MIMO Radar Ambiguity Functions: A Case Study

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***Abstract*—In recent years Multiple-Input Multiple-Output (MIMO) radar has been shown to offer enhanced performance. In traditional radar systems Woodward's ambiguity function (AF) is normally used to characterize the performance of the operating waveform. Due to their higher degrees of freedom (DOF) however, MIMO radars' performance cannot be sufficiently characterised by Woodward's AF. While many formulations of MIMO AF have been proposed addressing different necessities for system characterisation, most of them do not provide a complete tool covering all the DOFs. In this paper we examine how parameters such as array geometry, operating waveform, and target model can effect the performance of the overall system. Additionally we propose a MIMO AF definition based on Kullback-Leibler divergence and compare it with different proposed formulations.**

***Index Terms*—Multiple-Input Multiple-Output Ambiguity Function (MIMO AF); MIMO Radar ; Kullback-Leibler divergence (KLD)**

I. Introduction

MIMO (multiple-input multiple-output) radar systems have attracted the interest of the research community due to their capability to significantly increase their performance compared to the traditional monostatic and multistatic systems. While by general definition MIMO can be viewed as a type of multistatic radar, in this work the distinctive difference between the two arises from the distinction of waveforms attributed to each transmitter and the joint processing that MIMO is used on [1]. Following this definition, MIMO radars can be classified depending on the spatial allocation of their antennas with the two extremes of collocated and widely distributed configurations posing different advantages discussed in [2] and [3] respectively. Additionally as described in [2] and [4] the systems can also be categorised by the coherency of their operating waveforms with the special cases of fully orthogonal and coherent signals. Moreover the importance of the target model in MIMO systems was discussed in [1] and [5] where it was shown how the correlation of the transmitter-target-receiver channel matrix is dependent on the geometry of the system and the dimensions of the target.

Modern radar systems are required to operate with high accuracy for their intended applications. It is therefore very important to have a prior knowledge of the system's expected performance from the design stage. One of the mainly used tools by radar engineers is the ambiguity function (AF). Originally introduced by Woodward [6], the AF is a graphical representations of the received signal's response when a matched filter is applied for different delay and Doppler shifts. Using the AF it is therefore easy to extract valuable information such as the ambiguities and resolution expected for a particular configuration.

Due to the promising tendency of radar technology to extend in multi-sensor/multi-platform configurations, various formulations of AFs for MIMO systems have been proposed [2], [4], [5], [7]–[9]. In [4] and [7] the optimum detector concept was used and the MIMO AF was obtained by summing the matched filtered result from each receiver. Under a similar definition, a MIMO AF based on an ultrawideband signal model was derived in [10]. A different approach,was explored in [2] where the suggested MIMO AF is based on the log-likelihood function and the concept of information theory. In [5], the log-likelihood based MIMO AF definition is applied on a widely distributed MIMO system (DMRS) signal model. Lastly in, [9] a MIMO AF based on the Kullback-Leibler divergence (KLD) and a DMRS signal model is derived. While the approach of formulating the MIMO AF in [9] is very similar with the one presented in [5], the authors in [9] derive a formulation of the inverse covariance matrix of the expected signal, reducing computation complexity, and derive an upperbound to bound the MIMO AF in values between 0 and 1.

In this work a generalised MIMO AF definition base on the KLD is proposed to cover both widely distributed and collocated configurations. Moreover, the proposed formulation is parametrised by the auto-correlation and cross-correlation matrices of the expected and received with expected signal matrices, and the channel correlation matrices. This allows for more flexible modelling compared with previously proposed formulations.

The remainder of the paper is organised as follows. Section II introduces the MIMO radar signal that will be used. The proposed MIMO AF is derived in Section III while illustrations of its behavior under various configurations is shown in Section IV. Finally Section V summarises the outcomes of this work.

Comments on notation: Vectors and matrices are denoted by bold letters, e.g. $\boldsymbol{\ell}$. The transpose and conjugate transpose operators are denoted by $(\cdot)^T$ and $(\cdot)^\dagger$ respectively. The Euclidean distance operation is denoted by $|\cdot|$, $\delta(\cdot)$ denotes the Dirac delta function, and $\mathrm{j} = \sqrt{-1}$. Moreover, $\boldsymbol{I}_\ell$ denotes a $\ell \times \ell$ identity matrix, $\mathbf{1}_\ell$ denotes a $\ell \times \ell$ matrix populated by ones, and "$\otimes$" is the Kronecker product operation. Finally, for convenience and without loss of generality, in the rest of the paper a 2-D plane is assumed instead of a 3-D space, with the general format of coordinates and velocity being expressed as

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$\boldsymbol{x} = [x, y]^T$ and $\boldsymbol{v} = [v_x, v_y]^T$ respectively.

