

Maintenance scheduling for multi-component systems with hidden failures

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Abstract

This paper develops a maintenance policy for a multi-component system subject to hidden failures. Components of the system are assumed to suffer from hidden failures, which can only be detected at inspection. The objective of the maintenance policy is to determine the inspection intervals for each component such that the long-run cost rate is minimized. Due to the dependence among components, an exact optimal solution is difficult to obtain. Concerned with the intractability of the problem, a heuristic method named “base interval approach” is adopted to reduce the computational complexity. Performance of the base interval approach is analyzed and the result shows that the proposed policy can approximate the optimal policy within a small factor. Two numerical examples are presented to illustrate the effectiveness of the policy.

Key words: Hidden failure, multi-component system, maintenance optimization, inspection-replacement policy, base interval approach

1. Introduction

With the integration of technologies, modern systems are often complex systems consisting of multiple components. For multi-component systems, interactions among components cannot be

neglected when making maintenance decisions. Optimal maintenance strategy for an individual component may not necessarily remain optimal for a system consisting of multiple dependent components. If no dependence exists among components, maintenance strategy for multi-component systems is identical with that for a single-component systems (Wang, 2002). Dependency among components however brings a lot of challenges to multi-component maintenance optimization (Gustavsson et al, 2014; Phan & Zhu, 2015; Liu et al, 2015).

In the literature, two strategies have been widely used to deal with the multi-component maintenance issue, namely, group maintenance and opportunistic maintenance (Nicolai & Dekker, 2008). Group maintenance implies that multiple components are replaced in groups at predetermined times (Xiao et al, 2016). Vu et al (2014) proposed a dynamic group maintenance strategy for multi-component systems. Gao et al (2015) proposed a dynamic interval maintenance policy, which is more effective in reducing unnecessary maintenance actions compared with periodic preventive maintenance. Opportunistic maintenance occurs in the case that components are maintained when a maintenance opportunity arises, e.g., unexpected system shutdown. Pandey et al (2016) proposed a preventive maintenance scheduling model where maintenance actions are implemented during system breakdown. Recently, more advanced maintenance policies (e.g., condition-based maintenance) have been applied for multi-component systems (Camci, 2009). Liu et al (2014) proposed a value-based maintenance policy for multi-component systems with degrading components. In the work of Tian & Liao (2011), a condition based maintenance strategy with proportional hazards model was developed for multi-component systems.

Although numerous maintenance policies for multi-component systems have been proposed in literature, most of the models focus on catastrophic or sudden failures (failures that are self-announcing). However, in many cases, failures are not self-announcing, that a failure remains dormant before being exposed by inspection techniques or other system disturbances. This type of failure is referred to as hidden failure or soft failure (Ye et al, 2014; Ye & Xie, 2015; Wang et al, 2016). Hidden failure itself does not lead to system breakdown, but the existence of hidden failure may cause performance loss during system operation. Hidden failure is very common in modern systems, particularly in stand-by redundant systems and protection systems (Xiao & Ye, 2016). Yang et al (2006) reported that hidden failure in protection system may cause multiple

component outages and even spread power system disturbances. Cheng et al (2013) pointed out that hidden failure of relay protection system is the main contributor of cascading failures and blackouts.

A common practice to deal with hidden failure is to implement an inspection policy to detect the existence of hidden failures (Lam & Banjevic, 2015). Based on the interval between two consecutive inspections, inspection policy can be classified into two types: periodic inspection and sequential inspection (non-periodic). A common practice in industry is to implement periodic inspection at constant intervals, due to its feasibility and applicability. In literature, there exist plentiful research works on dealing with hidden failures. Wang & Pham (2011) investigated a multi-objective maintenance optimization for systems with dependent competing risks of degradation wear and random shocks. Ahmadi & Kumar (2011) studied cost based risk of system with hidden failures, where the optimal inspection and restoration intervals are determined based on the cost criterion. He et al (2015) considered an imperfect inspection policy for system subject to hidden failures, in which the interval of imperfect inspection interval and the number of imperfect inspections are determined so as to achieve an optimal maintenance strategy. Liu et al (2017) proposed a condition-based maintenance policy for a system with age- and state-dependent operating cost, where the side effect of degradation is investigated.

Although numerous studies have accommodated to hidden failures, most focus on maintenance/inspection policy for a single-component system. However, modern systems are usually subject to multiple hidden failures. For example, a protection system contains multiple components such as transducer and relays, and hidden failure may occur in any of the constituents. It is thus necessary to investigate hidden failure in a multi-component context. Yet few efforts have been devoted to multiple-component maintenance policy with hidden failures. Zhu et al (2016) established a multi-level maintenance model for a system with multiple failure modes. Taghipour & Banjevic (2011) developed a periodic inspection policy for a repairable system, where the components are subject to hidden failures. In that work, the optimal inspection interval was applied for all the components. Huynh et al (2014) proposed a multi-level maintenance policy, where the system was intervened at both the component and the system level. Long-run cost rate was minimized by jointly optimizing the inspection interval and preventive maintenance thresholds. One deficiency of the above methods is that all the

components are inspected simultaneously or a common metric is used for all the components. However, as the components in the system vary in degradation process and maintenance cost, it is more desirable to apply different maintenance metrics for heterogeneous components.

In this paper, we aim to develop a maintenance policy for a multi-component system subject to hidden failures. The objective of this study is to determine the inspection intervals for each component such that the long-run cost rate is minimized. Here, we focus on the effect of economic dependence since economic dependence is the most common interaction in a multi-component system. With respect to the maintenance optimization of the multi-component system, the difficulty lies in the computational complexity. Actually, optimization of multi-component maintenance scheduling with economic dependence has been proven to be an NP-complete problem (Levi et al, 2014). To deal with the intractability of this problem, we adopt a heuristic method named “base interval approach” on the basis of Laggoune et al (2009) and Hopp & Kuo (1998). The base interval approach reschedules the inspection interval of each component as an integer multiplier of a base interval, so as to share the common downtime cost and reduce maintenance cost. In addition, we investigate the performance of the base interval approach, where the effectiveness of proposed approach is demonstrated both analytically and numerically.

The remainder of this paper is organized as follows. Section 2 describes the problem and formulates the maintenance model. Maintenance policy using base interval approach is proposed in Section 3 and performance analysis of the approach is presented in Section 4. Numerical examples are presented in Section 5. Finally, concluding remarks and future discussions are given in Section 6.

Notations

C_I	Common downtime cost due to inspection
c_d^i	Downtime cost per unit time of component i
c_I^i	Inspection cost of component i
c_r^i	Replacement cost of component i
$T_d^i(t)$	Downtime of component i till time t

$G(\bullet)$	Long-run cost rate of the system
$G_i(\bullet)$	Long-run cost rate of component i , without considering the common cost
k_i	Integer multiplier, indicating the rescheduled inspection interval of component i
$i=1,2,\dots,n$	Component index
$N_d(t)$	Number of system shutdown due to inspection till time t
$N_I^i(t)$	Number of inspections of component i till time t
n	Number of components
$R_i(t)$	Reliability function of component i at time t
S_i	Length of a renewal cycle of component i
T_i	Time to failure of component i
μ_i	Mean time to failure of component i
τ	Base inspection interval
τ_i	Inspection interval of component i

2. Maintenance model formulation

2.1 General description and assumptions

Consider a he system consisting of multiple non-identical components. Each component suffers an underlying deteriorating process and fails randomly according to specific lifetime distribution. The components are non-repairable; if a component is found to have failed, it has to be replaced by a new one. Note that although the components are restored to the ‘as-good-as-new’ condition after replacement, the whole system is not perfectly maintained except that all the components are replaced simultaneously.

An inspection-replacement policy is implemented on each component. The components are inspected at periodic intervals with the individual inspection cost c_I^i . When an inspection is performed on a component, the whole system has to be shut down, which incurs a common downtime cost C_I . The common cost due to inspection can be shared when multiple inspections are carried out simultaneously. Without loss of generality, it is assumed that the set-up cost and system shutdown cost due to component replacement are included in C_I . Once a failure is

detected at inspection, the associated component is replaced immediately. During the period of hidden failures, additional cost is incurred due to performance loss of the component, denoted as c_d^i .

