Auction-based Network Selection in a Market-based Framework for Trading Wireless Communications Services

Jakub Konka, *Graduate Student Member, IEEE,* Ivan Andonovic, *Senior Member, IEEE,* Craig Michie, *Senior Member, IEEE,* and Robert Atkinson, *Senior Member, IEEE*

Abstract

Digital Marketplace is a market-based framework in which a network selection mechanism is facilitated through a variant of procurement first-price sealed-bid auction; that is, wireless network operators bid for the right to transport the subscriber's requested service over their infrastructure. In this paper, we create an economic model of this mechanism, and characterize the equilibrium under generic assumptions about the costs distributions of the network operators. Furthermore, the equilibrium is explicitly derived under more specific assumptions about the model; that is, two network operators and costs drawn from uniform distributions. In this case, we also characterize the expected prices the subscriber has to pay depending on their preferences about the service; for example, trading off quality for a lower price.

Index Terms

Heterogeneous wireless access networks; network selection; economics; auction theory; Digital Marketplace.

I. Introduction

The world of mobile communications is becoming increasingly diverse in terms of different wireless access technologies available: GSM, 3G, WiFi, and the cutting-edge 4G technology, LTE, gradually being rolled out in many countries including the USA [1], and the UK [2]. In an environment of such diversity and heterogeneity, where each wireless access technology has its own distinct characteristics, network

J. Konka, I. Andonovic, C. Michie, and R. Atkinson are with the Centre for Intelligent Dynamic Communications, Department of Electronic and Electrical Engineering, University of Strathclyde, Glasgow, G1 1XW, UK e-mail: jakub.konka@strath.ac.uk

selection mechanisms provide a resource efficient way of handling communications services by matching the services' required quality with the characteristics of a particular access technology [3]. The importance of these mechanisms is emphasized by the fact that multimode smartphones (iPhones, Android phones, BlackBerry phones) and tablets (iPads, Android tablets) currently dominate the market thus enabling users to connect to many of the available wireless access technologies.

This diversity opens exciting, new possibilities in both the technological and economic sense. The exclusive one-to-one mapping between network operators and subscribers need no longer hold; when requesting a bearer service, the network selection mechanism will be responsible for selecting the network operator (access technology) that best matches the required quality requirements of the service. From the subscribers' perspective, this permits the ability to seamlessly connect at any time, at any place, and to the technology offering the highest quality available for the best price: a paradigm referred to as *Always Best Connected* [4]. From the network operators' perspective, the integration of wireless access technologies will allow for more efficient usage of network resources, and hence, the most economic way of providing both universal coverage, and broadband access [3].

However, there also exists the possibility of a 'tussle' since there are many different actors with opposing interests involved [5]. For example, it is in the best interest of subscribers to obtain the highest quality of the service for the lowest price. Network operators, on the other hand, aim to maximize their profit by performing efficient load balancing. Furthermore, the situation may become even more complex should the service provision be decoupled from the network operators; that is, if the service provision is handled by a separate entity, service provider, while network operators are left with handling of the transport provision [6]. Therefore, the problem of network selection, which was considered to be technologically difficult, can also be considered to be the problem of economics where wireless access, traded on a per connection basis, are the electronic goods that are sold to the subscribers.

In this research, we analyze the network selection mechanism proposed in the Digital Marketplace—a market-based framework where network operators compete in a procurement auction-based setting for the right to transport the subscriber's requested service over their infrastructure [7]. Within this framework, the network selection mechanism is akin to a market selling mechanism where network operators assume the role of the sellers/bidders and subscribers are the buyers of the transport services offered by the network operators. In this way, the Digital Marketplace strives to address the tussle between the actors involved, and thus, attempts to address both technological as well as economic constraints of the problem.

We create a simple economic model of the auction, and characterize the equilibrium under generic assumptions about the costs distributions of the network operators. Furthermore, the equilibrium is explicitly derived under more specific assumptions about the model; that is, two network operators and costs drawn from uniform distributions. In this case, we also characterize the expected prices the subscriber has to pay depending on their preferences about the service; for example, trading off quality for a lower price.

The rest of this paper is organized as follows. In Section II, the historical context is given. Section III provides an overview of the Digital Marketplace, while Section IV presents the results of the analysis. Finally, Section V draws conclusions, and summarizes key learnings.

II. Historical Context

Over the last decade, several papers have explored the problem of intelligent network selection in heterogeneous wireless access networks. **Wang and Kuo provide an up-to-date survey of the mainstream approaches to the network selection problem covering: utility and game theory, fuzzy logic, multiple attribute decision making, combinatorial optimization, and Markov chains [8]. Liu *et al.*, propose an algorithm for optimal network selection which mainly aims at optimizing energy consumption of the user equipment [9]. Espi *et al.* present a machine learning approach to network selection; in particular, the authors utilize a Hopfield neural network to solve the underlying optimization problem [10].** Antoniou *et al.*, and Charilas *et al.* model the problem as a noncooperative game between wireless access networks with the aim of obtaining the best possible trade-off between the efficiency and the available capacity of networks, while, at the same time, satisfying the requested quality by the subscribers [11], [12]. Ormond *et al.* propose an algorithm for cost-oriented and performance-aware network selection that maximizes consumer surplus [13], [14]. Niyato *et al.* propose two algorithms based on evolutionary game theory for a network selection mechanism which performs intelligent load balancing so that network congestion and performance degradation can be avoided [15]. Addtionally, the same authors model the user churning behavior in heterogeneous wireless access networks using evolutionary game theory [16]. Khan *et al.* model the problem as a procurement second-price sealed-bid auction where network operators bid for the right to service the subscriber's request [17], [18]. Zhu *et al.* build upon the work reported in [15], and explore the dynamics of network selection, using Bayesian evolutionary game theory, in an environment where subscribers have only limited (incomplete) information about each others preferences [19]. Finally, Irvine *et al.* propose a market-based framework called the Digital Marketplace where network operators compete in a variant of a procurement first-price sealed-bid auction for the right to transport the subscriber's requested service over their infrastructure [7], [20], [21].

R3.1

III. The Digital Marketplace

The Digital Marketplace (DMP) is a market-based framework for trading wireless communications services. In its simplest form, there are three main groups of actors involved in the operation of the DMP: *subscribers*, *network operators*, and *market provider*. The subscribers are the end-users of the communications services, and they act as the buyers in the DMP. The network operators, on the other hand, act as the sellers/bidders. Lastly, the market provider is tasked with operating the DMP; thus providing common platform for all actors involved. It is left open-ended who should be the market provider; however, one of the following three choices is the most likely: a regulatory body, a consortium of network operators, or a single network operator on behalf of the regulatory body [21].

The process of negotiation (or the network selection mechanism) in the DMP is based on a procurement first-price sealed-bid auction. Unlike in a standard procurement first-price sealed-bid auction, the winning bid is a weighted (convex) combination of both the network operator's monetary bid and their reputation rating; we will refer to it as the *compound bid*. The network operator is elected as the winner of the auction if their compound bid is the lowest in value, and accrues their monetary bid minus the cost of supporting the service. The monetary bid is equivalent to the price of supporting the service by the network operator. The precise definition of the price is left open-ended; one possibility, for example, would be to charge the buyer per unit of bandwidth. The weights in the compound bid are set by the subscriber before each auction, and are announced to the network operators. This effectively gives the subscriber the freedom to choose any combination ranging from: a low price for the service but also poor quality; to a high quality but for a high price [7].