II. Signal Model

Consider a MIMO radar system with $N_{\mathcal{T}}$ transmitters and $N_{\mathcal{R}}$ receivers, and with all their antennas having an isotropic radiation pattern. The location and velocity of the i-th transmitter and the j-th receiver are denoted in the Cartesian plane by the column vectors $\boldsymbol{x}_{i,\mathcal{T}}$ and $\boldsymbol{u}_{i,\mathcal{T}}$ for $i = 1, \ldots, N_{\mathcal{T}}$, and $\boldsymbol{x}_{j,\mathcal{R}}$ and $\boldsymbol{u}_{j,\mathcal{R}}$ for $j = 1, \ldots, N_{\mathcal{R}}$ respectively. Moreover assume an extended target within the surveillance area comprises a finite number $N_{\mathcal{Q}}$ of independent isotropic scatterers with location and velocity described respectively by $\boldsymbol{x}_{q,\mathcal{Q}}$ and $\boldsymbol{u}_{q,\mathcal{Q}}$ for $q = 1, \ldots, N_{\mathcal{Q}}$. The reflectivity of the scatterer is modelled by an independent and identically distributed (i.i.d) complex random variable ζ_q with zero mean and variance $\mathcal{E}\{|\zeta_q|^2\} = \sigma_0^2/N_{\mathcal{Q}}$ where σ_0^2 is the average radar cross section (RCS) of the target. Additionally the target is assumed to follow the classic Swerling I model while its RCS centre of gravity is located at $\boldsymbol{x}_{0,\mathcal{Q}}$ and its velocity is $\boldsymbol{v}_{0,\mathcal{Q}}$.

Considering a stationary system, the delay of a signal emitted by i-th transmitter, reflected by a scatterer located at the target's centre of gravity and received by j-th receiver can be written as:

$$\tau_{j,i}^{(q)} = \frac{|\boldsymbol{D}_{i,\mathcal{T}}^{(q)}| + |\boldsymbol{D}_{j,\mathcal{R}}^{(q)}|}{c} \tag{1}$$

where $\boldsymbol{D}_{i,\mathcal{T}}^{(q)} = \boldsymbol{x}_{q,\mathcal{Q}} - \boldsymbol{x}_{i,\mathcal{T}}$ and $\boldsymbol{D}_{j,\mathcal{R}}^{(q)} = \boldsymbol{x}_{q,\mathcal{Q}} - \boldsymbol{x}_{j,\mathcal{R}}$ are the distance vectors from the q-th scatter of the target to the i-th transmitter and j-th receiver respectively, and c is the speed of light. Assuming a narrowband signal with complex envelope $s_i(t)$ at each transmitter with $\int_T |s_i(t)|^2 \mathrm{d}t = 1$ and carrier frequency f_c, the received signal for each transmitter-receiver pair can be expressed as:

$$r_{j,i}(t) = \sqrt{E_{j,i}} \sum_{q=1}^{N_{\mathcal{Q}}} \zeta_q e^{\phi_{j,i}^{(q)}} s_i\left(t - \tau_{j,i}\right) e^{\mathrm{j}\omega_{j,i} t} + n_j(t) \tag{2}$$

where $E_{j,i}$ is the signal energy parameter in the i-th transmitter-j-th receiver pair, $\omega_{j,i} = 2\pi f_c(a_{j,i} - 1)$ and $\phi_{j,i}^{(q)} = -\mathrm{j}2\pi f_c a_{j,i} \tau_{j,i}^{(q)}$ account respectively for the angular frequency and phase shifts applied to the signal due to the relative motion and delay in the i-th transmitter, q-th scatter, j-th receiver system, and $a_{j,i}$ is the time scaling factor defined as:

$$\alpha_{j,i} = 1 - \left((\boldsymbol{U}_{i,\mathcal{T}})^T \frac{\boldsymbol{D}_{i,\mathcal{T}}}{|\boldsymbol{D}_{i,\mathcal{T}}|} + (\boldsymbol{U}_{j,\mathcal{R}})^T \frac{\boldsymbol{D}_{j,\mathcal{R}}}{|\boldsymbol{D}_{j,\mathcal{R}}|} \right) / c \tag{3}$$