In terms of constructing a realistic model, the following assumptions are made:

1. Failure of a component is not self-announcing; it can only be revealed at inspection.
2. Inspection can be implemented on each component; inspection on a component is perfect. When a failure occurs, it can always be discovered at the next inspection.
3. A common cost is incurred at each inspection time, which can be shared when multiple inspections are carried out simultaneously. The assumption of constant common cost is reasonable when multiple maintenance crews are available to inspect the components.
4. Only economic dependence exists between components. Structural dependence and stochastic dependence are not considered.
5. The component is non-reparable; only replacement is applicable when a failure is revealed on a component. Replacement is instantaneous and restores the component to the 'as-good-as-new' condition.
6. Deterioration of a component remains unchanged during inspection.

2.2 Cost model

Consider a system consisting of n heterogeneous components. Let τ_i , $i=1,2,\dots,n$, be the inspection intervals for component i . The cost items can be divided into two categories. One is the component-level cost, which is related to specific component and varies for different components. The other category is the system-level cost, which is irrelevant to the component and can be shared by multiple maintenance actions.

Component-level cost: component inspection cost c_l^i , component replacement cost c_r^i , performance loss (due to hidden failures of components) cost per unit time c_d^i .

System-level cost: common downtime cost due to inspection C_l .

For an individual component i , one has to pay the associated cost at time t ,

$$\tilde{H}_i(t; \tau_i) = (C_l + c_l^i) N_l^i(t; \tau_i) + c_r^i N_r^i(t; \tau_i) + c_d^i T_d^i(t; \tau_i) \quad (1)$$

where τ_i is the inspection interval of component i , \tilde{H}_i is the total cost of component i incorporating the common downtime cost, N_I^i is the cumulative number of inspections for component i , $N_r^i(t; \tau_i)$ is the cumulative number of replacement for component i , and $T_d^i(t; \tau_i)$ is the cumulative downtime of component i . However, for a multi-component system, there may be cases that the common cost has already been shared by other components (e.g., due to inspection of other components). If this happens all the time, then we can neglect the common cost C_I . In such cases, the corresponding cost of a component at time t is

$$H_i(t; \tau_i) = c_I^i N_I^i(t; \tau_i) + c_r^i N_r^i(t; \tau_i) + c_d^i T_d^i(t; \tau_i) \quad (2)$$

As shown in Eq (1) and (2), the total maintenance cost can be reduced if multiple components are inspected simultaneously. As the common cost can be shared when performing inspections together, it is of interest to schedule inspections in groups so that multiple inspections can be implemented at one scheduled inspection time. However, on the other hand, rescheduling the inspection time incurs additional cost. As we will see in the following sections, each component has its optimal inspection interval so as to minimize the individual long-run cost rate. If the inspection interval is postponed or advanced in order to share common downtime cost C_I , additional cost is incurred compared with the minimal cost obtained at the optimal inspection interval. We have to consider this tradeoff when grouping multiple components to share the common downtime cost. An optimal policy should balance the potential benefit and the associated additional cost.

If the components are inspected with intervals $\tau_1, \tau_2, \dots, \tau_n$, the long-run cost rate of the system is expressed as

$$G^{(\tau_1, \tau_2, \dots, \tau_n)} = \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^n H_i(t; \tau_i) + N_d(t; \tau_1, \tau_2, \dots, \tau_n) C_I}{t} \quad (3)$$

where $N_d(t)$ is the number of system shutdown till time t . Obtaining an optimal solution of Eq (3) is difficult due to the economic dependence among components. Indeed, optimization of Eq (3) is an NP-hard problem as the number of combinations increases exponentially with the number of components. Due to the intractability of this problem, we have to resort to heuristic methods

that can provide a near optimal solution. In the following, we adopt base interval approach to deal with the computational complexity.

3. Maintenance policy with base interval approach

Base interval approach was initially proposed to solve the joint replacement issue of an aircraft engine (Hopp and Kuo, 1998). It was inspired by the power-of-two replenishment policy in inventory management (Jackson et al, 1985). Laggoune et al (2009) applied the base interval approach in opportunistic maintenance for a continuously operating system. The basic idea of base interval approach is to find a base interval τ such that the common cost is charged at every τ time units and the component is inspected at some integer multipliers of the base interval, $k_i\tau$. In other words, at least one component is inspected in each multiple of τ . Under this scenario, our objective is to find the optimal base interval τ and a set of integers $\{k_i, i = 1, 2, \dots, n\}$ such that the long-run cost rate of the system is minimized. The optimization problem is formulated into

$$\begin{aligned} \min_{\tau, k_1, k_2, \dots, k_n} \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^n H_i(t; k_i \tau)}{t} + \frac{C_f}{\tau} \\ \text{s.t. } \tau > 0, k_i \in Z^+, \forall i = 1, 2, \dots, n \end{aligned} \quad (4)$$

where Z^+ denotes the set of positive integers.

A natural way to calculate the long-run cost rate is to use the renewal cycle theorem and compute the expected cost rate in a renewal cycle of the system (Zhang et al, 2015). However, in the present case, a renewal cycle occurs only when all the components are replaced simultaneously, which takes extremely long time or even never occurs. Hence, it is difficult or even impossible to compute the long-run cost rate function based on renewal cycle theorem. However, by observing the cost structure of Eq (4), we can decompose the objective function from system level into component level, and then apply the renewal cycle theorem individually. As we can see, the objective function contains two terms: the first term, $\lim_{t \rightarrow \infty} \sum_{i=1}^n H_i(t; k_i \tau) / t$, is a summation of individual cost, while the second term, C_f / τ , is irrelevant to individual

component. By taking advantage of the cost structure, we proceed to study the individual optimal maintenance policy so as to reduce the computational complexity.

3.1 Individual optimal maintenance policy

In this section, we formulate a cost model for an individual component and obtain the optimal individual inspection interval. When a component is replaced at inspection, the component is restored to the ‘as-good-as-new’ state, which constitutes a renewal cycle (Liu et al, 2016). According to the renewal cycle theorem, the individual long-run cost rate is given as

$$G_i(\tau_i) = \lim_{t \rightarrow \infty} \frac{H_i(t; \tau_i)}{t} = \frac{E[H_i(S_i)]}{E[S_i]}$$

where S_i is the length of a renewal cycle of component i . Denote T_i as the failure time of component i . Let $R_i(t)$ be the reliability function of the component and $F_{T_i}(t)$ be the cumulative density function (cdf) of the failure time. For notational simplicity, we denote the number of inspection in a renewal cycle and the downtime in a renewal cycle for component i as N_i^i and T_d^i , instead of $N_i^i(S_i)$ and $T_d^i(S_i)$. N_i^i is related to the failure time and the inspection interval, which is denoted as $N_i^i = \lceil T_i / \tau_i \rceil$ (Badia et al, 2001). The expectation of N_i^i is calculated as

$$\begin{aligned} E[N_i^i] &= \sum_{j=1}^{\infty} j P\{(j-1)\tau_i < T_i \leq j\tau_i\} \\ &= \sum_{j=1}^{\infty} j (F_{T_i}(j\tau_i) - F_{T_i}((j-1)\tau_i)) = \sum_{j=0}^{\infty} R_i(j\tau_i) \end{aligned}$$

The length of a renewal cycle depends on N_i^i and τ_i , according to the assumption that replacement can only be carried out at inspection. We have $S_i = N_i^i \tau_i$ and the associated expectation $E[S_i] = \tau_i \sum_{j=0}^{\infty} R_i(j\tau_i)$ (Peng et al, 2010). The component downtime is the period between occurrence of a failure and replacement (end of a renewal cycle), $T_d^i = N_i^i \tau_i - T_i$. The expectation is $E[T_d^i] = \tau_i \sum_{j=0}^{\infty} R_i(j\tau_i) - \mu_i$, where μ_i is the mean time to failure (MTTF) of the component. Based on the above discussion, the expected cost in a renewal cycle can be obtained as

$$E[H_i(S_i)] = c_d^i \left(\tau_i \sum_{j=0}^{\infty} R_i(j\tau_i) - \mu_i \right) + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j\tau_i).$$

The long-run cost rate is then formulated as

$$G_i(\tau_i) = \frac{E[H_i(S_i)]}{E[S_i]} = c_d^i + \frac{-c_d^i \mu_i + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j\tau_i)}{\tau_i \sum_{j=0}^{\infty} R_i(j\tau_i)} \quad (5)$$

Proposition 1. If $c_d^i \mu_i - c_r^i > c_l^i$, there exists optimal inspection interval τ_i^* such that the long-run cost rate reaches its minimum. Otherwise, $\tau_i^* = \infty$. Mathematically,

$$\begin{cases} \tau_i^* \in \{ \tau_i : dG_i(\tau_i) / d\tau_i = 0 \}, & \text{if } c_d^i \mu_i - c_r^i > c_l^i \\ \tau_i^* = \infty, & \text{otherwise} \end{cases}$$

The detailed proof is shown in the Appendix.