Since the communications services are traded on an individual service level, it might be difficult for the subscriber to judge the overall quality of the services supplied by a particular network operator [21]. **Therefore, one of the fundamental assumptions governing the operation of the DMP is that, by** R1.1
registering in the DMP, network operators agree to report on their contract fulfillments to the R3.3
market provider; that is, they agree to report a binary value denoting the success in delivering the service to the subscriber within the agreed Quality of Service (QoS) bounds [7]. The value of 0 denotes a failure, while the value of 1 a success. The latest d ($d > 1$) reports are then used to compute the reputation rating of the network operator which will be used when a new service request arrives in the marketplace. Hence, assuming network operator i admitted t service requests,

the formula for computing a reputation rating update is as follows (cf. Section 3.2 in [7])

$$r_i^{t+1} = \sum_{k=t-d}^{d} \frac{1 - report_i^k}{d}, \tag{1}$$

where $report_i^k$ **denotes the** k^{th} **binary report of the network operator** i**. Note that Equation** (1) **implies** $r_i^{t+1} = 0$ **if the network operator** i **has successfully delivered** d **services to the subscriber, while** $r_i^{t+1} = 1$ **if has failed in all** d **attempts. Furthermore, Equation** (1) **implies that if the operator is consistently unreliable, their performance is reflected accordingly by their reputation rating history. Whilst, similarly, one failure in delivering the service does not immediately render a network operator unreliable; rather, it marginally affects their updated reputation rating. At the** R3.5 **same time, at the end of each contract, the subscriber may report on their satisfaction (or Quality of Experience, QoE) with the service, for example, by submitting a mean opinion score in case of real-time services, and achieved throughput for non-real-time ones. The reputation rating update formula in Equation** (1) **could then be modified to incorporate QoE, for instance, by taking an appriopriately weighted composition of both network operator's and subscriber's reports. Since this paper just barely scratches the surface of the reputation rating system maintained by the DMP, and the concept of QoE, the Readers are referred to [7], [20]–[24] for a more in-depth treatment of the former and to [25]–[28] of the latter.**

IV. Modeling and Analysis

Let $\Gamma^B = [N, \{S_i\}, \{u_i\}, \Theta, F]$ be a *Bayesian game with incomplete information*. Formally, in this type of games, each player $i \in N$ has a utility function $u_i(s_i, s_{-i}, \theta_i)$, where $s_i \in S_i$ denotes player i's action, $s_{-i} \in S_{-i} = \times_{j \neq i} S_j$ denotes actions of all other players different from i, and $\theta_i \in \Theta_i$ represents the type of player i. Letting $\Theta = \times_{i \in N} \Theta_i$, the joint probability distribution of the $\theta \in \Theta$ is given by $F(\theta)$, which is assumed to be common knowledge among the players [29].

In game Γ^B, a *pure strategy* for player i is a function $\psi_i : \Theta_i \to S_i$, where for each type $\theta_i \in \Theta_i$, $\psi_i(\theta_i)$ specifies the action from the feasible set S_i that type θ_i would choose. Therefore, player i's pure strategy set Ψ_i is the set of all such functions.

Player i's *expected utility* given a profile of pure strategies $(\psi_1, \ldots, \psi_N)$ is given by

$$\tilde{u}_i(\psi_1, \ldots, \psi_N) = E[u_i(\psi_1, \ldots, \psi_N, \theta_i)], \tag{2}$$

where the expectation is taken over the realizations of the players' types, $\theta \in \Theta$. Now, in game Γ^B, a strategy profile $(\psi_1^*, \ldots, \psi_N^*)$ is a *pure-strategy Bayesian Nash equilibrium* if it constitutes a Nash equilibrium of game $\Gamma^N = [N, \{\Psi_i\}, \{\tilde{u}_i\}]$; that is, if for each player $i \in N$,

$$\tilde{u}_i(\psi_i^*, \psi_{-i}^*) \geq \tilde{u}_i(\psi_i, \psi_{-i}^*) \tag{3}$$

for all $\psi_i \in \Psi_i$, where $\tilde{u}_i(\psi_i, \psi_{-i})$ is defined as in Equation (2).

A. *Problem Definition and Assumptions*

The formal description of the network selection mechanism employed in the DMP is as follows. The model is a modified version of procurement first-price sealed-bid auction. Thus, formally, it represents a Bayesian game of incomplete information, Γ^B. There are n network operators (n.o.s) who bid for the right to sell their product to the subscriber such that $n = |N|$ where N denotes the set of all n.o.s.

Let $\beta : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}_+$, defined by

$$\beta(b_i, r_i) = w_{price} \cdot b_i + w_{penalty} \cdot r_i \quad \text{for all } i \in N, \tag{4}$$

denote the compound bid, where w_{price} denotes the weight the subscriber attaches to the price of the service (price weight, in short), $w_{penalty}$ denotes the weight the subscriber attaches to the reputation of the n.o. (reputation weight, in short), b_i is the monetary bid (or offered price) submitted by the n.o. i, and r_i is the reputation of the n.o. i. Each n.o.s i is characterized by the utility function u_i such that R2.1

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \textbf{if } \beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j), \\ 0 & \textbf{if } \beta(b_i, r_i) > \min_{j \neq i} \beta(b_j, r_j), \end{cases} \tag{5}$$

where $b = (b_i, b_{-i})$ is an n-tuple of all n.o.s bids, $c = (c_i, c_{-i})$ is an n-tuple of all n.o.s costs, and $r = (r_i, r_{-i})$ is an n-tuple of all n.o.s reputations. The winner of the auction is determined as the n.o. whose compound bid is the lowest one; i.e., n.o. i is the winner if

$$\beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j).$$

In the event that there is a tie

$$\beta(b_i, r_i) = \min_{j \neq i} \beta(b_j, r_j),$$

the winner is randomly selected with equal probability.

It is, moreover, assumed that the price and reputation weights $(w_{price}, w_{penalty})$ are announced by the buyer to all n.o.s before the auction. Thus, there is no uncertainty in knowing how much the subscriber values the offered price of the service over the reputation of the n.o. (or vice versa). Furthermore,

$$w_{price} + w_{penalty} = 1, \quad 0 \leq w_{price}, w_{penalty} \leq 1.$$

In order to simplify the notation, it is assumed throughout the rest of this paper that $w = w_{price}$. This simplifies the definition of the compound bid in Equation (4) to

$$\beta(b_i, r_i) = wb_i + (1 - w)r_i \quad \text{for all } i \in N.$$

Note, however, that this assumption could potentially lead to a situation where n.o.s manipulate the knowledge of w to increase their profits by overcharging the subscriber. Therefore, in order to circumvent such an eventuality, the subscriber would only consider offers such that R3.2

$$v \geq wb_i + (1 - w)r_i \quad \textbf{for all } i \in N \tag{6}$$

where $v \in (0, 1]$ is the subscriber's valuation, and it is private knowledge [7]. However, in order to keep the analysis tractable, we do not incorporate this assumption in this research.

Following the standard assumptions from the auction literature [30], the set of n.o.s, N, is finite and the n.o.s are risk neutral; that is, they seek to maximize their expected profits. Furthermore, the subscriber is risk neutral and does not have any budget constraints; that is, the subscriber is prepared to accept any offer from the n.o.s.