To simplify (2) two factorisations are considered:

$$h_{j,i}^{(q)}(\theta) = \sqrt{E_{j,i}} \zeta_q e^{\phi_{j,i}^{(q)}} \tag{4}$$

$$y_{j,i}(t, \theta) = s_i(\alpha_{j,i}(t - \tau_{j,i})) e^{\mathrm{j}\omega_{j,i} t} \tag{5}$$

were $\theta = [\boldsymbol{x}_0, \boldsymbol{v}_0]^T$ and therefore (2) can be expressed as:

$$r_{j,i}(t, \theta) = \sum_{q=1}^{N_{\mathcal{Q}}} h_{j,i}^{(q)}(\theta) y_{j,i}(t, \theta) + n_j(t) \tag{6}$$

As the received signal is sampled, it is more practical to define it by using a $M \times 1$ column vector, where M is the number of captured samples. First we define $\boldsymbol{y}_{j,i}(\theta)$ as a $M \times 1$ column vector composed by the discrete samples of $y_i(t, \theta, j)$. The $M \times 1$ column vector describing the sampled $r_{j,i}(t)$ can be written as:

$$\boldsymbol{r}_{j,i}(\theta) = \boldsymbol{y}_{j,i}(\theta) h_{j,i}(\theta) + \boldsymbol{n}_j \tag{7}$$

where $\boldsymbol{n}_j$ is the $M \times 1$ column vector associated with the $n_j(t)$, while $h_{j,i}(\theta)$ defined as:

$$h_{j,i}(\theta) = \sum_{q=1}^{N_{\mathcal{Q}}} h_{j,i}^{(q)}(\theta) \tag{8}$$

The variable $h_{j,i}(\theta)$ can also be expressed as:

$$h_{j,i}(\theta) = \sqrt{E_{j,i}} \boldsymbol{k}_{j,i}(\theta) \boldsymbol{z} \tag{9}$$

where $\boldsymbol{k}_{j,i}(\theta)$ is the $1 \times N_{\mathcal{Q}}$ row vector matrix defined as:

$$\boldsymbol{k}_{j,i}(\theta) = \left[e^{\phi_{j,i}^{(1)}}, e^{\phi_{j,i}^{(2)}}, \ldots, e^{\phi_{j,i}^{(N_{\mathcal{Q}})}} \right] \tag{10}$$

and $\boldsymbol{z}$ is the $N_{\mathcal{Q}} \times 1$ column vector matrix given by:

$$\boldsymbol{z} = [\zeta_1, \zeta_2, \ldots, \zeta_{N_{\mathcal{Q}}}]^T \tag{11}$$

To examine the total signal at each receiver, we define $\boldsymbol{y}_j(\theta)$ as the $M \times N_{\mathcal{T}}$ matrix given by:

$$\boldsymbol{y}_j(\theta) = [\boldsymbol{y}_{j,1}(\theta), \boldsymbol{y}_{j,2}(\theta), \ldots, \boldsymbol{y}_{j,N_{\mathcal{T}}}(\theta)] \tag{12}$$

Additionally, we define $\boldsymbol{h}_j(\theta)$ as the $N_{\mathcal{T}} \times 1$ column vector given by:

$$\boldsymbol{h}_j(\theta) = \sqrt{\boldsymbol{E}_j(\theta)} \boldsymbol{k}_j(\theta) \boldsymbol{z} \tag{13}$$

where $\boldsymbol{E}_j(\theta)$ is the $N_{\mathcal{T}} \times N_{\mathcal{T}}$ diagonal matrix given by:

$$\boldsymbol{E}_j(\theta) = \mathrm{diag}(E_{1,j}, E_{2,j}, \ldots, E_{N_{\mathcal{T}},j}) \tag{14}$$

and $\boldsymbol{k}_j(\theta)$ is the $N_{\mathcal{T}} \times N_{\mathcal{Q}}$ matrix defined as:

$$\boldsymbol{k}_j(\theta) = [\boldsymbol{k}_{j,1}(\theta), \boldsymbol{k}_{j,2}(\theta), \ldots, \boldsymbol{k}_{j,N_{\mathcal{T}}}(\theta)]^T \tag{15}$$