Proposition 1 implies that if $c_d^i \mu_i - c_r^i > c_l^i$, it is beneficial to inspect the system with the inspection interval τ_i^* ; otherwise, inspection is unprofitable and we should resort to other maintenance policies. In this study, we consider the case that the condition of $c_d^i \mu_i - c_r^i > c_l^i$ can always be satisfied; otherwise, the maintenance problem is trivial and makes no sense. This assumption is plausible as the downtime cost is relatively high for modern complex systems and the inspection cost is becoming lower with the development of inspection techniques. The condition in Proposition 1 is equivalent to $\mu_i > (c_l^i + c_r^i) / c_d^i$, which implies that whether an optimal inspection interval exists or not can be easily judged by comparing the MTTF of a component with a constant value.

3.2 Selection of k_i

Based on the previous discussions, we can sketch $G_i(\tau_i)$ as shown in Fig 1. Since the system is inspected at some integer multiples of the base interval τ , the optimal k_i should be either $\lceil \tau_i^* / \tau \rceil$ or $\lfloor \tau_i^* / \tau \rfloor$, depending on which is more cost-effective. k_i can be expressed as

$$k_i = \begin{cases} \lfloor \tau_i^* / \tau \rfloor, & \text{if } \frac{-c_d^i \mu_i + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j \lfloor \tau_i^* / \tau \rfloor \tau)}{\lfloor \tau_i^* / \tau \rfloor \tau \sum_{j=0}^{\infty} R_i(j \lfloor \tau_i^* / \tau \rfloor \tau)} < \frac{-c_d^i \mu_i + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j \lceil \tau_i^* / \tau \rceil \tau)}{\lceil \tau_i^* / \tau \rceil \tau \sum_{j=0}^{\infty} R_i(j \lceil \tau_i^* / \tau \rceil \tau)} \\ \lceil \tau_i^* / \tau \rceil, & \text{otherwise} \end{cases} \quad (6)$$

Therefore, once the base interval is given, the optimal inspection intervals for each component can be easily obtained. The objective is reduce to find the optimal base interval τ^* such that the long-run cost rate of the system, $G(\tau)$, is minimized, *i.e.*,

$$\min_{\tau > 0} G(\tau) = \frac{C_I}{\tau} + \sum_{i=1}^n c_d^i + \sum_{i=1}^n \left[\frac{-c_d^i \mu_i + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j \lfloor \tau_i^* / \tau \rfloor \tau)}{\lfloor \tau_i^* / \tau \rfloor \tau \sum_{j=0}^{\infty} R_i(j \lfloor \tau_i^* / \tau \rfloor \tau)} \wedge \frac{-c_d^i \mu_i + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j \lceil \tau_i^* / \tau \rceil \tau)}{\lceil \tau_i^* / \tau \rceil \tau \sum_{j=0}^{\infty} R_i(j \lceil \tau_i^* / \tau \rceil \tau)} \right] \quad (7)$$

where the operator $(x \wedge y) = \min(x, y)$.

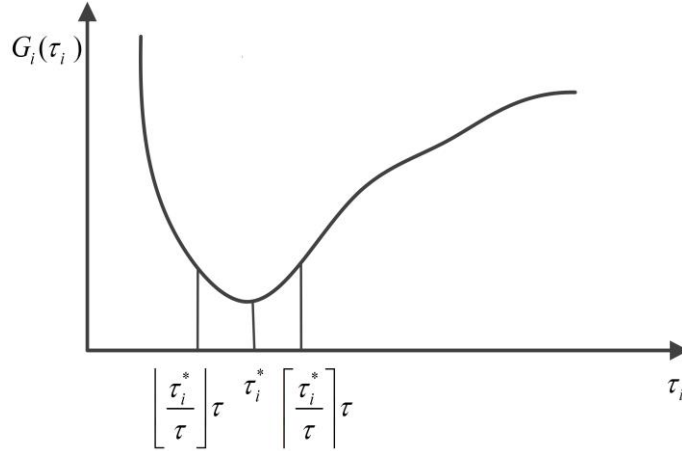


Fig 1 Sketch of $G_i(\tau_i)$

Optimal solution of Eq (7) can be obtained by a simple search algorithm, described as follows:

1. Initialization: For each component, compute the individual optimal inspection interval $\tau_i^*, i = 1, 2, \dots, n$.
2. Begin with a small τ .

3. Determine k_i for each component, according to Eq (6).
4. Compute the value of $G(\tau)$ according to Eq (7).
5. Increase τ for a small increment, repeat step 3-4. Stop until $\tau > \max\{\tau_i^* : i = 1, 2, \dots, n\}$.
6. Output the minimal $G^*(\tau)$ and the associated τ^* .

The time complexity of the algorithm is $O(n)$. Compared with the traditional approaches such as rolling horizon approach and brute-force search approach, the proposed base interval approach shows the advantage of computational effectiveness.

4. Performance analysis

Now that we have described the base interval policy, we proceed to analyze the effectiveness of the policy. A natural way to achieve this purpose is to compare the cost value obtained by our approach with the optimum value. However, for the present case, the optimal solution is difficult to obtain. Hence, instead, we demonstrate the effectiveness of our policy by comparing the upper bound of our approach with the lower bound of the optimal solution.

4.1 Lower bound of the optimal solution

The idea of obtaining a lower bound is to allocate the maintenance cost of the whole system on individual components. Let $\{t_l, l = 1, 2, \dots\}$ denote the time when the system is shut down due to inspection, where l is the number index of system shutdown. Let $x_i^{t_l}$ be the action for component i at time t_l , where

$$x_i^{t_l} = \begin{cases} 0, & \text{if no inspection is performed for component } i \text{ at } t_l \\ 1, & \text{if inspection is performed for component } i \text{ at } t_l \end{cases}$$

Let $\delta(\bullet)$ be the indicator function, where for any $\alpha \geq 0$,

$$\delta(\alpha) = \begin{cases} 0, & \text{if } \alpha = 0 \\ 1, & \text{if } \alpha > 0 \end{cases}$$

With the notations defined above, we can transform the problem of minimizing system long-run cost rate into finding the optimal $x_i^{t_l^*}$, $l = 1, 2, \dots$, $i = 1, 2, \dots, n$, that minimize

$$\min_{x_i^t} Z = \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^n H_i(t; \tilde{\tau}_i) + \delta \left(\sum_{i=1}^n x_i^t \right) C_I}{t} \quad (8)$$

It is straightforward to have (Atkins & Iyogun, 1987)

$$\delta \left(\sum_{i=1}^n x_i^t \right) \geq \sum_{i=1}^n \eta_i \delta(x_i^t)$$

where $\eta_i \geq 0$, $\forall i=1,2,\dots,n$, and $\sum_{i=1}^n \eta_i = 1$. Then, the lower bound of Z can be readily obtained as

$$\begin{aligned} \underline{Z} &= \lim_{t \rightarrow \infty} \frac{\sum_{i=1}^n H_i(t; \tilde{\tau}_i) + \sum_{i=1}^n \eta_i \delta(x_i^t) C_I}{t} \\ &= \sum_{i=1}^n \lim_{t \rightarrow \infty} \frac{H_i(t; \tilde{\tau}_i) + \eta_i \delta(x_i^t) C_I}{t} \end{aligned} \quad (9)$$