The costs, c_i, for each n.o. i are private knowledge. Thus, they are particular realizations of the random variables C_i for all $i \in N$. **Furthermore, it is assumed that each cost is scaled/normalized within the range $[0, 1]$, and identically and independently distributed (i.i.d.) over $[0, 1]$ according to some continuous (and atomless) probability distribution which admits a distribution function F_C and an associated density function f_C such that f_C is locally bounded away from zero over the interval $[0, 1]$. In order to keep the specification fairly generic, we do not explicitly decompose the costs into the underlying components such as interconnection charges, infrastructure fixed costs, etc. We refer the Reader to [31]–[33] for an in-depth coverage of the problem.** R3.4

The reputation ratings, r_i, for each n.o. i are common knowledge; i.e., their values are publicly disclosed to the subscriber and to every n.o. involved in the DMP. This makes the reputation system in the DMP similar to that found in eBay, for example, where the buyers of the goods leave sellers feedback which over time is perceived as reputation, and is also publicly available [34]. Furthermore, it is assumed that each $r_i \in [0, 1]$ such that the higher the reputation, the lower the rating r_i.

The bidding strategy functions $b_i : [0, 1] \to \mathbb{R}_+$ are nonnegative in value for all $i \in N$. The aim is to solve the game for pure-strategy Bayesian Nash equilibrium as defined in Equation (3).

B. Existing Results

The preliminary analysis of the problem has been conducted by Konka *et al.* [35], [36]. For brevity, we only include the important results, and refer the Reader to the aforementioned papers for full coverage of the initial analysis of the problem.

Theorem 1 (Existing results)**.** *Suppose c_i is i.i.d. over the interval $[0,1]$ for all $i \in N$ and $r_i \in [0,1]$ for all $i \in N$ is common knowledge.*

1) *Let $N_0 \subseteq N$ be the set of all those n.o.s with the lowest reputation rating. If $w = 0$, then every n.o. $j \in N_0$ will have an incentive to bid abnormally high, i.e., $b_j \to \infty$, while every remaining n.o. $k \in N \setminus N_0$ will be indifferent to the value of their bid.*
2) *If $w = 1$, then the (symmetric) equilibrium bidding strategy functions of the standard procurement first-price sealed-bid auction,*
$$b(c_i) = \frac{n-1}{(1-F_C(c_i))^{n-1}} \int_{c_i}^{1} t(1-F_C(t))^{n-2} f_C(t)dt \tag{7}$$
for all $i \in N$, constitute a (symmetric) pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction.
3) *If $r_i = r_j$ for all $i \neq j$ and $w \neq 0$, then the (symmetric) equilibrium bidding strategy functions in Equation (7) constitute a (symmetric) pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction.*

The formal proof of conclusion 2) in Theorem 1 can be found in [35] (cf. Proposition 2 in [35]), while the formal proof of conclusions 1) and 3) in [36] (cf. Proposition 1 and Corollary 4 in [36]).

C. Generic Case

To further elaborate upon the conclusions summarized in Theorem 1, it is possible to transform the problem from a bidding problem with symmetric type (or cost) distributions into a bidding problem with asymmetric type distributions. This type of bidding problem has already been researched by the economic community, both in a very specific setting (two bidders, specific type distributions) [37], [38], and in a very general setting (n bidders, arbitrary type distributions) [39], [40], and hence, there exist solutions that are applicable to the problem at hand.

In order to transform the problem, recall the utility function for each n.o. i
$$u_i(b,c,r) = \begin{cases} b_i - c_i & \text{if } wb_i + (1-w)r_i < \min\limits_{j\neq i}[wb_j + (1-w)r_j], \\ 0 & \text{if } wb_i + (1-w)r_i > \min\limits_{j\neq i}[wb_j + (1-w)r_j], \end{cases}$$

and let

$$\hat{b}_i = wb_i + (1-w)r_i \quad \text{for all } i \in N. \tag{8}$$

(Note that the new bid is just an alias for the compound bid, and in fact, they are equivalent; $\beta(b_i, r_i) \equiv \hat{b}_i$.) Solving Equation (8) for b_i yields

$$b_i = \frac{\hat{b}_i - (1-w)r_i}{w}, \quad w \neq 0. \tag{9}$$

Substituting Equation (9) back into the utility function yields

$$u_i(\hat{b}, c, r) = \begin{cases} \frac{1}{w}\left[\hat{b}_i - (wc_i + (1-w)r_i)\right] & \text{if } \hat{b}_i < \min_{j \neq i} \hat{b}_j, \\ 0 & \text{if } \hat{b}_i > \min_{j \neq i} \hat{b}_j. \end{cases}$$

If we further let

$$\hat{c}_i = wc_i + (1-w)r_i \quad \text{for all } i \in N, \tag{10}$$

the utility function simplifies to

$$u_i(\hat{b}, \hat{c}) = \begin{cases} \frac{1}{w}\left(\hat{b}_i - \hat{c}_i\right) & \text{if } \hat{b}_i < \min_{j \neq i} \hat{b}_j, \\ 0 & \text{if } \hat{b}_i > \min_{j \neq i} \hat{b}_j. \end{cases} \tag{11}$$

In order to avoid ambiguity, we shall refer to $\hat{c}_i$ as costs-hat and $\hat{b}_i$ as bids-hat, while still referring to c_i as costs and b_i as bids. Note, moreover, that since both w and r_i are assumed to be given to the n.o.s (i.e., they cannot directly modify their values), the costs-hat and bids-hat are simply convex (and hence, linear) combinations involving costs and bids respectively (Equations (8) and (10)). Therefore, an n.o. bidding their cost-hat is equivalent to bidding their cost.

As a result of this transformation, the costs-hat, $\hat{c}_i$, for each n.o. i are distributed over the interval

$$\hat{c}_i \in [(1-w)r_i, (1-w)r_i + w]$$

since $c_i \in [0, 1]$ for all $i \in N$. Note, moreover, that for all $i \in N$

$$[(1-w)r_i, (1-w)r_i + w] \subset [0, 1]$$

since $w \in (0, 1)$ and $r_i \in [0, 1]$, and in particular, if $w = 1$

$$[(1-w)r_i, (1-w)r_i + w] = [0, 1].$$

With these results at hand, we can proceed with the analysis of the game. It is sufficient to consider only the case when $w \in (0, 1)$, and at least one n.o. is characterized by a different reputation rating from the other n.o.s; that is, there exists $i \in N$ such that $r_i \neq r_j$ for all $i \neq j$ and $j \in N$. The remaining cases are already included in Theorem 1.

Firstly, note that under the generic assumptions specified in Section IV-A, the problem satisfies the following regularity conditions.

Proposition 2 (Regularity Conditions)**.** *Let F_i be the distribution function of $\hat{c}_i$ for all $i \in N$. Then,*

1) *the support of F_i is an interval*

$$[(1-w)r_i, (1-w)r_i + w],$$

2) *F_i is differentiable over*

$$((1-w)r_i, (1-w)r_i + w]$$

with a derivative f_i locally bounded away from zero over this interval, and

3) *F_i is atomless.*

The formal proof of Proposition 2 as well as any other proposition (unless stated otherwise) is given in Appendix A.

The regularity conditions in Proposition 2 correspond to the regularity assumptions on type distributions put forward by Lebrun [40] (cf. Assumptions A.1 in [40]). Therefore, since our problem satisfies Lebrun's assumptions, his results are applicable to our problem, and we conclude that:

Proposition 3 (Characterization of the Equilibrium)**.**

1) *There exists a pure-strategy Bayesian Nash equilibrium where n.o.s submit at least their costs, and*
2) *the pure-strategy Bayesian Nash equilibrium where n.o.s submit at least their costs is unique.*

However, even though the equilibrium exists and is unique, the establishment of a closed-form solution in a generic setting (n n.o.s and arbitrary cost distributions) is particularly challenging [30], [40].

