannot be compared directly, a scalar number is derived from them called cost functional (CF) f , defined as:

$$f = \frac{1}{2} \sum_{i=1}^n r_i^2(p); \quad n = \text{dim}(r(p)) \tag{6}$$

Genetic algorithms (GA) (Goldberg 1989) are applied to find the set of model parameters (p) that minimize the cost functional and provide the inverse problem optimal solution avoiding local minimum solutions (Rus et al. 2009). In contrast to gradient-based algorithms, for which the CF is defined as (6), when the minimization is carried out by GA, the CF is usually defined in an alternative way:

$$f^L = \log(f + \varepsilon) \tag{7}$$

where ε is a small non-dimensional value (here adopted $\varepsilon = 10e^{-6}$) that ensures the existence of f^L when f tends to zero. This definition of the CF increases the convergence speed of the algorithm. (Gallego and Rus 2004)

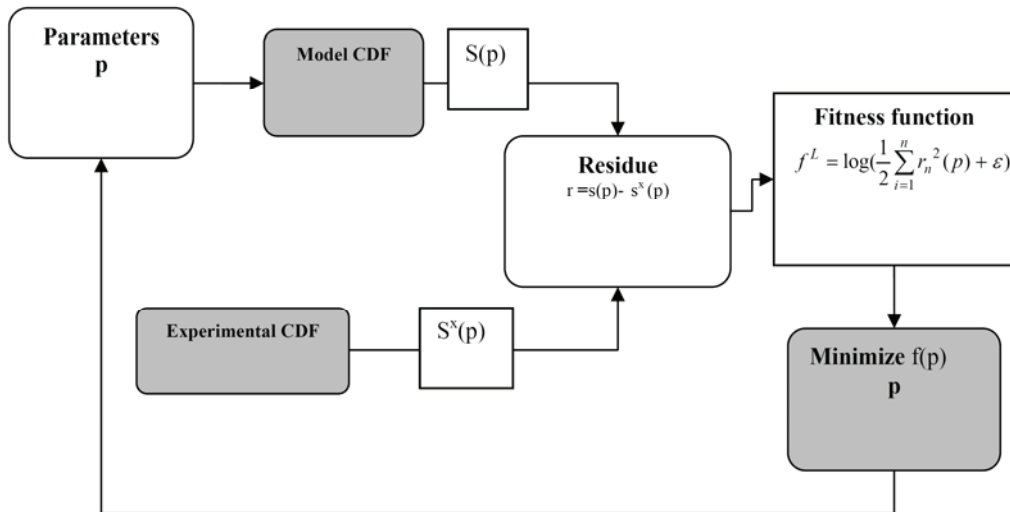


Figure 1: Procedure of MC model based inverse problem paradigm.

3. Numerical Examples

The proposed methodology has been validated against experimental data from literature to demonstrate its applicability and efficiency. Unfortunately a complete set of damage data for composite materials is not available in the open literature, so by now it has not been possible to know its efficiency in relation to different material lay-ups, variable amplitude fatigue loadings, etc.

Data for the reduction of the longitudinal Young's modulus is taken from (Rowat and Spanos 1998) for a six ply cross-plyed graphite-epoxy laminate subjected to a constant amplitude tension-tension fatigue loading with a stress ratio of $R=0.1$ and a maximum cyclic stress of $0.7S_T$. This data is normalized by the initial longitudinal laminate compliance and empirical cumulative distribution functions (EDF) of the number of duty cycle necessary to reach given increases in normalized longitudinal compliance are obtained.

The inverse procedure proposed in this work is applied to obtain the optimum model parameters for 5, 10, 12.5 and 15% increases in normalized compliance (*absorbing states*). The optimal model parameters found are compared in Table 1 with those obtained by the method of moments by means of the cost functional proposed in equation (8). As expected, the results show that even using a data set prepared to work properly with the method of moments, the accuracy of the inverse procedure proposed herein is far better than method of moments.

The agreement between the cumulative distribution functions obtained by method of moments (MM), the inverse problem based methodology (IP) and empirical distribution functions (EDF) is shown in Figure 3 for the example cases of 5%, 10%, 12.5% and 15% increase in normalized compliance respectively.

The fitness between IP-reconstructed CDF and empirical EDF appears to be fully convergent in all regions of the distribution with the proposed methodology, including the changes of curvature and the upper tails, which are relevant for high safety margins in reliability studies.