For notational simplicity, denote

$$Z_i(\eta_i, \tilde{\tau}_i) = \lim_{t \rightarrow \infty} \frac{H_i(t; \tilde{\tau}_i) + \eta_i \delta(x_i^t) C_I}{t}$$

so that $\underline{Z} = \sum_{i=1}^n Z_i(\eta_i, \tilde{\tau}_i)$. Eq (9) implies that the lower bound \underline{Z} can be decomposed into n single-unit cost items, with the common constraints $\sum_{i=1}^n \eta_i = 1$ and $\eta_i \geq 0$. For component i , at each inspection time, the individual inspection cost c_i^i and common shutdown cost $\eta_i C_I$ are charged; at the end of a renewal cycle of the component, the individual replacement cost c_r^i and component downtime cost $c_d^i \cdot T_d^i$ are incurred. Therefore, for a fixed η_i , $Z_i(\eta_i, \tilde{\tau}_i)$ is equivalent to the individual long-run cost rate of component i , with additional common downtime cost $\eta_i C_I$ at each inspection. The optimal inspection interval, $\tilde{\tau}_i^*(\eta_i)$, is obtained as

$$\begin{aligned}
\tilde{\tau}_i^*(\eta_i) &= \arg \min_{\tilde{\tau}_i} Z_i(\eta_i, \tilde{\tau}_i^*) \\
&= \arg \min_{\tilde{\tau}_i} c_d^i + \frac{-c_d^i \mu_i + c_r^i + (c_l^i + \eta_i C_l) \sum_{j=0}^{\infty} R_i(j \tilde{\tau}_i)}{\tilde{\tau}_i \sum_{j=0}^{\infty} R_i(j \tilde{\tau}_i)}
\end{aligned} \tag{10}$$

Eq (10) implies that if $\eta_i \geq 0$ is given for component i , an optimal inspection interval $\tilde{\tau}_i^*(\eta_i)$ can be readily obtained. Let $Z_i^*(\eta_i) = Z_i(\eta_i, \tilde{\tau}_i^*)|_{\tilde{\tau}_i = \tilde{\tau}_i^*(\eta_i)}$, the tightest lower bound becomes

$$\begin{aligned}
\max_{\eta_i} \underline{Z} &= \sum_{i=1}^n Z_i^*(\eta_i) \\
\text{s.t. } \sum_{i=1}^n \eta_i &= 1, \eta_i \geq 0, \forall i = 1, 2, \dots, n
\end{aligned}$$

Based on the above discussions, the optimization problem in Eq (8) is transformed into finding optimal allocation ($\eta_i C_l$) of the common downtime cost to each component so that the lower bound reaches its maximum. In the work of Hopp & Kuo (1998), it was observed that the lower bound increases when allocating higher percentages of the system intervention cost to components with smaller replacement interval. Here, we follow the similar procedure and assign higher proportion of the common cost to components that are inspected more frequently.

Without loss of generality, relabel the number of components according to their individual optimal inspection intervals. Component with a smaller inspection interval is associated with a smaller index. Assuming that the common downtime cost C_l is allocated to components $1, 2, \dots, \kappa$, with the corresponding fraction η_i , where $\sum_{i=1}^{\kappa} \eta_i = 1$. The inspection interval of component i is τ for $i = 1, 2, \dots, \kappa$ and τ_i^* for $i = \kappa + 1, \kappa + 2, \dots, n$. Then, the lower bound of the system's long-run cost rate is given as

$$LB = \frac{C_l}{\tau} + \sum_{i=1}^{\kappa} G_i(\tau) + \sum_{i=\kappa+1}^n G_i(\tau_i^*)$$

4.2 Upper bound of the base interval policy

Followed by the above discussions, we first analyze the upper bound of the base interval policy for an individual component, as shown in Lemma 1.

Lemma 1. For an individual component, the base interval policy for component i is guaranteed to have a cost at most $(\mu_i + \tau_i^*) / \mu_i$ times of the optimal solution.

The detailed proof is shown in the Appendix. Generally, the optimal individual inspection interval τ_i^* is much less than the MTTF of the component μ_i (as shown in the numerical examples), which indicates that the proposed base interval policy can perform quite well individually. Based on the individual analysis from Lemma 1, we can conclude the upper bound of the system long-run cost rate, as shown in Proposition 2.

Proposition 2. The upper bound of the base interval policy is at most $\sup\{(\mu_i + \tau_i^*) / \mu_i : i = 1, 2, \dots, n\}$ times of the lower bound.

The detailed proof is shown in the Appendix. Proposition 2 indicates that the base interval policy is guaranteed to approximate the optimal solution within a small factor.

For the problem of multi-component maintenance with hidden failures, the most intuitive approaches are the individual inspection policy and common inspection policy (Taghipour & Banjevic, 2011). It is interesting to compare the performance of the base interval approach and the extant two approaches. Obviously, the base interval approach always outperforms the common inspection policy. This is due to the fact that the base interval approach is actually a generalization of the common inspection policy. For any determined base interval τ , if all the k_i are set to 1, then the base interval approach is reduced to the common inspection policy.

On the other hand, the advantage of base interval approach over the individual inspection policy highly depends on the common downtime cost C_1 . If the common downtime cost is low, the additional cost of rescheduling the inspection intervals may overwhelm the shared cost, which implies the disadvantage of the base interval approach. This leads to another interesting issue that under what condition the base interval approach outperforms the individual inspection policy. In addition, for some components, the penalty cost due to rescheduling may exceed the common downtime cost, though not usual in reality, then it would be optimal to remain the inspection interval identical to that of the individual inspection policy, while the inspection

intervals of other components are rescheduled. The proposed base interval approach can be further improved by determining which components should be rescheduled and which should remain unchanged.

5. Numerical examples

In this section, two examples are presented to illustrate the effectiveness of the base interval approach. For the first example, we consider a 3-component system, where each component follows an exponential lifetime distribution. In the second example, the proposed maintenance policy is applied in a centrifugal compressor of a catalytic reforming unit.

5.1 An illustrative example with a 3-component system

A system consisting of three components is used to illustrate the base interval approach. The components are assumed to follow an exponential lifetime distribution, with the reliability function $R_i(t) = \exp(-\lambda_i t)$, where λ_i is the failure rate of component i , $i = 1, 2, 3$. At first, we analyze the optimal inspection policy for an individual component. Based on the discussion in Section 3.1, we have

$$E[N_i^i] = \sum_{j=0}^{\infty} R_i(j\tau_i) = \frac{e^{\lambda_i \tau_i}}{e^{\lambda_i \tau_i} - 1} \quad (11)$$

By substituting Eq (11) into Eq (5), long-run cost rate is expressed as

$$G_i(\tau) = \frac{E[H_i(S_i)]}{E[S_i]} = c_d^i - \frac{c_r^i - \frac{c_d^i}{\lambda_i}}{e^{\lambda_i \tau_i} \tau_i} + \frac{c_l^i + c_r^i - \frac{c_d^i}{\lambda_i}}{\tau_i} \quad (12)$$

The optimal inspection can be obtained by setting the derivative of to 0, *i.e.*,

$$\frac{\partial G_i(\tau_i)}{\partial \tau_i} = \frac{\left(c_r^i - \frac{c_d^i}{\lambda_i} \right) (\lambda_i \tau_i + 1)}{e^{\lambda_i \tau_i} \tau_i^2} - \frac{c_l^i + c_r^i - \frac{c_d^i}{\lambda_i}}{\tau_i^2} = 0 \quad (13)$$

The individual optimal inspection interval can be obtained by solving Eq (13). After some simplifications, we have

$$\tau_i^* \in \left\{ \tau_i : \left(c_r^i - \frac{c_d^i}{\lambda_i} \right) (\lambda_i \tau_i + 1) - \left(c_l^i + c_r^i - \frac{c_d^i}{\lambda_i} \right) e^{\lambda_i \tau_i} = 0 \right\}, \text{ where } \frac{c_d^i}{\lambda_i} - c_r^i > c_l^i$$

The parameters of the failure rate and cost items are listed in Table 1. By use of numerical methods, the individual optimal inspection interval is obtained as $\tau_1^* = 1.19$, $\tau_2^* = 1.39$, $\tau_3^* = 2.26$, and the corresponding optimal individual long-run cost rate is respectively $G_1^*(\tau_1) = 121.9$, $G_2^*(\tau_2) = 187.9$, $G_3^*(\tau_3) = 202.4$. In addition, we plot the variation of individual long-run cost rate $G_i(\tau_i)$ with respect to the individual inspection interval τ_i , as shown in Fig 2.

