Finite-time anti-synchronization of multi-weighted coupled neural networks with and without coupling delays

Jie Hou · Yanli Huang · Erfu Yang

Abstract The multi-weighted coupled neural networks (MWCNNs) models with and without coupling delays are investigated in this paper. Firstly, the finite-time anti-synchronization of MWCNNs with fixed topology and switching topology is analyzed respectively by utilizing Lyapunov functional approach as well as some inequality techniques, and several anti-synchronization criteria are put forward for the considered networks. Furthermore, when the parameter uncertainties appear in MWCNNs, some conditions for ensuring robust finite-time anti-synchronization are obtained. Similarly, we also consider the finite-time anti-synchronization and robust finite-time anti-synchronization for MWCNNs with coupling delays under fixed and switched topologies respectively. Lastly, two numerical examples with simulations are provided to confirm the effectiveness of these derived results.

1 Introduction

In recent years, neural networks (NNs) have become a hot topic because of their widespread applications, especially in optimization, associate memory,

Jie Hou
School of Computer Science and Technology, Tianjin Polytechnic University, Tianjin 300387, China

Yanli Huang (Corresponding author)
Tianjin Key Laboratory of Optoelectronic Detection Technology and System, School of Computer Science and Technology, Tianjin Polytechnic University, Tianjin 300387, China
E-mail: huangyanli@tjpu.edu.cn

Erfu Yang
Department of Design, Manufacture and Engineering Management, Faculty of Engineering, University of Strathclyde, Glasgow G1 1XJ, Scotland, UK

pattern recognition and so on [1–7]. Actually, these applications are heavily dependent on their dynamical behaviors. Hence, a lot of meaningful and important research results have been reported on stability and passivity of NNs [8–13]. Zhang et al. [8] studied the stability for a class of discrete-time NNs with time-varying delay via an extended reciprocally convex matrix inequality. The authors in [9] discussed the stability of complex-valued memristive recurrent NNs. By utilizing Lyapunov functional approach and some inequality techniques, the passivity and stability problems for NNs with reaction-diffusion terms were considered in [10].

As a particular sort of complex networks, coupled NNs (CNNs) is composed of many NNs which interact with each other. As we all known, synchronization is one of the most significant dynamic properties in CNNs, which has been applied in many areas such as image processing, secure communication [14,15]. Therefore, it is of great significance to investigate the synchronization of CNNs [16–22]. Wang et al. [18] studied the local synchronization problem for a sort of Markovian nonlinearly CNNs by applying the Lyapunov-Krasovskii functional as well as free-matrix-based integral inequality. Based on Lyapunov functions, Halanay inequality and stochastic analysis technique, the exponential synchronization problem of CNNs with time-varying delay was investigated in [19]. Several criteria for ensuring exponential synchronization for markovian stochastic CNNs were carried out via adaptive feedback control in [20]. The work in [22] analyzed the exponential synchronization and passivity of CNNs with reaction-diffusion terms by adopting appropriate pinning controllers. What is noteworthy is that anti-synchronization is also a fascinating phenomenon in the real world, which widely exists in memristive recurrent NNs, periodic oscillators, and so forth. Moreover, up to now, anti-synchronization has been successfully applied in many fields, for instance, image processing, information science and so on. Hence, it is highly meaningful to study anti-synchronization [23–28]. Some conditions for guaranteeing anti-synchronization of CNNs with mixed time-varying delays were set up via randomly occurring control in [25]. Liu et al. [26] devoted themselves to the anti-synchronization for the considered complex-valued memristive NNs by employing suitable Lyapunov functional and some inequality techniques. In [28], the anti-synchronization problem of a memristor-based bidirectional associate memory NN was studied by utilizing a robust feedback controller with an appropriate gain control matrix.

In many practical applications, it is usually required to realize synchronization within a limited time interval. Accordingly, a large number of literatures on finite-time synchronization have been reported [29–40]. In [29], several conditions for ensuring the finite-time synchronization of CNNs with time-varying delays were derived by designing a stochastic multiple Lyapunov-Krasovskii function and using weighted integral inequality. Some discontinuous or continuous controllers were constructed for the finite-time synchronization problem of switched CNNs in [30]. Sun et al. [32] investigated the finite-time synchronization for two complex-variable chaotic systems with unknown parameters via nonsingular terminal sliding mode control. However, only a few results have

been obtained about finite-time anti-synchronizaton up to now [41, 42]. In [41], the finite-time anti-synchronization for a class of time-varying delayed NNs was studied via feedback control with intermittent adjustment. The authors proposed some finite-time anti-synchronization criteria for memristive NNs by designing a nonlinear controller in [42]. As far as we know, a great many of existing networks can be described more accurately by multi-weighted complex dynamical networks (MWCDNs), for instance, transportation networks, social networks, communication networks and so on. Therefore, it is of great significance to study MWCDNs [43]. Qiu et al. [43] proposed several finite-time synchronization conditions for MWCDNs with and without coupling delays by means of Lyapunov functional approach and state feedback controllers. However, there are only a few results reported on the finite-time synchronization of MWCNNs [44]. In [44], several novel criteria for guaranteeing finite-time synchronization of MWCNNs with and without coupling delays were provided by exploiting new definitions of finite-time synchronization and designing appropriate controllers. It is a pity that the finite-time anti-synchronization of MWCNNs has not yet been considered in existing research results.

As is well known, the connection topology of most of networks mentioned above is always supposed to be fixed. Nevertheless, in practical applications, this requirement is very strict because of the impacts of limited communications and external disturbances. Therefore, more and more researchers devoted themselves to studying the synchronization of NNs with switching topology [45–48]. In [46], the authors investigated the finite-time synchronization of a sort of uncertain CNNs with switching topology by constructing appropriate Lyapunov-like functionals and using the average dwell time technique. The work in [47] proposed some conditions for guaranteeing pinning synchronization of directed networks with switching topologies by employing a multiple lyapunov functions approach. Based on Lyapunov functional approach and some inequality techniques, Qin et al. [48] studied the synchronization and $\mathcal{H}_\infty$ synchronization of MWCDNs with fixed and switching topologies. However, there is no research results reported on the finite-time anti-synchronization for MWCNNs with switching topology until now.

On the other hand, some uncertain factors may be occurred in NNs due to the existence of environmental noises and modeling errors, which might cause the exact parameter values of NNs could not be obtained. Therefore, it is important to consider the robust synchronization of CNNs which consisting of several identical uncertain NNs [49–52]. In [49], the authors investigated the robust synchronization of fractional-order CNNs by utilizing pinning control strategies. The robust synchronization problem for delayed CNNs with uncertain parameters was studied by intermittent pinning control in [50]. By using inequality techniques and constructing appropriate Lyapunov functional, Qin et al. [51] discussed the robust synchronization and $\mathcal{H}_\infty$ synchronization of MWCDNs with uncertain parameters. To our best knowledge, the robust finite-time anti-synchronization for MWCNNs with uncertain parameters has never been discussed.

On the basis of the above discussion, we study the finite-time anti-synchron ization and robust finite-time anti-synchronization of MWCNNs with fixed and switching topologies in this paper. Firstly, we present two MWCNN models. The first one is with constant coupling, and the second MWCNN is with delayed coupling. Then, several finite-time anti-synchronization criteria for these considered networks with fixed as well as switching topologies are derived respectively. Furthermore, some sufficient conditions for ensuring robust finite-time anti-synchronization of MWCNNs with fixed and switching topologies are also established.

In view of the above-mentioned discussion, the principal objective of this paper is to study the finite-time anti-synchronization of multi-weighted coupled neural networks (MWCNNs). The main contribution of this paper can be summarized as follows.

(1) The finite-time anti-synchronization of MWCNNs with fixed and switch-ing topologies is studied and several criteria are put forward by designing suitable controller and constructing appropriate Lyapunov functional.
(2) Some conditions for guaranteeing the robust finite-time anti-synchronization of MWCNNs with uncertain parameters under fixed and switching topolo-gies are established respectively.
(3) For the delayed MWCNNs, the finite-time anti-synchronization and robust finite-time anti-synchronization conditions are also derived similarly.

The outline of this paper is organized as follows. Several lemmas needed to be used throughout this paper are provided in Section 2. Section 3 is de-voted to analyzing finite-time anti-synchronization and the robust finite-time anti-synchronization for MWCNNs with fixed and switching topologies, respec-tively. In Section 4, the network model of MWCNNs with coupling delays is introduced, and then the finite-time anti-synchronization and the robust finite-time anti-synchronization for this kind of MWCNNs with fixed and switching topologies are investigated. Two simulation examples are presented in Section 5 to verify the effectiveness of the obtained theoretical results. Finally, this paper is concluded in Section 6.

2 Preliminaries

$0 \geqslant \chi \in \mathbb{R}^{n \times n}$ ($0 \leqslant \chi \in \mathbb{R}^{n \times n}$) symbols the matrix χ is semi-negative (semi-positive) definite and symmetric, $0 > \chi \in \mathbb{R}^{n \times n}$ ($0 < \chi \in \mathbb{R}^{n \times n}$) shows that the matrix χ is negative (positive) definite and symmetric. $\lambda_M(\cdot)$ and $\lambda_m(\cdot)$ respectively signify the maximum and minimum eigenvalues of the cor-responding matrix. In addition, for $e(t) = (e_1(t), e_2(t), \cdots, e_n(t))^T \in \mathbb{R}^n$, we have $\|e(t)\|_2 = \left(\sum\limits_{\iota=1}^{n} e_\iota^2(t) \right)^{\frac{1}{2}}$.