D. Restricted Case $n = 2$

It is possible to explicitly derive the equilibrium bidding strategy functions in a much restricted setting. Let $n = 2$, and assume costs, c_i, for both n.o.s are drawn from the uniform distribution. **Given the lack** R1.2 **of knowledge of the way the costs are distributed, we assume the probability of each cost to be equal, which is a standard practice under similar circumstances [41]. Furthermore, for technical**

reasons, we impose continuity on the distribution, and therefore, assume the costs are drawn from the uniform distribution.

Consider the utility function for each n.o.

$$u_i(\hat{b},\hat{c}) = \begin{cases} \frac{1}{w}\left(\hat{b}_i - \hat{c}_i\right) & \text{if } \hat{b}_i < \hat{b}_j, \\ \frac{1}{2w}\left(\hat{b}_i - \hat{c}_i\right) & \text{if } \hat{b}_i = \hat{b}_j, \\ 0 & \text{if } \hat{b}_i > \hat{b}_j. \end{cases}$$

Without loss of generality, suppose $r_i < r_j$. Since the distribution of costs, c_i, for each n.o. i is uniform with the support $[0,1]$, Equation (10) implies that the distribution of costs-hat, $\hat{c}_i$, for each n.o. i is uniform with the support $[\underline{\hat{c}}_i, \bar{\hat{c}}_i] = [(1-w)r_i, (1-w)r_i + w]$. Therefore, the distribution function of costs-hat satisfies the regularity conditions specified in Proposition 2, and by Proposition 3, we conclude that the pure-strategy Bayesian Nash equilibrium where n.o.s submit at least their costs exists and is unique.

The derivation of the equilibrium involves three stages: 1) deriving equilibrium inverse bidding strategy functions using the procedure described by Kaplan and Zamir [37]; 2) numerically estimating the equilibrium bidding strategy functions by inverting the inverses; and 3) transforming the problem back to the original domain (from costs-hat and bids-hat back to costs and bids). Since $r_i < r_j$, this implies that $\bar{\hat{c}}_i < \bar{\hat{c}}_j$. It is, moreover, assumed that bids are bounded from above which implies that no n.o. wins by bidding more than $\bar{\hat{c}}_j$ (since $\bar{\hat{c}}_i < \bar{\hat{c}}_j$). In particular, in equilibrium, there is no bid higher than $\bar{\hat{c}}_j$. Furthermore, it is assumed that, in equilibrium, an n.o. with zero probability of winning bids their cost-hat. (This assumption guarantees both the existence and uniqueness of the equilibrium by Proposition 3.)

If $\bar{\hat{c}}_i \leq 2\underline{\hat{c}}_j - \bar{\hat{c}}_j$, then any pure-strategy Bayesian Nash equilibrium must have n.o. i always bidding $\underline{\hat{c}}_j$, and hence, always winning the auction at price $\underline{\hat{c}}_j$. This case shall be referred to as trivial. Therefore, in the non-trivial case, we must have $\bar{\hat{c}}_i > 2\underline{\hat{c}}_j - \bar{\hat{c}}_j$ (cf. Lemma 1 in Kaplan and Zamir [37]). In this range, an equilibrium consists of strictly increasing, differentiable bidding strategy functions $\hat{b}_i(\hat{c})$ and $\hat{b}_j(\hat{c})$. Let the inverses of these bidding strategy functions be denoted as $\hat{c}_i(\hat{b})$ with support $[\underline{\hat{b}}_i, \bar{\hat{b}}_i]$, and $\hat{c}_j(\hat{b})$ with support $[\underline{\hat{b}}_j, \bar{\hat{b}}_j]$. As Kaplan and Zamir [37] establish, these supports are identical, and equal to the common support $[\underline{\hat{b}}, \bar{\hat{b}}]$.

Suppose, therefore, that $\bar{\hat{c}}_i > 2\underline{\hat{c}}_j - \bar{\hat{c}}_j$ holds, and consider the optimization problem for n.o. i

$$\max_b E\left[b - \hat{c}_i \,\middle|\, b < \hat{b}_j\right]$$
$$= \max_b (b - \hat{c}_i) \cdot P\{b < \hat{b}_j\}$$

$$= \max_b (b - \hat{c}_i) \cdot P\{\hat{c}_j(b) < \hat{C}_j\}$$

$$= \max_b (b - \hat{c}_i)(1 - F_j(\hat{c}_j(b))),$$

where F_j is the distribution function of costs-hat for n.o. j. The first-order condition yields

$$1 - F_j(\hat{c}_j(b)) - (b - \hat{c}_i) f_j(\hat{c}_j(b)) \cdot \frac{d}{db}\hat{c}_j(b) = 0$$
$$\Longleftrightarrow 1 - \frac{\hat{c}_j(b) - \underline{\hat{c}}_j}{\bar{\hat{c}}_j - \underline{\hat{c}}_j} = \frac{1}{\bar{\hat{c}}_j - \underline{\hat{c}}_j}(b - \hat{c}_i) \cdot \frac{d}{db}\hat{c}_j(b)$$
$$\Longleftrightarrow (\hat{c}_i(b) - b) \cdot \frac{d}{db}\hat{c}_j(b) = \hat{c}_j(b) - \bar{\hat{c}}_j, \tag{12}$$

where we have used the facts that F_j is the distribution function of the uniform distribution with the support $[\underline{\hat{c}}_j, \bar{\hat{c}}_j]$, and at an equilibrium $\hat{c}_i = \hat{c}_i(b)$. Similarly, for n.o. j we have

$$(\hat{c}_j(b) - b) \cdot \frac{d}{db}\hat{c}_i(b) = \hat{c}_i(b) - \bar{\hat{c}}_i. \tag{13}$$

The following boundary conditions must hold in equilibrium (cf. boundary conditions in Kaplan and Zamir [37]):

1) $\hat{c}_j(\bar{\hat{b}}) = \bar{\hat{b}}$, (the n.o. bids their cost-hat when their probability of winning is zero),
2) $\hat{c}_i(\bar{\hat{b}}) = \bar{\hat{c}}_i$, (the maximum cost-hat that gives n.o. i a positive probability of winning), and
3) $\hat{c}_i(\underline{\hat{b}}) = \underline{\hat{c}}_i$ and $\hat{c}_j(\underline{\hat{b}}) = \underline{\hat{c}}_j$ (the lowest bid-hat of each n.o. is reached for their lowest cost-hat).

Integrating Equations (12) and (13) bounded by the aforementioned boundary conditions results in the derivation of the equilibrium inverse bidding strategy functions. The derivation procedure is fully described in Kaplan and Zamir [37]; hence, we only provide the final result.