Table 1: Estimated model parameters from method of moments and proposed inverse problem respectively, for different values of changes in normalized longitudinal compliance

% compliance increase	Method of Moments (MM)			Inverse Problem (IP)						
	<i>b</i>	<i>r</i>	Cost Functional	<i>b</i>	<i>r</i>	\hat{f}_1	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_2$	Cost Functional
0.05	18	10.8068	3.1316	12	8.2458	0.2744	0.0163	0.6869	0.7069	2.002
0.1	3	230.8002	8.8913	4	164.863	0.195	0.0425	0.3660	0.4847	4.565
0.125	4	265.1121	8.3148	4	184.7058	0.2858	0.4093	0.0463	0.2798	5.872
0.15	6	233.9192	9.8299	2	382.9873	0.1726	0.4948	0.7916	0.4029	6.509

4. Conclusions and Further Works

A systematic description of the implementation of a stochastic damage accumulation model based on the inverse problem (IP) is presented, and an alternative time transformation method has been proposed. The accuracy and flexibility of the proposed method is discussed in conjunction with data acquired from literature. The model involves discrete Markov processes and numerical search techniques to find the optimal model parameters by minimizing a cost functional. This method is found more accurate than method of moments when confronting with experimental data of composite materials. In addition, the structure of the proposed method minimizes the amount of data needed for the model construction since model parameters are found by a numerical search technique, avoiding statistical process of data such as the method of moments does.

Testing more composites materials at different stacking sequences under real damage conditions, new cost functional designs and using ultrasonics as damage raw data, are future goals under development. In particular, the use of P-waves as damage raw stochastic data in conjunction with probabilistic inverse problem (Tarantola and Vallete 1982) is revealing as a promising methodology to improve significantly the accuracy of parameter estimation for damage evolution of composites under *real life* conditions. This new methodology will allow us to know prior probabilistic distributions of model parameters for damage evolution and will be able to reduce significantly the number of needed measurements since it uses all data contained within the signals.

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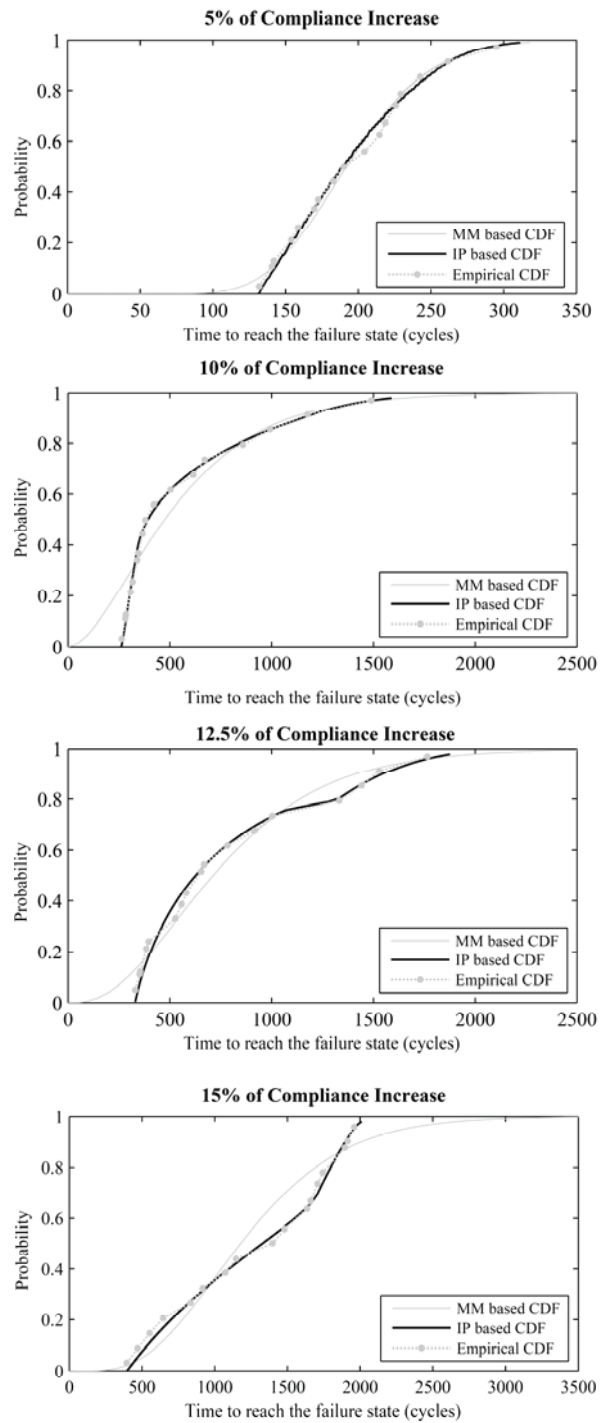


Figure 3. MM vs. IP agreement for 5%, 10%, 12.5% and 15% of compliance increase

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