Lemma 1 *(see [53]). Suppose that a continuous and positive-definite function $H(t)$ meets the following inequality:*

$$\dot{H}(t) \leqslant -\mu(H(t))^\omega, \ t \geqslant 0, \ H(0) \geqslant 0,$$

where $0 < \mu \in \mathbb{R}$ and $0 < \omega < 1$ are constants. Then,

$$(H(t))^{1-\omega} \leqslant (H(0))^{1-\omega} - \mu(1-\omega)t, \ 0 \leqslant t \leqslant T,$$

and

$$H(t) = 0, \ t \geqslant T,$$

with T given by

$$T = \frac{(H(0))^{1-\omega}}{\mu(1-\omega)}.$$

Lemma 2 *(see [54]). For $\nu_\xi \in \mathbb{R}$, $\xi = 1, 2, \cdots, n$, $0 < k \leqslant 1$, then*

$$|\nu_1|^k + |\nu_2|^k + \cdots + |\nu_n|^k \geqslant (|\nu_1| + |\nu_2| + \cdots + |\nu_n|)^k.$$

3 Finite-time anti-synchronization of MWCNNs

3.1 Anti-synchronization in finite time for MWCNNs

3.1.1 Anti-synchronization in finite time for MWCNNs with fixed topology

Consider the following MWCNNs with fixed topology:

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + c_1 \sum_{j=1}^{\kappa} G_{\iota j}^1 \Gamma_1 Y_j(t) + c_2 \sum_{j=1}^{\kappa} G_{\iota j}^2 \Gamma_2 Y_j(t) + \cdots$$

$$+ c_m \sum_{j=1}^{\kappa} G_{\iota j}^m \Gamma_m Y_j(t) + u_\iota(t), \quad \iota = 1, 2, \cdots, \kappa, \tag{1}$$

where $Y_\iota(t) = (Y_{\iota 1}(t), Y_{\iota 2}(t), \cdots, Y_{\iota n}(t))^T \in \mathbb{R}^n$ is the state vector of the ιth node; $A = \mathrm{diag}(a_1, a_2, \cdots, a_n) \in \mathbb{R}^{n \times n} > 0, D = (d_{\iota j})_{n \times n} \in \mathbb{R}^{n \times n}$ symbols a constant matrix; $g(Y_\iota(t)) = (g_1(Y_{\iota 1}(t)), g_2(Y_{\iota 2}(t)), \cdots, g_n(Y_{\iota n}(t)))^T \in \mathbb{R}^n$ and $\mathbb{R} \ni c_s > 0$ represents coupling strength for the sth coupling form; $u_\iota(t) \in \mathbb{R}^n$ means the control input; $\Gamma_s \in \mathbb{R}^{n \times n}$ is the inner coupling matrix of the sth coupling form; $G^s = (G_{\iota j}^s)_{\kappa \times \kappa} \in \mathbb{R}^{\kappa \times \kappa}$ expresses coupling weight between nodes in the sth coupling form, where $G_{\iota j}^s$ is defined as follows: $G_{\iota j}^s = G_{j\iota}^s > 0$ if and only if there exists a connection between node ι and node j for the sth coupling form; if not, $G_{\iota j}^s = G_{j\iota}^s = 0 \ (\iota \neq j)$; and

$$G_{\iota\iota}^s = -\sum_{\substack{j=1 \\ j \neq \iota}}^{\kappa} G_{\iota j}^s, \ \iota = 1, 2, \cdots, \kappa,$$

where $s = 1, 2, \cdots, m$.

Assumption 1 *The function $g_k(\cdot) \ (k = 1, 2, \cdots, n)$ satisfies*

$$|g_k(\alpha_1) + g_k(\alpha_2)| \leqslant \theta_k |\alpha_1 + \alpha_2|,$$

for any $\alpha_1, \alpha_2 \in \mathbb{R}$, where $0 < \theta_k \in \mathbb{R}$. Take $\Theta = \mathrm{diag}(\theta_1^2, \theta_2^2, \cdots, \theta_n^2) \in \mathbb{R}^{n \times n}$.

Suppose $Y_0(t) = (Y_{01}(t), Y_{02}(t), \cdots, Y_{0n}(t))^T \in \mathbb{R}^n$ is an arbitrary solution for the network (1), then

$$\dot{Y}_0(t) = -AY_0(t) + Dg(Y_0(t)). \tag{2}$$

Definition 1 For all $\iota = 1, 2, \cdots, \kappa$, if there exists a constant $T > 0$ such that

$$\lim_{t \to T} \|Y_\iota(t) + Y_0(t)\|_2 = 0,$$
$$and \ \|Y_\iota(t) + Y_0(t)\|_2 = 0, \ for \ t > T,$$

then the network (1) is called to be anti-synchronized in finite time.

Design the following controller for the network (1):

$$u_\iota(t) = -W_\iota(Y_\iota(t) + Y_0(t)) - \eta P^{\frac{a-1}{2}} \text{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a, \tag{3}$$

where $W_\iota \in \mathbb{R}^{n \times n}$, $0 < a < 1$, $\text{sign}(Y_\iota(t) + Y_0(t)) = \text{diag}(\text{sign}(Y_{\iota 1}(t) + Y_{01}(t)), \text{sign}(Y_{\iota 2}(t) + Y_{02}(t)), \cdots, \text{sign}(Y_{\iota n}(t) + Y_{0n}(t)))$, $|Y_\iota(t) + Y_0(t)|^a = (|Y_{\iota 1}(t) + Y_{01}(t)|^a, |Y_{\iota 2}(t) + Y_{02}(t)|^a, \cdots, |Y_{\iota n}(t) + Y_{0n}(t)|^a)^T$, $0 < \eta \in \mathbb{R}$, $0 < P = \text{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$, $P^{\frac{a-1}{2}} = \text{diag}(p_1^{\frac{a-1}{2}}, p_2^{\frac{a-1}{2}}, \cdots, p_n^{\frac{a-1}{2}})$. Then, we have

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^m \sum_{j=1}^\kappa c_s G_{\iota j}^s \Gamma_s Y_j(t) - W_\iota(Y_\iota(t) + Y_0(t))$$
$$- \eta P^{\frac{a-1}{2}} \text{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a.$$

Take $e_\iota(t) = Y_\iota(t) + Y_0(t)$. Then, one can get

$$\dot{e}_\iota(t) = -Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^m \sum_{j=1}^\kappa c_s G_{\iota j}^s \Gamma_s e_j(t)$$
$$- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \text{sign}(e_\iota(t))|e_\iota(t)|^a. \tag{4}$$

Theorem 1 *If there exist matrices $0 < P = \text{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$ and $W = \text{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ such that*

$$K_1 + K_2 + \sum_{s=1}^m c_s G^s \otimes (P\Gamma_s + \Gamma_s^T P) \leqslant 0, \tag{5}$$

where $K_1 = I_\kappa \otimes (-PA - AP + PDD^T P + \Theta)$, $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, then the network (1) is said to be finite-timely anti-synchronized under the controller (3). What's more, the settling time of anti-synchronization T satisfies $T \leqslant T_1 = \frac{(V_1(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Define the following Lyapunov functional for network (4):

$$V_1(t) = \sum_{\iota=1}^{\kappa} e_\iota^T(t) P e_\iota(t). \tag{6}$$

Then, one has

$$\dot{V}_1(t) = 2\sum_{\iota=1}^{\kappa} e_\iota^T(t) P\Big(-Ae_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s e_j(t)$$
$$+ Dg(Y_0(t)) - W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a \Big).$$

Obviously,

$$2e_\iota^T(t) PD(g(Y_\iota(t)) + g(Y_0(t))) \leqslant e_\iota^T(t)(PDD^T P + \Theta)e_\iota(t). \tag{7}$$

From (7), we can obtain

$$\dot{V}_1(t) \leqslant 2\sum_{s=1}^{m}\sum_{\iota=1}^{\kappa}\sum_{j=1}^{\kappa} c_s G_{\iota j}^s e_\iota^T(t) P\Gamma_s e_j(t) + \sum_{\iota=1}^{\kappa} e_\iota^T(t)(-PA - AP + PDD^T P$$
$$+ \Theta)e_\iota(t) - 2\sum_{\iota=1}^{\kappa} e_\iota^T(t) PW_\iota e_\iota(t) - 2\eta \sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a$$
$$= e^T(t)\Big[I_\kappa \otimes (-PA - AP + PDD^T P + \Theta) + \sum_{s=1}^{m} c_s G^s \otimes (P\Gamma_s$$
$$+ \Gamma_s^T P) - (I_\kappa \otimes P)W - W^T(I_\kappa \otimes P) \Big] e(t)$$
$$- 2\eta \sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a, \tag{8}$$

where $e(t) = (e_1^T(t), e_2^T(t), \cdots, e_\kappa^T(t))^T$. According to Lemma 2, we can easily obtain

$$\sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a = \sum_{\iota=1}^{\kappa}\sum_{j=1}^{n} p_j^{\frac{a+1}{2}} |e_{\iota j}(t)|^{a+1}$$
$$\geqslant \sum_{\iota=1}^{\kappa} \left(\sum_{j=1}^{n} p_j e_{\iota j}^2(t) \right)^{\frac{a+1}{2}}$$
$$= \sum_{\iota=1}^{\kappa} (e_\iota^T(t) P e_\iota(t))^{\frac{a+1}{2}}. \tag{9}$$

From (5), (8), (9) and Lemma 2, we can get

$$
\begin{aligned}
\dot{V}_1(t) \leqslant & e^T(t)\left[I_\kappa \otimes (-PA - AP + PDD^TP + \Theta) + \sum_{s=1}^{m} c_s G^s \otimes (P\Gamma_s + \Gamma_s^T P)\right. \\
& \left. - (I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)\right]e(t) - 2\eta \sum_{\iota=1}^{\kappa}(e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}} \\
\leqslant & - 2\eta \left(\sum_{\iota=1}^{\kappa} e_\iota^T(t)Pe_\iota(t)\right)^{\frac{a+1}{2}} \\
= & - 2\eta(V_1(t))^{\frac{a+1}{2}}.
\end{aligned}
$$

By Lemma 1, we can get $V_1(t) = 0$, $t > T_1$ with $T_1 = \frac{(V_1(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.
On the other hand,

$$
0 \leqslant \lambda_m(P) \sum_{\iota=1}^{\kappa} e_\iota^T(t)e_\iota(t) \leqslant V_1(t), \tag{10}
$$

where $\lambda_m(P) > 0$.

From (10), we obtain $\|e_\iota(t)\|_2 = 0$, $t > T_1$, where $\iota = 1, 2, \cdots, \kappa$. Therefore, the network (1) achieves finite-time anti-synchronization under the controller (3). The proof is completed.