Proposition 4. *Let there be $n = 2$ n.o.s, and suppose c_i is independently drawn from uniform distribution over the interval $[0, 1]$ for all $i \in N$. Let for all $i \in N$*

$$\hat{b}_i = wb_i + (1 - w)r_i, \quad \textit{and} \quad \hat{c}_i = wc_i + (1 - w)r_i,$$

with

$$\underline{\hat{c}}_i = (1 - w)r_i, \quad \bar{\hat{c}}_i = (1 - w)r_i + w. \tag{14}$$

Then the equilibrium inverse bidding strategy functions for all $i \in N$ and $w \in (0,1)$ are given by

$$\hat{c}_i(b) = \bar{\hat{c}}_i + \frac{(\bar{\hat{c}}_j - \bar{\hat{c}}_i)^2}{(\bar{\hat{c}}_j + \bar{\hat{c}}_i - 2b) d_i \exp\left(\dfrac{\bar{\hat{c}}_j - \bar{\hat{c}}_i}{\bar{\hat{c}}_j + \bar{\hat{c}}_i - 2b}\right) + 4(\bar{\hat{c}}_j - b)}, \tag{15}$$

$$\hat{c}_j(b) = \bar{\hat{c}}_j + \frac{(\bar{\hat{c}}_i - \bar{\hat{c}}_j)^2}{(\bar{\hat{c}}_i + \bar{\hat{c}}_j - 2b) d_j \exp\left(\dfrac{\bar{\hat{c}}_i - \bar{\hat{c}}_j}{\bar{\hat{c}}_i + \bar{\hat{c}}_j - 2b}\right) + 4(\bar{\hat{c}}_i - b)}, \tag{16}$$

where

$$d_i = \frac{\dfrac{(\bar{\hat{c}}_j - \bar{\hat{c}}_i)^2}{\underline{\hat{c}}_i - \bar{\hat{c}}_i} + 4(\underline{\hat{b}} - \bar{\hat{c}}_j)}{-2(\underline{\hat{b}} - \bar{\hat{b}})} \exp\left(\frac{\bar{\hat{c}}_j - \bar{\hat{c}}_i}{2(\underline{\hat{b}} - \bar{\hat{b}})}\right), \tag{17}$$

$$d_j = \frac{\dfrac{(\bar{\hat{c}}_i - \bar{\hat{c}}_j)^2}{\underline{\hat{c}}_j - \bar{\hat{c}}_j} + 4(\underline{\hat{b}} - \bar{\hat{c}}_i)}{-2(\underline{\hat{b}} - \bar{\hat{b}})} \exp\left(\frac{\bar{\hat{c}}_i - \bar{\hat{c}}_j}{2(\underline{\hat{b}} - \bar{\hat{b}})}\right), \tag{18}$$

and

$$\underline{\hat{b}} = \frac{\underline{\hat{c}}_i \underline{\hat{c}}_j - \dfrac{(\bar{\hat{c}}_i + \bar{\hat{c}}_j)^2}{4}}{\underline{\hat{c}}_i - \bar{\hat{c}}_i + \underline{\hat{c}}_j - \bar{\hat{c}}_j}, \quad \bar{\hat{b}} = \frac{\bar{\hat{c}}_i + \bar{\hat{c}}_j}{2}. \tag{19}$$

Note that the equilibrium inverse bidding strategy functions are inconvenient to work with: for a particular bid-hat value, they map into a particular cost-hat for either n.o. It would be more intuitive to work with their inverses, where for a particular cost-hat, we would get a particular bid-hat. Since it is difficult to analytically invert the equilibrium inverse bidding strategy functions in Equations (15) and (16), we resort to numerical methods for estimating the inverses for a particular set of cost-reputation pairs with respect to the price weights for both n.o.s. Without loss of generality, assume that $r_i < r_j$. The numerical procedure is then as follows.

1) For a particular price weight w and reputation ratings r_i and r_j, calculate the costs-hat supports for both n.o.s; that is, the endpoints of the interval $[\underline{\hat{c}}_i, \bar{\hat{c}}_i]$ for n.o. i, and $[\underline{\hat{c}}_j, \bar{\hat{c}}_j]$ for n.o. j (Equation (14)).
2) If $\bar{\hat{c}}_i \leq 2\underline{\hat{c}}_j - \bar{\hat{c}}_j$, then the equilibrium is trivial. N.o. i bids the lower endpoint of the cost-hat support of n.o. j; that is, n.o. i bids $\underline{\hat{c}}_j$ for all $\hat{c} \in [\underline{\hat{c}}_i, \bar{\hat{c}}_i]$. N.o. j, on the other hand, bids their cost-hat; that is, n.o. j bids $\hat{c}$ for all $\hat{c} \in [\underline{\hat{c}}_j, \bar{\hat{c}}_j]$.
3) If $\bar{\hat{c}}_i > 2\underline{\hat{c}}_j - \bar{\hat{c}}_j$, then the equilibrium is nontrivial. Hence,
   a) Calculate the common bids-hat support $[\underline{\hat{b}}, \bar{\hat{b}}]$ (Equation (19)).
   b) For all $\hat{b} \in [\underline{\hat{b}}, \bar{\hat{b}}]$, calculate the corresponding costs-hat for both n.o.s using Equations (15) and (16).

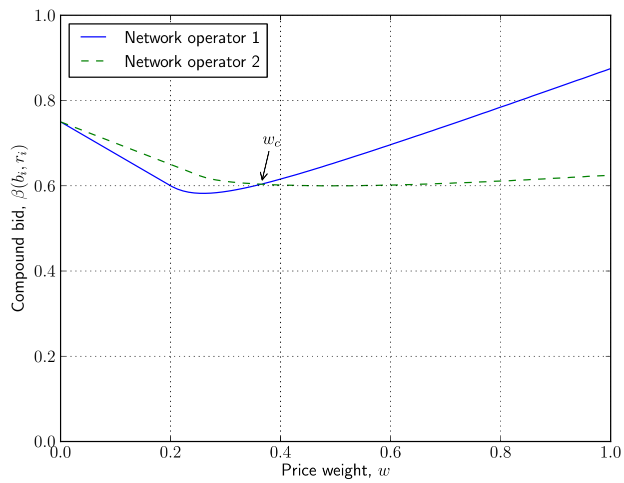

Fig. 1. Compound bid plotted against the price weight

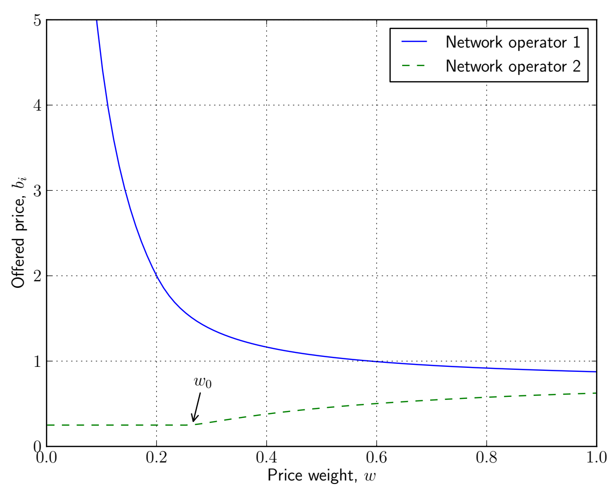

Fig. 2. Offered prices (bids) plotted against the price weight

c) Since by assumption $r_i < r_j$, it follows that $\bar{\hat{c}}_i \leq \bar{\hat{c}}_j$, and hence, $\bar{\hat{b}} \leq \bar{\hat{c}}_j$. Thus, n.o. j bids their cost-hat, $\hat{c}_j(\hat{b}) = \hat{c}_j$ for all $\hat{b} \in [\bar{\hat{b}}, \bar{\hat{c}}_j]$.