Using (12) and (13), the $M \times 1$ column vector of the overall received signal on the j-th receiver can be expressed as:

$$\boldsymbol{r}_j(\theta) = \boldsymbol{y}_j(\theta) \boldsymbol{h}_j(\theta) + \boldsymbol{n}_j \tag{16}$$

To examine now the complete MIMO system, $\boldsymbol{Y}(\theta)$ is defined as the $N_{\mathcal{R}} M \times N_{\mathcal{T}} N_{\mathcal{R}}$ block diagonal matrix given by:

$$\boldsymbol{Y}(\theta) = \mathrm{diag}(\boldsymbol{y}_1(\theta), \boldsymbol{y}_2(\theta), \ldots, \boldsymbol{y}_{N_{\mathcal{R}}}(\theta)) \tag{17}$$

Moreover the $N_{\mathcal{R}} N_{\mathcal{T}} \times 1$ block matrix $\boldsymbol{H}(\theta)$ is defined as:

$$\boldsymbol{H}(\theta) = \sqrt{\boldsymbol{E}(\theta)} \boldsymbol{K}(\theta) \boldsymbol{Z} \tag{18}$$

where $\boldsymbol{K}(\theta)$ is the $N_{\mathcal{R}} N_{\mathcal{T}} \times N_{\mathcal{Q}} N_{\mathcal{R}}$ block matrix defined as:

$$\boldsymbol{K}(\theta) = \mathrm{diag}(\boldsymbol{k}_1(\theta), \boldsymbol{k}_2(\theta), \ldots, \boldsymbol{k}_{N_{\mathcal{R}}}(\theta)) \tag{19}$$

and $\boldsymbol{Z}$ is the $N_{\mathcal{R}} N_{\mathcal{Q}} \times 1$ block matrix given by:

$$\boldsymbol{Z} = \boldsymbol{1}_{N_{\mathcal{R}}} \otimes \boldsymbol{z} \tag{20}$$

Moreover, $\boldsymbol{E}(\theta)$ is the $N_{\mathcal{R}} N_{\mathcal{T}} \times N_{\mathcal{T}} N_{\mathcal{R}}$ diagonal matrix given by:

$$\boldsymbol{E}(\theta) = \mathrm{diag}(\boldsymbol{E}_1(\theta), \boldsymbol{E}_2(\theta), \ldots, \boldsymbol{E}_{\mathcal{R}}(\theta)) \tag{21}$$

$$I(\theta_0:\theta)=\frac{1}{2}\Big(-\operatorname{tr}\left[\boldsymbol{\Psi}(\theta_0,\theta)^\dagger\frac{\boldsymbol{C}(\theta_0)}{\sigma_n^2}\boldsymbol{\Psi}(\theta_0,\theta)\frac{\boldsymbol{C}(\theta)}{\sigma_n^2}[\boldsymbol{\Phi}(\theta)\frac{\boldsymbol{C}(\theta)}{\sigma_n^2}+\boldsymbol{I}_{N_\mathcal{T}N_\mathcal{R}}]^{-1}\right]+\operatorname{tr}\left[\boldsymbol{\Phi}(\theta_0)\frac{\boldsymbol{C}(\theta_0)}{\sigma_n^2}\right]$$
$$-\operatorname{tr}\left[\boldsymbol{\Phi}(\theta)\frac{\boldsymbol{C}(\theta)}{\sigma_n^2}[\boldsymbol{\Phi}(\theta)\frac{\boldsymbol{C}(\theta)}{\sigma_n^2}+\boldsymbol{I}_{N_\mathcal{T}N_\mathcal{R}}]^{-1}\right]+\ln\left|\boldsymbol{\Phi}(\theta)\frac{\boldsymbol{C}(\theta)}{\sigma_n^2}+\boldsymbol{I}_{N_\mathcal{T}N_\mathcal{R}}\right|$$
$$-\ln\left|\boldsymbol{\Phi}(\theta_0)\frac{\boldsymbol{C}(\theta_0)}{\sigma_n^2}+\boldsymbol{I}_{N_\mathcal{T}N_\mathcal{R}}\right|\Big) \tag{27}$$