3.1.2 Anti-synchronization in finite time for MWCNNs with switching topology

In this section, we consider the following MWCNNs with switching topology:

$$
\begin{aligned}
\dot{Y}_\iota(t) = & - AY_\iota(t) + Dg(Y_\iota(t)) + c_1 \sum_{j=1}^{\kappa} G_{\iota j}^{1,\omega(t)}\Gamma_1 Y_j(t) + c_2 \sum_{j=1}^{\kappa} G_{\iota j}^{2,\omega(t)}\Gamma_2 Y_j(t) + \cdots \\
& + c_m \sum_{j=1}^{\kappa} G_{\iota j}^{m,\omega(t)}\Gamma_m Y_j(t) + u_\iota(t), \quad \iota = 1, 2, \cdots, \kappa, \tag{11}
\end{aligned}
$$

where $Y_\iota(t)$, $g(\cdot)$, A, D, $u_\iota(t)$, c_s, $\Gamma_s(s = 1, 2, \cdots, m)$ are defined similarly as those in subsection 3.1.1, $\omega(t) : [0, \infty) \to I = \{1, 2, \cdots, i\}$ symbols switching signal. For each $\varsigma \in I$, $G^{s,\varsigma} = (G_{\iota j}^{s,\varsigma})_{\kappa \times \kappa}$ represents the coupling configuration matrix in the sth coupling form for the ςth topology, which satisfies $G_{\iota j}^{s,\varsigma} = G_{j\iota}^{s,\varsigma} > 0$ if nodes ι and j are connected for the ςth topology; otherwise, $G_{\iota j}^{s,\varsigma} = G_{j\iota}^{s,\varsigma} = 0$ ($\iota \neq j$); and

$$
G_{\iota\iota}^{s,\varsigma} = -\sum_{\substack{j=1 \\ j \neq \iota}}^{\kappa} G_{\iota j}^{s,\varsigma}, \quad \iota = 1, 2, \cdots, \kappa.
$$

For the network (11), design the same controller as (3). Then, we can obtain

$$\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma}\varGamma_s Y_j(t) - W_\iota(Y_\iota(t) + Y_0(t))$$
$$- \eta P^{\frac{a-1}{2}}\text{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a.$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$, then

$$\dot{e}_\iota(t) = - Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma}\varGamma_s e_j(t)$$
$$- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}}\text{sign}(e_\iota(t))|e_\iota(t)|^a. \tag{12}$$

Theorem 2 *If there exist matrices $0 < P = \text{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n\times n}$ and $W = \text{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ such that*

$$K_1 + K_2 + \sum_{s=1}^{m} c_s G^{s,\varsigma} \otimes (P\varGamma_s + \varGamma_s^T P) \leqslant 0, \tag{13}$$

where $\varsigma = 1, 2, \cdots, i$, $K_1 = I_\kappa \otimes (-PA - AP + PDD^T P + \varTheta)$ and $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, then the network (11) is finite-timely anti-synchronized under the controller (3). Moreover, the settling time of anti-synchronization T satisfies $T \leqslant T_1 = \frac{(V_1(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Select the same Lyapunov functional as (6) for the network (12). Then, we have

$$D^+ V_1(t) = 2\sum_{\iota=1}^{\kappa} e_\iota^T(t) P\Big(- Ae_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma}\varGamma_s e_j(t) + Dg(Y_0(t))$$
$$- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}}\text{sign}(e_\iota(t))|e_\iota(t)|^a \Big)$$
$$\leqslant 2\sum_{s=1}^{m}\sum_{\iota=1}^{\kappa}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} e_\iota^T(t) P\varGamma_s e_j(t) + \sum_{\iota=1}^{\kappa} e_\iota^T(t)(-PA - AP + PDD^T P$$
$$+ \varTheta)e_\iota(t) - 2\sum_{\iota=1}^{\kappa} e_\iota^T(t) PW_\iota e_\iota(t) - 2\eta \sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}}\text{sign}(e_\iota(t))|e_\iota(t)|^a$$
$$= e^T(t)\Big[I_\kappa \otimes (-PA - AP + PDD^T P + \varTheta) + \sum_{s=1}^{m} c_s G^{s,\varsigma} \otimes (P\varGamma_s$$
$$+ \varGamma_s^T P) - (I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)\Big] e(t)$$
$$- 2\eta \sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}}\text{sign}(e_\iota(t))|e_\iota(t)|^a.$$

It follows from (9), (13) and Lemma 2 that

$$
\begin{aligned}
D^+V_1(t) \leqslant &- 2\eta \sum_{\iota=1}^{\kappa}(e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}} \\
\leqslant &- 2\eta \left(\sum_{\iota=1}^{\kappa} e_\iota^T(t)Pe_\iota(t) \right)^{\frac{a+1}{2}} \\
= &- 2\eta(V_1(t))^{\frac{a+1}{2}}.
\end{aligned}
$$

Similar to the proof of Theorem 1, we can derive $\|e_\iota(t)\|_2 = 0$, $t > T_1$, where $\iota = 1, 2, \cdots, \kappa$. Thus, the network (11) reaches finite-time anti-synchronization under the controller (3). The proof is completed.

3.2 Robust anti-synchronization in finite time for MWCNNs

3.2.1 Robust anti-synchronization in finite time for MWCNNs with fixed topology

During the modeling process of CNNs, the existence of environmental noise, limitations of equipment and external interference may lead to the parameters varying within bounded ranges in some circumstances. Therefore, we consider a MWCNNs with parameteric uncertainties consisting of κ identical nodes in this section which can be described as

$$
\begin{aligned}
\dot{Y}_\iota(t) = &- AY_\iota(t) + Dg(Y_\iota(t)) + c_1 \sum_{j=1}^{\kappa} G_{\iota j}^1 \Gamma_1 Y_j(t) + c_2 \sum_{j=1}^{\kappa} G_{\iota j}^2 \Gamma_2 Y_j(t) + \cdots \\
&+ c_m \sum_{j=1}^{\kappa} G_{\iota j}^m \Gamma_m Y_j(t) + u_\iota(t), \quad \iota = 1, 2, \cdots, \kappa,
\end{aligned}
\tag{14}
$$

where $Y_\iota(t)$, $g(\cdot)$, c_s, $G_{\iota j}^s$, Γ_s, $s = 1, 2, \cdots, m$, $u_\iota(t)$ have the same meanings as those in subsection 3.1.1, and the parameters A and D change in the following given ranges:

$$
\begin{cases}
A_I :=\{A = \text{diag}(a_r) : A^- \leqslant A \leqslant A^+, i.e., 0 < a_r^- \leqslant a_r \leqslant a_r^+, \\
\quad r = 1, 2, \cdots, n, \forall A \in A_I\}, \\
D_I :=\{D = (d_{rj})_{n\times n} : D^- \leqslant D \leqslant D^+, i.e., d_{rj}^- \leqslant d_{rj} \leqslant d_{rj}^+, r, \\
\quad j = 1, 2, \cdots, n, \forall D \in D_I\}.
\end{cases}
\tag{15}
$$

For convenience, we define

$$
\tilde{d}_{rj} = \max\{|d_{rj}^-|, |d_{rj}^+|\}, \ r = 1, 2, \cdots, n, \ j = 1, 2, \cdots, n.
$$

Definition 2 For all $\iota = 1, 2, \cdots, \kappa$, $A \in A_I$ and $D \in D_I$, if there exists a constant $T > 0$ satisfying

$$\lim_{t \to T} \|Y_\iota(t) + Y_0(t)\|_2 = 0,$$
$$and \; \|Y_\iota(t) + Y_0(t)\|_2 = 0, \; for \; t > T,$$

then the network (14) with the parameter ranges defined by (15) is said to be robustly anti-synchronized in finite time.

For the network (14), design the same controller as (3). Then, we can obtain

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s Y_j(t) - W_\iota(Y_\iota(t) + Y_0(t))$$
$$- \eta P^{\frac{a-1}{2}} \mathrm{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a.$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$, then

$$\dot{e}_\iota(t) = -Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s e_j(t)$$
$$- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \mathrm{sign}(e_\iota(t))|e_\iota(t)|^a, \tag{16}$$

where A and D belong to the parameter ranges defined by (15).

Theorem 3 *If there exist matrices $0 < P = \mathrm{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$ and $W = \mathrm{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ such that*

$$K_3 + K_2 + \sum_{s=1}^{m} c_s G^s \otimes (P\Gamma_s + \Gamma_s^T P) \leqslant 0, \tag{17}$$

where $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, $K_3 = I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta)$, $\varrho_D = \sum_{r=1}^{n} \sum_{j=1}^{n} \tilde{d}_{rj}^2$, then the network (14) with the parameter ranges defined by (15) is robust finite-timely anti-synchronized under the controller (3). What's more, the settling time of anti-synchronization T satisfies $T \leqslant T_1 = \frac{(V_1(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Define the same Lyapunov functional as (6) for network (16). Then, one has

$$\dot{V}_1(t) = 2 \sum_{\iota=1}^{\kappa} e_\iota^T(t) P \Big(-Ae_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s e_j(t) + Dg(Y_0(t))$$
$$- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \mathrm{sign}(e_\iota(t))|e_\iota(t)|^a \Big).$$

Obviously,

$$2\sum_{\iota=1}^{\kappa} e_\iota^T(t)PD(g(Y_\iota(t))+g(Y_0(t))) \leqslant \sum_{\iota=1}^{\kappa} e_\iota^T(t)(PDD^TP+\Theta)e_\iota(t)$$

$$\leqslant \sum_{\iota=1}^{\kappa} e_\iota^T(t)(\varrho_D P^2 + \Theta)e_\iota(t). \qquad (18)$$

From (18), we can obtain

$$\dot{V}_1(t) \leqslant 2\sum_{s=1}^{m}\sum_{\iota=1}^{\kappa}\sum_{j=1}^{\kappa} c_s G_{\iota j}^s e_\iota^T(t)P\Gamma_s e_j(t) + \sum_{\iota=1}^{\kappa} e_\iota^T(t)(-PA^- - A^-P)e_\iota(t)$$

$$- 2\sum_{\iota=1}^{\kappa} e_\iota^T(t)PW_\iota e_\iota(t) - 2\eta\sum_{\iota=1}^{\kappa} e_\iota^T(t)P^{\frac{a+1}{2}}\mathrm{sign}(e_\iota(t))|e_\iota(t)|^a$$

$$+ \sum_{\iota=1}^{\kappa} e_\iota^T(t)(\varrho_D P^2 + \Theta)e_\iota(t)$$

$$= e^T(t)\left[I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta) + \sum_{s=1}^{m} c_s G^s \otimes (P\Gamma_s\right.$$

$$\left. + \Gamma_s^T P) - (I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)\right]e(t)$$

$$- 2\eta\sum_{\iota=1}^{\kappa} e_\iota^T(t)P^{\frac{a+1}{2}}\mathrm{sign}(e_\iota(t))|e_\iota(t)|^a. \qquad (19)$$

It follows from (9), (17) and Lemma 2 that

$$\dot{V}_1(t) \leqslant -2\eta\sum_{\iota=1}^{\kappa}(e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}}$$

$$\leqslant -2\eta\left(\sum_{\iota=1}^{\kappa} e_\iota^T(t)Pe_\iota(t)\right)^{\frac{a+1}{2}}$$

$$= -2\eta(V_1(t))^{\frac{a+1}{2}}.$$

Similar to the proof of Theorem 1, we can derive $\|e_\iota(t)\|_2 = 0$, $t > T_1$, where $\iota = 1, 2, \cdots, \kappa$. Hence, the network (14) with the parameter ranges defined by (15) achieves robust finite-time anti-synchronization under the controller (3). The proof is completed.