The result of the steps described above is the tabulation of the costs-hat and their corresponding equilibrium bids-hat for a particular price weight w, and reputation ratings r_i and r_j for both n.o.s, in the ranges $[\underline{\hat{c}}_i, \bar{\hat{c}}_i]$ for n.o. i and $[\underline{\hat{c}}_j, \bar{\hat{c}}_j]$ for n.o. j. Denote by

$$\hat{b}_i(\hat{c}_i) = \hat{b}_i \quad \text{for all } \hat{c}_i \in [\underline{\hat{c}}_i, \bar{\hat{c}}_i], \tag{20}$$

TABLE I

AN EXEMPLARY SET OF COST-REPUTATION PAIRS FOR TWO NETWORK OPERATORS

	Cost, c_i	**Reputation rating**, r_i
Network operator 1	0.75	0.25
Network operator 2	0.25	0.75

and

$$\hat{b}_j(\hat{c}_j) = \hat{b}_j \quad \text{for all } \hat{c}_j \in [\underline{\hat{c}}_j, \bar{\hat{c}}_j] \tag{21}$$

the resultant equilibrium bidding strategy functions.

The problem can be transformed back into the original domain by substituting Equations (8) and (10) into Equations (20) and (21); that is,

$$\hat{b}_i(\hat{c}_i) = \hat{b}_i \iff b_i = \frac{\hat{b}_i(wc_i + (1-w)r_i) - (1-w)r_i}{w}$$

for all $c_i \in [0, 1]$, and

$$\hat{b}_j(\hat{c}_j) = \hat{b}_j \iff b_j = \frac{\hat{b}_j(wc_j + (1-w)r_j) - (1-w)r_j}{w}$$

for all $c_j \in [0, 1]$. Keeping costs and reputation ratings fixed, one can then estimate the equilibrium bidding strategy functions with respect to the price weights by sliding the value of $w \in (0, 1)$.

By way of example, the equilibrium bidding strategy functions were estimated for the set of cost-reputation pairs depicted in Table I. Figure 1 shows the value of the compound bid, $\beta(b_i, r_i)$, for different values of w for both n.o.s, while Figure 2 depicts the value of the monetary bid (or offered price), b_i, for different values of w for both n.o.s. The numerical data in Table I suggests that n.o. 2 should be the winner for the values of $w \to 1$ since n.o. 2's cost is strictly lower than that of their opponent's. On the other hand, n.o. 1 should be winner for the values of $w \to 0$ since n.o. 1's reputation rating is strictly lower than that of their opponent's (which implies that n.o. 1's reputation is in fact strictly higher than that of their opponent's). This prediction agrees with the numerical output shown in Figure 1. Let w_c denote the value of w for which an intersection between the compound bids of both n.o.s occurs (if it exists). In Figure 1, $w_c \approx 0.365$. Hence, n.o. 2 wins the auction for $w_c < w < 1$, while n.o. 1 for $0 < w < w_c$.

Note, furthermore, that since we have explicitly required the n.o.s to bid their own costs when their probability of winning is zero, the monetary bid of n.o. 2 is capped at their cost, $b_2 = 0.25$, for $0 < w \leq w_0$ where $w_0 \approx 0.265$ (Figure 2). In the same range of w, as w decreases, n.o. 1's bid increases

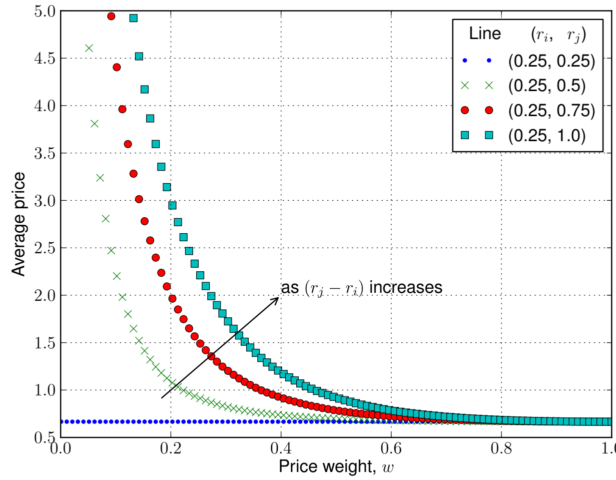

Fig. 3. Average prices plotted against the price weight for different pairs of reputation ratings

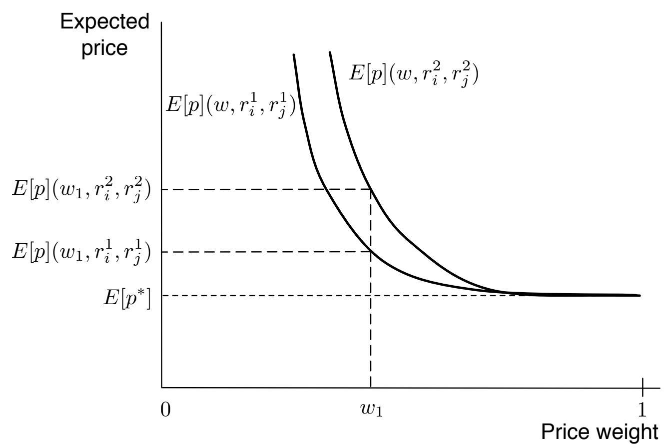

Fig. 4. Sensitivity of the price weight to the expected prices

in an exponential-like fashion, to finally culminate in $b_1 \to \infty$ at $w = 0$ in accordance with Theorem 1, part 1). As $w \to 1$, on the other hand, the monetary bids of both n.o.s tend to the values specified in part 2) of Theorem 1, that is, $b_1 = 0.875$ and $b_2 = 0.625$, to finally attain those values at $w = 1$.

E. Discussion

Having derived the equilibrium bidding strategy functions in the restricted case, it is possible to examine the expected prices the subscriber will have to pay for different values of the price weight given the reputation ratings of the n.o.s. To this end, suppose all of the assumptions of Section IV-D hold; that is, there are two n.o.s, and costs are uniformly distributed over the interval $[0, 1]$. The expected price is equivalent to the expected value of the winning bid; that is, with some abuse of notation,

$$E[p](w, r_i, r_j) = E[b_i \mid \arg\min_{i \in N} \beta(w, b_i, r_i)], \tag{22}$$

where b_i is the equilibrium bid, and $\beta(w, b_i, r_i) = \beta(b_i, r_i)$ evaluated for a particular value of w for all $i \in N$.

If both n.o.s have equal reputation ratings, $r = r_i = r_j$ say, then Theorem 1, part 3) holds for all $w \in [0, 1]$. Therefore, regardless of the choice of the price weight, the subscriber expects to pay the price of

$$E[p^*] \equiv E[p](w, r, r) = E\left[\min_{i \in N} \frac{1 + c_i}{2}\right] \quad \text{for all } w \in [0, 1], \tag{23}$$

which is equivalent to Equation (7) evaluated at $n = 2$. In particular, for costs, c_i, uniformly distributed over the interval $[0, 1]$, $E[p^*] = \frac{2}{3}$.

If, on the other hand, both n.o.s are characterized by different reputation ratings, then an analytical derivation of the expected prices for each value of the price weight given a pair of reputation ratings is cumbersome. This is due to the fact that n.o.s bid according to a pair of inverse equilibrium bidding functions specified in Proposition 4, which are not easily invertible. Hence, we resort to numerical methods for estimating average (sample mean) prices for selected values of the price weight given a pair of reputation ratings.

To this end, for any given pair of reputation ratings, the costs are pseudo-randomly drawn from the uniform distribution over the discretized interval $[0, 1]$. For each selected price weight, the average price is averaged over 10,000 i.i.d. observations. The Strong Law of Large Numbers implies that as the number of observations tends to infinity, the average (sample mean) of the observations approaches the real mean of the distribution of the random variable in question. Therefore, an average of 10,000 observations of the price for each selected price weight should provide a reasonable approximation of the expected price for that price weight. Without loss of generality, suppose further that $r_i \leq r_j$. Figure 3 shows the result of the estimation for four pairs of reputation ratings: $(r_i, r_j) = (0.25, 0.25)$, $(0.25, 0.5)$, $(0.25, 0.75)$, and $(0.25, 1.0)$.