The total MIMO system's output can now be defined as the $N_\mathcal{R}M\times 1$ block matrix $\boldsymbol{r}(\theta)$ populated by the samples of the discrete signal captured in all receivers given by:

$$\boldsymbol{r}(\theta)=[\boldsymbol{r}_1(\theta),\boldsymbol{r}_2(\theta),\ldots,\boldsymbol{r}_{N_\mathcal{R}}(\theta)]^T \tag{22}$$

or

$$\boldsymbol{r}(\theta)=\boldsymbol{Y}(\theta)\boldsymbol{H}(\theta)+\boldsymbol{n} \tag{23}$$

where $\boldsymbol{n}$ is a $N_\mathcal{R}M\times 1$ block diagonal matrix stated as:

$$\boldsymbol{n}=[\boldsymbol{n}_1,\boldsymbol{n}_2,\ldots,\boldsymbol{n}_{N_\mathcal{R}}]^T \tag{24}$$

In the next section the proposed signal system will be used to derive the MIMO AF.

III. Ambiguity Function Formulation

In this section the Kullback-Leibler Divergence and how it can be used to define the ambiguity in the context of a radar system is described. It should be noted that this ambiguity definition was firstly proposed for the mono-static radar case in [11] and later extended for a distributed MIMO radar system (DMRS) model [9]. This work is mainly focus on examining how this definition can be applied for a more generalised MIMO system and therefore the ambiguity definition will not be analytically derived here. We refer the reader to [11] for a detailed ambiguity formulation.

The KLD is a measure of similarity between probability densities [12]. In the model described in Section II the received signal $\boldsymbol{r}(\theta)$ was described as the summation of products between i.i.d random variables in $\boldsymbol{H}(\theta)$ multiplied by the deterministic signals in $\boldsymbol{Y}(\theta)$. For a large number of scatters $\boldsymbol{N}_\mathcal{Q}$ and according to the central limit theorem, the received signal follows a Gaussian distribution $\boldsymbol{r}\sim\mathcal{CN}(0,\boldsymbol{R}_\theta)$. Moreover, the covariance matrix $\boldsymbol{R}_\theta$ of the received signal can be calculated as:

$$\begin{aligned}\boldsymbol{R}_\theta&=\mathcal{E}\{\boldsymbol{r}(\theta)\boldsymbol{r}(\theta)^H\}\\&=\mathcal{E}\{(\boldsymbol{Y}(\theta)\boldsymbol{H}(\theta)+\boldsymbol{n})(\boldsymbol{Y}(\theta)\boldsymbol{H}(\theta)+\boldsymbol{n})^\dagger\}\\&=\boldsymbol{Y}(\theta)\mathcal{E}\{\boldsymbol{H}(\theta)\boldsymbol{H}(\theta)^\dagger\}\boldsymbol{Y}(\theta)^\dagger+\sigma_n^2\boldsymbol{I}_{MN_\mathcal{R}}\\&=\boldsymbol{Y}(\theta)\boldsymbol{C}(\theta)\boldsymbol{Y}(\theta)^\dagger+\sigma_n^2\boldsymbol{I}_{MN_\mathcal{R}}\end{aligned} \tag{25}$$

where $\boldsymbol{C}(\theta)=\mathcal{E}\{\boldsymbol{H}(\theta)\boldsymbol{H}(\theta)^\dagger\}$. The KLD between two $MN_\mathcal{R}$ normal densities with zero mean and covariance matrices R_{θ_0} and R_θ is [11]:

$$I(\theta_0:\theta)=\frac{1}{2}\left[\operatorname{tr}[\boldsymbol{R}_\theta^{-1}\boldsymbol{R}_{\theta_0}]-MN_\mathcal{R}-\ln|\boldsymbol{R}_\theta^{-1}\boldsymbol{R}_{\theta_0}|\right] \tag{26}$$

In this case the two normal densities are those described by the return from the target being at the spatial/velocity location θ_0 and the expected location θ respectively. In [9] it was shown that the KLD in (26) can be further expanded as in (27) where $\boldsymbol{\Phi}(\theta)$ and $\boldsymbol{\Psi}(\theta_1,\theta_2)$ are the auto-correlation and cross-correlation matrices defined as:

$$\boldsymbol{\Phi}(\theta)=\boldsymbol{Y}(\theta)^\dagger\boldsymbol{Y}(\theta) \tag{28}$$
$$\boldsymbol{\Psi}(\theta_1,\theta_2)=\boldsymbol{Y}(\theta_1)^\dagger\boldsymbol{Y}(\theta_2) \tag{29}$$