3.2.2 Robust anti-synchronization in finite time for MWCNNs with switching topology

In this subsection, we consider the switched MWCNNs with parameter uncertainties:

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + c_1 \sum_{j=1}^{\kappa} G_{\iota j}^{1,\varsigma} \Gamma_1 Y_j(t) + c_2 \sum_{j=1}^{\kappa} G_{\iota j}^{2,\varsigma} \Gamma_2 Y_j(t) + \cdots$$

$$+ c_m \sum_{j=1}^{\kappa} G_{\iota j}^{m,\varsigma} \Gamma_m Y_j(t) + u_\iota(t), \quad \iota = 1, 2, \cdots, \kappa, \tag{20}$$

where $Y_\iota(t)$, $g(\cdot)$, $u_\iota(t)$, c_s, $\Gamma_s(s = 1, 2, \cdots, m)$ are defined similarly as those in subsection 3.1.1, $G_{\iota j}^{s,\varsigma}$ has the same definition as in subsection 3.1.2, and A, D belong to the parameter ranges given by (15).

For the network (20), design the same controller as (3). Then, we can obtain

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s Y_j(t) - W_\iota(Y_\iota(t) + Y_0(t))$$

$$- \eta P^{\frac{a-1}{2}} \text{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a.$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$, then

$$\dot{e}_\iota(t) = -Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s e_j(t)$$

$$- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \text{sign}(e_\iota(t))|e_\iota(t)|^a, \tag{21}$$

in which the parameters A and D change in some given parameter ranges defined by (15).

Theorem 4 *If there exist matrices $0 < P = \text{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$ and $W = \text{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ such that*

$$K_3 + K_2 + \sum_{s=1}^{m} c_s G^{s,\varsigma} \otimes (P\Gamma_s + \Gamma_s^T P) \leqslant 0, \tag{22}$$

where $\varsigma = 1, 2, \cdots, i$, $K_3 = I_\kappa \otimes (-PA^- - A^- P + \varrho_D P^2 + \Theta)$, $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, $\varrho_D = \sum_{r=1}^{n} \sum_{j=1}^{n} \tilde{d}_{rj}^2$, then the network (20) with the parameter ranges defined by (15) is robust finite-timely anti-synchronized under the controller (3). Moreover, the settling time of anti-synchronization T satisfies $T \leqslant T_1 = \frac{(V_1(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Select the same Lyapunov functional as (6) for the network (21). Then, one has

$$
\begin{aligned}
D^+V_1(t) =& 2\sum_{\iota=1}^{\kappa} e_\iota^T(t) P\Big(-Ae_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s e_j(t) + Dg(Y_0(t)) \\
& - W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a \Big) \\
\leqslant & 2\sum_{s=1}^{m}\sum_{\iota=1}^{\kappa}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} e_\iota^T(t) P\Gamma_s e_j(t) + \sum_{\iota=1}^{\kappa} e_\iota^T(t)(-PA^- - A^-P)e_\iota(t) \\
& - 2\sum_{\iota=1}^{\kappa} e_\iota^T(t) PW_\iota e_\iota(t) - 2\eta\sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a \\
& + \sum_{\iota=1}^{\kappa} e_\iota^T(t)(\varrho_D P^2 + \Theta)e_\iota(t) \\
=& e^T(t)\Big[I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta) + \sum_{s=1}^{m} c_s G^{s,\varsigma} \otimes (P\Gamma_s \\
& + \Gamma_s^T P) - (I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)\Big] e(t) \\
& - 2\eta\sum_{\iota=1}^{\kappa} e_\iota^T(t) P^{\frac{a+1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a.
\end{aligned}
$$

It follows from (9), (22) and Lemma 2 that

$$
\begin{aligned}
D^+V_1(t) \leqslant & -2\eta\sum_{\iota=1}^{\kappa}(e_\iota^T(t) Pe_\iota(t))^{\frac{a+1}{2}} \\
\leqslant & -2\eta\left(\sum_{\iota=1}^{\kappa} e_\iota^T(t) Pe_\iota(t)\right)^{\frac{a+1}{2}} \\
=& -2\eta(V_1(t))^{\frac{a+1}{2}}.
\end{aligned}
$$

Similar to the proof of Theorem 1, we can derive $\|e_\iota(t)\|_2 = 0$, $t > T_1$, where $\iota = 1, 2, \cdots, \kappa$. Therefore, the network (20) with the parameter ranges defined by (15) achieves robust finite-time anti-synchronization under the controller (3). The proof is completed.

Remark 1 In recent years, some meaningful research results have been reported on the dynamical behaviors of Markovian jump systems [55–61]. Shen et al. [56] investigated the finite-time event-triggered H_∞ control problem for Takagi-Sugeno Markov jump fuzzy systems. As is well known, Markov jump systems can be used to model some real-life systems experiencing random changes in their parameters or structures. Therefore, it would be very interesting to apply a Markov process into the MWCNNs considered in our paper and investigate the finite-time anti-synchronization of MWCNNs with Markovian jump topology. This would be a meaningful problem for our future research.

4 Finite-time anti-synchronization of MWCNNs with coupling delays

4.1 Anti-synchronization in finite time for delayed MWCNNs

4.1.1 Anti-synchronization in finite time for delayed MWCNNs with fixed topology

In this section, we consider the following network model of MWCNNs with coupling delays:

$$\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s Y_j(t - \tau_s(t)) + u_\iota(t), \quad (23)$$

where $\iota = 1, 2, \cdots, \kappa$, $Y_\iota(t)$, $g(\cdot)$, A, D, c_s, $G_{\iota j}^s$, Γ_s, $u_\iota(t)$ are defined similarly as those in subsection 3.1.1, $\tau_s(t)$ is the time-varying delay with $0 \leqslant \tau_s(t) \leqslant \tau$, $s = 1, 2, \cdots, m$.

Construct the controller for the network (23) as follows:

$$u_\iota(t) = - \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} (Y_\iota(h) + Y_0(h))^T Q_\iota^s (Y_\iota(h) + Y_0(h)) dh \right)^{\frac{a+1}{2}}$$

$$\times \frac{Y_\iota(t) + Y_0(t)}{\|Y_\iota(t) + Y_0(t)\|_2^2} - \eta P^{\frac{a-1}{2}} \text{sign}(Y_\iota(t) + Y_0(t)) |Y_\iota(t) + Y_0(t)|^a$$

$$- W_\iota (Y_\iota(t) + Y_0(t)). \quad (24)$$

where $0 < Q_\iota^s \in \mathbb{R}^{n \times n}$, $Q_s = \text{diag}(Q_1^s, Q_2^s, \cdots, Q_\kappa^s)$, $s = 1, 2, \cdots, m$, W_ι, a, P, η, $\text{sign}(Y_\iota(t) + Y_0(t))$, $|Y_\iota(t) + Y_0(t)|^a$ have the same meanings as in (3). Then, we have

$$\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s Y_j(t - \tau_s(t)) - W_\iota(Y_\iota(t) + Y_0(t))$$

$$- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} (Y_\iota(h) + Y_0(h))^T Q_\iota^s (Y_\iota(h) + Y_0(h)) dh \right)^{\frac{a+1}{2}}$$

$$\times \frac{Y_\iota(t) + Y_0(t)}{\|Y_\iota(t) + Y_0(t)\|_2^2} - \eta P^{\frac{a-1}{2}} \text{sign}(Y_\iota(t) + Y_0(t)) |Y_\iota(t) + Y_0(t)|^a.$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$. Obviously, we can obtain

$$\dot{e}_\iota(t) = - Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s e_j(t - \tau_s(t)) - W_\iota e_\iota(t)$$

$$- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} e_\iota^T(h) Q_\iota^s e_\iota(h) dh \right)^{\frac{a+1}{2}} \frac{e_\iota(t)}{\|e_\iota(t)\|_2^2}$$

$$- \eta P^{\frac{a-1}{2}} \text{sign}(e_\iota(t)) |e_\iota(t)|^a, \quad (25)$$

where $\iota = 1, 2, \cdots, \kappa$.

Theorem 5 *Suppose $\dot{\tau}_s(t) \leqslant \gamma_s < 1$. The network (23) realizes finite-time anti-synchronization under the controller (24) if there exist matrices $0 < P = \mathrm{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$, $W = \mathrm{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ and $0 < Q_s = \mathrm{diag}(Q_1^s, Q_2^s, \cdots, Q_\kappa^s) \in \mathbb{R}^{n\kappa \times n\kappa}$, such that*

$$K_1 + K_2 + \sum_{s=1}^{m} c_s \left(F_s^T Q_s^{-1} F_s + \frac{1}{1 - \gamma_s} Q_s \right) \leqslant 0, \qquad (26)$$

where $K_1 = I_\kappa \otimes (-PA - AP + PDD^T P + \Theta)$, $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, $F_s = G^s \otimes (\Gamma_s^T P)$. In addition, the settling time of anti-synchronization T satisfies $T \leqslant T_2 = \frac{(V_2(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Construct a Lyapunov functional for network (25):

$$V_2(t) = \sum_{\iota=1}^{\kappa} e_\iota^T(t) P e_\iota(t) + \sum_{s=1}^{m} \frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} e^T(h) Q_s e(h) dh. \qquad (27)$$

Then,

$$\dot{V}_2(t) \leqslant 2 \sum_{\iota=1}^{\kappa} e_\iota^T(t) P \left(-A e_\iota(t) + D g(Y_\iota(t)) + D g(Y_0(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s e_j(t - \tau_s(t)) \right.$$
$$\left. - W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \mathrm{sign}(e_\iota(t)) |e_\iota(t)|^a \right) - \sum_{s=1}^{m} c_s e^T(t - \tau_s(t)) Q_s e(t - \tau_s(t))$$
$$- 2\eta \sum_{\iota=1}^{\kappa} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} e_\iota^T(h) Q_\iota^s e_\iota(h) dh \right)^{\frac{a+1}{2}} + \sum_{s=1}^{m} \frac{c_s}{1 - \gamma_s} e^T(t) Q_s e(t),$$

where $e(t - \tau_s(t)) = (e_1^T(t - \tau_s(t)), e_2^T(t - \tau_s(t)), \cdots, e_\kappa^T(t - \tau_s(t)))^T$. Obviously,

$$2 \sum_{s=1}^{m} \sum_{\iota=1}^{\kappa} \sum_{j=1}^{\kappa} c_s G_{\iota j}^s e_\iota^T(t) P \Gamma_s e_j(t - \tau_s(t))$$
$$= 2 \sum_{s=1}^{m} c_s e^T(t) \left[G^s \otimes (P\Gamma_s) \right] e(t - \tau_s(t))$$
$$\leqslant \sum_{s=1}^{m} c_s e^T(t) \left[G^s \otimes (P\Gamma_s) \right] Q_s^{-1} \left[G^s \otimes (\Gamma_s^T P) \right] e(t)$$
$$+ \sum_{s=1}^{m} c_s e^T(t - \tau_s(t)) Q_s e(t - \tau_s(t)). \qquad (28)$$