It can be observed that regardless of the values of the reputation ratings, the expected prices, $E[p](w, r_i, r_j)$, are bounded from below by $E[p^*]$ for each price weight; this is depicted in Figure 3. Hence, it can be concluded that regardless of the values of the reputation ratings, the lowest expected price is achieved for $w = 1$, and will not decrease as w decreases; in fact, it can only either increase or remain constant.

Furthermore, as the difference $(r_j - r_i)$ increases, the expected prices, $E[p](w, r_i, r_j)$, increase as the price weight decreases; this is depicted in Figure 3. Therefore, it can be inferred that the smaller the difference $(r_j - r_i)$, the less (expected) price sensitive the price weight; that is, for any $w_1 \in [0, 1]$, if $(r_j^2 - r_i^2) > (r_j^1 - r_i^1)$ for all $r_i^1, r_j^1, r_i^2, r_j^2 \in [0, 1]$, then $E[p](w_1, r_i^2, r_j^2) \geq E[p](w_1, r_i^1, r_j^1)$ (Figure 4). In other words, for any expected price, as the difference $(r_j - r_i)$ between the reputation ratings of the n.o.s increases, the price weight has to increase (or remain constant) in order to keep the expected price fixed.

V. CONCLUSIONS

In this paper, we have presented the results of the analysis of the network selection mechanism in the Digital Marketplace. This framework offers a market-based approach to the problem of intelligent network selection in heterogeneous wireless networks where the selection mechanism is based on a procurement first-price sealed-bid auction; network operators represent the sellers/bidders, while the subscriber is the buyer. The mechanism gives the subscriber an opportunity to influence the bidding process so that the network operator who matches the subscriber's needs is chosen; for example, the operator who offers low price for the service but at the expense of the quality, or the opposite, high quality of the service but for a high price. This is accomplished through the choice of the so-called price weight, w, by the subscriber before each auction.

We have shown that in a generic setting, i.e., with n network operators and arbitrary cost distributions, the pure-strategy Bayesian Nash equilibrium where network operators submit at least their costs exists and is unique (Proposition 3, Section IV-C). However, it is very difficult, if at all possible, to derive the closed-form expression of the equilibrium. After restricting the problem to a more specific setting, however, i.e., two network operators and uniform cost distributions, we have derived the pure-strategy Bayesian Nash equilibrium bidding functions for all values of the price weight (Proposition 4, Section IV-D). It was shown that for the values of the price weight approaching $w = 0$ the mechanism selects the network operator offering high quality of the service but for a high price, while for the values of the price weight approaching $w = 1$ the mechanism selects the network operator offering low price for the service but of a low quality.

In the restricted setting, we have also numerically established that, for any choice of the price weight by the subscriber and given any pair of reputation ratings, the expected prices should be bounded from below by $E[p^*] = \frac{2}{3}$, which corresponds to the expected price for the choice of $w = 1$ or when both network operators are characterized by equal reputation ratings (Section IV-E). Furthermore, we have inferred that the smaller the difference between the reputation ratings, the less (expected) price sensitive the price weight.

APPENDIX A
PROOFS

Proof of Proposition 2: Proof of 1) is trivial. To prove 2) and 3), we note that for all $x \in [(1-w)r_i, (1-w)r_i + w]$,

$$\begin{aligned} F_i(x) &= P\{\hat{C}_i \leq x\} \\ &= P\{wC + (1-w)r_i \leq x\} \\ &= P\left\{C \leq \frac{x-(1-w)r_i}{w}\right\} \end{aligned}$$

since $\hat{c}_i = wc_i + (1-w)r_i$ and $w \neq 0$. Hence, $F_i(x) = F_C\left(\frac{x-(1-w)r_i}{w}\right)$ and $\frac{x-(1-w)r_i}{w} \in [0,1]$ for all $x \in [(1-w)r_i, (1-w)r_i+w]$. Therefore, since F_C is differentiable over $(0,1]$ with a derivative f_C locally bounded away from zero over this interval, by extension, F_i is differentiable over $((1-w)r_i, (1-w)r_i+w]$ with a derivative f_i locally bounded away from zero over this interval, and this proves 2). Moreover, since F_C is atomless, by extension, F_i is atomless, and this proves 3). ■

Proof of Proposition 3: Since both w and r_i are assumed to be given to the n.o.s (i.e., the n.o.s cannot directly modify their values), the costs-hat and bids-hat are simply convex (and hence, linear) combinations involving costs and bids respectively (Equations (8) and (10)). Therefore, an n.o. bidding their cost-hat is equivalent to bidding their cost. Hence, to prove 1) we note that Lebrun [40] proves the existence of a pure-strategy Bayesian Nash equilibrium where n.o.s submit at least their costs-hat (cf. C.5 Characterization with Possibly Different Lower and Upper Extremities in [40]). But since costs-hat are equivalent to costs, this also proves the existence of a pure-strategy Bayesian Nash equilibrium where n.o.s submit at least their costs.

To prove 2) we note that since $r_i \neq r_j$ for at least one n.o. $i \in N$ such that $i \neq j$ and $j \in N$, then either $r_i < r_j$ or $r_i > r_j$. Without loss of generality, assume $r_i < r_j$ which implies that $(1-w)r_i < (1-w)r_j$ for all $w \in (0,1)$. By Theorem 1 in Lebrun [40], the additional condition (ii) holds, and hence, the considered first-price auction has one and only one pure-strategy Bayesian Nash equilibrium where n.o.s bid at least their costs-hat. But since costs-hat are equivalent to costs, this also proves the uniqueness of a pure-strategy Bayesian Nash equilibrium where n.o.s submit at least their costs. ■

Proof of Proposition 4: The proof is analogous to the proof of Proposition 1 in Kaplan and Zamir [37]. ■

REFERENCES

[1] Verizon Wireless, "LTE Information Center." [Online]. Available: http://news.verizonwireless.com/LTE/Overview.html [Accessed Jan. 31, 2012], 2012.

[2] BBC, "O2 begins '4G' LTE mobile data trial in London." [Online]. Available: http://www.bbc.co.uk/news/technology-15717913 [Accessed Jan. 31, 2012], 2011.

[3] R. Beaubrun, "Integration of Heterogeneous Wireless Access Networks," in *Heterogeneous Wireless Access Networks: Architectures and Protocols* (E. Hossain, ed.), ch. 1, pp. 1–18, Springer, 2009.

[4] E. Gustafsson and A. Jonsson, "Always Best Connected," *Wireless Communications, IEEE*, vol. 10, no. 1, pp. 49–55, 2003.

[5] D. D. Clark and J. Wroclawski, "Tussle in Cyberspace: Defining Tomorrow's Internet," in *SIGCOMM'02*, (Pittsburgh, Pennsylvania, USA), 19–23 August 2002.

[6] J. Bush, J. Irvine, and J. Dunlop, "A Digital Marketplace for Tussle in Next Generation Wireless Networks," in *Vehicular Technology Conference Fall (VTC 2009-Fall), 2009 IEEE 70th*, pp. 1–5, Sept. 2009.

[7] G. Le Bodic, D. Girma, J. Irvine, and J. Dunlop, "Dynamic 3G Network Selection for Increasing the Competition in the Mobile Communications Market," in *Vehicular Technology Conference, 2000. IEEE VTS-Fall VTC 2000. 52nd*, vol. 3, pp. 1064–1071, 2000.