Having define the KLD, its upper bound $I(\theta_0)_{\mathrm{ub}}$ can be calculated as in [9]. Finally, the MIMO AF can be defined as:

$$\mathcal{A}(\theta_0,\theta)\triangleq 1-\frac{I(\theta_0:\theta)}{I_{\mathrm{ub}}(\theta_0)}\leq 1 \tag{30}$$

where, as it can be seen from (27), the equality holds for $\theta=\theta_0\rightarrow I(\theta_0:\theta)=0$.

A. Channel Correlation matrix

As it was shown in Section II, the channel matrix $\boldsymbol{H}(\theta)$ accounts for phase and amplitude shifts of the received signal. In [13] a framework on estimating the value of the elements of the matrix $\boldsymbol{C}(\theta)$ based on a similar signal model as the one presented in Section II is derived. Using this formulation $\boldsymbol{C}(\theta)$ can be simplified as:

$$\boldsymbol{C}(\theta)=\sqrt{\boldsymbol{E}(\theta)}\boldsymbol{K}_0(\theta)\mathcal{E}\left\{\boldsymbol{\Omega}(\theta)\right\}\boldsymbol{K}_0(\theta)^\dagger\sqrt{\boldsymbol{E}(\theta)} \tag{31}$$

where $\boldsymbol{K}_0(\theta)$ is the $N_\mathcal{R}N_\mathcal{T}\times N_\mathcal{T}N_\mathcal{R}$ diagonal matrix populated by the steering vectors of each transmitter receiver pair:

$$\boldsymbol{K}_0(\theta)=\operatorname{diag}(e^{\phi_{1,1}},e^{\phi_{1,2}},\ldots,e^{\phi_{N_\mathcal{R},N_\mathcal{T}}}) \tag{32}$$

with $\phi_{j,i}=-\mathrm{j}2\pi f_c\alpha_{j,i}\tau_{j,i}$ and $\boldsymbol{\Omega}(\theta)$ being the $N_\mathcal{R}N_\mathcal{T}\times N_\mathcal{T}N_\mathcal{R}$ matrix populated by real values depended on the correlation of each transmitter receiver pair. In [13] this correlation degree is expressed as a function of the spacial location of the transmitter and receiver, their distance from the target and the dimension of the target. On the extremes of a collocated system the matrix $\boldsymbol{\Omega}(\theta)$ will be populated by ones, meaning that all the channels are correlated, while if the system is widely distributed it will be a diagonal matrix denoting that all the channels are decorrelated.

B. Relationship with Other MIMO AFs

The proposed definition follows a very similar approach to the one presented in [11]. However in here the product of the covariance and inverse covariance matrix, see (26) for similar definition to [11], is further decomposed and factorised into meaningful terms as auto-correlation, correlation signal matrices and channel correlation matrices. This approach offers

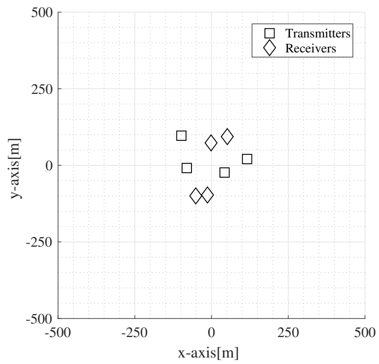
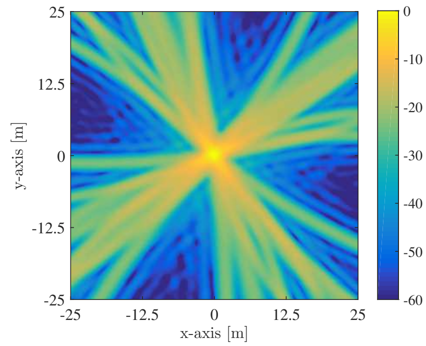
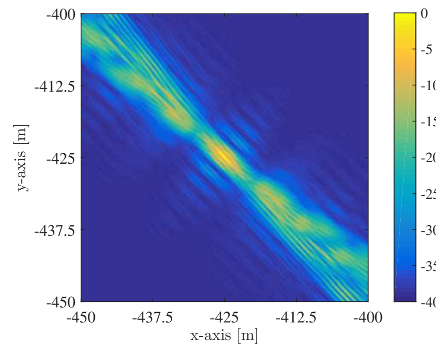