Table 1 Parameters of component-specific failure rate and cost items

Component	Failure rate λ_i	MTBF μ_i	Replacement cost c_r^i	Inspection cost c_l^i	Downtime cost c_d^i
1	0.02	50	200	70	5000
2	0.05	20	420	115	2500
3	0.1	10	640	150	750

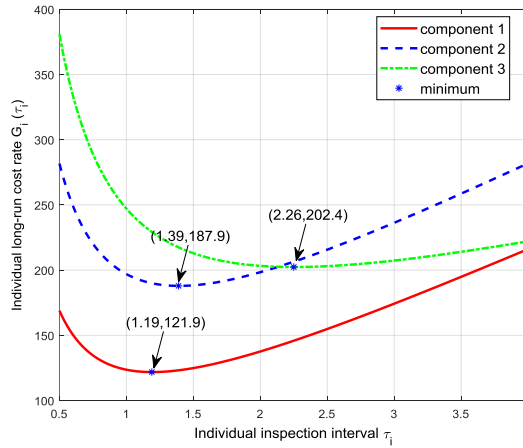


Fig 2 Variation of $G_i(\tau_i)$ with τ_i

With regard to the whole system, the common downtime cost is set as $C_l = 60$. The relationship between $G(\tau)$ and τ is given in Eq (7). Fig 3 shows how the long-run cost rate $G(\tau)$ changes with the base interval τ . As we can see, when the base interval is $\tau = 1.4$, the long-run cost rate $G(\tau)$ reaches its minimum, $G^*(\tau) = 559.1$. The associated k_i is given as

$k_1 = 1, k_2 = 1, k_3 = 2$. The result implies that based on the base interval approach, when we inspect component 1 every $k_1\tau = 1.4$ unit time, component 2 every $k_2\tau = 1.4$ unit time, and component 3 every $k_3\tau = 2.8$ unit time, we have the optimal maintenance policy. Table 2 shows the comparison of the base interval approach, the individual inspection policy and the common inspection policy (Taghipour & Banjevic, 2011). The result shows that the proposed base interval approach dominates the other policies.

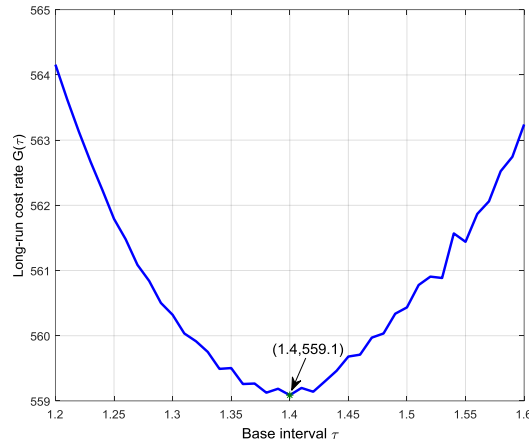


Fig 3 Variation of $G(\tau)$ with respect to τ

Table 2 Comparison of the rescheduled inspection policy, the individual inspection policy and common inspection policy

Component inspection interval	τ_1	τ_2	τ_3	Cost rate
Individual inspection policy	1.19	1.39	2.26	692.2
Common inspection policy	1.69	1.69	1.69	562.9
Rescheduled inspection policy	1.4	1.4	2.8	559.1

In addition, Proposition 2 implies that the upper bound of the base interval policy is at most $\sup\left\{\left(\mu_i + \tau_i^*\right) / \mu_i : i = 1, 2, 3\right\} = 1.226$ times of the lower bound. Here, the lower bound is calculated as $LB = 556.4$. The long-run cost rate obtained by the base interval approach is actually 1.005 times of the lower bound, which indicates that the base interval approach performs quite effectively.

We are interested in the value of the common downtime cost C_I , since C_I characterizes the economic dependence among components and affects the necessity of performing base interval approach. Sensitivity analysis is carried out on C_I to find out how C_I influences the optimal base interval and the optimal long-run cost rate. Fig 4 shows how the optimal inspection interval τ^* and the associated long-run cost rate $G^*(\tau)$ vary with different C_I . As can be observed, $G^*(\tau)$ increases monotonically with C_I and τ^* shows a non-decreasing trend. This can be explained by the fact that the long-run cost rate $G(\tau)$ consists of two items: one is related to the common cost, and the other is determined by the individual cost rate. When the common cost C_I increases, the cost item C_I / τ has an increasing effect on $G(\tau)$. Therefore, the optimal base interval τ increases with C_I to reduce the number of system shutdowns.