From (7), (9), (26), (28) and Lemma 2, we can obtain

$$
\begin{aligned}
\dot{V}_2(t) \leqslant & e^T(t)\big[I_\kappa \otimes (-PA - AP + PDD^TP + \Theta)\big]e(t) + \sum_{s=1}^{m} \frac{c_s}{1-\gamma_s} e^T(t)Q_s e(t) \\
& + \sum_{s=1}^{m} c_s e^T(t)\big[G^s \otimes (P\Gamma_s)\big]Q_s^{-1}\big[G^s \otimes (\Gamma_s^T P)\big]e(t) - 2\sum_{\iota=1}^{\kappa} e_\iota^T(t)PW_\iota e_\iota(t) \\
& - 2\eta \sum_{\iota=1}^{\kappa} (e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}} - 2\eta \sum_{s=1}^{m} \left(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^{t} e^T(h)Q_s e(h)dh \right)^{\frac{a+1}{2}} \\
= & e^T(t)\bigg\{ I_\kappa \otimes (-PA - AP + PDD^TP + \Theta) + \sum_{s=1}^{m} \frac{c_s}{1-\gamma_s}Q_s - (I_\kappa \otimes P)W \\
& - W^T(I_\kappa \otimes P) + \sum_{s=1}^{m} c_s \big[G^s \otimes (P\Gamma_s)\big]Q_s^{-1}\big[G^s \otimes (\Gamma_s^T P)\big] \bigg\}e(t) \\
& - 2\eta \sum_{s=1}^{m} \left(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^{t} e^T(h)Q_s e(h)dh \right)^{\frac{a+1}{2}} \\
& - 2\eta \sum_{\iota=1}^{\kappa} (e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}} \\
\leqslant & -2\eta \sum_{\iota=1}^{\kappa} (e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}} - 2\eta \sum_{s=1}^{m} \left(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^{t} e^T(h)Q_s e(h)dh \right)^{\frac{a+1}{2}} \\
\leqslant & -2\eta \left(\sum_{\iota=1}^{\kappa} e_\iota^T(t)Pe_\iota(t) + \sum_{s=1}^{m} \frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^{t} e^T(h)Q_s e(h)dh \right)^{\frac{a+1}{2}} \\
= & -2\eta(V_2(t))^{\frac{a+1}{2}}.
\end{aligned}
$$

Similar to the proof of Theorem 1, it is easy to obtain $\|e_\iota(t)\|_2 = 0$, $\iota = 1, 2, \cdots, \kappa$, $t > T_2$. Obviously, the network (23) is finite-time anti-synchronization. The proof is completed.

4.1.2 Anti-synchronization in finite time for delayed MWCNNs with switching topology

Describe the network model of MWCNNs with coupling delays and switching topology in this section as follows:

$$
\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s Y_j(t - \tau_s(t)) + u_\iota(t), \quad (29)
$$

where $\iota = 1, 2, \cdots, \kappa$, $Y_\iota(t)$, $g(\cdot)$, A, D, c_s, $u_\iota(t)$, Γ_s represent the same definitions as in subsection 3.1.1, $\tau_s(t)$ is the time-varying delay as that in subsection 4.1.1, and $G_{\iota j}^{s,\varsigma}$ has the same meaning as that in subsection 3.1.2.

Construct the same controller as (24) for the network (29). Then, one has

$$
\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s Y_j(t - \tau_s(t)) - W_\iota(Y_\iota(t) + Y_0(t))
$$

$$
- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} (Y_\iota(h) + Y_0(h))^T Q_\iota^s (Y_\iota(h) + Y_0(h)) dh \right)^{\frac{a+1}{2}}
$$

$$
\times \frac{Y_\iota(t) + Y_0(t)}{\|Y_\iota(t) + Y_0(t)\|_2^2} - \eta P^{\frac{a-1}{2}} \operatorname{sign}(Y_\iota(t) + Y_0(t)) |Y_\iota(t) + Y_0(t)|^a.
$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$. Then, we have

$$
\dot{e}_\iota(t) = - Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s e_j(t - \tau_s(t)) - W_\iota e_\iota(t)
$$

$$
- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} e_\iota^T(h) Q_\iota^s e_\iota(h) dh \right)^{\frac{a+1}{2}} \frac{e_\iota(t)}{\|e_\iota(t)\|_2^2}
$$

$$
- \eta P^{\frac{a-1}{2}} \operatorname{sign}(e_\iota(t)) |e_\iota(t)|^a, \tag{30}
$$

where $\iota = 1, 2, \cdots, \kappa$.

Theorem 6 *Suppose $\dot{\tau}_s(t) \leqslant \gamma_s < 1$. The network (29) realizes finite-time anti-synchronization under the controller (24) if there exist matrices $0 < P = \operatorname{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$, $W = \operatorname{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ and $0 < Q_s = \operatorname{diag}(Q_1^s, Q_2^s, \cdots, Q_\kappa^s) \in \mathbb{R}^{n\kappa \times n\kappa}$, such that*

$$
K_1 + K_2 + \sum_{s=1}^{m} c_s \left(F_{s,\varsigma}^T Q_s^{-1} F_{s,\varsigma} + \frac{1}{1 - \gamma_s} Q_s \right) \leqslant 0, \tag{31}
$$

where $\varsigma = 1, 2, \cdots, i$, $K_1 = I_\kappa \otimes (-PA - AP + PDD^T P + \Theta)$, $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$ and $F_{s,\varsigma} = G^{s,\varsigma} \otimes (\Gamma_s^T P)$. What's more, the settling time of anti-synchronization T satisfies $T \leqslant T_2 = \frac{(V_2(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Construct the same Lyapunov functional as (24) for network (30). Then,

$$
\begin{aligned}
D^+V_2(t) \leqslant& 2\sum_{\iota=1}^{\kappa} e_{\iota}^T(t)P\bigg(-Ae_{\iota}(t)+Dg(Y_{\iota}(t))+Dg(Y_0(t))+\sum_{s=1}^{m}\sum_{j=1}^{\kappa}c_sG_{\iota j}^{s,\varsigma}\Gamma_s e_j(t\\
&-\tau_s(t))-W_{\iota}e_{\iota}(t)-\eta P^{\frac{a-1}{2}}\mathrm{sign}(e_{\iota}(t))|e_{\iota}(t)|^a\bigg)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}e^T(t)Q_s e(t)\\
&-2\eta\sum_{\iota=1}^{\kappa}\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e_{\iota}^T(h)Q_{\iota}^s e_{\iota}(h)dh\right)^{\frac{a+1}{2}}-\sum_{s=1}^{m}c_s e^T(t\\
&-\tau_s(t))Q_s e(t-\tau_s(t))\\
\leqslant& e^T(t)\big[I_{\kappa}\otimes(-PA-AP+PDD^TP+\Theta)\big]e(t)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}e^T(t)Q_s e(t)\\
&+\sum_{s=1}^{m}c_s e^T(t)\big[G^{s,\varsigma}\otimes(P\Gamma_s)\big]Q_s^{-1}\big[G^{s,\varsigma}\otimes(\Gamma_s^TP)\big]e(t)-2\sum_{\iota=1}^{\kappa}e_{\iota}^T(t)PW_{\iota}e_{\iota}(t)\\
&-2\eta\sum_{\iota=1}^{\kappa}(e_{\iota}^T(t)Pe_{\iota}(t))^{\frac{a+1}{2}}-2\eta\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}}\\
=& e^T(t)\bigg\{I_{\kappa}\otimes(-PA-AP+PDD^TP+\Theta)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}Q_s-(I_{\kappa}\otimes P)W\\
&-W^T(I_{\kappa}\otimes P)+\sum_{s=1}^{m}c_s\big[G^{s,\varsigma}\otimes(P\Gamma_s)\big]Q_s^{-1}\big[G^{s,\varsigma}\otimes(\Gamma_s^TP)\big]\bigg\}e(t)\\
&-2\eta\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}}\\
&-2\eta\sum_{\iota=1}^{\kappa}(e_{\iota}^T(t)Pe_{\iota}(t))^{\frac{a+1}{2}}\\
\leqslant& -2\eta\sum_{\iota=1}^{\kappa}(e_{\iota}^T(t)Pe_{\iota}(t))^{\frac{a+1}{2}}-2\eta\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}}\\
\leqslant& -2\eta\left(\sum_{\iota=1}^{\kappa}e_{\iota}^T(t)Pe_{\iota}(t)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}}\\
=& -2\eta(V_2(t))^{\frac{a+1}{2}}.
\end{aligned}
$$

Similar to the proof of Theorem 1, it is easy to obtain $\|e_{\iota}(t)\|_2 = 0$, $\iota = 1, 2, \cdots, \kappa$, $t > T_2$. Obviously, the network (29) achieves finite-time anti-synchronization. The proof is completed.

4.2 Robust anti-synchronization in finite time for delayed MWCNNs

4.2.1 Robust anti-synchronization in finite time for delayed MWCNNs with fixed topology

The MWCNNs model with coupling delays and parameter uncertainties is described by:

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s Y_j(t - \tau_s(t)) + u_\iota(t), \quad (32)$$

where $\iota = 1, 2, \cdots, \kappa$, $Y_\iota(t)$, $g(\cdot)$, c_s, $G_{\iota j}^s$, Γ_s, $u_\iota(t)$ are defined similarly as those in subsection 3.1.1, A, D are intervalized as those in (15), $\tau_s(t)$ is the time-varying delay as that in subsection 4.1.1.