Figure 1. Sensors geometry in the surveillance area



(a) Distributed



(b) Collocated

Figure 2. Zoomed part of the MIMO AF when the target's centre of gravity is positioned in (a) $\boldsymbol{x}_{0,\mathcal{Q}} = [0, 0]^T$ and (b) $\boldsymbol{x}_{0,\mathcal{Q}} = [-425, -425]^T$, and orthogonal waveforms and constant energy parameter is considered.

lower computational complexity [9] and, more important, a better understanding of how the MIMO AF behaves under different configurations. Moreover the proposed framework is more flexible and accommodates collocated configurations.

In [2] a similar definition to [11] is proposed. Similar to this work, the authors also derive the inverse of the covariance matrix of the expected received signal to simplify the proposed definition. In the definition of the inverse matrix however the matrix $\boldsymbol{C}(\theta)$ needs to be firstly decomposed as $\boldsymbol{C}(\theta) = \boldsymbol{Q}(\theta)\boldsymbol{Q}(\theta)^\dagger$. While the authors model the size of $\boldsymbol{Q}(\theta)$ as function of the correlation in the elements of $\boldsymbol{C}(\theta)$, no close definition on how $\boldsymbol{Q}(\theta)$ should be constructed is derived. In this work the MIMO AF is described as a function of $\boldsymbol{C}(\theta)$, of which an estimation process for different configurations can be derived [13]. Additionally, the proposed MIMO AF is bounded for values between 0 and 1 providing a definition more similar to the traditional AF.

IV. Exampes and Illustrations

In this section the behavior of the MIMO AF will be illustrated for different target placements. Figure 1 illustrates the spatial allocation of a 4 transmitters and 4 receivers composing the MIMO system. Moreover, we consider a constant energy parameter for all resolution bins i.e $\sqrt{\boldsymbol{E}(\theta)} = \boldsymbol{I}_{N_\mathcal{R} N_\mathcal{T}}$. The used carrier frequency is $f_c = 500$ MHz while the sequences used in the system are linear frequency modulated (LFM) waveforms shifted in different frequencies described as:

$$s_i(t) = e^{\mathrm{j}\pi \mathrm{BW}(\frac{1}{T}t + i - 1)t} \tag{33}$$

where the waveforms' bandwidth and period are $\mathrm{BW} = 100$ MHz and $T = 10\mu$s. The targets average RCS is $\sigma_0^2 = 1$ while the noise variance is $\sigma_n^2 = 16 \times 10^{-4}$.

Figure 2 illustrates the proposed MIMO AF in logarithmic scale for an area close to a target with velocity $\boldsymbol{u}_{0,\mathcal{Q}} = [0, 0]^T$ and centre of gravity is positioned at (a) $\boldsymbol{x}_{0,\mathcal{Q}} = [0, 0]^T$ and (b) $\boldsymbol{x}_{0,\mathcal{Q}} = [-425, -425]^T$. For the first target placement the system is closer to the distributed geometry while in the second case it is modelled better by the collocated. In both cases the MIMO AF is populated by 16 ellipsoid shaped ridges corresponding to each bistatic transmitter-target-receiver system. Upon closer inspection it can be seen that in the distributed case these ridges are added constructively to form a peak at the position of the target while in the collocated case the diagonal stripes of fluctuations are present. This phenomenon is presented due to the constructive or destructive correlation of the different channels associated with the collocated systems.

V. Conclusion

In this work a MIMO AF is presented based on the Kullback-Leibler divergence and applied in a MIMO radar system framework. Theoretical analysis showed that the proposed MIMO AF can be factorised in auto-correlation and cross-correlation signal matrices, and channel correlation matrices. Moreover, the MIMO AF maximally stretched between 0 and 1 while also being flexible for various system spatial configuration assumptions. The relationship of the proposed MIMO AF with other proposed definition is also examined. Lastly, the behaviour of the proposed MIMO AF for different target placements is investigated.

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