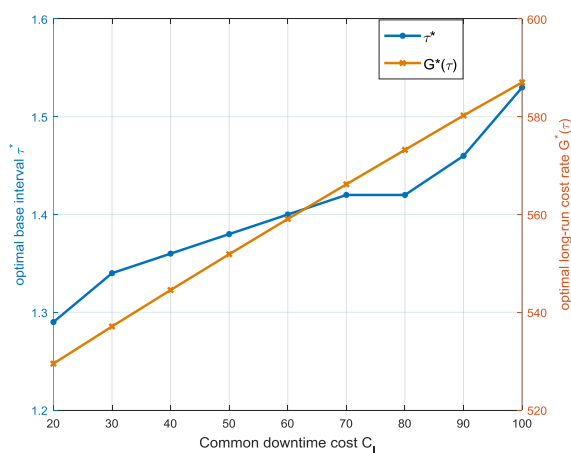


Fig 4 Sensitivity analysis on C_I

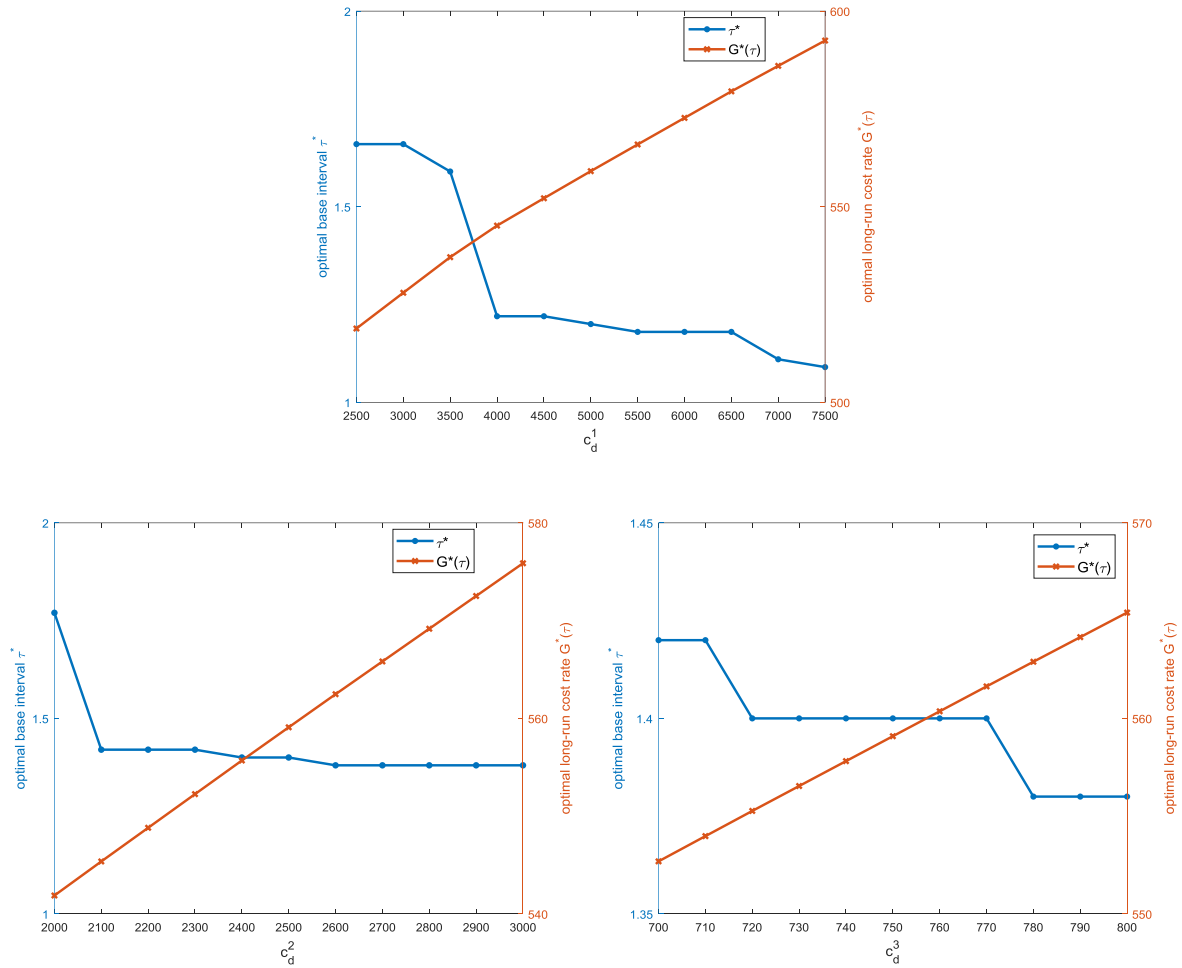


Fig 5 Variation of τ^* and $G^*(\tau)$ in terms of individual downtime cost C_d^i

(a) component 1 C_d^1 (b) component 2 C_d^2 (c) component 3 C_d^3

Additionally, Fig 5 presents the variation of τ^* and $G^*(\tau)$ in terms of the individual downtime cost C_d^i . Obviously, the optimal long-run cost rate $G^*(\tau)$ shows a monotone increasing trend with C_d^i . The optimal inspection interval τ^* , however, presents a non-increasing trend. This is due to the fact that a more frequent inspection takes effect in reducing the downtime cost. With the increase of the individual downtime cost, a smaller inspection interval is expected to balance the effect of increasing downtime cost.

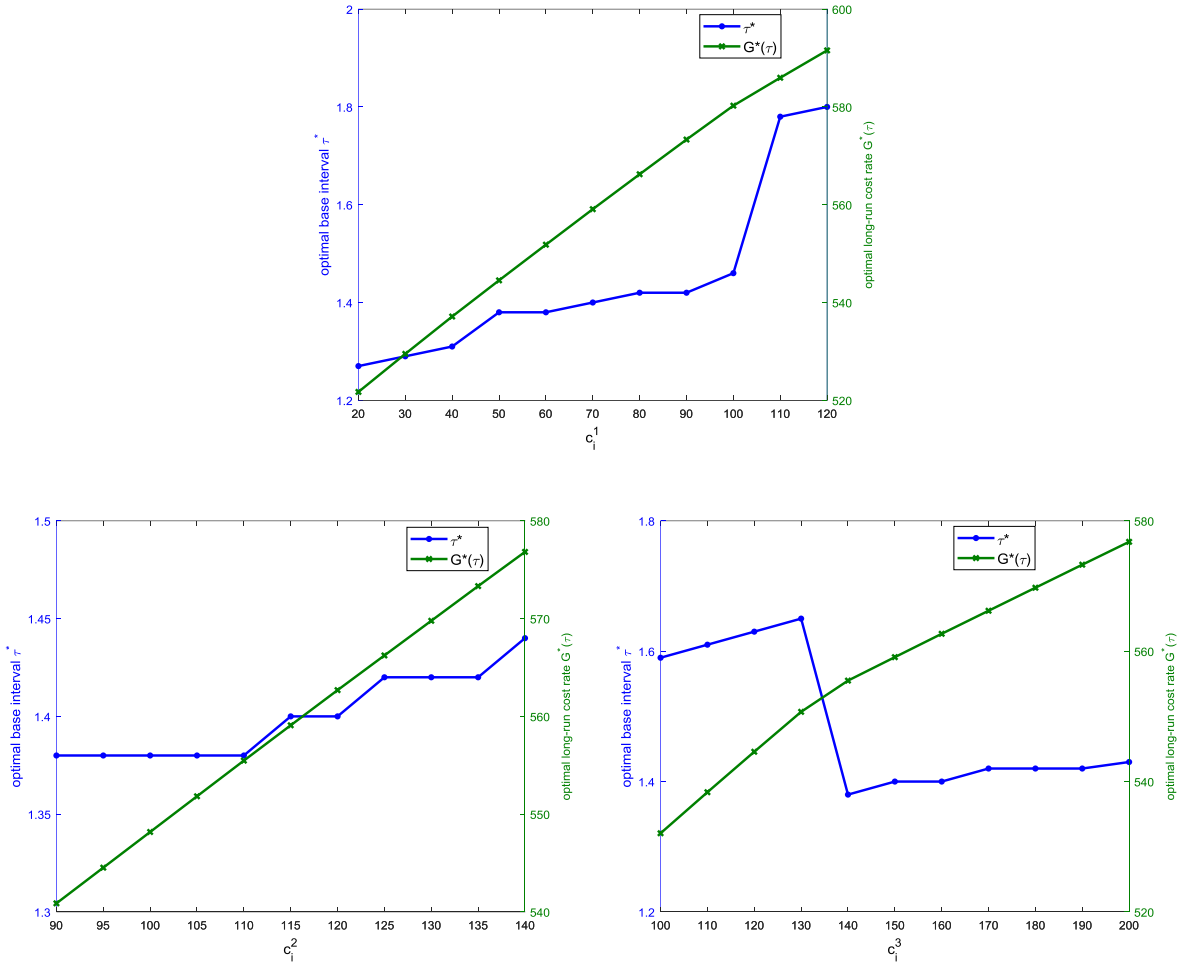


Fig 6 Variation of τ^* and $G^*(\tau)$ in terms of individual inspection cost C_i^i

(a) component 1 C_i^1 (b) component 2 C_i^2 (c) component 3 C_i^3

Fig 6 investigates the effect of individual inspection cost on the optimal maintenance policy. For component 1 and component 2, the optimal inspection interval τ^* shows a non-decreasing trend. The optimal maintenance policy calls for a larger inspection interval so as to balance the tradeoff of an increased inspection cost. However, component 3 shows a non-conform trend. τ^* increases for $C_i^3 \in (100, 130)$ and $C_i^3 \in (140, 200)$ respectively. However, there is a drop-off for $C_i^3 \in (130, 140)$. This is due to the fact that the integer multiplier k_3 varies for $C_i^3 \in (130, 140)$. In the present setting, k_3 increases from 1 to 2 when C_i^3 changes from 130 to 140. Although the optimal base interval decreases, the inspection interval of component 3 still increases, due to the increase of k_3 .

5.2 Application in refinery centrifugal compressor

In this section, we apply the base interval inspection policy to a centrifugal compressor of a catalytic reforming unit. A centrifugal compressor is an essential constitute of a catalytic reforming unit, which plays a vital role in oil refinery. As reported in Laggoune et al (2009), a compressor dysfunction may cause pressure loss in the reforming unit and consequently lead to performance loss of the system. In addition, the centrifugal compressor is responsible for pre-heating and providing air to guarantee that the unit operates under proper condition. A centrifugal compressor comprises multiple components, such as sheathing, tightness and so on. Here, we assume that the components of a compressor operate in a redundant mode, where failure of a component is not self-evident. The base interval inspection policy is adopted to reduce the maintenance cost and enhance system performance.