Design the same controller for the network (32) as (24). Then,

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s Y_j(t - \tau_s(t)) - W_\iota(Y_\iota(t) + Y_0(t))$$

$$- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} (Y_\iota(h) + Y_0(h))^T Q_\iota^s (Y_\iota(h) + Y_0(h)) dh \right)^{\frac{a+1}{2}}$$

$$\times \frac{Y_\iota(t) + Y_0(t)}{\|Y_\iota(t) + Y_0(t)\|_2^2} - \eta P^{\frac{a-1}{2}} \text{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a.$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$. Then, we have

$$\dot{e}_\iota(t) = -Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m}\sum_{j=1}^{\kappa} c_s G_{\iota j}^s \Gamma_s e_j(t - \tau_s(t)) - W_\iota e_\iota(t)$$

$$- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} e_\iota^T(h) Q_\iota^s e_\iota(h) dh \right)^{\frac{a+1}{2}} \frac{e_\iota(t)}{\|e_\iota(t)\|_2^2}$$

$$- \eta P^{\frac{a-1}{2}} \text{sign}(e_\iota(t))|e_\iota(t)|^a, \quad (33)$$

where $\iota = 1, 2, \cdots, \kappa$, A and D are uncertain parameters which belong to the ranges given in (15).

Theorem 7 *Suppose $\dot{\tau}_s(t) \leqslant \gamma_s < 1$. The network (32) with the parameter ranges defined by (15) achieves robust finite-time anti-synchronization under the controller (24) if there exist matrices $0 < P = \text{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$, $W = \text{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ and $0 < Q_s = \text{diag}(Q_1^s, Q_2^s, \cdots, Q_\kappa^s) \in \mathbb{R}^{n\kappa \times n\kappa}$, such that*

$$K_3 + K_2 + \sum_{s=1}^{m} c_s \left(F_s^T Q_s^{-1} F_s + \frac{1}{1 - \gamma_s} Q_s \right) \leqslant 0, \quad (34)$$

where $K_3 = I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta)$, $\varrho_D = \sum_{r=1}^n \sum_{j=1}^n \tilde{d}_{rj}^2$, $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, $F_s = G^s \otimes (\Gamma_s^T P)$. In addition, the settling time of anti-synchronization T satisfies $T \leqslant T_2 = \frac{(V_2(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. The Lyapunov functional is constructed similarly as that in Theorem 5 for network (33). Then,

$$
\dot{V}_2(t) \leqslant 2 \sum_{\iota=1}^\kappa e_\iota^T(t) P \Big(-Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^m \sum_{j=1}^\kappa c_s G_{\iota j}^s \Gamma_s e_j(t - \tau_s(t))
$$

$$
- W_\iota e_\iota(t) - \eta P^{\frac{a-1}{2}} \mathrm{sign}(e_\iota(t)) |e_\iota(t)|^a \Big) - \sum_{s=1}^m c_s e^T(t - \tau_s(t)) Q_s e(t - \tau_s(t))
$$

$$
- 2\eta \sum_{\iota=1}^\kappa \sum_{s=1}^m \Big(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^t e_\iota^T(h) Q_s^s e_\iota(h) dh \Big)^{\frac{a+1}{2}} + \sum_{s=1}^m \frac{c_s}{1-\gamma_s} e^T(t) Q_s e(t)
$$

$$
\leqslant e^T(t) \big[I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta) \big] e(t) + \sum_{s=1}^m \frac{c_s}{1-\gamma_s} e^T(t) Q_s e(t)
$$

$$
+ \sum_{s=1}^m c_s e^T(t) \big[G^s \otimes (P\Gamma_s) \big] Q_s^{-1} \big[G^s \otimes (\Gamma_s^T P) \big] e(t) - 2 \sum_{\iota=1}^\kappa e_\iota^T(t) P W_\iota e_\iota(t)
$$

$$
- 2\eta \sum_{\iota=1}^\kappa (e_\iota^T(t) P e_\iota(t))^{\frac{a+1}{2}} - 2\eta \sum_{s=1}^m \Big(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^t e^T(h) Q_s e(h) dh \Big)^{\frac{a+1}{2}}
$$

$$
= e^T(t) \Big\{ I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta) + \sum_{s=1}^m \frac{c_s}{1-\gamma_s} Q_s - (I_\kappa \otimes P)W
$$

$$
- W^T(I_\kappa \otimes P) + \sum_{s=1}^m c_s \big[G^s \otimes (P\Gamma_s) \big] Q_s^{-1} \big[G^s \otimes (\Gamma_s^T P) \big] \Big\} e(t)
$$

$$
- 2\eta \sum_{s=1}^m \Big(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^t e^T(h) Q_s e(h) dh \Big)^{\frac{a+1}{2}}
$$

$$
- 2\eta \sum_{\iota=1}^\kappa (e_\iota^T(t) P e_\iota(t))^{\frac{a+1}{2}}
$$

$$
\leqslant - 2\eta \sum_{\iota=1}^\kappa (e_\iota^T(t) P e_\iota(t))^{\frac{a+1}{2}} - 2\eta \sum_{s=1}^m \Big(\frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^t e^T(h) Q_s e(h) dh \Big)^{\frac{a+1}{2}}
$$

$$
\leqslant - 2\eta \Big(\sum_{\iota=1}^\kappa e_\iota^T(t) P e_\iota(t) + \sum_{s=1}^m \frac{c_s}{1-\gamma_s} \int_{t-\tau_s(t)}^t e^T(h) Q_s e(h) dh \Big)^{\frac{a+1}{2}}
$$

$$
= - 2\eta (V_2(t))^{\frac{a+1}{2}}.
$$

Similar to the proof of Theorem 1, it is easy to obtain $\|e_\iota(t)\|_2 = 0$, $\iota = 1, 2, \cdots, \kappa$, $t > T_2$. Obviously, the network (32) with the parameter ranges

defined by (15) achieves robust finite-time anti-synchronization under the controller (24). The proof is completed.

4.2.2 Robust anti-synchronization in finite time for delayed MWCNNs with switching topology

In this subsection, the network model of delayed MWCNNs with the parameter uncertainties and switching topology is described by:

$$\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s Y_j(t - \tau_s(t)) + u_\iota(t), \quad (35)$$

where $\iota = 1, 2, \cdots, \kappa$, $Y_\iota(t)$, $g(\cdot)$, c_s, $u_\iota(t)$, Γ_s represent the same definitions as those in subsection 3.1.1, A, D change in some given precision described by (15), $\tau_s(t)$ is the time-varying delay as that in subsection 4.1.1, and $G_{\iota j}^{s,\varsigma}$ has the same meaning as that in subsection 3.1.2.

Construct the same controller as (24) for the network (35). Then,

$$\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s Y_j(t - \tau_s(t)) - W_\iota(Y_\iota(t) + Y_0(t))$$

$$- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} (Y_\iota(h) + Y_0(h))^T Q_\iota^s(Y_\iota(h) + Y_0(h))dh \right)^{\frac{a+1}{2}}$$

$$\times \frac{Y_\iota(t) + Y_0(t)}{\|Y_\iota(t) + Y_0(t)\|_2^2} - \eta P^{\frac{a-1}{2}} \operatorname{sign}(Y_\iota(t) + Y_0(t))|Y_\iota(t) + Y_0(t)|^a.$$

Let $e_\iota(t) = Y_\iota(t) + Y_0(t)$. Then, we have

$$\dot{e}_\iota(t) = - Ae_\iota(t) + Dg(Y_\iota(t)) + Dg(Y_0(t)) + \sum_{s=1}^{m} \sum_{j=1}^{\kappa} c_s G_{\iota j}^{s,\varsigma} \Gamma_s e_j(t - \tau_s(t)) - W_\iota e_\iota(t)$$

$$- \eta P^{-1} \sum_{s=1}^{m} \left(\frac{c_s}{1 - \gamma_s} \int_{t-\tau_s(t)}^{t} e_\iota^T(h) Q_\iota^s e_\iota(h)dh \right)^{\frac{a+1}{2}} \frac{e_\iota(t)}{\|e_\iota(t)\|_2^2}$$

$$- \eta P^{\frac{a-1}{2}} \operatorname{sign}(e_\iota(t))|e_\iota(t)|^a, \tag{36}$$

where $\iota = 1, 2, \cdots, \kappa$, A and D are uncertain parameters which belong to the ranges given in (15).

Theorem 8 *Suppose $\dot{\tau}_s(t) \leqslant \gamma_s < 1$. The network* (35) *with the parameter ranges defined by* (15) *achieves robust finite-time anti-synchronization under the controller* (24) *if there exist matrices $0 < P = \operatorname{diag}(p_1, p_2, \cdots, p_n) \in \mathbb{R}^{n \times n}$, $W = \operatorname{diag}(W_1, W_2, \cdots, W_\kappa) \in \mathbb{R}^{n\kappa \times n\kappa}$ and $0 < Q_s = \operatorname{diag}(Q_1^s, Q_2^s, \cdots, Q_\kappa^s) \in \mathbb{R}^{n\kappa \times n\kappa}$, such that*

$$K_3 + K_2 + \sum_{s=1}^{m} c_s \left(F_{s,\varsigma}^T Q_s^{-1} F_{s,\varsigma} + \frac{1}{1 - \gamma_s} Q_s \right) \leqslant 0, \tag{37}$$

where $\varsigma = 1, 2, \cdots, i$, $K_3 = I_\kappa \otimes (-PA^- - A^-P + \varrho_D P^2 + \Theta)$, $K_2 = -(I_\kappa \otimes P)W - W^T(I_\kappa \otimes P)$, $\varrho_D = \sum_{r=1}^{n}\sum_{j=1}^{n} \tilde{d}_{rj}^2$, $F_{s,\varsigma} = G^{s,\varsigma} \otimes (\Gamma_s^T P)$. What's more, the settling time of anti-synchronization T satisfies $T \leqslant T_2 = \frac{(V_2(0))^{\frac{1-a}{2}}}{\eta(1-a)}$.