[8] L. Wang and G.-S. Kuo, "Mathematical modeling for network selection in heterogeneous wireless networks – a tutorial," *Communications Surveys Tutorials, IEEE*, vol. 15, no. 1, pp. 271–292, 2013.

[9] H. Liu, C. Maciocco, V. Kesavan, and A. Low, "Energy efficient network selection and seamless handovers in mixed networks," in *World of Wireless, Mobile and Multimedia Networks Workshops, 2009. WoWMoM 2009. IEEE International Symposium on a*, pp. 1–9, June 2009.

[10] J. Espi, R. Atkinson, D. Harle, and I. Andonovic, "An Optimum Network Selection Solution for Multihomed Hosts Using Hopfield Networks," in *Networks (ICN), 2010 Ninth International Conference on*, pp. 249–254, April 2010.

[11] J. Antoniou and A. Pisillides, "4G Converged Environment: Modeling Network Selection as a Game," in *16th IST Mobile and Wireless Communication Summit*, (Budapest), July 2007.

[12] D. Charilas, O. Markaki, and E. Tragos, "A Theoretical Scheme for Applying Game Theory and Network Selection Mechanisms in Access Admission Control," in *Wireless Pervasive Computing, 2008. ISWPC 2008. 3rd International Symposium on*, pp. 303–307, May 2008.

[13] O. Ormond, G.-M. Muntean, and J. Murphy, "Economic Model for Cost Effective Network Selection Strategy in Service Oriented Heterogeneous Wireless Network Environment," in *NOMS'06*, 2006.

[14] O. Ormond, G.-M. Muntean, and J. Murphy, "Evaluation of an Intelligent Utility-Based Strategy for Dynamic Wireless Network Selection," in *MMNS'06*, pp. 158–170, 2006.

[15] D. Niyato and E. Hossain, "Dynamics of Network Selection in Heterogeneous Wireless Networks: An Evolutionary Game Approach," *Vehicular Technology, IEEE Transactions on*, vol. 58, pp. 2008–2017, May 2009.

[16] D. Niyato and E. Hossain, "Modeling User Churning Behavior in Wireless Networks Using Evolutionary Game Theory," in *Wireless Communications and Networking Conference, 2008. WCNC 2008. IEEE*, pp. 2793–2797, 31 2008-april 3 2008.

[17] M. A. Khan, U. Toseef, S. Marx, and C. Goerg, "Game-Theory Based User Centric Network Selection with Media Independent Handover Services and Flow Management," in *Communication Networks and Services Research Conference (CNSR), 2010 Eighth Annual*, pp. 248–255, May 2010.

[18] M. Khan, U. Toseef, S. Marx, and C. Goerg, "Auction-based Interface Selection with Media Independent Handover Services and Flow Management," in *Wireless Conference (EW), 2010 European*, pp. 429–436, April 2010.

[19] K. Zhu, D. Niyato, and P. Wang, “Network Selection in Heterogeneous Wireless Networks: Evolution with Incomplete Information,” in *Wireless Communications and Networking Conference (WCNC), 2010 IEEE*, pp. 1–6, april 2010.

[20] J. Irvine, C. McKeown, and J. Dunlop, “Managing Hybrid Mobile Radio Networks with the Digital Marketplace,” in *Vehicular Technology Conference, 2001. VTC 2001 Fall. IEEE VTS 54th*, vol. 4, pp. 2542–2546, 2001.

[21] J. Irvine, “Adam Smith Goes Mobile: Managing Services Beyond 3G with the Digital Marketplace,” in *Wireless Conference (EW), 2002 European. Invited paper to*, 2002.

[22] G. Le Bodic, *A Multiagent System for Application of Market Concepts to Emerging Mobile Communication Services*. PhD thesis, University of Strathclyde, Glasgow, UK, 2000.

[23] R. Mathur, J. Irvine, and J. Dunlop, “Estimating Commitment in Digital Marketplace Environment,” in *European Wireless, 2002*, 2002.

[24] A. McDiarmid and J. Irvine, “Commitment-aware Reputation System for the Digital Marketplace,” *Electronics Letters*, vol. 42, pp. 102 – 103, jan. 2006.

[25] K. Kilkki, “Quality of Experience in Communications Ecosystem,” *Journal of Universal Computer Science*, vol. 14, pp. 615–624, Mar. 2008.

[26] P. Brooks and B. Hestnes, “User measures of quality of experience: why being objective and quantitative is important,” *Network, IEEE*, vol. 24, pp. 8–13, March-April 2010.

[27] M. Fiedler, T. Hossfeld, and P. Tran-Gia, “A generic quantitative relationship between quality of experience and quality of service,” *Network, IEEE*, vol. 24, pp. 36–41, March-April 2010.

[28] J. Shaikh, M. Fiedler, and D. Collange, “Quality of experience from user and network perspectives,” *Annals of Telecommunications*, vol. 65, pp. 47–57, 2010.

[29] R. Gibbons, *A Primer in Game Theory*. Financial Times/Prentice Hall, 1992.

[30] V. Krishna, *Auction Theory*. Academic Press, second ed., 2010.

[31] P. Njoroge, A. Ozdaglar, N. Stier, and G. Weintraub, “Competition, Market Coverage, and Quality Choice in Interconnected Platforms,” in *NetEcon’09: Workshop on the Economics of Networks, Systems and Computation*, July 2009.

[32] H. Le Cadre, M. Bouhtou, and B. Tuffin, “A Pricing Model for a Mobile Network Operator Sharing Limited Resource with a Mobile Virtual Network Operator,” in *ICQT 2009*, 2009.

[33] T. Hau and W. Brenner, “Price Setting in Two-Sided Markets for Internet Connectivity,” in *ICQT 2009*, 2009.

[34] eBay Inc., “eBay.com: All about Feedback.” [Online]. Available: http://pages.ebay.com/help/feedback/allaboutfeedback.html [Accessed Dec. 4, 2012].

[35] J. Konka, R. Atkinson, and J. Irvine, “Economic Aspects of Intelligent Network Selection: A Game-Theoretic Approach,” in *Mobile Ubiquitous Computing, Systems, Services and Technologies, 2011. UBICOMM ’11. The Fifth International Conference on*, 20-25 Nov. 2011.

[36] J. Konka, J. Irvine, and R. Atkinson, “Economics of Intelligent Selection of Wireless Access Networks in a Market-based Framework: A Game-Theoretic Approach,” *International Journal on Advances in Networks and Services*, vol. 5, no. 3 & 4, pp. 210–224, 2012.

[37] T. R. Kaplan and S. Zamir, “Asymmetric First-Price Auctions with Uniform Distributions: Analytic Solutions to the General Case,” *Economic Theory*, (forthcoming).

[38] E. Maskin and J. Riley, “Asymmetric Auctions,” *The Review of Economic Studies, Ltd.*, vol. 67, pp. 413–438, Jul. 2000.

[39] B. Lebrun, “First Price Auctions in the Asymmetric N Bidder Case,” *International Economic Review*, vol. 40, pp. 125–142, Feb. 1999.

[40] B. Lebrun, "Uniqueness of the equilibrium in first-price auctions," *Games and Economic Behavior*, vol. 55, no. 1, pp. 131–151, 2006.

[41] C. A. Chung, *Simulation Modeling Handbook: A Practical Approach*, ch. 5. CRC Press, 2004.