Table 3 Component-specific Weibull parameters and cost items

Component	code	Shape parameter β_i	Scale parameter θ_i	MTBF μ_i (day)	Replacement cost c_r^i (€)	Inspection cost c_l^i (€) (assumed)	Downtime cost c_d^i (€) (assumed)
Sheathing	C286	1.73	486	483	14868	1200	350
Sheathing	C285	1.88	507	475	39204	320	1100
Tightness	C275	2.43	286	240	44880	330	1000
Stub bearing	C230	2.53	898	787	57876	1180	1600
Tightness ring	C460	2.14	905	844	73860	420	1900
Carrying bearing	C419	3.55	736	636	46752	360	1600
Stub bearing	C401	2.68	1094	888	48568	1670	1700
Labyrinth support	C780	2.09	1388	1047	74232	1700	1800

The compressor consists of eight components, wherein each component is associated with a code, as shown in Table 3. The components are assumed to follow a two-parameter Weibull distribution. The reliability function of component i is denoted as $R_i(t) = \exp\left[-\left(t/\theta_i\right)^{\beta_i}\right]$,

where θ_i is the scale parameter and β_i is the shape parameter. Table 3 gives the value of Weibull parameters and the associated cost items of each component. The shape parameter β_i and the scale parameter θ_i are obtained from the study of Laggoune et al (2009), where failure data were sampled to estimate the two parameters. The cost parameters related to inspection and hidden failure are introduced for illustration purpose, including the individual inspection cost c_i^i and the individual downtime cost c_d^i .

Based on the discussions in Section 3.2.1, we obtain the optimal inspection interval τ_i^* and the associated long-run cost rate $G_i^*(\tau_i)$ for each component, as shown in Table 4. For the purpose of illustration, we also plot the variation of $G_i(\tau_i)$ with respect to τ_i . Four components are selected for illustration: tightness (C275), stub bearing (C230), carrying bearing (C419) and stub bearing (C401). Fig 7 shows how $G_i(\tau_i)$ varies with τ_i of the four components.

Table 4 Optimal inspection policy for individual component

Component	code	Optimal individual inspection interval τ_i^* (day)	Optimal individual long-run cost rate $G_i^*(\tau_i)$ (€)
Sheathing	C286	58	37
Sheathing	C285	17	65
Tightness	C275	15	275
Stub bearing	C230	36	159
Tightness ring	C460	19	36
Carrying bearing	C419	18	175
Stub bearing	C401	48	269
Labyrinth support	C780	54	390

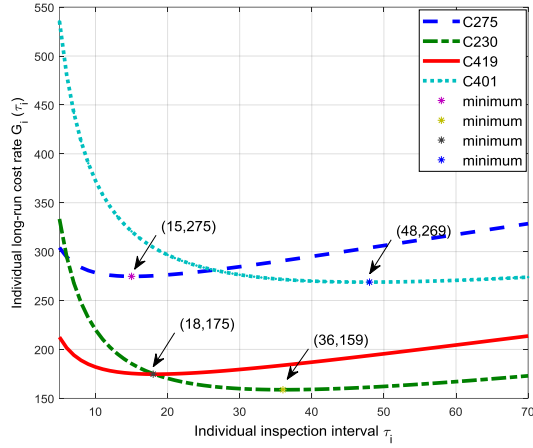


Fig 7 Plot of individual long-run cost rate

The common downtime cost is given as $C_I = \text{€}2500$. According to Eq (7), the variation of long-run cost rate $G(\tau)$ with respect to the base interval τ is plotted in Fig 8. From Fig 8, the minimum long-run cost rate of $\text{€}1517$, obtained at the optimal base interval $\tau^* = 29$ days. After obtaining the optimal base interval τ^* , we can determine the integral multiple k_i for each component. The result is shown in Table 5. By comparing the results of Table 5 with the optimal individual inspection interval, it is observed that the inspection period of stub bearing (C230) is shifted in advance, the inspection period of sheathing (C286) remains unchanged, and the inspection period of other components are postponed. Proposition 2 implies that the upper bound of the base interval policy is at most $\sup\left\{\left(\mu_i + \tau_i^*\right) / \mu_i : i=1,2,\dots,8\right\} = 1.201$ times of the lower bound. In this section, the lower bound can be computed as $LB = 1501$. The long-run cost rate obtained by base interval approach is actually 1.011 times of the lower bound.

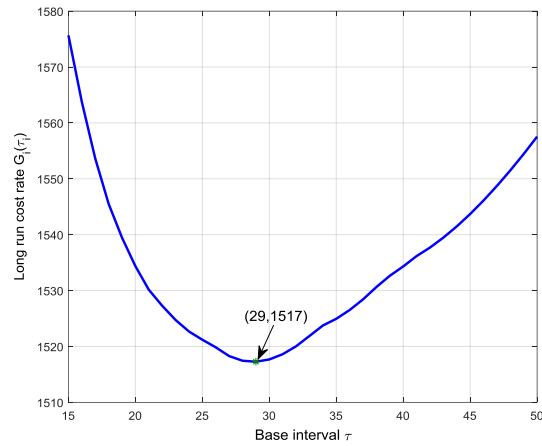


Fig 8 Variation of $G(\tau)$ with respect to base interval τ

Table 5 Optimal inspection interval for base interval policy

Component	code	k_i associated with optimal base interval	Optimal inspection interval $k_i\tau$ (days)
Sheathing	C286	2	58
Sheathing	C285	1	29
Tightness	C275	1	29
Stub bearing	C230	1	29
Tightness ring	C460	1	29
Carrying bearing	C419	1	29
Stub bearing	C401	2	58
Labyrinth support	C780	2	58

Furthermore, in order to investigate how the components are grouped with various base interval τ , we plot k_i with respect to τ in Fig 9. When τ increases from 15 to 50 days, k_i of the components show a non-increasing trend. When τ is no less than 41 days, k_i reduces to 1 for $i=1,2,\dots,8$, which implies that all the components are inspected at the same time. Note that we only plot k_i of four components (C286, C230, C401 and C780). k_i of the other four components are not shown in Fig 6, as they always remain 1 when τ varies within the range of [15, 50].

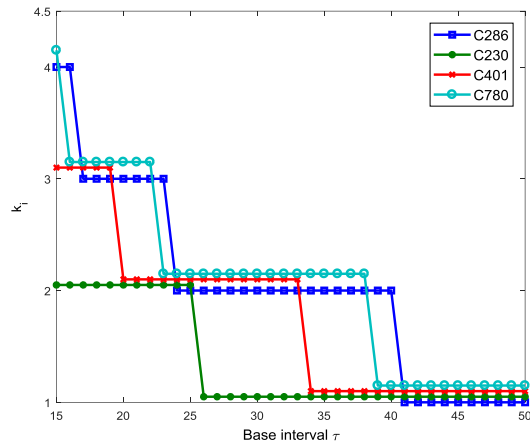


Fig 9 Plot of k_i with respect to base interval τ

Fig 10 shows how the optimal base interval τ^* and the optimal long-run cost rate $G^*(\tau)$ vary with the common downtime cost C_I . When C_I increases from €500 to €4500, $G^*(\tau)$ increase monotonically from €1434 to €1582. In addition, the optimal base interval τ^* shows an increasing trend with C_I , changing from 9 to 26 days. The result indicates that common downtime cost has a significant influence on the optimal inspection policy. It is therefore suggested that maintenance engineers or managers should take into account the common cost (interactions among components) when making maintenance decisions for a multi-component system.

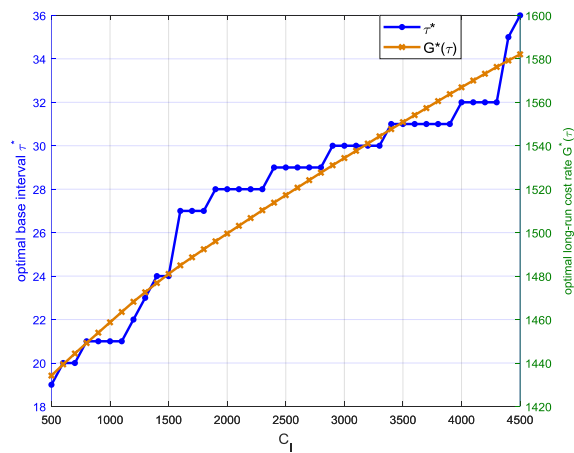


Fig 10 Sensitivity analysis on C_I

6. Conclusions

This paper develops a maintenance policy for multi-component systems subject to hidden failures. Failure of a component can only be detected at inspection and a failed component is replaced once the failure is revealed. Different from previous works where an optimal inspection interval is applied for the whole system, we obtain the optimal inspection interval for each component. Concerned with the difficulty of the optimization problem, a heuristic method named base interval approach is adopted to reduce the computational complexity. Upper bound of the policy is analyzed, followed by two numerical examples. It is illustrated that the base interval policy can approximate the optimal policy within a small factor. The result of a numerical example shows that our approach outperforms the individual inspection policy and the common inspection policy.