Proof. Construct the same Lyapunov functional as in Theorem 5 for network (36). Then,

$$
\begin{aligned}
D^+V_2(t) \leqslant & 2\sum_{\iota=1}^{\kappa} e_\iota^T(t)P\Big(-Ae_\iota(t)+Dg(Y_\iota(t))+Dg(Y_0(t))+\sum_{s=1}^{m}\sum_{j=1}^{\kappa}c_s G_{\iota j}^{s,\varsigma}\Gamma_s e_j(t \\
& -\tau_s(t))-W_\iota e_\iota(t)-\eta P^{\frac{a-1}{2}}\text{sign}(e_\iota(t))|e_\iota(t)|^a\Big)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}e^T(t)Q_s e(t) \\
& -2\eta\sum_{\iota=1}^{\kappa}\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e_\iota^T(h)Q_\iota^s e_\iota(h)dh\right)^{\frac{a+1}{2}}-\sum_{s=1}^{m}c_s e^T(t \\
& -\tau_s(t))Q_s e(t-\tau_s(t)) \\
\leqslant & e^T(t)\big[I_\kappa\otimes(-PA^- - A^-P+\varrho_D P^2+\Theta)\big]e(t)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}e^T(t)Q_s e(t) \\
& +\sum_{s=1}^{m}c_s e^T(t)\big[G^{s,\varsigma}\otimes(P\Gamma_s)\big]Q_s^{-1}\big[G^{s,\varsigma}\otimes(\Gamma_s^T P)\big]e(t)-2\sum_{\iota=1}^{\kappa}e_\iota^T(t)PW_\iota e_\iota(t) \\
& -2\eta\sum_{\iota=1}^{\kappa}(e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}}-2\eta\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}} \\
= & e^T(t)\Big\{I_\kappa\otimes(-PA^- - A^-P+\varrho_D P^2+\Theta)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}Q_s-(I_\kappa\otimes P)W \\
& -W^T(I_\kappa\otimes P)+\sum_{s=1}^{m}c_s\big[G^{s,\varsigma}\otimes(P\Gamma_s)\big]Q_s^{-1}\big[G^{s,\varsigma}\otimes(\Gamma_s^T P)\big]\Big\}e(t) \\
& -2\eta\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}} \\
& -2\eta\sum_{\iota=1}^{\kappa}(e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}} \\
\leqslant & -2\eta\sum_{\iota=1}^{\kappa}(e_\iota^T(t)Pe_\iota(t))^{\frac{a+1}{2}}-2\eta\sum_{s=1}^{m}\left(\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}} \\
\leqslant & -2\eta\left(\sum_{\iota=1}^{\kappa}e_\iota^T(t)Pe_\iota(t)+\sum_{s=1}^{m}\frac{c_s}{1-\gamma_s}\int_{t-\tau_s(t)}^{t}e^T(h)Q_s e(h)dh\right)^{\frac{a+1}{2}} \\
= & -2\eta(V_2(t))^{\frac{a+1}{2}}.
\end{aligned}
$$

Similar to the proof of Theorem 1, it is easy to obtain $\|e_\iota(t)\|_2 = 0$, $\iota = 1, 2, \cdots, \kappa$, $t > T_2$. Obviously, the network (35) with the parameter ranges defined by (15) achieves robust finite-time anti-synchronization under the controller (24). The proof is completed.

Remark 2 In this paper, by designing suitable state feedback controllers, several sufficient conditions which guarantee the finite-time anti-synchronization of MWCNNs with and without coupling delays are obtained. To the best of knowledge, this is the first paper toward to studying anti-synchronization for MWCNNs in finite time. Recently, a few scholars established some novel finite-time synchronization criteria for complex networks via intermittent feedback control scheme [35,36]. Incorporating this new control approach and techniques into the research on finite-time anti-synchronization of MWCNNs is a very interesting and challenging problem, which would become one of research topics of our future work.

5 Numerical Examples

Example 4.1. Consider the switched MWCNNs with parameter uncertainties as follows:

$$\dot{Y}_\iota(t) = -AY_\iota(t) + Dg(Y_\iota(t)) + 0.5 \sum_{j=1}^{6} G_{\iota j}^{1,\varsigma} \Gamma_1 Y_j(t) + 0.8 \sum_{j=1}^{6} G_{\iota j}^{2,\varsigma} \Gamma_2 Y_j(t)$$

$$+ 1.1 \sum_{j=1}^{6} G_{\iota j}^{3,\varsigma} \Gamma_3 Y_j(t) + u_\iota(t), \tag{38}$$

where $\iota = 1, 2, \cdots, 6$, $\varsigma = 1, 2$, $g_k(\sigma) = \frac{|\sigma+1|-|\sigma-1|}{4}$ $(k = 1, 2, 3)$, $G^{1,1}$ and $G^{1,2}$ are two possible topologies which are switched as $G^{1,1} \to G^{1,2} \to G^{1,1} \to G^{1,2} \to \cdots$, and each graph is active for 1s, $G^{2,1}$, $G^{2,2}$, $G^{3,1}$, $G^{3,2}$ are switched in the same way. The inner coupling matrices Γ_1, Γ_2, Γ_3 and the coupling matrices $G^{1,1} = (G_{\iota j}^{1,1})_{6\times6}$, $G^{1,2} = (G_{\iota j}^{1,2})_{6\times6}$, $G^{2,1} = (G_{\iota j}^{2,1})_{6\times6}$, $G^{2,2} =$

$(G^{2,2}_{\iota j})_{6\times6}$, $G^{3,1} = (G^{3,1}_{\iota j})_{6\times6}$, $G^{3,2} = (G^{3,2}_{\iota j})_{6\times6}$ are chosen as, respectively

$$
\Gamma_1 = \begin{pmatrix} 0.6 & 0.1 & 0 \\ 0 & 0.4 & 0 \\ 0 & 0.2 & 0.5 \end{pmatrix}, \; \Gamma_2 = \begin{pmatrix} 0.7 & 0 & 0.2 \\ 0 & 0.4 & 0 \\ 0.3 & 0 & 0.7 \end{pmatrix}, \; \Gamma_3 = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.7 & 0.2 \\ 0.1 & 0 & 0.6 \end{pmatrix},
$$

$$
G^{1,1} = \begin{pmatrix} -0.7 & 0.1 & 0.1 & 0.3 & 0.2 & 0 \\ 0.1 & -0.8 & 0 & 0.2 & 0.2 & 0.3 \\ 0.1 & 0 & -0.4 & 0.2 & 0 & 0.1 \\ 0.3 & 0.2 & 0.2 & -1.2 & 0.3 & 0.2 \\ 0.2 & 0.2 & 0 & 0.3 & -0.9 & 0.2 \\ 0 & 0.3 & 0.1 & 0.2 & 0.2 & -0.8 \end{pmatrix},
$$

$$
G^{1,2} = \begin{pmatrix} -0.6 & 0.2 & 0 & 0.2 & 0.1 & 0.1 \\ 0.2 & -0.5 & 0.1 & 0.1 & 0.1 & 0 \\ 0 & 0.1 & -0.7 & 0 & 0.4 & 0.2 \\ 0.2 & 0.1 & 0 & -0.5 & 0.2 & 0 \\ 0.1 & 0.1 & 0.4 & 0.2 & -0.9 & 0.1 \\ 0.1 & 0 & 0.2 & 0 & 0.1 & -0.4 \end{pmatrix},
$$

$$
G^{2,1} = \begin{pmatrix} -0.8 & 0.2 & 0.1 & 0.2 & 0.3 & 0 \\ 0.2 & -0.6 & 0 & 0.2 & 0 & 0.2 \\ 0.1 & 0 & -1.0 & 0.3 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0.3 & -1.2 & 0.1 & 0.4 \\ 0.3 & 0 & 0.4 & 0.1 & -1.0 & 0.2 \\ 0 & 0.2 & 0.2 & 0.4 & 0.2 & -1.0 \end{pmatrix},
$$

$$
G^{2,2} = \begin{pmatrix} -0.5 & 0.1 & 0 & 0.1 & 0.1 & 0.2 \\ 0.1 & -0.7 & 0.4 & 0.1 & 0.1 & 0 \\ 0 & 0.4 & -1.2 & 0 & 0.5 & 0.3 \\ 0.1 & 0.1 & 0 & -0.5 & 0.3 & 0 \\ 0.1 & 0.1 & 0.5 & 0.3 & -1.1 & 0.1 \\ 0.2 & 0 & 0.3 & 0 & 0.1 & -0.6 \end{pmatrix},
$$

$$
G^{3,1} = \begin{pmatrix} -0.7 & 0.3 & 0.1 & 0 & 0.1 & 0.2 \\ 0.3 & -0.9 & 0.3 & 0.1 & 0 & 0.2 \\ 0.1 & 0.3 & -0.9 & 0.2 & 0.3 & 0 \\ 0 & 0.1 & 0.2 & -0.8 & 0.4 & 0.1 \\ 0.1 & 0 & 0.3 & 0.4 & -0.9 & 0.1 \\ 0.2 & 0.2 & 0 & 0.1 & 0.1 & -0.6 \end{pmatrix},
$$

$$
G^{3,2} = \begin{pmatrix} -0.6 & 0.2 & 0 & 0.1 & 0.2 & 0.1 \\ 0.2 & -0.8 & 0.2 & 0.2 & 0.2 & 0 \\ 0 & 0.2 & -0.7 & 0 & 0.1 & 0.4 \\ 0.1 & 0.2 & 0 & -0.6 & 0.3 & 0 \\ 0.2 & 0.2 & 0.1 & 0.3 & -1.0 & 0.2 \\ 0.1 & 0 & 0.4 & 0 & 0.2 & -0.7 \end{pmatrix}.
$$

The parameters $A = \operatorname{diag}(a_1, a_2, \cdots, a_n)$, $D = (d_{rj})_{n \times n}$ in the network (38) can be changed in the following given precisions:

$$
\begin{cases}
A_I := \{A = \operatorname{diag}(a_r) : A^- \leqslant A \leqslant A^+, i.e., 0 < 0.5r \leqslant a_r \leqslant 0.6r, \\
\quad r = 1, 2, \cdots, n, \forall A \in A_I\}, \\
D_I := \{D = (d_{rj})_{n \times n} : D^- \leqslant D \leqslant D^+, i.e., \dfrac{0.2j}{r+j} \leqslant d_{rj} \leqslant \dfrac{0.3j}{r+j}, \\
\quad r, j = 1, 2, \cdots, n, \forall D \in D_I\}.
\end{cases}
\tag{39}
$$

Obviously, $g_k(\cdot)$ $(k = 1, 2, 3)$ satisfies **Assumption 1** with $\theta_k = 0.5$. Take $W = \operatorname{diag}(0.2I_3, 0.4I_3, 0.3I_3, 0.6I_3, 0.7I_3, 0.5I_3)$. The following matrix P satisfying (22) can be computed by utilizing MATLAB Toolbox:

$$
P = \begin{pmatrix}
0.3252 & 0 & 0 \\
0 & 0.2505 & 0 \\
0 & 0 & 0.1992
\end{pmatrix}.
$$

According to Theorem 4, the network (38) with the parameter ranges defined by (39) achieves robust finite-time anti-synchronization under the controller (3) and the time estimation of achieving anti-synchronization is $T_1 = 9.32$. The simulation result is displayed in Figure 1.