The proposed maintenance strategy can be applied in various complex systems such as power generators and processing facilities, especially for systems with a high common downtime cost. One advantage of the proposed maintenance policy lies in its feasibility and applicability in industry. Maintenance crews only need to inspect the components at fixed intervals and replace the failed components, while no other tricky implementations are required.

Future research can be conducted by extending the present inspection-replacement policy into a more general maintenance context. For example, consider a multi-component repairable system where various maintenance actions such as minimal repair and imperfect maintenance are allowed. In addition, if the components of the system follow a continuously degrading process and the degrading state can be observed by inspection devices, one can integrate the concept of base interval approach into condition-based maintenance and develop a more efficient maintenance policy.

Acknowledgement

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Appendix

1. Proof of Proposition 1.

$G_i(\tau_i)$ can be rewritten as

$$G_i(\tau_i) = c_d^i + \frac{U(\tau_i)}{\tau_i},$$

where $U(\tau_i) = \left(-c_d^i \mu_i + c_r^i + c_l^i \sum_{j=0}^{\infty} R_i(j\tau_i) \right) / \sum_{j=0}^{\infty} R_i(j\tau_i)$.

After some simplifications, $U(\tau_i)$ is expressed as $U(\tau_i) = c_l^i - (c_d^i \mu_i - c_r^i) / \sum_{j=0}^{\infty} R_i(j\tau_i)$.

Obviously, $\sum_{j=0}^{\infty} R_i(j\tau_i)$ is a decreasing function of τ_i and $\lim_{\tau_i \rightarrow \infty} \sum_{j=0}^{\infty} R_i(j\tau_i) = 1$. Suppose

that $c_d^i \mu_i - c_r^i < c_l^i$, then we have $G_i(\tau_i) > c_d^i, \forall \tau_i \in (0, \infty)$. Moreover, $\lim_{\tau_i \rightarrow \infty} \sum_{j=0}^{\infty} R_i(j\tau_i) = 1$ implies

that $\lim_{\tau_i \rightarrow \infty} G_i(\tau_i) = c_d^i$, leading to the conclusion that $\tau_i^* = \infty$.

If the condition $c_d^i \mu_i - c_r^i > c_l^i$ holds, we can conclude that $U(\tau_i)$ is decreasing with τ_i . Also, we have

$$\begin{aligned}\lim_{\tau_i \rightarrow 0} U(\tau_i) &= c_l^i > 0 \\ \lim_{\tau_i \rightarrow \infty} U(\tau_i) &= c_l^i - (c_d^i \mu_i - c_r^i) < 0\end{aligned}$$

The monotonicity of $U(\tau_i)$ and the above limit property imply that there exists a τ_i^0 such that

$$\begin{aligned}U(\tau_i) &> 0, \forall \tau_i \in (0, \tau_i^0) \\ U(\tau_i) &< 0, \forall \tau_i \in (\tau_i^0, \infty)\end{aligned}$$

Hence, $G_i(\tau_i)$ has the following properties:

$$\begin{aligned}G_i(\tau_i^0) &= c_d^i \\ \lim_{\tau_i \rightarrow \infty} G_i(\tau_i) &= c_d^i \\ G_i(\tau_i) &> c_d^i, \forall \tau_i \in (0, \tau_i^0) \\ G_i(\tau_i) &< c_d^i, \forall \tau_i \in (\tau_i^0, \infty)\end{aligned}$$

The above properties imply that there exists an optimal $\tau_i^* \in (\tau_i^0, \infty)$ such that $G_i(\tau_i)$ reaches its minimum. The optimal inspection interval τ_i^* can be obtained by setting the derivative to zero, i.e., $\tau_i^* \in \{\tau_i : dG_i(\tau_i)/d\tau_i = 0\}$. \square

2. Proof of lemma 1.

For notational simplicity, define $Q(\tau_i) = \sum_{j=0}^{\infty} R_i(j\tau_i)$. By definition, we have

$$\frac{\mu_i}{\tau_i} \leq \left\lceil \frac{\mu_i}{\tau_i} \right\rceil < \frac{\mu_i}{\tau_i} + 1$$

Taking expectation of the above expression leads to

$$\frac{\mu_i}{\tau_i} \leq Q(\tau_i) = E \left[\left\lceil \frac{\mu_i}{\tau_i} \right\rceil \right] < \frac{\mu_i}{\tau_i} + 1$$

Thus, we have

$$\mu_i \leq \tau_i Q(\tau_i) < \mu_i + \tau_i$$

The individual optimal inspection interval τ_i^* is limited within the range

$$(k_i - 1)\tau < \tau_i^* \leq k_i\tau$$

Then, we can obtain

$$G_i(k_i\tau) = \frac{-c_d^i \mu_i + c_r^i + c_l^i Q(k_i\tau)}{k_i\tau S(k_i\tau)} \leq \frac{-c_d^i \mu_i + c_r^i + c_l^i Q(k_i\tau)}{\mu_i} \leq \frac{-c_d^i \mu_i + c_r^i + c_l^i Q(\tau_i^*)}{\mu_i}$$

The first inequality is due to the expression that $\mu_i \leq \tau_i Q(\tau_i)$; the second inequality is due to the fact that $\tau_i^* \leq k_i\tau$ and that $Q(\tau_i)$ is a non-increasing function. The follows the inequality

$$G_i(\tau_i^*) = \frac{-c_d^i \mu_i + c_r^i + c_l^i Q(\tau_i^*)}{\tau_i^* Q(\tau_i^*)} > \frac{-c_d^i \mu_i + c_r^i + c_l^i Q(\tau_i^*)}{\mu_i + \tau_i^*}$$

Therefore, we have

$$\frac{G_i(k_i\tau)}{G_i(\tau_i^*)} < \frac{\mu_i + \tau_i^*}{\mu_i}$$

which completes the proof. \square

3. Proof of Proposition 2.

Let the inspection interval of component i is τ for $i = 1, 2, \dots, \kappa$ and $k_i\tau$ for $i = \kappa + 1, \kappa + 2, \dots, n$. As shown in lemma 1,

$$\frac{G_i(k_i\tau)}{G_i(\tau_i^*)} < \frac{\mu_i + \tau_i^*}{\mu_i}$$

the upper bound is

$$\begin{aligned}
UB &= \frac{C_L}{\tau} + \sum_{i=1}^{\kappa} G_i(\tau) + \sum_{i=\kappa+1}^n G_i(k_i\tau) \\
&< \frac{C_L}{\tau} + \sum_{i=1}^{\kappa} G_i(\tau) + \sum_{i=\kappa+1}^n \frac{\mu_i + \tau_i^*}{\mu_i} G_i(\tau_i^*) \\
&< \frac{C_L}{\tau} + \sum_{i=1}^{\kappa} G_i(\tau) + \sup \left\{ \frac{\mu_i + \tau_i^*}{\mu_i} : i = 1, 2, \dots, n \right\} \sum_{i=\kappa+1}^n G_i(\tau_i^*) \\
&< \sup \left\{ \frac{\mu_i + \tau_i^*}{\mu_i} : i = 1, 2, \dots, n \right\} \left(\frac{C_L}{\tau} + \sum_{i=1}^{\kappa} G_i(\tau) + \sum_{i=\kappa+1}^n G_i(\tau_i^*) \right) \\
&= \sup \left\{ \frac{\mu_i + \tau_i^*}{\mu_i} : i = 1, 2, \dots, n \right\} LB
\end{aligned}$$

which concludes the proof. \square

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