Example 4.2. Consider the following switched MWCNNs with the parameter uncertainties and coupling delays:

$$
\dot{Y}_\iota(t) = - AY_\iota(t) + Dg(Y_\iota(t)) + 0.2 \sum_{j=1}^{6} G_{\iota j}^{1;\varsigma} \Gamma_1 Y_j(t - \tau_1(t)) + 0.1 \sum_{j=1}^{6} G_{\iota j}^{2;\varsigma} \Gamma_2 Y_j(t
$$

$$
- \tau_2(t)) + 0.3 \sum_{j=1}^{6} G_{\iota j}^{3;\varsigma} \Gamma_3 Y_j(t - \tau_3(t)) + u_\iota(t),
\tag{40}
$$

where $\iota = 1, 2, \cdots, 6$, $\varsigma = 1, 2$, $g_k(\sigma) = \frac{|\sigma+1| - |\sigma-1|}{8}$ $(k = 1, 2, 3)$, $\tau_1(t) = \frac{1}{10} - \frac{1}{30}e^{-t}$, $\tau_2(t) = \frac{1}{10} - \frac{1}{40}e^{-t}$, $\tau_3(t) = \frac{1}{10} - \frac{1}{50}e^{-t}$, $\tau = \frac{1}{10}$, $\gamma_1 = \frac{1}{30}$, $\gamma_2 = \frac{1}{40}$, $\gamma_3 = \frac{1}{50}$, $G^{1,1}$ and $G^{1,2}$ are two possible topologies which are switched as $G^{1,1} \to G^{1,2} \to G^{1,1} \to G^{1,2} \to \cdots$, and $G^{2,1}$, $G^{2,2}$, $G^{3,1}$, $G^{3,2}$ are switched in the same way with activation time of 1s for each graph. The inner coupling matrices Γ_1, Γ_2, Γ_3 and the coupling matrices $G^{1,1} = (G_{\iota j}^{1,1})_{6 \times 6}$, $G^{1,2} = (G_{\iota j}^{1,2})_{6 \times 6}$, $G^{2,1} = (G_{\iota j}^{2,1})_{6 \times 6}$, $G^{2,2} = (G_{\iota j}^{2,2})_{6 \times 6}$, $G^{3,1} = (G_{\iota j}^{3,1})_{6 \times 6}$, $G^{3,2} = (G_{\iota j}^{3,2})_{6 \times 6}$ are chosen as, respectively

$$\Gamma_1 = \begin{pmatrix} 0.4 & 0.1 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0.2 & 0.6 \end{pmatrix}, \ \Gamma_2 = \begin{pmatrix} 0.6 & 0 & 0.3 \\ 0 & 0.2 & 0 \\ 0.1 & 0 & 0.4 \end{pmatrix}, \ \Gamma_3 = \begin{pmatrix} 0.3 & 0 & 0 \\ 0.1 & 0.4 & 0.2 \\ 0 & 0 & 0.5 \end{pmatrix},$$

$$G^{1,1} = \begin{pmatrix} -0.5 & 0.2 & 0.1 & 0 & 0.1 & 0.1 \\ 0.2 & -1.0 & 0.2 & 0.3 & 0.1 & 0.2 \\ 0.1 & 0.2 & -0.6 & 0.2 & 0 & 0.1 \\ 0 & 0.3 & 0.2 & -0.9 & 0.3 & 0.1 \\ 0.1 & 0.1 & 0 & 0.3 & -0.5 & 0 \\ 0.1 & 0.2 & 0.1 & 0.1 & 0 & -0.5 \end{pmatrix},$$

$$G^{1,2} = \begin{pmatrix} -1.0 & 0.3 & 0 & 0.2 & 0.3 & 0.2 \\ 0.3 & -0.8 & 0.1 & 0.2 & 0.2 & 0 \\ 0 & 0.1 & -0.8 & 0 & 0.5 & 0.2 \\ 0.2 & 0.2 & 0 & -0.5 & 0.1 & 0 \\ 0.3 & 0.2 & 0.5 & 0.1 & -1.4 & 0.3 \\ 0.2 & 0 & 0.2 & 0 & 0.3 & -0.7 \end{pmatrix},$$

$$G^{2,1} = \begin{pmatrix} -0.9 & 0.3 & 0.2 & 0 & 0.3 & 0.1 \\ 0.3 & -1.0 & 0 & 0.3 & 0.1 & 0.3 \\ 0.2 & 0 & -0.8 & 0.1 & 0.5 & 0 \\ 0 & 0.3 & 0.1 & -0.7 & 0.1 & 0.2 \\ 0.3 & 0.1 & 0.5 & 0.1 & -1.0 & 0 \\ 0.1 & 0.3 & 0 & 0.2 & 0 & -0.6 \end{pmatrix},$$

$$G^{2,2} = \begin{pmatrix} -0.7 & 0.2 & 0 & 0.2 & 0.1 & 0.2 \\ 0.2 & -0.8 & 0.2 & 0.2 & 0.2 & 0 \\ 0 & 0.2 & -0.8 & 0 & 0.4 & 0.2 \\ 0.2 & 0.2 & 0 & -0.7 & 0.3 & 0 \\ 0.1 & 0.2 & 0.4 & 0.3 & -1.3 & 0.3 \\ 0.2 & 0 & 0.2 & 0 & 0.3 & -0.7 \end{pmatrix},$$

$$G^{3,1} = \begin{pmatrix} -0.7 & 0.1 & 0.1 & 0.3 & 0.2 & 0 \\ 0.1 & -0.8 & 0.2 & 0 & 0.3 & 0.2 \\ 0.1 & 0.2 & -1.0 & 0.2 & 0.3 & 0.2 \\ 0.3 & 0 & 0.2 & -0.6 & 0 & 0.1 \\ 0.2 & 0.3 & 0.3 & 0 & -0.9 & 0.1 \\ 0 & 0.2 & 0.2 & 0.1 & 0.1 & -0.6 \end{pmatrix},$$

$$G^{3,2} = \begin{pmatrix} -0.8 & 0.2 & 0 & 0.4 & 0.1 & 0.1 \\ 0.2 & -0.8 & 0.3 & 0.2 & 0.1 & 0 \\ 0 & 0.3 & -0.8 & 0 & 0.4 & 0.1 \\ 0.4 & 0.2 & 0 & -0.8 & 0.2 & 0 \\ 0.1 & 0.1 & 0.4 & 0.2 & -1.0 & 0.2 \\ 0.1 & 0 & 0.1 & 0 & 0.2 & -0.4 \end{pmatrix}.$$

The parameters $A = \mathrm{diag}(a_1, a_2, \cdots, a_n)$, $D = (d_{rj})_{n \times n}$ in the network (40) can be changed same as those in (39). Obviously, $g_k(\cdot)$ ($k = 1, 2, 3$) satisfies
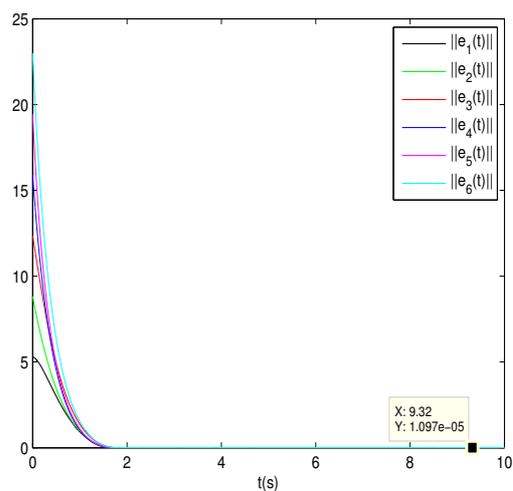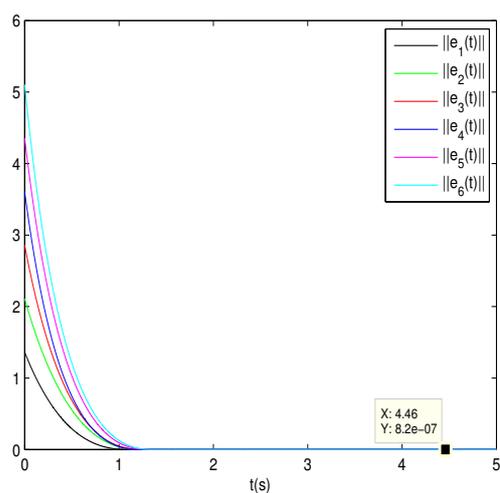
Fig. 1 $\|e_\iota(t)\|$, $\iota = 1, 2, \cdots, 6$.

Fig. 2 $\|e_\iota(t)\|$, $\iota = 1, 2, \cdots, 6$.

Assumption 1 with $\theta_k = 0.25$. Take $W = \mathrm{diag}(0.2I_3, 0.1I_3, 0.3I_3, 0.6I_3, 0.4I_3, 0.5I_3)$. By employing MATLAB Toolbox, the following matrices P, Q_1, Q_2

and Q_3 satisfying (37) can be obtained:

$$P = \begin{pmatrix} 0.4613 & 0 & 0 \\ 0 & 0.3981 & 0 \\ 0 & 0 & 0.3020 \end{pmatrix},$$

$$Q_1 = I_6 \otimes \begin{pmatrix} 0.2083 & 0 & 0 \\ 0 & 0.2107 & 0 \\ 0 & 0 & 0.2110 \end{pmatrix},$$

$$Q_2 = I_6 \otimes \begin{pmatrix} 0.1050 & 0 & 0 \\ 0 & 0.1053 & 0 \\ 0 & 0 & 0.1054 \end{pmatrix},$$

$$Q_3 = I_6 \otimes \begin{pmatrix} 0.3067 & 0 & 0 \\ 0 & 0.3145 & 0 \\ 0 & 0 & 0.3157 \end{pmatrix}.$$

According to Theorem 8, the network (40) with the parameter ranges defined by (39) achieves robust finite-time anti-synchronization under the controller (24) and the time estimation of achieving anti-synchronization is $T_2 = 4.46$. The simulation result is displayed in Figure 2.

6 Conclusion

In this paper, the finite-time anti-synchronization and robust finite-time anti-synchronization for MWCNNs with and without coupling delays have been respectively studied. By making use of Lyapunov functional method as well as inequality techniques, some finite-time anti-synchronization and robust finite-time anti-synchronization conditions have been derived for those network models. Furthermore, we have also considered the finite-time anti-synchronization and robust finite-time anti-synchronization of MWCNNs with switching topology. Finally, several simulation examples have been provided to verified the correctness of our results.

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